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DOCTORAL THESIS

**Stochastic Model Predictive Control for Robust  
Operation of Distribution Systems**

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# Abstract

There are many systems in which uncertainties are present in their model, either in the same system description or as disturbances. Among many random variables we can mention the electrical demand of a generation network, the amount of rainfall in an irrigation system, the number of people occupying a room in a system of heating. Among others, they are examples of stochastic systems, in which the idea of scenarios can be considered for their solution. Specifically, the stochastic model predictive control seeks to generate a solution for several scenarios that can be established under a probabilistic condition. In this work is carried out an analysis and comparison regarding performance among the three well-known stochastic MPC approaches, namely, multi-scenario, tree-based, and chance-constrained model predictive control. The possibility of application in several distribution sectors is also analyzed. Moreover, some improvements are proposed in terms of robustness. To this end, the stochastic MPC controllers are designed and implemented in a real renewable-hydrogen-based microgrid as well as to the drinking water network of Barcelona via simulation. Finally, an application of CC-MPC to inventory management in a hospital pharmacy, is also presented.

Stochastic MPC controllers are applied in a hierarchical and distributed fashion. In this sense, a scenario-based hierarchical and distributed MPC is applied for water resources management by considering dynamical uncertainty. In addition, a multicriteria optimal operation of a microgrid considering risk analysis and MPC is shown. For all applications, their design has considered the important role that uncertainty plays in these kind of systems.

Finally, in order to analyze different types of the so-called insider attacks in a DMPC scheme is presented. In particular, the situation where one of the local controllers sends false information to others is considered to manipulate costs for its own advantage. Then, some mechanisms based on stochastic MPC techniques are proposed to protect or, at least, relieve the consequences of the attack in a typical DMPC negotiation procedure is addressed along this work.



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# Chapter 1

## Introduction

Model Predictive Control (MPC) is a control strategy widely used in the industry compared with other control techniques. MPC provides a control framework capable of dealing with delays, nonlinearities, constraints on the states as well as on the input variables, moreover, it can be easily extended to multi-variable systems, to name a few advantages of this technique [1, 2]. The main idea of MPC is to obtain a control signal by solving, at each time step, a finite-horizon optimization problem (FHOP) that takes into account a model of the system to predict its evolution and to steer it in accordance to given objectives. The first component of the obtained control sequence is applied to the system at the current time step and the problem is solved again at the next time step, following a receding horizon strategy [3].

However, the classical formulation of MPC does not allow considering systems with uncertainties, although some MPC schemes have been proposed to ensure stability and compliance with constraints in the presence of disturbances [4]. As summarized in [5], alternative approaches of MPC for stochastic systems are based on *min-max* MPC, *tube-based* MPC, and *stochastic* MPC (SMPC). The first two approaches are oriented to ensure worst-case robustness and consequently are conservative, while the third approach relies on stochastic programming (SP) techniques to offer a probabilistic constraint fulfillment [6]. Since some violations are allowed with some stochastic approaches, the solutions obtained are less conservative and hence the performance is better in terms of cost from the objective function. In this way, disturbances are modeled as random variables, and the control problem is stated by using the expected value of the system variables, i.e., states and control inputs. A less conservative approach is the stochastic one, which is based on the design of predictive controllers for dynamical systems subject to disturbances and/or uncertainty in terms of the *probability* that a

certain solution is feasible [7], mainly because it is not strictly possible to speak about guaranteed feasibility in this context.

Nevertheless, there exist other MPC schemes reported in the literature that aim to ensure robust stability and compliance with constraints in the presence of stochastic disturbances, see e.g., [4, 5].

The stochastic approach is a mature theory in the field of optimization [8], but renewed attention has been given to it due to its great potential in control applications, see e.g., [9] and references therein. From the wide range of SMPC methods, this work is focused on three specific techniques, namely: *multiple-scenario MPC* (MS-MPC), also called Multiple MPC in [10], *tree-based MPC* (TB-MPC), and *chance-constrained MPC* (CC-MPC).

MS-MPC consists in calculating a single control sequence that takes into account different possible evolutions of the process disturbances. Hence, the control sequence calculated has a certain degree of robustness against the potential realizations of the uncertainties. This approach is used for example in [10] for water systems and in [11, 12] within the context of the control of smart grids. One of its advantages is that it is possible to calculate bounds on the probability of constraint violation as a function of the number of scenarios considered [13].

An alternative to model the uncertainty that is faced by this type of systems is to use rooted trees. The rationale behind this approach is that uncertainty spreads with time, i.e., it is possible to predict –more accurately– both the energy demand and energy production by a renewable source in a short horizon than in a large one. For this reason, the possible evolutions of the disturbances can be confined to a tree. In the tree, there is a bifurcation point whenever the disturbances branch into two possible trajectories. Consequently, the outcome, the so-called TB-MPC, is a rooted tree of control actions. This approach is used for example in [14] for a semi-batch reactor example, in [15] for the energy management of a renewable hydrogen-based microgrid, and in [16] in the context of water systems.

CC-MPC uses an explicit probabilistic modeling of the system disturbances to calculate explicit bounds on the system constraint satisfaction. For instance, [17] presents a chance-constrained two-stage stochastic program for unit commitment with uncertain wind power output and [18] shows an autoregressive-moving-average (ARMA) type prediction model for the underlying uncertainties (load/generation) into chance-constrained finite-horizon optimal control. An application of this technique in the context of the drinking water network of the city of Barcelona is reported in [19]. In addition, [20] shows a comparison between TB-MPC and CC-MPC approaches applied to drinking water systems. Further, this subject has drawn significant interest; a stochastic optimization model implemented in the context of the control of microgrids can be seen in [21–24] and references therein.

An important aspect here is the way the control can be implemented from a decentralized viewpoint. Some systems – e.g., power dispatch system, water and navigation system, logistic systems, among others– often spread over large-scale areas. The whole system may be divided into smaller ones that can be governed by different local entities. If the local controllers do not communicate at all, the control architecture is said to be decentralized. By other side, when all the controllers are equally important and share information to find the most appropriate control actions from a global perspective, it is using a distributed control scheme, see e.g., [25, 26]. Moreover, a possibility is to use a hierarchical structure where an upper control layer provides instructions to the lower control layers, the latter are in charge of the regulation of smaller regions controlled by local agents. In this way, coordination is attained [27–29].

Since the uncertainties and disturbances can be presented on different geographically disperse subsystems, their structures are different and need to be reviewed in the hierarchy for sending reliable information to multi-subsystem. At this point, this work shows a scenario based Hierarchical and Distributed MPC (HD-MPC) in one of its chapters. The overall control architecture is composed of two layers. On the one hand, the top layer collects global forecast information and sends to the local agents a set of different scenarios one per each subsystem to deal with the uncertainty. On the other hand, the bottom layer solves the optimal control problem in a distributed fashion by using a distributed scheme, where the role played by the uncertainties is carried out by multi-subsystem scenario based MPC.

Moreover, many approaches for DMPC schemes have been developed in recent years, as described in [30]. A topic that deserves attention is the regular exchange of information during the negotiation process among the controllers. In this sense, DMPC schemes have been carried out by considering a coordinated negotiation process where all controllers work in a reliable way. However, a malicious controller could exploit the vulnerabilities of the network by sharing false information with other controllers, producing an undesirable behavior in the optimization process. At this point, it is possible to speak about cyber-security in the context of DMPC. At this point, cyber-security issues have not been considered in the DMPC literature. Hence, in this work it is analyzed one of the most popular schemes, Lagrange based DMPC. In particular, it is shown how a malicious controller in the network can take advantage of the vulnerabilities of the scheme to increase its own benefit at the cost of other controllers. Also, these issues are addressed by considering some stochastic based MPC techniques to ensure robustness within the DMPC network.

## 1.1 Objectives of the thesis

The main objective of this thesis is to carry out an analysis and comparison regarding performance among the three stochastic MPC approaches, namely, multi-scenario, tree-based, and chance-constrained model predictive control. The possibility of application in several distribution sectors has been also analyzed. Finally, some improvements have been proposed in terms of robustness. To this end, the stochastic MPC controllers have been designed and implemented in first place in a real renewable-hydrogen-based microgrid. Moreover, on the comparison of these stochastic techniques is applied to the drinking water network of Barcelona. Finally, an application of CC-MPC to inventory management in hospitalary pharmacy, is also presented.

As a second objective in this work is to apply stochastic MPC controllers in a hierarchical and distributed fashion. In this sense, a scenario-based hierarchical and distributed MPC is applied for water resources management by considering dynamical uncertainty. In addition, a multicriteria optimal operation of a microgrid considering risk analysis and MPC is shown. For all applications, their design has considered the important role that uncertainty plays in these kind of systems.

To analyze different types of the so-called insider attacks in a DMPC scheme is presented as the last objective of this thesis. In particular, the situation where one of the local controllers sends false information to others is considered to manipulate costs for its own advantage. Then, some mechanisms based on stochastic MPC technique are proposed to protect or, at least, relieve the consequences of the attack in a typical DMPC negotiation procedure is addressed along this work.

## 1.2 Thesis outline

The remainder of this thesis is organized as follows. To cope with uncertainty present in the most kind of distribution systems, the use of three stochastic MPC approaches is proposed: multiple-scenario MPC, tree-based MPC, and chance-constrained MPC. A comparative assessment of these approaches is performed when they are applied to real case studies, specifically, a hydrogen based microgrid situated at the University of Seville, a sector and an aggregate version of the Barcelona drinking water network, and the stock management in a hospital pharmacy using chance-constrained model predictive control, are shown in the Chapter 2.

Chapter 3 is focused on the hierarchical stochastic MPC. On the one hand, the optimal power dispatch taking into account risk management and renewable resources in the real laboratory-scale plant is addressed. To this end, identification of potential risks has been performed and two MPCs are designed: one for risk mitigation and another

for the optimal control of the microgrid. The proposed algorithm considers an external loop where information about risk evaluation is updated. The risk mitigation policy may change setting points and constraints as well as execute actions. Results show improvements in terms of costs and demand satisfaction. On the other hand, a tree based hierarchical and distributed Model Predictive Control (HD-MPC) is applied to deal with operational water management problems under dynamical uncertainty. A two layer-hierarchical structure is proposed, the higher layer collects and coordinates forecast information for sending different scenarios that take into account the uncertainties to the local agents. The lower layer, comprised of local agents, solves an optimization problem in a distributed fashion. The HD-MPC method is tested on a real-world case, the North Sea Canal system. At this point, results show the benefits of the proposed method regarding control performance of large-scale systems.

Chapter 4 provides an analysis of the vulnerability of a distributed model predictive control scheme in the context of cyber-security. A distributed system can be easily attacked by a malicious agent that modifies the reliable information exchange. We consider different types of so-called insider attacks. In particular, it is analyzed a controller that is part of the control architecture that sends false information to others to manipulate costs for its own advantage. In addition, mechanisms to protect or, at least, relieve the consequences of attack in a typical DMPC negotiation procedure is proposed. More specifically, a consensus approach that dismisses the extreme control actions is presented as a way to protect the distributed system from potential threats. In this sense, secure dual decomposition techniques based on stochastic MPC is developed to mitigate the impact that an attacker can cause to the other controllers. A distributed local electricity grid of households is considered as case study to illustrate both the consequences of the attacks and the defense mechanisms.

Finally, conclusions are drawn in Chapter 5.



## Chapter 2

# Centralized Stochastic Model Predictive Control

In this chapter, three popular stochastic MPC techniques (MS-MPC, TB-MPC, and CC-MPC) are briefly introduced. Moreover, these stochastic approaches have been applied on the framework of centralized distribution applications. In first place, stochastic MPC controllers have been designed to power dispatch in a real hydrogen-based microgrid. Moreover, a comparison among these controllers is carried out on a second case study, the Barcelona drinking water network. Finally, CC-MPC is applied to stock management in a hospital pharmacy. These three applications have been presented previously in [31–33], respectively.

### 2.1 Model Predictive Control (MPC)

MPC is a strategy based on the explicit use of a dynamical model of the plant to predict the state/output evolution of the process in future time instants along a prediction horizon  $N_p$  [2]. The set of future control signals is calculated by the optimization of a criterion or objective function. Only the control signal calculated for the time instant  $k$  is applied to the process, whereas the others are withdrawn. One of the advantages of MPC over other control methods includes the easy extension to the multivariable case.

The optimization problem to be solved at each time instant  $k$  is formulated as

$$\min_{\{u(k), \dots, u(k+N_p-1)\}} \sum_{i=0}^{N_p-1} J(x(k+i), u(k+i)), \quad (2.1)$$

subject to

$$x(i+1) = Ax(i) + Bu(i) + D\omega(i), \quad (2.2a)$$

$$x(0) = x(k), \quad (2.2b)$$

$$x(i+1) \in \mathcal{X}, \quad (2.2c)$$

$$u(i) \in \mathcal{U}, \quad \forall i \in \mathbb{Z}_0^{N_p-1}, \quad (2.2d)$$

where  $x \in \mathbb{R}^{n_x}$ ,  $u \in \mathbb{R}^{n_u}$ , and  $\omega \in \mathbb{R}^{n_d}$  represent the state vector of the system, the manipulated variables, and the system disturbances, respectively. Moreover,  $A \in \mathbb{R}^{n_x \times n_x}$ ,  $B \in \mathbb{R}^{n_x \times n_u}$ , and  $D \in \mathbb{R}^{n_x \times n_d}$  are the matrices that defines the linear dynamic system. The sequence of inputs that must be applied to the system along the horizon is denoted by  $\{u(k), \dots, u(k+N_p-1)\}$ . Note that only  $u(k)$  is actually applied.

A common approach used to cope with perturbed systems is to rely on the so-called certainty equivalence property [34], which in the MPC framework leads to a perturbed nominal deterministic MPC strategy, also named *certainty-equivalent* MPC (CE-MPC) in [35]. This strategy addresses perturbed systems by considering nominal models that do not include the uncertainty. Hence, the expected value of system inputs will lead to an average performing system. In the case of linear systems with uniformly distributed scenarios, the certainty equivalence property holds [36] and this strategy is optimal. Nevertheless, this may not be the case due to factors such as the presence of nonlinearities. Hence, the CE-MPC is usually complemented with a (de)tuning of the controller. Although, in one hand, a frequent violation of soft constraints can occur, on the other hand, infeasible solutions would result if the constraints were hard due to the ignored effects of future uncertainty.

Next, the description of the stochastic MPC techniques designed and implemented is presented.

