Pygmy resonances in artificial nuclei: far-infrared absorption by electron-hole droplets

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The structure of E1 resonances is examined in a microscopic random phase approximation calculation for neutral, symmetric, closed shell, electron-hole systems in a quantum dot. The number of electron-hole pairs, N, is varied from 6 to 42. The ocurrence of small, but distinct, E1 peaks in the far infrared spectra located in the low energy tail of the giant dipole resonance and consisting of highly coherent electron-hole excitations is predicted. These pigmy resonances account for about 2 % of the dipole energy-weighted photoabsorption sum rule. A very weak dependence of the average pigmy resonance energy on the number of electron-hole pairs is found.
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A phenomena well studied in Nuclear Physics is the presence of giant multipolar resonance modes related to collective excitations in nuclei. Particularly, collective states corresponding to giant electric dipole oscillations are known in nuclei long ago [1,2]. They show themselves as very high peaks in photoabsorption at energies $\sim\ 80A^{-1/3}$ MeV, where A is the mass number. Both the excitation energy and the total E1 strength are rather well understood from a collective [3] as well as from a microscopic point of view [4]. It is shown that the giant dipole resonance (GDR) is a state coming from the splitting of a degenerate set of 1^- states, which takes almost the whole strength of the 1^- transition. Collective states like GDR have been extensively studied, both experimentally and theoretically, in Atomic Physics. They are at the base of giant and broad atomic resonances in the photoionisation continuum [5]. The photoabsorption cross section of metallic clusters [5,6] exhibits also the dominance of a collective oscillation mode at 2 - 3 eV [7]. Theoretical calculations based on the Random Phase Approximation (RPA) [5,8] are shown to reproduce the resonance position and dipole strength distribution. In a previous paper [9], we showed that the far infrared absorption probability in neutral electron-hole systems confined in a quantum dot is also dominated by a GDR. This result is mainly related to the existence of positively and negatively charged particles, and not to a particular dot geometry. The electron-hole system in a quantum dot shares many similarities with a real nucleus: a self-consistent confinement potential, attractive and repulsive channels in the residual interactions, etc. Therefore we want to verify if, for such symmetric systems, we can also observe concentration of the E1 strength in a low excitation energy region, very similar to the so called *pygmy resonances* in nuclear physics.

Recently, the study of electric dipole transitions from high-lying bound states in nuclei gained much interest. The E1 strength distribution in spherical nuclei near shell closures seems to display quite some fine structure in this energy region and modulations of the GDR tail occur. Experimental efforts have been made on ^{138}Ba [10], ^{116,124}Sn [11], ^{140}Ce [12] and the odd ^{89}Y [13], using the photon scattering technique. A concentration of E1 strength turned up clearly around 6.5 MeV in these nuclei. This pygmy resonance, named as such in correspondence to the GDR, was first observed in heavier nuclei in the Pb region [14] at 5.5 MeV. Recent experiments on Ba and Ce isotopes [15] , on ^{56}Fe and ^{58}Ni nuclei [16], and in the $N = 50$ nuclei [17] also show signatures of similar phenomena. Finally, for the ^{92}Mo nucleus a recent LINAC experiment [18] established the presence and the E1 character of this pygmy resonance centered at 6.5 MeV. The observation of a fairly large dipole strength($B(E1)$) value, at relatively low excitation energy, might be due to admixtures of the GDR into these low-lying 1^- states. [19,20] Theoretically, attention has been given to the observed local concentration of E1 strength by, e.g., Iachello [21], and Van Isacker *et al* [22]. Iachello, discussing the examples of clustering in nuclei and of permanent octupole deformation effects, has suggested that with isospin as a local symmetry rather important concentrations of E1 strength might well show up at low excitation energies. Van Isacker *et al* suggest that nuclei with a reasonable neutron skin may exhibit pygmy-E1 resonances below the GDR. The same phenomena was explained by Oros *et al* [23] using a two-group schematic random phase approximation (RPA) model, suggesting that concentrations of strength remain distributed among the unperturbed $1p$-$1h$ states when the rest of the strength is pulled up into the GDR. A similar interpretation is given by the Dubna group in a microscopic quasiparticle phonon model (QPM) [24].