### 2.1.1 Multiple-scenarios MPC approach (MS-MPC)

The optimization based on scenarios provides an intuitive way to approximate the solution to the stochastic optimization problem. In order to design the MS-MPC, it is required to know several scenarios with possible evolutions of the energy demand and



generation. The scenario forecasts can be obtained either from historical data or by introducing a random scenario generation. The idea behind this approach is that a general control sequence that optimizes all the considered scenarios is calculated, obtaining in this way a certain robustness against the different possible evolutions of the disturbances. The scenario-based approach is computationally efficient since its solution is based on a deterministic convex optimization, even when the original problem is not [37]. One advantage of this approach is that it does not assume a prior knowledge of the statistical properties that characterize the uncertainty (e.g., a certain probability function) as generally required in stochastic optimization.

The main idea for optimization with a finite number of scenarios is to consider the same system for each one of the known disturbance realizations. The problem consists in solving

$$\min_{\{u(k), \dots, u(k+N_p-1)\}} \sum_{j=1}^{N_s} \left( \sum_{i=0}^{N_p-1} J(x_j(k+i), u(k+i)) \right), \quad (2.3)$$

subject to

$$x_j(i+1) = Ax_j(i) + Bu(i) + D\omega_j(i), \quad (2.4a)$$

$$x_j(0) = x(k), \quad (2.4b)$$

$$\omega_j(i) = \hat{\omega}_j(i), \quad (2.4c)$$

$$x_j(i+1) \in \mathcal{X}, \quad (2.4d)$$

$$u(i) \in \mathcal{U} \quad \forall i \in \mathbb{Z}_0^{N_p-1}, \quad \forall j \in \mathbb{Z}_1^{N_s}, \quad (2.4e)$$

where  $N_s \in \mathbb{Z}_+$  is the finite number of scenarios considered and  $\hat{\omega}_j(k)$  is the disturbance forecast for scenario  $j \in \mathbb{Z}_1^{N_s}$ .

Due to the stochastic nature of the disturbances, the number of scenarios considered  $N_s$  deserves special attention to ensure compliance with the state constraints with a certain confidence degree, i.e.,

$$\mathbb{P}[x_j(i+1) \in \mathcal{X}] > 1 - \delta_x,$$

where  $\mathbb{P}[\cdot]$  denotes the probability operator and  $\delta_x \in (0, 1)$  is the *risk acceptability level* of constraint violation for the states. The number of scenarios needed to achieve this goal can be calculated as a function of  $\delta_x$ , the number of variables in the optimization problem ( $z$ ), and a quite small confidence level ( $\beta \leq 10^{-6}$ ), as indicated

in [38]

$$N_s \geq \frac{z + 1 + \ln(\frac{1}{\beta}) + \sqrt{2(z + 1) \ln(\frac{1}{\beta})}}{\delta_x}. \quad (2.5)$$

Furthermore, the sample scenarios must meet the following assumptions, as pointed out in [37]:

1. The uncertainties  $\hat{\omega}_j$ ;  $\forall j \in \mathbb{Z}_1^{N_s}$  are independent and identically distributed (IID) random variables on a probability space.
2. A “sufficient number” of IID samples of  $\hat{\omega}_j$  can be obtained, either empirically or by a random-number generator.

In this manner, a control sequence is optimized for the system given by (2.4a), which includes different possible evolutions of the original one. The calculation of the controller will result in a unique robust control action that satisfies all the potential realizations of the disturbances with a certain probability.

### 2.1.2 Tree-based MPC (TB-MPC)

This technique consists of transforming the different possible evolutions of the disturbances into a rooted tree that, through its evolution, diverges and generates a reduced number of scenarios. The points of divergence are called *bifurcations* and they represent moments in time in which the potential evolution of the disturbances is uncertain enough to consider more than one trajectory, as shown in Figure 2.1. The formulation of the control problem involves making tree-based optimization scenarios, where only the most relevant disturbance patterns are modeled, starting with a common root that corresponds to the current disturbance at each time instant. It must be noted that TB-MPC formulates the optimization problem by means of Multistage Stochastic Programming [39, 40]. The number of scenarios used to build the tree should be coherent with the computational capability of the controller and the risk probability,  $\delta_x$ .

Being a scenario-based approach, it is possible to determine  $\delta_x$  by taking into account the number of discarded scenarios  $R$  from the initial  $N_s$  scenarios for any violation level  $v \in [0, 1]$ , as seen in [37]. The probability of satisfying the state constraints is given by

$$\mathbb{P}[x_{i+1} \in \mathcal{X}] \geq 1 - \delta_x,$$

where

$$\delta_x = \int_0^1 U(v) dv, \quad (2.6a)$$

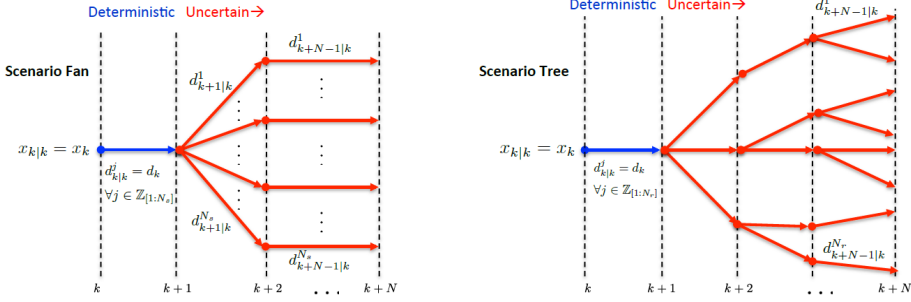


Figure 2.1: Scenario fan and scenario tree over the prediction horizon.

and

$$U(v) = \min \left\{ 1, \binom{R+z-1}{R} \sum_{j=0}^{R+z-1} \binom{N_s}{j} v^j (1-v)^{N_s-j} \right\}. \quad (2.6b)$$

In this way, the amount of  $N_r$  used in the optimization problem is calculated as  $N_r = N_s - R$ .

Unlike the MS-MPC approach, each scenario into the tree has its own control signal, which means that more optimization variables are needed. However, given that the control signal cannot anticipate events beyond the next bifurcation point, control sequences for different scenarios must be equal as long as the scenarios do not branch out. As a consequence, the solution of this control problem is a rooted-tree of control inputs. Notice that only the first component of this tree, which is equal for all the scenarios, is actually applied. For the design of this controller, the bifurcation points of the tree are checked: if they are equal, then the control actions are the same so that both the number of variables and the computational time can be reduced significantly.

The TB-MPC problem formulation to be solved at each time instant is represented by

$$\min_{\{u_j(k), \dots, u_j(k+N_p-1)\}} \sum_{j=1}^{N_r} \left( \sum_{i=0}^{N_p-1} J(x_j(k+i), u_j(k+i)) \right), \quad (2.7)$$

subject to

$$x_j(i+1) = Ax_j(i) + Bu_j(i) + D\omega_j(i), \quad (2.8a)$$

$$x_j(0) = x(k), \quad (2.8b)$$

$$\omega_j(i) = \hat{\omega}_j(i), \quad (2.8c)$$

$$x_j(i+1) \in \mathcal{X}, \quad \forall i \in \mathbb{Z}_0^{N_p-1}, \quad (2.8d)$$

$$u_j(i) \in \mathcal{U}, \quad \forall j \in \mathbb{Z}_1^{N_r}. \quad (2.8e)$$

In addition, it is necessary to introduce non-anticipative constraints to force the controller to compute the control inputs only considering the observed uncertainty before the bifurcation points [40]. These constraints are given by

$$u_i(k) = u_j(k) \quad \text{if} \quad \hat{\omega}_i(k) = \hat{\omega}_j(k); \quad \forall i \neq j. \quad (2.8f)$$

One way to satisfy (2.8f) is to introduce equality constraints into the optimization problem and solving it with a number of optimization variables defined as  $z = N_p \times N_r \times n_u$ . Nevertheless, constraints in (??) can be used to reduce the number of optimization variables by removing the redundancy to lower the computational burden.

As said before, a control sequence is optimized for the extended system with a disturbance tree, and only the first component of the input tree is applied to the system. The problem is repeated at each time instant  $k \in \mathbb{Z}_+$ .

### 2.1.3 Chance-Constrained MPC (CC-MPC)

Given that disturbances are stochastic, another way of addressing this problem is using CC-MPC. The stochastic behavior from the disturbances can be addressed by formulating hard constraints into probabilistic constraints related to a risk of constraint violation that determines the degree of the conservatism when computing the control inputs. Also, the cost function is expressed as its expected value in the formulation of the optimization problem. A major advantage of this approach is that the computational burden is not increased as in the scenario-based techniques.

Given that the disturbances in the dynamic model (2.45) are stochastic, the state constraints (2.19) must be formulated in a probabilistic manner, i.e.,

$$\mathbb{P}[x(i+1) \in \mathcal{X} \mid Gx \leq g] > 1 - \delta_x. \quad (2.9)$$

Here,  $G \in \mathbb{R}^{n_r \times n_x}$  and  $g \in \mathbb{R}^{n_r}$ . The probabilistic constraints (2.9), also called *chance constraints*, can be written in two different manners [19]:

- *Individual chance constraints* that express a probabilistic equivalent for each constraint. They are formulated as

$$\mathbb{P}[G_{(m)}x < g_{(m)}] > 1 - \delta_{x,m}, \quad \forall m \in \mathbb{Z}_1^{n_x}, \quad (2.10)$$

where  $G_{(m)}$  and  $g_{(m)}$  are the  $m^{\text{th}}$  row of  $G$  and  $g$ , respectively. Each  $m^{\text{th}}$  row satisfies its respective  $\delta_{x,m}$ .

- *Joint chance constraints*, which take into account an unique risk of constraint violation for all stochastic constraints. They are written as

$$\mathbb{P}[G_{(m)}x < g_{(m)}, \quad \forall m \in \mathbb{Z}_1^{n_x}] > 1 - \delta_x. \quad (2.11)$$

All rows jointly satisfy the unique  $\delta_x$ .

The application of (2.11) along  $N_p$  is necessary to implement the controller. To this end, it is assumed that the disturbances behave as Gaussian random variables, with a known cumulative distribution function. The deterministic equivalent of these chance constraints can be formulated as follows:

$$\begin{aligned} \mathbb{P}[G_{(m)}x(k+1) < g_{(m)}] > 1 - \delta_x \\ \Leftrightarrow F_{G_{(m)}D\omega(k)}(g_{(m)} - G_{(m)}(Ax(k) + Bu(k))) > 1 - \delta_x \\ \Leftrightarrow G_{(m)}(Ax(k) + Bu(k)) < g_{(m)} - F_{G_{(m)}D\omega(k)}^{-1}(1 - \delta_x). \end{aligned} \quad (2.12)$$

Here,  $F_{G_{(m)}D\omega(k)}(\cdot)$  represents the cumulative distribution function of the random variable  $G_{(m)}D\omega(k)$ , and  $F_{G_{(m)}D\omega(k)}^{-1}(\cdot)$  is its inverse cumulative distribution function.

Note that the expression (2.12) is the deterministic equivalent of the chance constraints and is built based on historical data.

The optimization problem formulation related to the design of the CC-MPC controller is stated as

$$\min_{\{u(k), \dots, u(k+N_p-1)\}} \sum_{i=0}^{N_p-1} \mathbb{E}[J(x(k+i), u(k+i))], \quad (2.13)$$

subject to

$$x(i+1) = Ax(i) + Bu(i) + D\omega(i), \quad (2.14a)$$

$$x(0) = x(k), \quad (2.14b)$$

$$\omega(i) = \hat{\omega}(i), \quad (2.14c)$$

$$G_{(m)}(Ax(k) + Bu(k)) < g_{(m)} - F_{G_{(m)}D\omega(k)}^{-1}(1 - \delta_x), \quad (2.14d)$$

$$u(i) \in \mathcal{U}, \quad \forall i \in \mathbb{Z}_1^{N_p-1}, \quad (2.14e)$$

where  $\mathbb{E}[\cdot]$  denotes the expected value of the cost function.

## 2.2 Case Study: A Hydrogen-based Microgrid

A microgrid is a network of electric generators that may take advantage of several renewable energy sources: solar panels, wind mini-generators, micro-turbines, fuel cells, among others, to meet the consumer demand by working together with the centralized grid or autonomously [41]. In a microgrid, the energy is generated only at certain times, being necessary to provide continuous service to meet the demand at any time of the day. Challenges arise from the natural intermittency of renewable energy sources and the requirements to satisfy the user energy demand [42]. Thereby, storage devices become very important in the operation of this type of systems. Among well-established energy storage technologies, there are batteries, super-capacitors, conventional capacitors, etc. In this case study, the use of hydrogen as an energy vector for energy storage is focussed. Hydrogen, combined with other renewable energy sources, is a safe and viable option to mitigate the problems associated with hydrocarbon combustion because the entire system can be developed as an efficient, clean, and sustainable energy source, as mentioned in [43]. The hydrogen is converted into electrical energy by using fuel cells; the reverse process, i.e., the transformation of electric energy into hydrogen, is conducted by electrolysis [44], or ethanol reforming [45], among other techniques.

The control problem in a microgrid is to satisfy the electricity demand under economical and optimal conditions despite the uncertainties and disturbances that might appear in the processes. Taking into account that there are mathematical models available that represent the main dynamics and the load of these systems [46], and that the control problem here requires the simultaneous handling of constraints, delays, and disturbances, model predictive control (MPC) emerges as a solution to this problem. In this, sense, uncertainty in the load and generation profiles has been mainly addressed indirectly in the dispatch problem by using the MPC approach [47].

### 2.2.1 Hydrogen-based Microgrid Description

The microgrid under study is the lab-scale microgrid called *HyLab* [48]. The microgrid test bench used in this study is an experimental platform specifically designed for testing control strategies. HyLab is composed of a modular system equipped with various components that allow experimentation and simulation of several types of renewable energy sources. In the Figure 2.2, a picture of the experimental Hylab platform is shown.

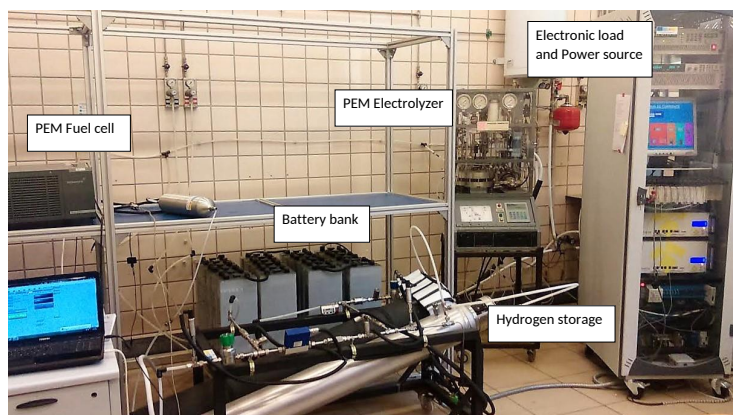


Figure 2.2: Experimental HyLab Plant.

The system consists of a solar field, emulated by an electronic power source, which produces electricity to supply the electronic load. Any excess of power can be either stored in a battery bank or derived to the electrolyzer. If the power obtained from renewable energy is not enough, both the fuel cell and the battery bank can support the load, which is emulated electronically. This type of hybrid storage operation allows implementing strategies in separated times scales: the battery can either absorb or contribute to balance small amounts of energy in fast transient periods while the hydrogen path complements larger variations [49]. The microgrid can work either connected to the utility network or as an isolated system. The Hydrogen Path is composed of three subsystems: the electrolyzer, which is proton exchange membrane (PEM) type [50], for producing hydrogen; a metal-hydride hydrogen storage tank; and finally a PEM fuel cell [51, 52] that provides power to the loads/batteries. It is important to notice that both subsystems –electrolyzer and fuel cell– cannot work simultaneously. DC/DC power converters are used as power interfaces that allow energy transfer between different distributed generation units. The equipment is connected to 48  $V_{DC}$  bus that is held by the battery bank. Table 2.1 presents the nominal values of the HyLab equipment.

### Microgrid linear model and constraints

As it can be inferred, behind the experimental setup there is a set of complex nonlinear subsystems. The detailed description of sub-models and the physical equations are out

Table 2.1: HyLab equipment.

Equipment	Nominal Value
Electronic power source	6 kW
Electronic load source	2.5 kW
PEM fuel cell	1.2 kW
PEM electrolyzer	0.23 Nm <sup>3</sup> h <sup>-1</sup> @5barg
	1 kW
Metal hydrides tank	7 Nm <sup>3</sup>
	5 bar
Battery bank	$C_{120} = 367$ Ah
DC/DC converters	1.5 kW, 1 kW

of the scope of this work. The complete non-linear model of the plant, its simulation, and validation are presented in [53].

**Remark 1** *To apply linear MPC techniques is required to find a linear model of the system around a working point  $(x^*, u^*)$ . The identification process for obtaining the linear model of the plant is developed in [54]. The continuous linear system was discretized using Tustin’s method with a sampling time of 30 s. Also, the working point is given by  $u^* = [0 \text{ kW}, 1.75 \text{ kW}]^T$  and  $x^* = [50 \%, 50 \%^T$ .*

The linear discrete-time model of the plant consists of two input variables,  $P_{H_2}(k)$  and  $P_{\text{grid}}(k)$ , which are measured in kilowatts (kW). Here,  $P_{H_2}(k)$  represents the power of the electrolyzer and the power of the fuel cell: when it is greater than zero, the PEM fuel cell is working ( $P_{\text{fc}}(k)$ ), and when  $P_{H_2}(k)$  is negative, it indicates that the electrolyzer is operating ( $P_{\text{ez}}(k)$ ). Both the electronic load and the electronic power source can either deliver or absorb power from the utility power grid (UPG). The connection with the electric network is “virtual”, since it is emulated by the source and electronic load. Moreover,  $P_{\text{grid}}(k)$  represents the power of UPG, which is positive when the power is imported by the microgrid from the UPG, and it is negative when exporting power to the UPG. The system is subject to uncertainties from the power produced by a renewable energy source; in this case, it is the power from the solar field, ( $P_{\text{res}}(k)$ ) and the power demanded by the consumers ( $P_{\text{dem}}(k)$ ); the difference between them can be considered as disturbances ( $P_{\text{net}}(k)$ ) to the system. Moreover, the plant counts with an additional variable, the power of the batteries ( $P_{\text{batt}}$ ), which is



controlled indirectly, resulting of the power balance. The states are given by the state of charge of the batteries ( $SOC(k)$ ), and the metal hydrides level ( $MHL(k)$ ) of the storage tank, both measured in percentage (%). A scheme of the power variables is shown in Figure 2.3. The discrete-time linear model of the plant, for each time instant  $k \in \mathbb{Z}_+$ , around a working point ( $u^*, x^*$ ), can be written as

$$x(k+1) = Ax(k) + Bu(k) + D\omega(k), \quad (2.15a)$$

that is,

$$x(k+1) = x(k) + \begin{bmatrix} 8.1360 & 5.958 \\ -15.2886 & 0 \end{bmatrix} u(k) + \begin{bmatrix} 5.958 \\ 0 \end{bmatrix} \omega(k). \quad (2.15b)$$

In this model,  $u(k) = [P_{H_2}(k), P_{grid}(k)]^T$  represents the vector of manipulated variables,  $x(k) = [SOC(k), MHL(k)]^T$  is the state vector of the system and  $\omega(k) = P_{net}(k) \in \mathbb{R}^{n_d}$  represents the system disturbance, where  $n_d = 1$ .

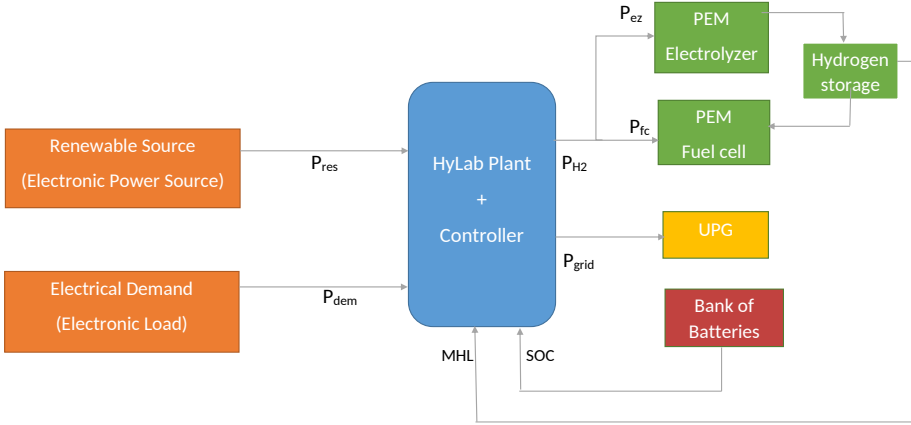


Figure 2.3: HyLab variables scheme.

The system is subject to constraints that avoid equipment damage and guarantee its safe operation. In particular, the Hydrogen Path –both the electrolyzer and the fuel cell– has constraints for limiting the values of  $P_{H_2}(k)$  since its power capacity is limited to 0.9 kW; this value reflects some conservatism and it ensures that the hydrogen path does not work at its nominal value to protect the equipment. In this way, a longer lifespan is expected. Also, the Hydrogen Path has a dead zone between  $-0.1$  kW

and 0.1 kW that ensures a minimum production of power from both the electrolyzer and the fuel cell. The constraints for  $P_{\text{grid}}(k)$  correspond to physical limitations of the electronic units. Furthermore, it is necessary to include constraints on their incremental signals  $\Delta P_{\text{H}_2}(k)$  and  $\Delta P_{\text{grid}}(k)$ , to guarantee the physical safety of the equipment. These constraints are mathematically expressed as follows:

$$-0.9 \text{ kW} \leq P_{\text{H}_2}(k) \leq 0.9 \text{ kW}, \quad (2.16a)$$

$$-2.5 \text{ kW} \leq P_{\text{grid}}(k) \leq 2 \text{ kW}, \quad (2.16b)$$

$$-20 \text{ W s}^{-1} \leq \Delta P_{\text{H}_2}(k) \leq 20 \text{ W s}^{-1}, \quad (2.16c)$$

$$-2.5 \text{ kW s}^{-1} \leq \Delta P_{\text{grid}}(k) \leq 2 \text{ kW s}^{-1}. \quad (2.16d)$$

Overall constraints have to be considered as *hard constraints*, since the equipment lifespan could be drastically reduced. Both the battery bank and the metal hydrides storage tank have limited capacity to prevent any plant damage by overcharge or undercharge. Constraints on  $SOC(k)$  guarantee suitable voltage levels in the 48  $V_{DC}$  bus. Also, they protect the battery bank of strong load voltage variations. These state constraints are written as

$$40 \% \leq SOC(k) \leq 90 \%, \quad (2.17a)$$

$$10 \% \leq MHL(k) \leq 90 \%. \quad (2.17b)$$

The input constraints given by (2.16) can be properly rewritten as

$$u(k) \in \mathcal{U} \subseteq \mathbb{R}^{n_u}, \quad (2.18)$$

with  $n_u = 2$ , while the state constraints defined by (2.17) are expressed as

$$x(k) \in \mathcal{X} \subseteq \mathbb{R}^{n_x}, \quad (2.19)$$

with  $n_x = 2$ . Furthermore, the total power delivered to the load, in order to satisfy the consumer demand, must satisfy the energy balance

$$P_{\text{dem}}(k) = P_{\text{H}_2}(k) - P_{\text{batt}}(k) + P_{\text{grid}}(k) + P_{\text{res}}(k). \quad (2.20)$$

The multi-objective cost function to be minimized is given by

$$\begin{aligned} J(x(k), u(k)) &= a_1(SOC(k) - SOC_{\text{ref}})^2 \\ &+ a_2(MHL(k) - MHL_{\text{ref}})^2 \\ &+ b_1 P_{\text{H}_2}^2(k) + b_2 P_{\text{grid}}^2(k). \end{aligned} \quad (2.21)$$

Here,  $SOC_{\text{ref}} = 65\%$  and  $MHL_{\text{ref}} = 40\%$  are the references given for the state of charge of the batteries and the metal hydride level, respectively. The tuning of the cost function weights seeks for a soft tracking of the output variables towards the given references and an efficient use of the energy. More specifically, the controller is designed such that the batteries are the first way of energy storage. If there exists a big difference between the demanded energy and the produced energy by the renewable sources, it proceeds to the production of hydrogen. These prioritization weights  $a_i, b_i$  have been adjusted by trial and error approach carried out on simulation tests reported in previous works with this plant, see, e.g. [44, 54, 55]. In this sense, they have been established as  $a_1 = a_2 = 10$ ,  $b_1 = 5000$ , and  $b_2 = 8000$ . As can be seen, the weight associated with the hydrogen production is lower than the weight related to the power of the grid in order to minimize the power interchange with the UPG. The weights associated with the outputs take low values compared with the others to give flexibility to the smart grid. However, these values can be modified in the multi-objective function (2.21) for tracking the reference. In this work, the energy management is the main objective, therefore the weights associated with the hydrogen path and the grid are higher than those associated with the outputs of the system.

### 2.2.2 Results and Discussion

The experiments were conducted in the microgrid described in Section 2.2.1 during a trial period of eight hours for each experiment. The controller receives the measured variables  $SOC(k)$  and  $MHL(k)$ , which are used to compute the optimal control signals  $P_{H_2}(k)$  and  $P_{\text{grid}}(k)$  by means of Simulink Real-Time workshop toolbox. The control signals are sent to the SCADA via the OPC Matlab Library and finally the PLC carries out these control actions.

The prediction horizon was  $N_p = 5$  and the sampling time was 30 s. The selected weather and load profiles for verifying the performance of the three proposed controllers were the scaled difference between the real solar generation and the demand registered by the Spanish National Electricity Network (SNEN)<sup>1</sup> on May 23, 2014. These values were sampled each 3 s and scaled for the microgrid allowable power values, which are shown in Figure 2.4(a). The initial conditions for all experiments were  $SOC(0) = 70\%$  and  $MHL(0) = 50\%$ .

An issue that deserves particular attention is the amount of scenarios to be considered into the optimization problem. This number should be selected by taking into account a trade-off between robustness and computational burden. In this sense, it is possible to establish the number of scenarios that guarantees a particular risk level,

<sup>1</sup>SNEN demand data can be obtained at: <https://demanda.ree.es/movil/peninsula/demanda/total>

according to (2.5), as shown Table 2.2.

Table 2.2: Number of scenarios ( $N_s$ ) that fulfills an specific risk level ( $\delta_x$ ).

$\delta_x$	0.20	0.15	0.10	0.05	0.01
$N_s$	152	203	316	611	3005

MS-MPC was performed by using the electricity demand and the solar generation registered during  $N_s = 316$  different days of one year from historical data, obtained from the SNEN. For these scenarios, it is expected a risk of violation of constraints less than  $\delta_x \leq 10\%$ . This number of scenarios offers an acceptable risk and ensures a reasonable computational burden when solving the optimization problem. Furthermore, this set of scenarios considers days with enough solar energy generation as well as cloudy days, which makes the controllers more robust and somehow relieves the need for increasing the number of scenarios used. TB-MPC was performed by using an original number of  $N_s = 316$  scenarios, which were reduced to  $N_r = 250$  scenarios forming a tree using GAMS [56]. This reduction tried to replicate the main dynamics of all original disturbances considered in a small disturbance tree. This reduction introduces a boundary that guarantees  $\delta_x \leq 10\%$ , according to (2.6). Finally, CC-MPC approach was performed considering the failure probability  $\delta_x = 10\%$ . The disturbances were considered as a random function with a cumulative distribution function (cdf), which were obtained from the historical daily data registered in 2014.

The scheme of the microgrid operation, from a general point of view, follows similar patterns for the three proposed controllers. At this point, given that the energy from the renewable source is not sufficient to meet the energy demand, the fuel cell turns on, the battery *SOC* and the *MHL* decrease gradually without going below their forbidden levels. Also, energy is imported from the grid to meet the load beyond demand. When the energy from the renewable source greatly exceeds demand, the electrolyzer is switched on, the batteries are fully charged, and the excess of energy is stored in the hydrogen tank, and the remaining power that cannot be stored in the form of hydrogen is exported to the grid. However, each stochastic MPC approach shows particular differences, as reported below, which are highlighted to offer a suitable comparison among them.

In order to compare all the considered strategies, Figure 2.4(b) shows the battery power for the three aforementioned stochastic predictive controllers. As can be noticed, CC-MPC controller performs a deep cycle using the batteries. It strives for using the full capacity, reaching the upper and lower levels. In contrast, TB-MPC controller,

although partly discharges and recharges the batteries, it does in a softer way. This implies that the excess of energy must be balanced through either the electrolyzer or the grid. It is observed that MS-MPC technique behaves between the two other approaches. Therefore, MS-MPC controller achieves a trade-off between using the full capacity of the batteries and the energy derived either to the electrolyzer or the grid.

Figure 2.4(c) presents the fuel cell and electrolyzer power along the test duration. The fuel cell performance signals obtained are similar for all the three controllers, except for CC-MPC controller, which shows a peak between the first and second hour, to satisfy an increase in the energy demand at that time. When there is an excess of energy from the renewable source, the electrolyzer starts its operation. Results show a clear difference in the electrolyzer operation. On the one hand, with CC-MPC technique, the electrolyzer presents a larger use of the power, as expected. On the contrary, the electrolyzer utilization is restricted quite more with TB-MPC approach, reaching only a peak of 200 W, while CC-MPC controller sets the electrolyzer power to nearly 600 W. Regarding TB-MPC approach, it also shows a small ripple; this is explained because the controller seeks to primary satisfy the demand and compensate any power unbalance in the system. As it has been shown through experimental tests, there are clear differences in the way each controller manages the power signals of the electrolyzer and the fuel cell.