In this paper we will study the existence and behavior of pygmy resonances in neutral quantum dot systems using a large harmonic oscillator basis ($\sim 20\hbar\omega$). A simplified model of disk-shaped dot is employed, in which the lateral confinement is ideally parabolic [25]. N electron-hole pairs are supposed to be created in the dot by means of, e.g., laser pumping with a typical ~ 1 eV energy, as corresponding to a semiconductor band gap. The created particles live hundreds of picoseconds and even more [26]. We will study the linear absorption of a second far infrared, ~ 10 meV, light wave by the N-pair cluster. The period of this signal is $\sim 10^{-13}$s, thus the cluster may be considered as stable when studying absorption. We will use the dipole approximation for the interaction hamiltonian, and will consider in-plane light polarization..

In our model, there are only one electron and one hole bands, and $m_e = m_h$ [27]. Electron-hole exchange will be neglected. The second-quantized Hamiltonian for the two-dimensional motion of electrons and holes is given elsewhere [9]. The important quantity is the ratio of the Coulomb characteristic energy to the confinement energy of the dot, denoted as $\beta = \sqrt{(\frac{me^4}{\kappa^2\hbar^2})/(\hbar\omega)}$, where κ is the dielectric constant, and ω is the dot frequency. We consider the dot in the strong-confinement regime, where the one-particle energy dominates the total energy. This is a good starting picture for small dots with large N values. We notice that both the quasiplanar shape and the strong-confinement regime are common conditions met in experiments on quantum dot luminescence [28]. We study closed-shell quantum dots, thus we have a close analogy with near magic nuclear systems where pygmy resonances were observed. The number of electrons is equal to the number of holes, and it is given by the expression $N = N_s(N_s + 1)$, where N_s is the last two-dimensional harmonic-oscillator filled shell. Hartree-Fock shells are distorted harmonic oscillator shells. The total angular momentum, total electron spin and total hole spin are all equal to zero for the ground state.

The lowest excitations against the ground state are one-particle excitations from the last filled shell to the next empty shell. The states with total angular momentum projection, $M = \pm 1$, and electron and hole total spins, $S_e = S_h = 0$, are the candidates among which we shall look for the GDR as well as for pygmy resonances. To be definite, we will consider $M = 1$. For $\beta = 0$ GDR and pygmy resonances are all degenerated in energy. Notice that there are N_s orbitals in the last shell, that is $2N_s$ electrons and, thus, $2N_s$ wave functions with $M = 1$. There are also $2N_s$ functions corresponding to hole excitations. It makes a total of $4N_s$ degenerate states. The Coulomb interaction breaks the degeneracy. The collective nature of the resonances is manifested in the fact that the electric dipole transition probability from the ground state is strongly enhanced [4]. We will show that this property also holds for pygmy resonances using RPA calculations.

In the RPA [29], we allow a general correlated ground state, $|RPA\rangle$, and the excited states are looked for in the form $\Psi = Q^\dagger|RPA\rangle$, where the $Q^\dagger$ operator contains "up" and "down" transitions

$$Q^\dagger = \sum_{\sigma,\lambda}(X^e_{\sigma\lambda} e^\dagger_\sigma e_\lambda + X^h_{\sigma\lambda} h^\dagger_\sigma h_\lambda - Y^e_{\sigma\lambda} e^\dagger_\lambda e_\sigma - Y^h_{\sigma\lambda} h^\dagger_\lambda h_\sigma). \tag{1}$$

The $Y_{\sigma\lambda}$ coefficients are nonzero only for λ and σ satisfying $m_\lambda - m_\sigma = 1$, where the m_λ are magnetic quantum numbers. The physical solutions annihilate the RPA ground state $Q|RPA\rangle = 0$, and satisfy the normalization condition $1 = \sum_{\sigma,\lambda}\{(X_{\sigma\lambda})^2 - (Y_{\sigma\lambda})^2\}$. The far infrared radiation to be absorbed is assumed to be linearly polarized, with the electric field vibrating along the y direction. The dipole matrix elements are computed from

$$\begin{aligned}\langle\Psi|D|RPA\rangle = \sum_{\sigma,\lambda}\{&(X^h_{\sigma\lambda} - X^e_{\sigma\lambda})\langle\sigma|y|\lambda\rangle \\ &+ (Y^h_{\sigma\lambda} - Y^e_{\sigma\lambda})\langle\lambda|y|\sigma\rangle\}.\end{aligned} \tag{2}$$