Figure 2.4(d) shows the grid power signal generated by applying the stochastic MPC controllers. From the point of view of the network operators (DSO<sup>2</sup>/TSO<sup>3</sup>), the use of the UPG is minimized with the CC-MPC approach. In this manner, the impact in the electrical system generated by the renewable sources present in the microgrid is reduced. On the other hand, for the consumer point of view, it might be convenient not to force the equipment to a deep duty cycle and take advantage of the grid to smooth the power profiles.

Figure 2.4(e) shows the evolution of the *SOC* and *MHL* for each proposed controllers along the test period. In general, for all the implemented controllers, the batteries are discharged until the fuel cell turns off at the first time, and then they raise their charge level lower than 85% for MS-MPC and CC-MPC controllers. Regarding TB-MPC controller, it holds a charge level around 75% for a longer period compared with the other ones. Then, the *SOC* starts to decrease again for all controllers under study. The *MHL* presents a minor variation, and it reduces its level below 40% until the renewable source can contribute with power to the load. After this, the *MHL* seeks to track its reference.

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<sup>2</sup>DSO: Distributed System Operator

<sup>3</sup>TSO: Transmission System Operator

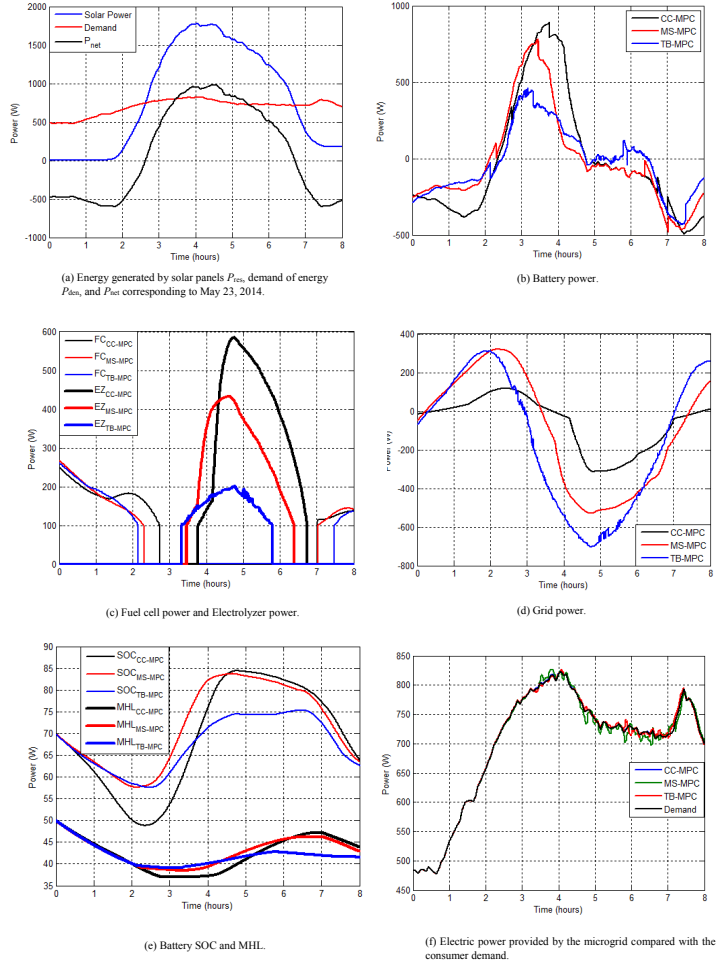


Figure 2.4: Experimental results applying the proposed stochastic MPC approaches.

Figure 2.4(f) shows the comparison among the different powers delivered to the load by applying the controllers. As seen, the demand is satisfied by the power from the microgrid for all the controllers as imposed in their design. Notice that, in some situations, using the “elasticity” of the consumer; it might be possible to momentarily

unbalance the power demand to satisfy other microgrid objectives [57]. Nevertheless, demand response is out of the scope of this work.

In order to quantitatively assess the performances of these three stochastic approaches that have been implemented in the HyLab microgrid laboratory, several KPIs have been defined as follows:

- KPI<sub>1</sub> defines the final cumulative cost given by (2.21) (in cost units).
- KPI<sub>2</sub> is the computational time to solve the optimization problem (in s).
- KPI<sub>3</sub> counts the average unmet demand with respect to the overall power demand (in %).
- KPI<sub>4</sub> is the time that the fuel cell is operating (in hours).
- KPI<sub>5</sub> is the time that the electrolyzer is operating (in hours).
- KPI<sub>6</sub> indicates the final value of *SOC* (in %).
- KPI<sub>7</sub> indicates the final value of *MHL* (in %).

Table 3 summarizes the numerical results of the KPIs. As it can be seen the highest value for KPI<sub>1</sub> is obtained when using MS-MPC. The rationale behind this value is that the controller optimizes a sequence of control actions valid for the most favorable scenarios as well as the least favorable ones. In this sense, an over-conservative control action is carried out. This issue can be relaxed by calculating a tree of control actions that is subject to non-anticipatory equality constraints. In this way, the control actions are calculated in a closed-loop fashion, i.e., the controller can adapt the future control actions to the evolution of the disturbances. As can be seen, TB-MPC reduces its cumulative cost by increasing the number of control variables involved into the optimization problem. Hence, its computational time is the biggest within this comparative study. Regarding CC-MPC, it has the lowest cumulative cost without increasing the number of control variables. For reference purposes, the final cumulative cost for an MPC with a perfect forecast (PF-MPC), obtained via simulation, is  $2.05 \times 10^{12}$ . The computational time comparison is provided by KPI<sub>2</sub>.

The three tested controllers are able to meet the overall demand in a satisfactory way, as indicated by the performance comparison given by KPI<sub>3</sub>. At this point, we must remark that the three tested controllers solve their optimization problems faster than the sampling time. Therefore, it is possible to select the approach that has the best performance in terms of the demand satisfaction and the use of the hydrogen path.

Table 2.3: Comparison of the MS-MPC, TB-MPC, and CC-MPC controllers applied to HyLab microgrid by means of  $KPI_i$ ,  $i = 1, \dots, 7$ .

<b>Controller</b>	<b>KPI<sub>1</sub></b> (cost units)	<b>KPI<sub>2</sub></b> (s)	<b>KPI<sub>3</sub></b> (%)	<b>KPI<sub>4</sub></b> (hours)	<b>KPI<sub>5</sub></b> (hours)	<b>KPI<sub>6</sub></b> (%)	<b>KPI<sub>7</sub></b> (%)
MS-MPC	$3.89 \times 10^{12}$	7.76	0.12	3.26	2.90	63.53	42.91
TB-MPC	$2.75 \times 10^{12}$	18.15	0.11	2.63	2.45	62.51	41.58
CC-MPC	$2.44 \times 10^{12}$	1.04	0	3.70	2.97	63.71	43.85

The comparisons between the proposed controllers regarding the time when both the fuel cell and the electrolyzer are operating are given by  $KPI_4$  and  $KPI_5$ , respectively. In this sense, TB-MPC shows the lowest time for the fuel cell and the electrolyzer. It offers a larger conservatism when working with the hydrogen path, which is obtained at the expense of a higher computational time since TB-MPC meets the current demand and reformulates its disturbance tree at each time step. Notice that the main difference is at the time that the hydrogen path is working.

The final values of  $SOC$  and  $MHL$ , which present similar values for the three controllers, are around 63% and 42%, respectively. These values are provided by  $KPI_6$  and  $KPI_7$ .

Table 2.4 presents a comparison among the total energy produced by the fuel cell ( $E_{fc}$ ), the electrolyzer ( $E_{ez}$ ), the batteries ( $E_{batt}$ ), and the grid ( $E_{grid}$ ) during the test period. The negative sign in  $E_{grid}$  indicates that the amount of energy sold to UPG is greater than the energy purchased. The total energy of the batteries indicates the difference between the stored energy and the delivered energy to the load: the negative value means that the stored energy predominates over the delivered energy.

Table 2.4: Energy produced by the fuel cell, electrolyzer, batteries, and grid during the test period by applying the proposed stochastic MPC controllers.

<b>Controller</b>	$E_{fc}$ (Wh)	$E_{ez}$ (Wh)	$E_{batt}$ (Wh)	$E_{grid}$ (Wh)
MS-MPC	302	481	-62.2	-418
TB-MPC	261	217	-110.23	-661
CC-MPC	348	642	-43.09	-268



Notice that the absolute value of the energy amounts are taken to achieve a reliable comparison in terms of energy consumption for each component of the system. In this sense, CC-MPC has better performance regarding energy efficiency. CC-MPC achieves less exchange with UPG, and the batteries provide enough power to supply the load. Also, both the fuel cell and electrolyzer use energy in a wider range when compared to the MS-MPC and TB-MPC approaches. Note also that TB-MPC and MS-MPC handled more cautiously hydrogen energy from the path while performing more exchanges with the UPG, specially TB-MPC.

Another KPI to compare the performance of the controllers for energy management in a smartgrid is the number of start-ups for both equipment, the fuel cell and the electrolyzer. From the results obtained from the experimental setup, the number of start-ups is the same for all the controllers. However, it is a major factor that could reduce the lifespan of the hydrogen path.

Finally, Table 2.5 shows the range of values of each variable obtained during the experiments by applying the proposed approaches. As seen, the control actions satisfy the constraints given by (2.16) and (2.17).

Table 2.5: Range of values for the states and control inputs obtained during the test period by applying the proposed stochastic MPC controllers.

<b>Variable</b>	<b>MS-MPC</b>	<b>TB-MPC</b>	<b>CC-MPC</b>
$SOC$ (%)	[57.61, 83.73]	[57.59, 75.30]	[48.82, 84.42]
$MHL$ (%)	[38.51, 50]	[39.07, 50]	[37.06, 50]
$P_{fc}$ (W)	[100, 268.13]	[100, 259.69]	[100, 250.44]
$P_{ez}$ (W)	[100, 432.9]	[100, 202.13]	[100, 584.94]
$P_{grid}$ (W)	[-529.4, 320.3]	[-705.02, 314.0]	[-312.8, 117.5]

In order to extend the comparative analysis to general results and taking into account that the experimental setup of the plant is limited, the non-linear simulation model developed in [53] is used to compare the controllers in other situations and the same circumstances. This simulation model replicates the main dynamics of the real plant with enough accuracy. An additional case study for testing the three stochastic MPC controllers and a PF-MPC controller is introduced to enhance the results and obtain conclusions.

Figure 2.5 shows the evolution of the signals by applying the three stochastic MPC controllers and a PF-MPC controller for a cloudy day in the simulation model of the

HyLab microgrid. All controllers present the same evolution to satisfy the demand. The fuel cells are turned on when the power from the renewable sources is not enough to meet the electric demand. Hence, *SOC* and *MHL* decrease gradually to supply power to the load. For this particular day, the microgrid imports energy power from the UPG. Given that the excess of renewable energy production over the demand is not enough, the batteries are charged, and the electrolyzer stays off.

To compare the behavior of these MPC controllers, Table 2.6 shows the results from aforementioned KPIs. The results obtained from the comparison are similar to the previous experimental case study. As expected, the lowest value of  $KPI_1$  is presented by standard MPC controller with perfect information; this value gives a target for the comparison. In this sense, CC-MPC controller results in a lower cumulative cost as well as the computational time compared with MS-MPC and TB-MPC controllers. The electrical demand is satisfied by all controllers. Regarding  $KPI_3$ , MS-MPC controller uses the hydrogen path longer than the other two approaches. Finally,  $KPI_6$  shows very similar values for all controllers, the battery *SOC* is reduced until its lower constrained level. The lowest value of  $KPI_7$  is presented by CC-MPC controller because this controller delivers a bigger amount of energy from the fuel cell. Finally, the electrolyzer stays off over the simulation period; therefore  $KPI_5$  is zero for all controllers.

Table 2.6: Comparison of the MS-MPC, TB-MPC, CC-MPC, and PF-MPC controllers applied to the simulation model of HyLab microgrid for a cloudy day by means of  $KPI_i$ ,  $i = 1, \dots, 7$ .

<b>Controller</b>	<b><math>KPI_1</math></b> (cost units)	<b><math>KPI_2</math></b> (s)	<b><math>KPI_3</math></b> (%)	<b><math>KPI_4</math></b> (hours)	<b><math>KPI_6</math></b> (%)	<b><math>KPI_7</math></b> (%)
MS-MPC	$6.24 \times 10^{12}$	7.76	0.10	6.00	40.56	20.43
TB-MPC	$5.33 \times 10^{12}$	18.20	0.11	5.67	40.64	23.46
CC-MPC	$4.22 \times 10^{12}$	1.04	0.10	5.68	40.46	14.08
PF-MPC	$4.05 \times 10^{12}$	0.98	0	5.90	40.01	19.77

Table 2.7 compares the stochastic MPC controllers regarding energy for a cloudy day via simulation. The CC-MPC controller results in higher energy consumption from the fuel cell. The energy from the renewable sources is not enough at the time to turn on the electrolyzer for storing energy as hydrogen. The batteries are used to provide energy to the load; both, MS-MPC and TB-MPC controllers, show a similar use of the energy of the batteries. A remarkable difference is shown in the energy exchanged with

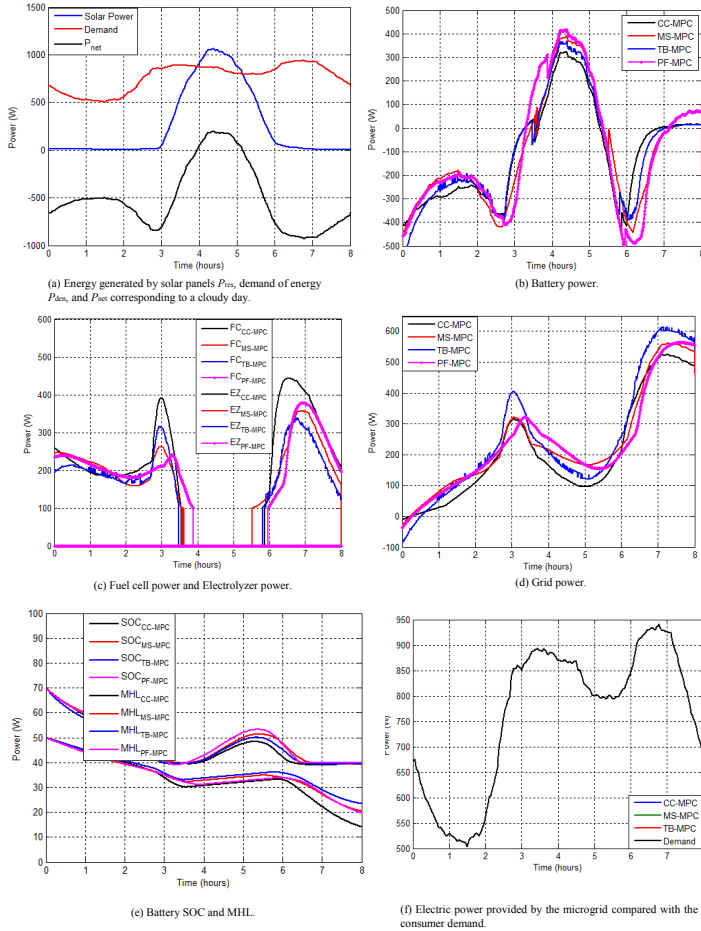


Figure 2.5: Simulation results for a cloudy day applying the proposed stochastic MPC approaches and a standard PF-MPC.

the grid, in this case, the TB-MPC controller presents the highest value.

All in all, Table 2.8 shows priority factors for each one of the proposed stochastic

Table 2.7: Energy produced by the fuel cell, electrolyzer, batteries, and grid during the test period for the simulation model by applying the proposed stochastic MPC controllers.

<b>Controller</b>	$E_{fc}$ (Wh)	$E_{ez}$ (Wh)	$E_{batt}$ (Wh)	$E_{grid}$ (Wh)
MS-MPC	472	0	-285	688
TB-MPC	437	0	-287	721
CC-MPC	547	0	-295	604

Table 2.8: Priority factors for selecting one of the proposed stochastic MPC controllers.

<b>Priority</b>	<b>MS-MPC</b>	<b>TB-MPC</b>	<b>CC-MPC</b>
Maximization of hydrogen path lifespan		✓	
Minimization of energy exchanged with the UPG			✓
Cumulative cost			✓
Computational burden			✓
Demand satisfaction	✓	✓	✓
Availability of historical data	✓	✓	

MPC controllers based on the overall analysis at the time of selecting one of them.

Other factors that are important to take into account are the initial conditions for *SOC* and *MHL*. These values will determine the evolution of the variables. Besides, the final value of these variables will take an additional meaning of comparison after a longer time of use of the plant. However, they have been employed in a smaller period to show how they finish after the experiments.

These results have been published in [31].

## 2.3 Case study: Drinking Water Network

Drinking water networks (DWNs) transport water from sources to consumers ensuring the quality of service [58]. Nevertheless, limited water sources, conservation and sustainability policies, as well as the infrastructure complexity for meeting consumer demands with appropriate flow pressure and quality levels make water management a challenging problem [1]. Water demand forecasting based on historical data is com-

monly used for the operational control of water supply along a given prediction horizon. However, the optimality of such scheduling is affected by the one associated to water demand forecasts. Therefore, the scheduling of control inputs must be continuously adjusted. This leads to consider the DWNs as dynamical systems and their operation as optimal control problems, with the objective of satisfying water demands in an optimal manner despite the presence of disturbances and uncertainties, and considering additionally issues such as constraints on the manipulated and output variables and multiple conflicting control goals.

The MPC approaches presented in this work are assessed with two representative case studies based on the Barcelona DWN.

A DWN must satisfy water demands and guarantee service reliability by making optimal use of water sources and network components in order to minimize economic costs. The water network operates as a full-interconnected system driven by endogenous and exogenous flow demands. In the Barcelona DWN, water is taken from both superficial and underground sources. Flows coming from sources are regulated by pumps or valves. After being extracted from sources, water is purified up to levels suitable for human consumption in four water treatment plants (WTP). The water flow from any of the sources is limited and has costs associated to the extraction and the treatment required. The DWN is divided in two management layers: the *transport network*, which links the water treatment plants with the reservoirs located all over the city, and the *distribution network*, which is sectorized in sub-networks, linking reservoirs directly to consumers. In this work, each sector of the distribution network is considered as a pooled demand to be satisfied by the transport network.

The two systems under study have been extracted from the Barcelona transport network. The first case study consists in a sector model and the second one is an aggregate model of the whole network. They differ mainly in the size of the network flow problems and the number of constitutive elements:

- The sector network considers only a small-scale subsystem related to a portion of the overall DWN (see Fig. 2.6). This case study considers 2 water sources, 3 tanks, 6 flows controlled by valves and pumps, 4 demand nodes and 2 intersection nodes.
- The aggregate network represents a simplification from the original DWN, where sets of elements are aggregated in a single element in order to reduce the size of the original model (see Fig. 2.7). It consists of 9 water sources, 17 tanks, 61 flows controlled by valves and pumps, 25 demand nodes and 11 intersection nodes.

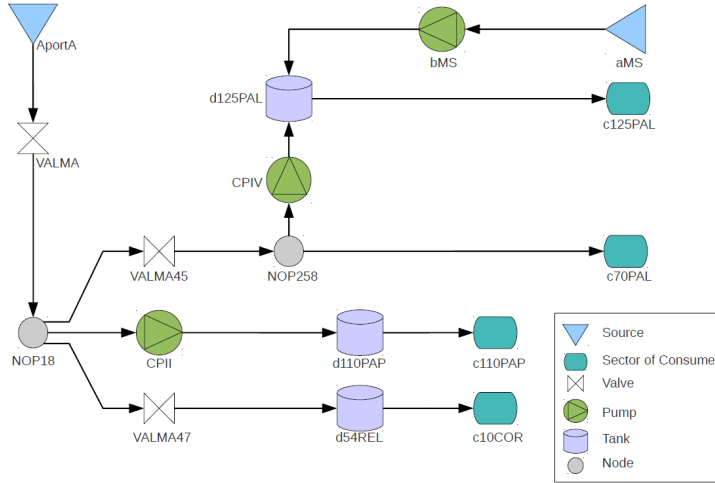


Figure 2.6: Sector diagram of the Barcelona DWN

### Demand Modelling and Scenario Generation

In DWNs, the uncertainty is generally introduced by the stochastic behavior of water consumers. Therefore, a proper demand modeling is required to achieve an acceptable water supply service level. For the case studies of this work, time series forecasting based on auto-regressive integrated moving average (ARIMA) models are used due to its ability to capture complex linear dynamics from historical data [59]. In this way, it is possible to generate artificial scenarios with similar statistical properties to those obtained from historical data.

A correct sampling of scenarios is essential for developing the proposed SMPC approaches. For the CC-MPC approach, ARIMA models are used to generate a large number of possible demand scenarios by Monte Carlo sampling for a given time horizon  $N_p \in \mathbb{Z}_+$ ; the mean demand is then used for the controller design. For the MS-MPC approach, a set of  $N_s \in \mathbb{Z}_+$  water demand scenarios is generated and used. Increasing the number of scenarios allows the controller to gain robustness but at the expense of additional computational effort and economic performance losses. MS-MPC is generally over-conservative, because it does not consider the controller capacity to adapt to the new observations of the uncertainty and reformulate its controller structure at each time instant. To cope with this drawback, a representative subset of scenarios might be chosen using scenario reduction algorithms [39, 60]. Moreover, the

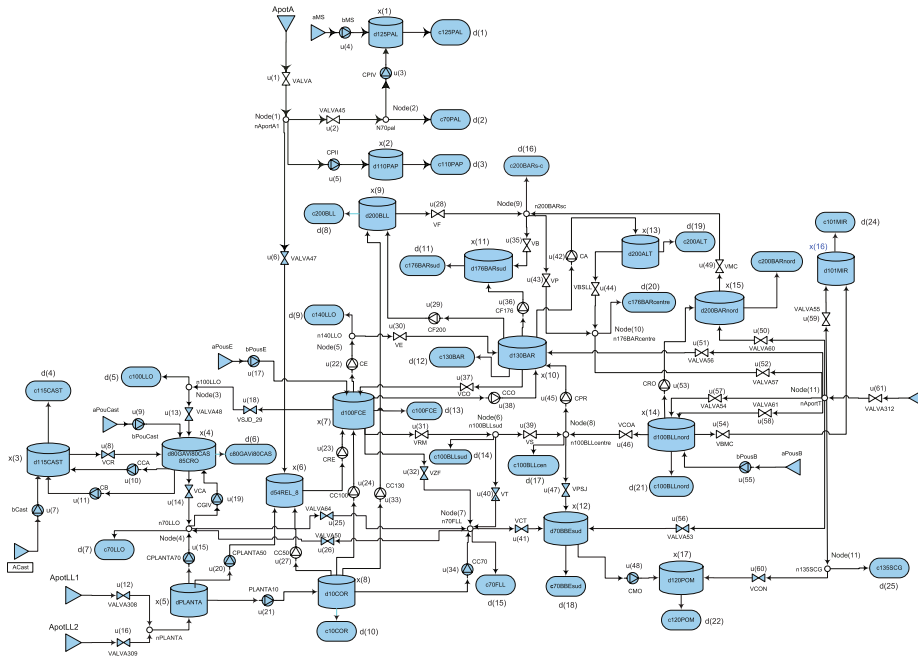


Figure 2.7: Aggregate model of the Barcelona DWN

reduced set of scenarios can be transformed into a rooted tree of possible evolutions of the demand [61], so that, it can be used with the TB-MPC approach, see, e.g., [14, 62]. More specifically, a reduction of the initial number of scenarios into a rooted tree of  $N_r$  scenarios, obtained by generating an ensemble forecast tree, only reduces the number of scenarios that have similar features with their adjacent scenarios. The disturbances tree remains the dominant scenarios. The rationale behind this approach is that uncertainty spreads with time, i.e., it is possible to predict more accurately the demand evolution in a short time horizon than in a large one. Besides, TB-MPC takes into account, within the optimization problem, the MPC capacity to adapt, i.e., a control input sequence is calculated for each branch of the tree at each time step, by implementing the so-called Multistage Stochastic Programming, as pointed in [40].

### 2.3.1 DWN Control Problem Statement

This section introduces the CE-MPC formulation, including the system defined by time-invariant state-space linear model in discrete time, its goals and constraints.

#### Control-oriented Model

The system model may be described considering the volume of water in tanks as the state variables  $x \in \mathbb{R}^n$ , the flow through the actuators as the manipulated inputs  $u \in \mathbb{R}^m$ , and the demanded flows as additive measured disturbances  $d \in \mathbb{R}^p$ . The control-oriented model of the network is described by the following equations for all time instant  $k \in \mathbb{Z}_+$ :

$$x_{k+1} = Ax_k + Bu_k + B_p d_k, \quad (2.22a)$$

$$0 = E_u u_k + E_d d_k, \quad (2.22b)$$

where (2.22a) and (2.22b) describe the mass balance equations for storage tanks and intersection nodes, respectively. Moreover,  $A \in \mathbb{R}^{n_x \times n_x}$ ,  $B \in \mathbb{R}^{n_x \times n_u}$ ,  $B_p \in \mathbb{R}^{n_x \times n_d}$ ,  $E_u \in \mathbb{R}^{n_q \times n_u}$  and  $E_d \in \mathbb{R}^{n_q \times n_d}$ , are time-invariant matrices dictated by the network topology.

**Assumption 1** *The states in  $x$  and the demands in  $d$  are measured at any time instant  $k \in \mathbb{Z}_+$ .*

**Assumption 2** *The pair  $(A, B)$  is controllable and (2.22b) is reachable<sup>4</sup>, provided that  $n_q \leq n_u$  with  $\text{rank}(E_u) = n_q$ .*

**Assumption 3** *The realization of disturbances at the current time instant  $k$  may be decomposed as*

$$d_k = \bar{d}_k + e_k, \quad (2.23)$$

where  $\bar{d}_k \in \mathbb{R}^{n_e}$  is the vector of expected disturbances, and  $e_k \in \mathbb{R}^{n_e}$  is the vector of forecasting errors with non-stationary uncertainty and a known (or approximated) quasi-concave probability distribution  $\mathcal{D}$ . Therefore, the stochastic nature of each  $j^{\text{th}}$  row of  $d_k$  is described by  $d_{(j),k} \sim \mathcal{D}_i(\bar{d}_{(j),k}, \Sigma(e_{(j),k}))$ , where  $\bar{d}_{(j),k}$  denotes its mean, and  $\Sigma(e_{(j),k})$  its variance.

**Assumption 4** *The demands are bounded, i.e.,  $d_k \in \mathbb{D}$ , for all  $k \in \mathbb{Z}_+$ , and input-disturbance dominance constraints hold, i.e.,  $B_d \mathbb{D} \subseteq -B \mathbb{U}$  and  $E_d \mathbb{D} \subseteq -E_u \mathbb{U}$ .*

<sup>4</sup>If  $n_q < n_u$ , then multiple solutions exist, so  $u_k$  should be selected by means of an optimization problem. Equation (2.22b) implies the possible existence of uncontrollable flows  $d_k$  at the junction nodes. Therefore, a subset of the control inputs will be restricted by the domain of some flow demands.



The system is subject to storage and flow capacity hard constraints considered here in the form of convex polyhedra defined as

$$x_k \in \mathbb{X} \triangleq \{x \in \mathbb{R}^{n_x} \mid Gx \leq g\}, \quad (2.24a)$$

$$u_k \in \mathbb{U} \triangleq \{u \in \mathbb{R}^{n_u} \mid Hu \leq h\}, \quad (2.24b)$$

for all  $k$ , where  $G \in \mathbb{R}^{r_x \times n_x}$ ,  $g \in \mathbb{R}^{r_x}$ ,  $H \in \mathbb{R}^{r_u \times n_u}$ ,  $h \in \mathbb{R}^{r_u}$ , being  $r_x \in \mathbb{Z}_+$  and  $r_u \in \mathbb{Z}_+$  the number of state and input constraints, respectively.

Note that in (2.22b) a subset of controlled flows are directly related with a subset of uncontrolled flows. Hence,  $u$  does not take values in  $\mathbb{R}^{n_u}$  but in a linear variety. This latter observation, in addition to Assumption 2, can be exploited to develop an affine parametrization of control variables in terms of a minimum set of disturbances as shown in [19, Appendix A], mapping control problems to a space with a smaller decision vector and with less computational burden due to the elimination of the equality constraints. Thus, the system (2.32) can be rewritten as

$$x_{k+1} = Ax_k + \tilde{B}\tilde{u}_k + \tilde{B}_d d_k, \quad (2.25)$$

and the input constraint (2.24b) replaced with a time-varying restricted set defined as

$$\tilde{\mathbb{U}}_k \triangleq \{\tilde{u} \in \mathbb{R}^{n_u - n_q} \mid H\tilde{P}_1\tilde{u} \leq h - H\tilde{P}_2 d_k\} \quad \forall k \in \mathbb{Z}_+, \quad (2.26)$$

where  $\tilde{B} \in \mathbb{R}^{n_x \times (n_u - n_q)}$ ,  $\tilde{B}_d \in \mathbb{R}^{n_x \times n_d}$ ,  $\tilde{P}_1 \in \mathbb{R}^{n_u \times (n_u - n_q)}$  and  $\tilde{P}_2 \in \mathbb{R}^{n_u \times n_d}$ , are selection and permutation matrices (see [19, Appendix A] for details). The sets  $\tilde{\mathbb{U}}_k$  are non-empty for all  $k$  due to Assumption 4.