The Hartree-Fock energies and one-particle wave functions are obtained selfconsistently by means of an iterative scheme [9]. Excitation energies and dipole strength distributions were computed for N ranging from 6 to 42, and β equal to 1. Therefore, our Fermi levels were always much lower than cut-off energy of the oscillator basis $\sim 20\hbar\omega$. The calculated dipole elements fulfill the energy-weighted sum rule [29]

$$\sum_{\Psi} \Delta E\ |\langle\Psi|D|RPA\rangle|^2 = \sum_{\Psi_0} \Delta E_0 |\langle\Psi_0|D|0\rangle|^2 = N/2. \tag{3}$$

where 0 indexes refer to the noninteracting $\beta = 0$ case.

It should be stressed that for larger values of β, pairing becomes very important [30], and should be taken into account more properly. We draw in Fig. **??** the GDR and pygmy resonance excitation energies at $\beta = 1$ as a function of $N^{1/4}$. Notice the approximate dependence $\Delta E_{GDR} - 1 \approx 0.7 + 0.4\ N^{1/4}$ for a GDR energy and $\Delta E_{pygmy} - 1 \approx 0.7 - 0.1\ N^{1/4}$ for an average pygmy resonance energy. At $\beta \neq 1$, the absolute energy shift $(\Delta E - 1)$ has a linear dependence on β. A remarkable feature is that the total number of pygmy resonances is equal to $N_s - 1$. Indeed, we checked that no pygmy resonances are obtained for the case of two confined pairs ($N = 2$), where the whole dipole strength goes to the GDR state.

It can be seen that the average pygmy resonance energy has a very weak dependence on the number of electron-hole pairs N, confined in the system. It is interesting to note that such weak dependence for average pygmy resonance energy behavior is obtained in highly symmetric quantum dot systems, equivalent to atomic nuclei with total isospin equal 0 ($N = Z$). This conclusion is in agreement with the results for ^{204,206,208}Pb nuclei [31], where the comparison of available experimental data below 6.5 MeV suggests that the E1 strength in this region can not be attributed to an oscillation of the excess neutrons with respect to the remaining nucleus core.

The similarity with the nuclear system is confirmed by inspecting the experimental E1 strength distribution, measured up to endpoint energy of 6.75 MeV, in ^{204,206,208}Pb nuclei [31]. The GDR energy for these nuclear systems is about 13.5 MeV and the average energy for observed pygmy resonances is about 6 MeV (~ 0.45 times the GDR energy). The dipole strength and the number of pygmy resonances increases when the number of neutron pairs increase by 2 from ^{204}Pb to ^{208}Pb nucleus. The results for closed shell quantum dots with $N = 6, 12, 20, 30$, and 42 confined electron-hole pairs at $\beta = 1$ are presented in Fig. **??**. In analogy with the lead nuclei, the number of pygmy resonance states increases when the total number of pairs is increased in the system. Thus, the overall behavior of both systems is quite similar.

For the quantum dot systems, the fraction of the sum rule (3) accounted for the GDR state is close to 98 %. The remaining 2 % of the E1 strength lies at much lower energy (about 0.5-0.6 times the GDR energy) corresponding to the pygmy resonance states, in analogy with the behavior of the nuclear systems. Notice that only collective (highly coherent) states, for which $X^e = -X^h$, are represented in our calculations (see Fig. **??**). The rest of the states show a threefold degeneracy and give practically zero dipole matrix elements. Let us note that the energy gap is increased ($\Delta E > 1$) after the Coulomb interaction is switched on. Energy of the collective dipole states increases when coulomb interaction is considered. This is a signal that the overall interaction among particles is repulsive.

We have shown that the far infrared absorption spectrum of neutral confined systems of electrons and holes is dominated by collective dipole resonances, including GDR as well as pygmy resonances. The latter account for about 2% of the energy-weighted photoabsorption sum rule, lying at average energy about $0.5 - 0.6$ times the GDR energy. Calculations were done in a two-band model with $m_e = m_h$ and a disk-shaped parabolic dot. The qualitative conclusions are, however, expected to be valid for realistic systems since they are mainly related to the existence of positively and negatively charged particles in the system, which causes the enhancement of dipole oscillations.