### Control Problem Statement

The goal is to design a control law that minimises a (possibly multi-objective) convex stage cost  $\ell(k, x, \tilde{u}) : \mathbb{Z}_+ \times \mathbb{X} \times \tilde{\mathbb{U}}_k \rightarrow \mathbb{R}_+$ , which bears a functional relationship to the economics of the system operation. Let  $x_k \in \mathbb{X}$  be the current state and let  $\mathbf{d}_k = \{d_{k+i|k}\}_{i \in \mathbb{Z}_{[0, N_p-1]}}$  be the sequence of disturbances over a given prediction horizon  $N_p \in \mathbb{Z}_{\geq 1}$ . The first element of this sequence is measured, i.e.,  $d_{k|k} = d_k$ , while the rest of the elements are estimates of future disturbances computed by an exogenous system and are available at each time step  $k \in \mathbb{Z}_+$ . Hence, the MPC controller design is based on the solution of the following finite-horizon optimization problem:

$$\min_{\mathbf{u}_k} \sum_{i=0}^{N_p-1} \ell(k+i, x_{k+i|k}, \tilde{u}_{k+i|k}), \quad (2.27a)$$

subject to:

$$x_{k+i+1|k} = Ax_{k+i|k} + \tilde{B}\tilde{u}_{k+i|k} + \tilde{B}_d d_{k+i|k}, \quad \forall i \in \mathbb{Z}_{[0, N_P-1]} \quad (2.27b)$$

$$x_{k+i|k} \in \mathbb{X}, \quad \forall i \in \mathbb{Z}_{[1, N_P]} \quad (2.27c)$$

$$\tilde{u}_{k+i|k} \in \tilde{\mathbb{U}}_{k+i}, \quad \forall i \in \mathbb{Z}_{[0, N_P-1]} \quad (2.27d)$$

$$x_{k|k} = x_k. \quad (2.27e)$$

Assuming that (2.27) is feasible, i.e., there exists a non-empty control input sequence  $\tilde{\mathbf{u}}_k = \{\tilde{u}_{k+i|k}\}_{i \in \mathbb{Z}_{[0, N_P-1]}}$ , then the receding horizon philosophy and the model back-transformation commands to apply the control input

$$u_k = \kappa_N(k, x_k, \mathbf{d}_k) = \tilde{P}_1 \tilde{u}_{k|k}^* + \tilde{P}_2 d_k. \quad (2.28)$$

This procedure is repeated at each time instant  $k$ , using the current measurements of states and disturbances and the most recent forecast of these latter over the next future horizon.

### 2.3.2 Results

The formulation of the SMPC problems for the case studies considered in this work addresses the design of a control law that (i) minimizes the economic operational cost, (ii) guarantees the availability of enough water to satisfy the demand and (iii) operates the network with smooth variations of the flow through actuators. These objectives can be expressed quantitatively by the following performance indicators<sup>5</sup> for all time steps  $k \in \mathbb{Z}_+$ :

$$\ell_E(x_k, \tilde{u}_k; c_{u,k}) \triangleq c_{u,k}^\top W_e \tilde{u}_k \Delta t, \quad (2.29a)$$

$$\ell_S(x_k; s_k) \triangleq \begin{cases} (x_k - s_k)^\top W_s (x_k - s_k) & \text{if } x_k \leq s_k \\ 0 & \text{otherwise,} \end{cases} \quad (2.29b)$$

$$\ell_\Delta(\Delta \tilde{u}_k) \triangleq \Delta \tilde{u}_k^\top W_{\Delta \tilde{u}} \Delta \tilde{u}_k. \quad (2.29c)$$

The first objective,  $\ell_E(x_k, \tilde{u}_k; c_{u,k}) \in \mathbb{R}_{\geq 0}$ , represents the economic cost of network operation at time step  $k$ , which depends on a time-of-use pricing scheme driven by a time-varying price of the flow through the actuators  $c_{u,k} \triangleq (c_1 + c_{2,k}) \in \mathbb{R}_+^{n_u - n_q}$ , which in this application takes into account a fixed water production cost  $c_1 \in \mathbb{R}_+^{n_u}$

<sup>5</sup>The performance indicators considered in this work may vary or be generalized with the corresponding manipulation to include other control objectives.

and a water pumping cost  $c_{2,k} \in \mathbb{R}_+^{n_u}$  that changes according to the electricity tariff (assumed periodically time-varying). All prices are given in economic units per cubic meter (e.u./m<sup>3</sup>). The second objective,  $\ell_S(x_k; s_k) \in \mathbb{R}_{\geq 0}$  for all  $k$ , is a performance index that penalizes the amount of water volume going below a given safety threshold  $s_k \in \mathbb{R}^{n_x}$  in m<sup>3</sup>, which is desired to be stored in tanks and satisfies the condition  $x_{\min} \leq s_k \leq x_{\max}$ . Note that this safety objective is a piecewise continuous function, but it can be redefined as  $\ell_S(\xi_k; x_k, s_k) \triangleq \xi_k^\top W_s \xi_k$ , accompanied with two additional convex constraints, i.e.,  $x_k \geq s_k - \xi_k$  and  $\xi_k \in \mathbb{R}_+^{n_x}$ , for all  $k$ , being  $\xi_k$  a slack variable. The minimal volume of water required in a tank is given by its net demand, hence,  $s_k = -B_p d_k$  for all  $k$ . The last objective,  $\ell_\Delta(\Delta\tilde{u}_k) \in \mathbb{R}_{\geq 0}$ , represents the penalization of control signal variations  $\Delta\tilde{u}_k \triangleq \tilde{u}_k - \tilde{u}_{k-1} \in \mathbb{R}^{n_u - n_q}$ . The inclusion of this latter objective aims to extend actuator's life and assure a smooth operation of the dynamic network flows. Furthermore,  $W_e \in \mathbb{S}_{++}^{n_u - n_q}$ ,  $W_s \in \mathbb{S}_{++}^{n_x}$  and  $W_{\Delta\tilde{u}} \in \mathbb{S}_{++}^{n_u - n_q}$  are matrices that weight each decision variable in their corresponding cost function.

To achieve the control task, the above predefined objectives are aggregated in a multi-objective stage cost function, which depends explicitly on time due to the time-varying parameters of the involved individual objectives. The overall stage cost is defined for all  $k \in \mathbb{Z}_+$  as

$$\ell(k, x_k, \tilde{u}_k, \xi_k) \triangleq \lambda_1 \ell_E(x_k, \tilde{u}_k; c_{u,k}) + \lambda_2 \ell_\Delta(\Delta\tilde{u}_k) + \lambda_3 \ell_S(\xi_k; x_k, s_k), \quad (2.30)$$

where  $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}_+$  are scalar weights that allow to prioritize the impact of each objective involved in the overall performance of the network. These weights are assumed to be fixed by the managers of the DWN.

Numerical results of applying the three different SMPC approaches (CC-MPC, TB-MPC and MS-MPC) to the Barcelona DWN case studies are summarized in Tables 2.9, 2.10 and 2.11.

Simulations have been carried out over a time horizon of eight days, i.e.,  $n_s = 192$  hours, with a sampling time of one hour. The patterns of the water demand in this work were synthesized from real values measured in the considered demand of the Barcelona DWN between July 23<sup>th</sup> and July 27<sup>th</sup>, 2007. Initial conditions, i.e., source capacities, initial volume of water at tanks and starting demands, are set a priori according to real data. The weights of the cost function (2.30) are  $\lambda_1 = 100$ ,  $\lambda_2 = 10$ , and  $\lambda_3 = 1$ ; these values allow to prioritize the impact of each objective involved in the overall performance of the network. The prediction horizon is selected as  $N_p = 24$  hours, due to the periodicity of disturbances. The formulation of the optimization problems and the closed-loop simulations have been carried out using MATLAB R2012a (64 bits) and CPLEX solver, running in a PC Core i7 at 3.2 GHz with 16 GB of RAM.

The key performance indicators used to assess the aforementioned controllers are defined as follows:

$$\Phi_1 \triangleq \frac{24}{n_s} \sum_{k=1}^{n_s} \ell_k, \quad (2.31a)$$

$$\Phi_2 \triangleq |\{k \in \mathbb{Z}_{[1, n_s]} \mid x_k < -B_p d_k\}|, \quad (2.31b)$$

$$\Phi_3 \triangleq \sum_{k=1}^{n_s} \sum_{i=1}^{n_x} \max\{0, -B_{p(i)} d_k - x_{k(i)}\}, \quad (2.31c)$$

$$\Phi_4 \triangleq \frac{1}{n_s} \sum_{k=1}^{n_s} t_k, \quad (2.31d)$$

where  $\Phi_1$  is the average daily multi-objective cost with  $\ell_k$  given by (2.30),  $\Phi_2$  is the number of time instants where water demands are not satisfied,  $\Phi_3$  is the accumulated volume of water demand that was not satisfied over the simulation horizon  $n_s$ , and  $\Phi_4$  is the average time in seconds required to solve the MPC problem at each time instant  $k \in \mathbb{Z}_{[1, n_s]}$ .

The effect of considering different levels of joint risk acceptability was analyzed for the CC-MPC approach using  $\delta = \{0.3, 0.2, 0.1\}$ . In the same way, the size of the set of scenarios selected for the MS-MPC is established by using (2.6) to guarantee the same risk levels of the CC-MPC approach. In this way, the total number of scenarios that represents the evolution of the water demand in the considered simulation time for the MS-MPC was  $N_s = 192$ . Likewise, the TB-MPC approach was applied considering different sizes for the set of reduced scenarios, i.e.,  $N_r = \{5, 10, 19, 38, 75, 107, 129, 149\}$ . The last three  $N_r$  scenarios allow to compare the behavior between MS-MPC and TB-MPC, while the remaining scenarios were used to analyze the performance with respect to a small number of scenarios.

Table 2.9 summarizes the results of applying the SMPC approaches to the sector model of the Barcelona DWN presented in Fig. 2.6. The different values of  $\delta$  in the CC-MPC approach highlight that both reliability and control performance are conflicting objectives, i.e., the inclusion of safety mechanisms in the controller increases the reliability of the DWN in terms of demand satisfaction, but also the cost of its operation. The main advantage of the CC-MPC is its formal methodology, which leads to obtain optimal safety constraints that tackle uncertainties and allow to achieve a specified global service level in the DWN. Moreover, the CC-MPC robustness is achieved with a low computational burden given that the only extra load is the computation of the stochastic characteristics of disturbances propagated along the prediction horizon

Table 2.9: Comparison of the CC-MPC, TB-MPC and MS-MPC applied to the sector model of the DWN.

CC-MPC					TB-MPC					MS-MPC					
$\delta$	$\Phi_1$	$\Phi_2$	$\Phi_3$	$\Phi_4$	$\Phi_1$	$\Phi_2$	$\Phi_3$	$\Phi_4$	$N_r$	$\Phi_1$	$\Phi_2$	$\Phi_3$	$\Phi_4$	$N_s$	$\delta$
0.3	58535.80	0	0	0.0919	58397.14	0	0	1.2548	5	60831.33	0	0	16.1547	107	0.3
					58515.40	1	0.4813	1.9701	10						
					58589.15	1	4.155	3.0145	14						
0.2	58541.19	0	0	0.707	58515.37	0	0	3.2218	19	69342.48	0	0	17.2401	129	0.2
					58678.12	2	0.7443	7.5362	38						
					58647.27	4	4.0329	27.0914	75						
0.1	58558.29	0	0	0.716	58705.28	1	0.2136	35.6572	107	66011.29	0	0	21.7521	149	0.1
					58713.15	0	0	42.7024	129						
					58761.98	0	0	54.4587	149						

$N_p$ . In this way, the CC-MPC approach is suitable for real-time control of large-scale DWNs.

Regarding the TB-MPC and MS-MPC approaches, numerical results show that considering a large number of scenarios, increments (in average) the stage cost while reducing the volume of unsatisfied water demand. This might be influenced by the quality of the information that remains after the reduction algorithms, consequently, it affects the robustness of the approach being subject of further research.

Results show that all the proposed methods take less than 1 minute to solve the optimization problem in each case, being much shorter than the sampling time of 1 hour. Hence, it is possible to select an approach that may show the best performance in terms of demand satisfaction (in practice).

The main drawback of the TB-MPC approach is the solution average time and the computational burden. The implementation for all cases taking scenarios greater than  $N_r = 149$  was not possible due to memory issues. Hence, some simplification assumptions as those used in [63] or parallel computing techniques might be useful. Another way to address the problem generated by the computer effort is to use a MS-MPC based on the three-scenarios approach. At this point, the best, the worst and the average disturbance scenarios were obtained by generating 100 different possible evolutions of the disturbance, then they were lumped and averaged the 10 lowest, 10 highest, and 80 middle, respectively. It means that the occurrence probabilities were established as 0.1, 0.1, and 0.8 for the best, the worst, and the average disturbance scenario, respectively, as proposed in [10].

Additionally, Table 2.10 summarizes the simulation results of applying the studied SMPC approaches again to the sector model DWN. The configuration of the controllers in this case is as follows: the CC-MPC with a probability of risk of 5%, the TB-MPC reducing to  $N_r = 3$  branched disturbances, the MS-MPC based on the three

scenarios approach (i.e., best, worst and average), and the CE-MPC with an average disturbance. On the one hand, the CE-MPC approach presents the lower cost but on the other hand it has problems with the demand satisfaction. The TB-MPC approach presents a lower accumulated volume of unsatisfied water demand compared with respect to the CE-MPC approach. The MS-MPC and CC-MPC approaches are able to satisfy the water demand required by the consumers. The CC-MPC approach presents a better performance regarding cost and computational time compared to the MS-MPC approach.

Table 2.10: Comparison of the MS-MPC, TB-MPC, CC-MPC and CE-MPC applied to the sector model DWN.

<i>Controller</i>	$\Phi_1$	$\Phi_2$	$\Phi_3$	$\Phi_4$
CC-MPC	585401.16	0	0	0.1069
TB-MPC	58425.59	1	1.2229	1.4235
MS-MPC	60567.62	1	0.6965	0.5314
CE-MPC	58327.55	1	0.7411	0.1041

As for the second case study, Table 2.11 presents the results obtained after the application of the SMPC approaches to the aggregate model DWN, as a way to show the strengths and weaknesses of each of the aforementioned approaches applied to a larger system. For this reason, TB-MPC and MS-MPC with a large number of scenarios, could not be applied due to memory issues. TB-MPC was implemented with a reduction to  $N_r = 3$  branched scenarios. MS-MPC has been designed considering the three scenarios (minimum, average and maximum) as explained in the previous case study. CC-MPC is applied to this system with a risk probability of 5%. As it can be seen from the results, the TB-MPC approach does not offer benefits in terms of satisfaction of water demand and computational time with this limited amount of scenarios for the DWN aggregate model. MS-MPC presents encouraging results regarding demand satisfaction and computational time well below that TB-MPC. MS-MPC approach presents a higher average daily multi-objective cost and a computational time required to solve the FHOP around three times more regarding CC-MPC. Furthermore, MS-MPC and CC-MPC, have a good performance with respect to water demand satisfaction. Based on the obtained results, the CC-MPC approach offers better performance in terms of demand satisfaction, computational time and, it presents the best behavior with respect to the average daily multi-objective cost ( $\phi_1$ ) compared with the same indicator obtained with TB-MPC and MS-MPC approaches.

Table 2.11: Comparison of the MS-MPC, TB-MPC, CC-MPC and CE-MPC applied to the DWN case study for the aggregate DWN model.

<i>Controller</i>	$\Phi_1$	$\Phi_2$	$\Phi_3$	$\Phi_4$
CC-MPC	$1.4064 \cdot 10^4$	0	0	0.9056
TB-MPC	$1.4497 \cdot 10^4$	20	18.81	8.48
MS-MPC	$1.5267 \cdot 10^4$	0	0	3.24
CE-MPC	$1.2038 \cdot 10^4$	23	5.211	0.8442

As expected, the SMPC approaches have a better performance indicators with respect to CE-MPC approach, which does not take into account the stochastic nature of water demand in the formulation of FHOP.

Simulation results show that all the considered methods require less than 1 min to solve the optimization problem in each case, much shorter than the sampling time of 1 h. Hence, it is possible to choose an approach that shows the best performance in terms of demand satisfaction, which is given by the number of time instants where water demands were not satisfied and by the cumulated volume of non-satisfied water demand. In this sense, the results in this work show that CC-MPC is more appropriate when requiring a low probability of constraint violation because the use of TB-MPC and MS-MPC implies the inclusion of a higher number of scenarios, which hinders the application of these control strategies to large DWNs. However, the use of these scenario-based approaches may be very demanding in terms of computational time.

These results have been published in [32].

## 2.4 Case Study: Stock management in a hospital pharmacy

Stock management is a common problem that is present in almost all the companies and organizations. The solution for this problem is given by a policy that determines how and when the orders should be placed. However, there are different difficulties associated to the problem. In the first place, there are uncertainties in the demand and delays in the deliveries, which make the problem not deterministic and require a degree of conservatism to avoid stockouts. It is needless to say that the lack of certain drugs in a hospital may endanger the life of the inpatients and, in the worst case, may have catastrophic consequences in the form of human losses. In order to avoid this situation, it is preferred to increase stock levels, but this is not always possible due

to economical constraints. Actually, the pharmacy is a major source of expenses in hospitals. In [64], it is estimated that about 35% of the total budget of a public hospital comes from the pharmacy department. In a wider sense, the limitations imposed by the budget are also translated into the human resources in the pharmacy and the room available for storing drugs, which introduce additional constraints for the management. Hence, it may not be possible to place and receive orders too often due to the lack of pharmacists. Moreover, space constraints are important for example in drugs that must be stored in a fridge. Therefore, there is a need to develop advanced cost-efficient safe policies for stock management in hospitals capable of dealing with many different type of constraints.

In general, the typical method used to solve inventory control problems is simple. An usual policy is a point of reorder one  $(s, S)$ , that is, whenever the stock is below the level  $s$ , an order is placed to increase the stock up to the value  $S$ . Another option is to fix a size for the orders,  $Q$ , and submit an order once the stock is at level  $s$ . Other related policies about how to solve this problem are given in [65] and [66]. The major drawback with these techniques is that they are not able to take into account all the factors involved in the decision problem. For example, [67] presents an analytical model for the coordination of inventory and transportation in supply-chain systems considering a vendor realizing a sequence of random demands. Also in [68], a supply chain network model consisting of manufacturers and retailers, where the demand is random, is developed. More strategies are presented in [69], where a competitive and cooperative selection of inventory policies in a supply chain with stochastic demand are studied. On the other hand, [70] develops a model to design a supply chain network with deterministic demand. In this sense, CC-MPC is applied to cope with the uncertainty that involves the drug demand in a hospital pharmacy.

### 2.4.1 Pharmacy Inventory Management

In this section, the mathematical background needed to build the optimization problem to be solved by the CC-MPC is presented.

#### System Definition

In general, it will be assumed that there are  $N_i$  different drugs in the pharmacy inventory. The stock level of each one follows an evolution depending on the orders and on the demand. This evolution is represented by a discrete linear model, which for the



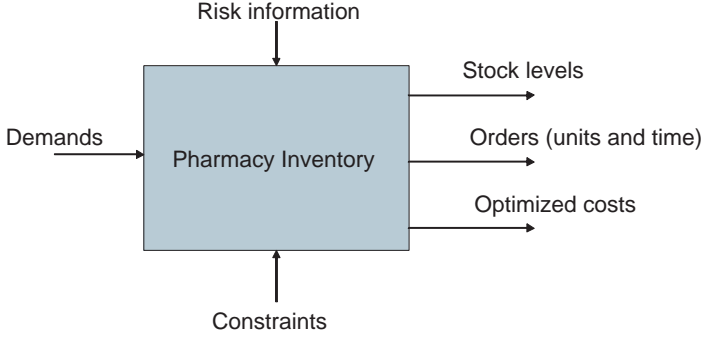


Figure 2.8: Block System

particular case of drug  $i$  is

$$s_i(t + 1) = s_i(t) + \sum_{j=1}^{np_i} o_i^j(t - \tau_i^j) - d_i(t), \quad (2.32)$$

where  $s_i \in \mathbb{Z}$  is the stock of drug  $i$ ,  $o_i^j \in \mathbb{Z}$  is the number of ordered items to the  $j$ -th of the  $np_i$  providers of the drug  $i$ ,  $\tau_i^j$  is its corresponding transport delay, and  $d_i(t)$  represents the aggregate demand of drug  $i$ .

The number of ordered items can be modeled as  $o_i^j = \delta_i^j(t - \tau_i^j) o_i^j(t - \tau_i^j)$ , where  $\delta_i^j(t)$  is a Boolean variable, that is,  $\delta_i^j(t) = 1$  if an order of drug  $i$  to provider  $j$  is placed during time  $t$ , otherwise  $\delta_i^j(t) = 0$ , and  $o_i^j \in \mathbb{Z}$  represents the number of ordered items of drug  $i$  to provider  $j$ , only in those cases where  $\delta_i^j(t) = 1$ .

### Single Hospital Optimization Problem

The system can be represented according to Figure 2.8. In this figure, the inputs represent the information that the pharmacy managers have available in order to make the decisions about the order placement. The information consists of the estimated drug demands, the information about the disturbances and the constraints. The outputs are the optimal stock levels, minimum costs and data about when and how many orders should be placed.

Every time an order is placed, the following costs are associated to it:

- $p_i^j$ , the  $j$ -th provider offers this price for drug  $i$ . It is supposed in this paper that it does not depend on the number of ordered items.
- $C_{sh,i}^j$ , asking drug  $i$  to provider  $j$  has associated this shipping cost.
- $C_{op,i}$  is the cost of ordering drug  $i$ .
- $C_{os,i}$  is the cost associated with having less stock of drug  $i$  in the pharmacy than the minimum stock level allowed for drug  $i$ . This situation is particularly dangerous since there is a high risk for the hospital of running out of drug  $i$  and not satisfy the clinical needs of the patients. In this case, it would be possible to ask the drugs to other hospitals, but these kind of loans may have a high cost.
- $C_{s,i}$  is the cost of storage of drug  $i$ .
- $C_{o,i}$  is the opportunity cost of having drug  $i$  in the pharmacy storage.

Note that both stock levels and costs are direct consequence of the orders placed. Finally, the goals that the managers reach are the following: *i*) the demand has to be satisfied; *ii*) the fixed assets must be reduced; and *iii*) the number of orders placed has to be minimized. This goals are considered in the optimization problem. In particular, the performance index considered in this work involves a multicriteria weighted function where demand satisfaction, expenses and number of orders are included, i.e.,

$$\min_o J := \beta_1 J_1(o, t) + \beta_2 J_2(o, t) + \beta_3 J_3(o, t), \quad (2.33)$$

where  $J_1$ ,  $J_2$  and  $J_3$  are the terms associated to demand satisfaction, costs and orders, respectively. Weights  $\beta_i$  prioritize the different terms, being the solution of the problem strongly dependable on them.

The terms in the objective function (2.33) are described next in decreasing priority:

1.  $J_1$ : Demand satisfaction. The main objective is the minimization of stockout probability. The main issue here is that the demand is not known in advance, i.e., it is stochastic. There may also be uncertainty in the transport delays. For this reason, it is usual in practice to set a safety stock to mitigate the impact of uncertainty. There are two possibilities in the way the safety stock is set: it can be either fixed or variable. The former proposes that the minimum stock level is introduced as a fixed parameter in the optimization problem. The latter treats the safety stock as an optimization variable. The mathematical condition is expressed as

$$\min_{\delta_i^j, \sigma_i^j \forall i,j} \sum_{k=0}^N \sum_{i=1}^{N_i} C_{os,i} \lambda_{\text{stockout}}^i, \quad (2.34)$$

with

$$\lambda_{\text{stockout}}^i = \begin{cases} 1 & \text{if } s_i(t+k) < S_{\min}^i, \\ 0 & \text{if } s_i(t+k) > S_{\min}^i, \end{cases} \quad (2.35)$$

where  $\lambda_{\text{stockout}}^i$  is a variable that tells whether or not the safety stock has been violated,  $S_{\min}^i$  is the minimum stock level allowed for the drug  $i$ ,  $N$  is the length of the time horizon in which the condition has to be satisfied and  $N_i$  is the number of different drugs.

2.  $J_2$ : Expenses. This term of the objective function (2.33) deals with the minimization of the expenses in the orders of drugs and the inventory levels, i.e.,

$$\begin{aligned} \min_{\delta_i^j, o_i^j \forall i, j} & \sum_{k=0}^N \sum_{i=1}^{N_i} \sum_{j=1}^{np_i} \delta_i^j(t+k) (p_i^j o_i^j(t+k) + C_{sh,i}^j) \\ & + \sum_{k=0}^N \sum_{i=1}^{N_i} C_{s,i} s_i(t+k) + \sum_{k=0}^N \sum_{i=1}^{N_i} C_{o,i} s_i(t+k). \end{aligned} \quad (2.36)$$

3.  $J_3$ : Orders. With the inclusion of this term in the objective function, the number of placed orders is trying to be minimized. That is useful because each order has associated certain costs. For example, in a hospital such as Reina Sofía (Cordoba, Spain) more than twelve thousand orders are placed during a year. This condition can be written as the following minimization problem:

$$\min_{\delta} \sum_{k=0}^N \sum_{i=1}^{N_i} \sum_{j=1}^{np_i} C_{op,i} \delta_i^j(t+k). \quad (2.37)$$

Furthermore, the problem considers the following constraints:

- **Storage constraints.** As explained before, the level of the stock of drug  $i$  has to be greater than a safety stock  $S_{\min}^i$ , to reduced the probability of running out of the drug. Furthermore, the space restrictions in the storage room must also be considered, which limits the maximum number of drugs that can be stored in the pharmacy. Therefore,

$$s_i \in [S_{\min}^i, S_{\max}^i]. \quad (2.38)$$

- **Order constraints.** In order to formulate these constraints, two type of variables are going to be used. The first one is a Boolean variable  $\delta_i^j(t) \in [0, 1]$ , where the value 1 means that an order of drug  $i$  has been placed to provider  $j$  during time  $t$ , and the value 0 means that no order has been placed. In case of placing an

order ( $\delta_i^j(t) = 1$ ), the other variable, that represents the ordered number of items, is used. This variable should be bounded by both a minimum and a maximum values, i.e.,

$$o_i^j \in [\min_{o_i^j}, \max_{o_i^j}]. \quad (2.39)$$

There are also some considerations about the minimum number of items to order:

- There is a minimum of items to order at each time,  $\min_{\text{item}_i^j}$ . That is because the distributors do not work if the number of items is too small. Hence, there is a threshold for the number of items to order.
- The pharmaceutical laboratories do not provide the drugs unless a minimum quantity of money is spent. That is translated into a minimum order size,  $\min_{\text{lab}_i^j}$ . Taking into account these quantities,

$$\min_{o_i^j} = \max(\min_{\text{item}_i^j}, \min_{\text{lab}_i^j}).$$

- There is also another consideration to take into account related to the *non-working days* of the laboratory (e.g., Sundays, holidays). The controller will have to be synthesized in such a way this constraint is considered. That leads into the following constraint:

$$\delta_i^j(t) = 0, \quad \forall t \notin \{\text{working days}\}. \quad (2.40)$$

- **Operational constraints.** These constraints take into account the limited capacity of the pharmacy for placing orders and receiving shipments. This fact limits the number of orders placed along a time horizon of length  $N$ , i.e.,

$$\sum_{k=0}^N \sum_{j=1}^{np_i} \delta_i^j(t+k) \leq \Delta_i, \quad (2.41)$$

where  $\Delta_i$  is the maximum number of orders of drug  $i$  that can be placed along  $N$ .

- **Economical constraints.** The money spent during the time horizon  $N$  has to be also limited, being  $\max_s$  the maximum amount. Therefore, the mathematical constraint is expressed as

$$\sum_{k=0}^N \sum_{i=1}^{N_i} \sum_{j=1}^{np_i} \delta_i^j(t+k) (p_i^j o_i^j(t+k) + C_{sh,i}^j + C_{op,i}) \leq \max_s. \quad (2.42)$$

### Multiple-hospitals Extension

In order to reduce the minimal stock of drugs required in a hospital pharmacy, different hospitals could collaborate between them, e.g., if they are close to each other and the consumption of certain drugs is uncorrelated between them. Consequently, the expenses derived from loans between them are getting lower or can even be neglected. This way, the hospitals can focus on the joint stockout probability instead of the individual one, which should be higher, resulting in lower safety and average stock levels. In the simplest case, each hospital would keep its original optimization problem only with different constraints. Likewise, it could also be possible to pose this problem as a distributed control one, where the hospitals are agents that have to reach a consensus on the safety levels. The optimization problem is

$$\min_o \sum_{h=1}^H J_h, \quad (2.43)$$

where  $J_h$  stands for the cost of each hospital and  $H$  is the number of collaborating hospitals. The overall objective function taking into account all considered hospitals is given by

$$J_h = \sum_{i=1}^3 \sum_{j=1}^H \beta_{i,j} J_{i,j}(o, t),$$

where the demand, expenses and orders terms are like in (2.34)-(2.37). The difference here is that, in the demand term, the probability  $Pr(s_i^h(t+k) < 0)$  can be greater.