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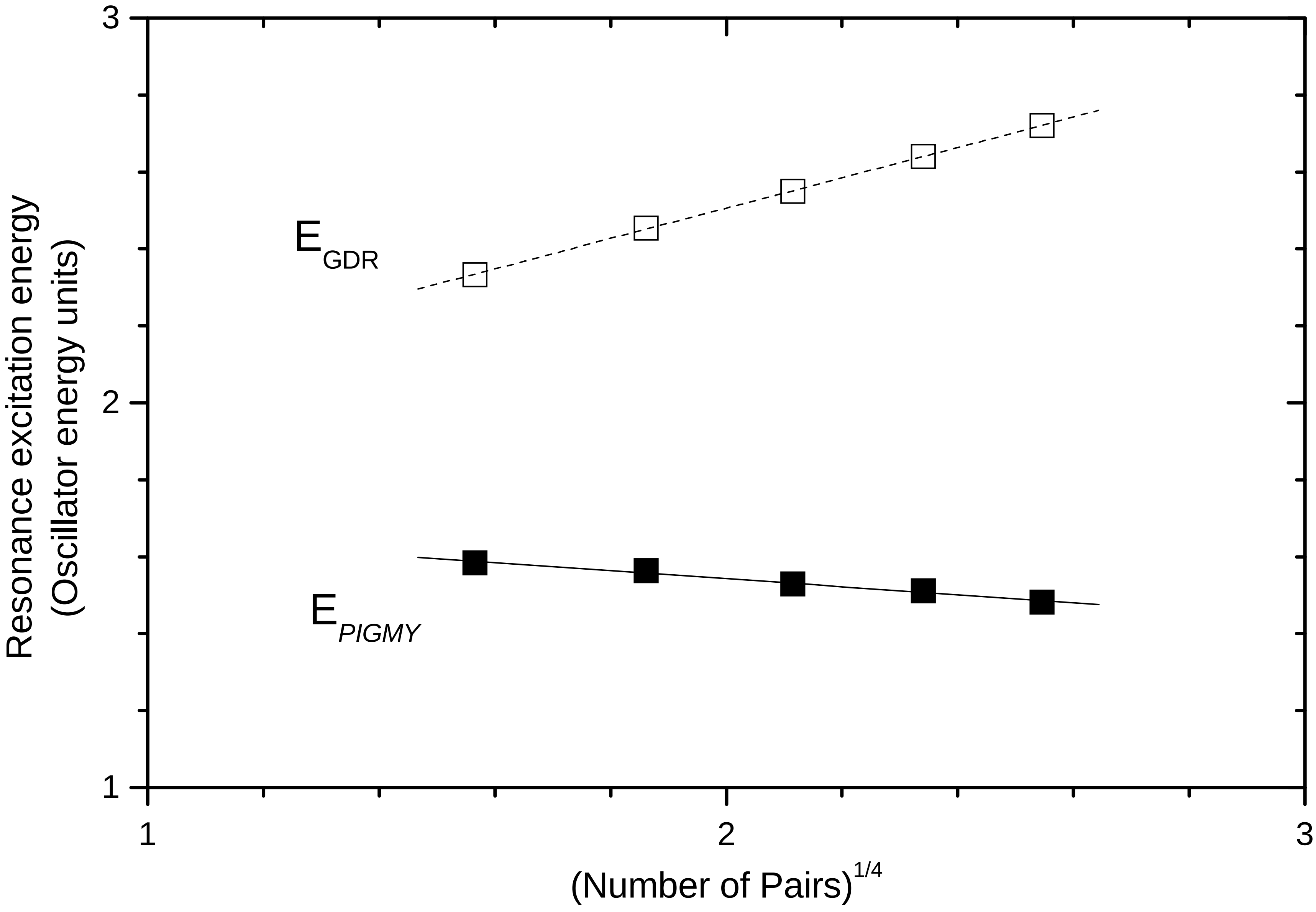
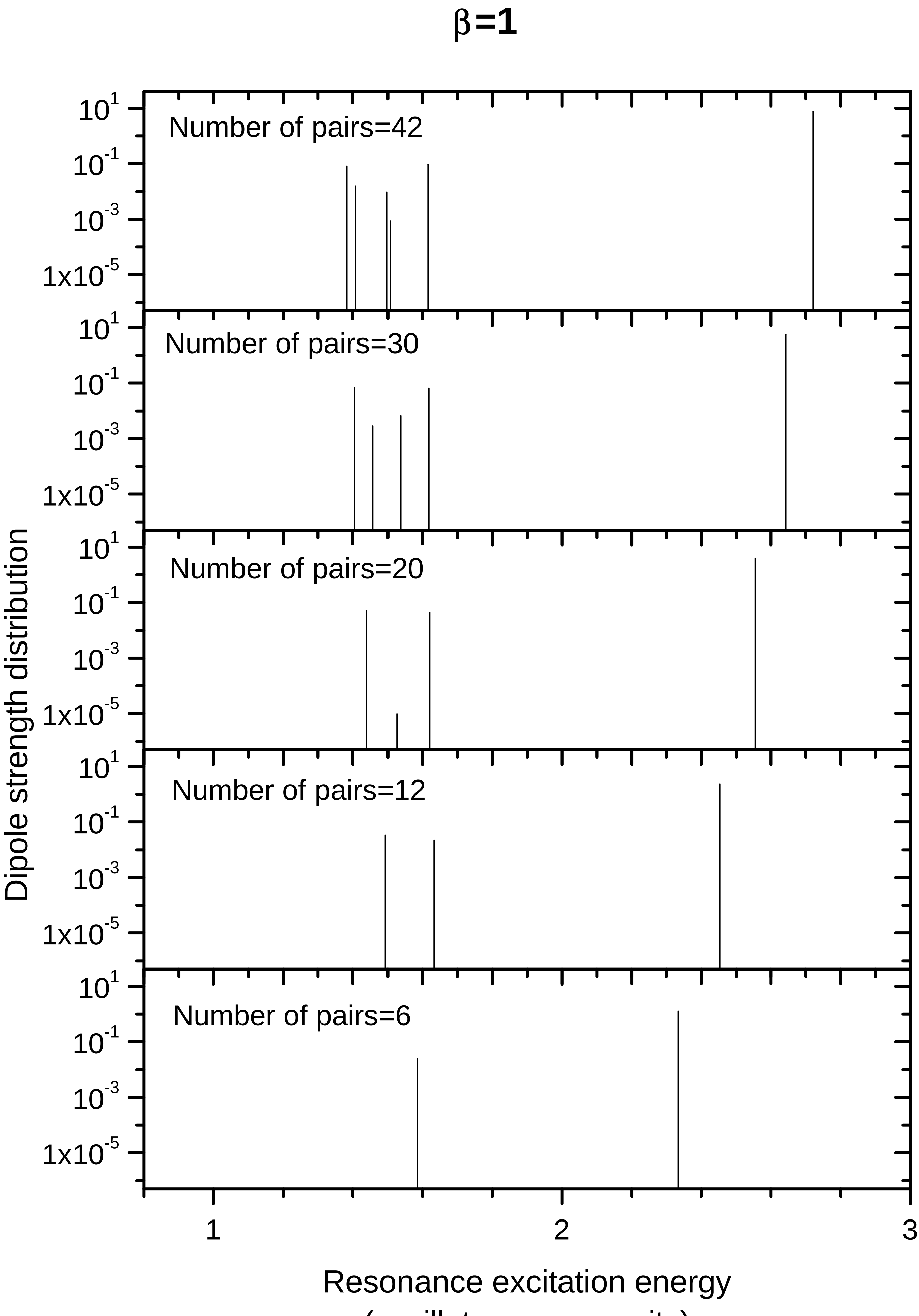
A. Figure captions:

Figure 1. GDR and *pigmy resonance* excitation energies at $\beta = 1$ as a function of the number of pairs, N, in the dot. Dashed and solid line correspond to the linear fit for GDR and average *pigmy resonance* energy as a function of N. The linear fit y=b*X+a parameters for GDR energy are a=1,75+/-0,01 and b=-0,105+/-0,007 and for average *pigmy resonance* energy are a=1,717+/-0,005 and b=0,394+/-0,003 correspondingly.

Figure 2. Dipole matrix elements squared at $\beta = 1$ as a function of the number of electron-hole pairs, N.



Resonance excitation energy
(Oscillator energy units)
E_{GDR}
E_{PIGMY}
(Number of Pairs)$^{1/4}$
1
2
3



β=1
Number of pairs=42
Number of pairs=30
Number of pairs=20
Number of pairs=12
Number of pairs=6
10^1
10^-1
10^-3
1x10^-5
Dipole strength distribution
1
2
3
Resonance excitation energy
(oscillator energy units)