The constraints are, like in the previous case, (2.38)-(2.42), and:

$$s_i^h(t+1) = s_i^h(t) + \sum_{j=1}^{np_i} o_i^{j,h}(t - \tau_i^j) - d_i^h(t), \quad \forall h \in \{1, \dots, H\}. \quad (2.44)$$

### 2.4.2 MPC Setup

The implementation of the control problem will be detailed, adding some consideration to make it easier. The objective is the minimization of the objective function, which will minimize the number of orders. Consider the system defined by

$$s(t+1) = s(t) + o(t - \tau) - d(t), \quad (2.45)$$

where  $s(t) = [s_1(t), \dots, s_{N_i}(t)]$ ,  $d(t) = [d_1(t), \dots, d_{N_i}(t)]$  and  $o(t - \tau) = \sum_{j=1}^{np_i} \delta_i^j(t - \tau_i^j) u_i^j(t - \tau_i^j)$  represent the total number of items ordered. Note that system (2.45) is equivalent to (2.32).

The control variables taken into account in this problem are  $\delta_i^j(t)$  and  $\sigma_i^j(t)$ , both components of the control variable  $o(t)$ . Solving the optimization problem by using directly the control variable  $o(t)$  (i.e.,  $\delta_i^j(t)$  and  $\sigma_i^j(t)$  together) is a difficult task, since they have different nature because  $\delta_i^j(t)$  is a Boolean variable. The way to proceed will be the use of an exhaustive search algorithm, which will solve the problem as many times as possible scenarios depending on the value of  $\{\delta_i^j(t), \dots, \delta_i^j(t + N_p)\}$ , i.e.,  $2^{n_{p_i} \times N_i(N_p+1)}$  times. In that way, the optimization problem can be solved with respect to the variable  $\sigma_i^j(t)$ .

It is straightforward to see that if  $\delta_i^j(t+k) = 0$ , for  $k \in \{0, 1, \dots, N_p\}$ , the quantity of ordered items is  $\sigma_i^j(t+k) = 0$ . Furthermore, the variable  $\sigma_i^j(t)$  can be considered as a real one, in order to simplify the problem, since the obtained solution is an integer one because of the problem features. Besides, that fact also accelerates the problem resolution. Moreover, the vector of control variables  $\{\sigma_i^j(t), \dots, \sigma_i^j(t + N_p)\}$  is reduced by eliminating the components  $\sigma_i^j(t+k)$  that are equal to zero. Hence,

$$\forall \delta_i^j(t+k) = 0, \quad k \in \{0, 1, \dots, N_p\},$$

then

$$\underbrace{\begin{bmatrix} \sigma_i^j(t) \\ \vdots \\ \sigma_i^j(t+k) \\ \vdots \\ \sigma_i^j(t+N_p) \end{bmatrix}}_{\mathbf{o}_i^j(t)} \rightarrow \begin{bmatrix} \sigma_i^j(t) \\ \vdots \\ \sigma_i^j(t+k-1) \\ \sigma_i^j(t+k+1) \\ \vdots \\ \sigma_i^j(t+\tilde{N}_p) \end{bmatrix},$$

$$\underbrace{\hspace{15em}}_{\tilde{\mathbf{o}}_i^j(t)}$$

where  $\mathbf{o}_i^j(t) \in \mathbb{R}^{N_p+1}$  and  $\tilde{\mathbf{o}}_i^j(t) \in \mathbb{R}^{\tilde{N}_p+1}$  is a reduced vector of non-zero orders, where

$$\tilde{N}_p = N_p - \sum_{k=0}^{N_p} (1 - \delta_i^j(t+k)),$$

is the number of non-zero orders.

The vector reduction from  $\mathbf{o}_i^j(t)$  to  $\tilde{\mathbf{o}}_i^j(t)$  can be represented by the following change of variable:

$$\mathbf{o}_i^j(t) = M \tilde{\mathbf{o}}_i^j(t),$$

where  $M \in \mathbb{R}^{(N_p+1) \times (\tilde{N}_p+1)}$ .

Matrix  $M$  is used to reduce the dimension of the order vector  $\mathbf{o}_i^j(t)$  depending on the value of  $\delta_i^j(t)$ . Therefore,  $M$  is defined as

$$M(i, j) = \begin{cases} 1 & \text{if } \delta_i^j(t) = 1 \wedge i = j, \\ 0 & \text{if } \delta_i^j(t) = 0 \vee i \neq j. \end{cases} \quad (2.46)$$

As direct consequence,  $\tilde{\mathbf{o}}_i^j(t)$  contains only non-null components, i.e., the orders that are non-zero.

As it was explained before, this optimization problem will be solved as many times as many possible combinations with the values of  $\{\delta_i^j(t), \dots, \delta_i^j(t + N_p)\}$ . This way, this variable can be avoided in the optimization and then the same number of values of the objective function will be obtained. Of course, the optimal solution corresponds with the combination of the values of  $\{\delta_i^j(t), \dots, \delta_i^j(t + N_p)\}$  that provides the minimal value of the objective function.

Taking into account the integer nature of the variable  $\delta_i^j(t)$ , the resulting optimization problem is a mixed integer one (MIP). There are different techniques to solve them like branch and bound [71], genetic algorithms [72] or the cutting-plane method [73].

**Remark 2** *It is necessary to pay special attention to the constraints while solving this problem. It is not possible to impose the whole matrix of constraints to the reduced vector  $\tilde{\mathbf{o}}_i^j(t)$ , so it is necessary to also apply the change matrix  $M$  to the matrix of constraints to impose them only to the considered control components.*

### CC-MPC

The aggregate demand  $d(t)$  in (2.45) includes a stochastic disturbance component given its uncertain nature. Due to the presence of these uncertainties, the constraints have a stochastic nature, i.e., they can not be written as deterministic ones. Therefore, the constraints can be expressed as

$$P(s(t+k) \geq S_{\min}) \geq 1 - \delta_s, \quad \forall k \in \{1, \dots, N_p\},$$

where  $\delta_s$  is the probability of having less stock than  $S_{\min}$ . This expression can be developed along  $N_p$  and obtain the mean and standard deviation of the state.

**Remark 3** *It is also possible to assume that the behavior of the disturbances can be adjusted as a function of a certain probability distribution. In [74], a normal distribution is used to characterize the behavior of the perturbations, with mean  $\mu$  and standard deviation  $\sigma$ , i.e.,  $d(t) = \mathcal{N}(\mu, \sigma)$ . This assumption could be extended to other patterns or even work directly with historical data, like in this case.*

For the first time instant along  $N$  (i.e.,  $k = 1$ ), it yields

$$\begin{aligned} P(s(t+0) + o(t+0) - d(t+0) \geq S_{\min}) &\geq 1 - \delta_s, \\ P(-d(t+0) \geq S_{\min} - s(t+0) - o(t+0)) &\geq 1 - \delta_s, \\ P(\underbrace{d(t+0)}_{\text{Random}} \leq \underbrace{-S_{\min} + s(t+0) + o(t+0)}_{\text{Deterministic}}) &\geq 1 - \delta_s, \end{aligned}$$

which can be rewritten as

$$\phi_0(-S_{\min} + s(t+0) + o(t+0)) \geq 1 - \delta_s,$$

where  $\phi_0(\cdot)$  is the cumulative distribution function (cdf) of the random variable  $d(t+0)$ .

The deterministic equivalent for this chance constraint is

$$\begin{aligned} -S_{\min} + s(t+0) + o(t+0) &\geq \phi_0^{-1}(1 - \delta_s), \\ -o(t+0) &\leq -S_{\min} + s(t+0) - \phi_0^{-1}(1 - \delta_s). \end{aligned}$$

For the next time instant along  $N$  (i.e.,  $k = 2$ ), it yields

$$\begin{aligned} P(s(t+2) \geq S_{\min}) &\geq 1 - \delta_s \\ P(s(t+1) + o(t+1) - d(t+1) \geq S_{\min}) &\geq 1 - \delta_s \\ P((s(t+0) + o(t+0) - d(t+0)) + o(t+1) - d(t+1) \geq S_{\min}) &\geq 1 - \delta_s \\ P(-d(t+0) - d(t+1)) \geq S_{\min} - s(t+0) - o(t+0) - o(t+1)) &\geq 1 - \delta_s, \\ P(d(t+0) + d(t+1) \leq -S_{\min} + s(t+0) + o(t+0) + o(t+1)) &\geq 1 - \delta_s. \end{aligned}$$

Defining  $\phi_1(\cdot)$  as the cumulative distribution function of the variable  $d(t+0) + d(t+1)$  yields

$$\begin{aligned} \phi_1(-S_{\min} + s(t+0) + o(t+0) + o(t+1)) &\geq 1 - \delta_s, \\ -S_{\min} + s(t+0) + o(t+0) + o(t+1) &\geq \phi_1^{-1}(1 - \delta_s), \\ o(t+0) + o(t+1) \geq S_{\min} - s(t+0) + \phi_1^{-1}(1 - \delta_s), \\ -o(t+0) - o(t+1) &\leq -S_{\min} + s(t+0) - \phi_1^{-1}(1 - \delta_s). \end{aligned}$$

Iteratively (e.g.,  $k = 3$ ) and according to the previous development, it can written as

$$-o(t+0) - o(t+1) - o(t+2) \leq -S_{\min} + s(t+0) - \phi_2^{-1}(1 - \delta_s),$$

where  $\phi_2(\cdot)$  denotes the cumulative distribution function of the variable  $d(t+0) + d(t+1) + d(t+2)$ . Generalizing for a prediction horizon  $N_p$ ,

$$-\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ \vdots & & & & \vdots \\ 1 & 1 & & \cdots & 1 \end{bmatrix} \begin{bmatrix} o(t+0) \\ o(t+1) \\ o(t+2) \\ \vdots \\ o(t+N_p-1) \end{bmatrix} \leq \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ \vdots & \vdots \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s(t+0) \\ -S_{\min} \end{bmatrix} - \begin{bmatrix} \phi_0^{-1}(1 - \delta_s) \\ \phi_1^{-1}(1 - \delta_s) \\ \phi_2^{-1}(1 - \delta_s) \\ \vdots \\ \phi_{N_p-1}^{-1}(1 - \delta_s) \end{bmatrix},$$



where  $\phi_{N_p-1}^{-1}(1 - \delta_s)$  is the cumulative distribution function of the random variable  $d(t+0) + d(t+1) + d(t+2) + \dots + d(t+N_p-1)$ .

### 2.4.3 Results

Due to CC-MPC offers a significant advantage regarding computational burden, robustness, and performance; moreover, taking into account that it is possible to obtain a cumulative distribution function from historical data of demand from particular drugs, CC-MPC is going to be applied to manage the orders of two drugs available in the hospitals San Juan de Dios and Universitario Reina Sofía (both located at Córdoba, Spain). These drugs are not only expensive because of their prices but also because of their maintenance costs, since they must be stored in a fridge. Due to this fact, the reduction of their stock levels is a priority.

Regarding the controller, a prediction horizon  $N_p=8$  days has been considered. The evolution of the stock is modeled by using the discrete-time linear model in (2.45). The orders of these drugs have a minimum amount of 4 units and the maximum has been set to 1000. The prices of the drugs are respectively 227 and 298 euros per unit, respectively, and each order placed implies an additional cost of 2 euros. The deliveries of these drugs usually have a delay of 2 days with respect to the moment in which the order was placed. The initial values of the stock levels are 500 and 1520, respectively. Finally, the demand term of (2.45) is non deterministic. A probabilistic characterization of their behavior has been calculated for these drugs based on historical data.

For simplicity, neither storage cost nor storage limits have been considered at this stage of the proposed work. The only implemented constraint with respect to the stock is that the probability of stockout event has to be lower than 0.001 (i.e., it is requested a reliability level of 99.999 %).

The 360-days simulation scenario considered here is shown in Figures 2.9 and 2.10 for each drug. In blue, the evolution of the stocks using CC-MPC is shown. In red, the real evolution of the stock according to the hospital data is shown. Tables 2.4.3 and 2.4.3 show a comparison of the behavior of these two drugs applying CC-MPC with the results register by the hospitals in this period, considering the average level, standard deviation and the number of orders. In this period the hospitals placed 9 orders for the drug 1 and 14 for the second one.

These results clearly show that the CC-MPC policy provides better results than the policy that is currently implemented in the hospital pharmacies. For the first drug, more than 1000 euros on average could be used for purposes other than having stock at the pharmacy with the same clinical results. In the case of drug 2 this amount is 27118

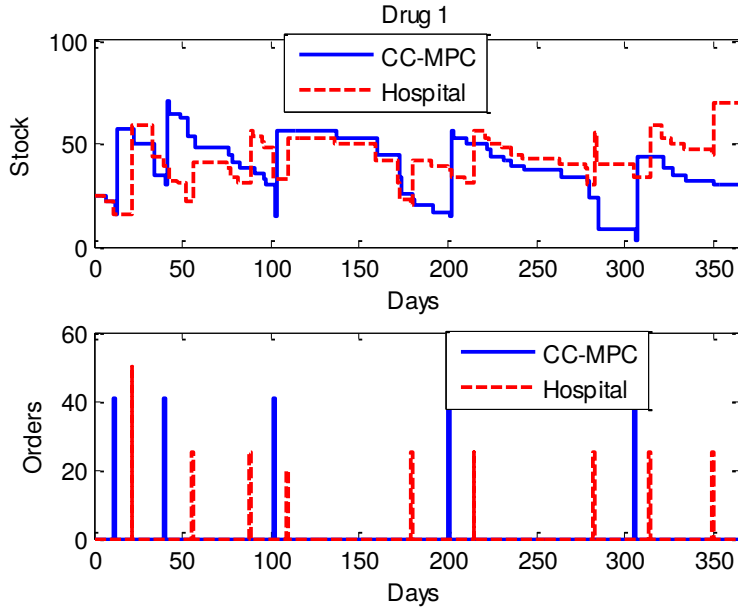


Figure 2.9: Real and simulated stock evolution and placed orders for drug 1.

euros. Another noteworthy point is that the staff at the pharmacy department is freed partially from the duties related to the placement and reception of orders. In both cases the CC-MPC placed 40% less orders than the policy followed by the hospital. Finally, note that a more aggressive tuning of the controller could be used to reduce these values at the cost of higher stockout risks.

Table 2.12: Comparison of the behavior of the drug 1 applying CC-MPC and hospital policy.

Approach	Orders	Stock-out	Mean	Desviation
CC-MPC	5	0	39	14
Hospital historical data	9	0	43	11

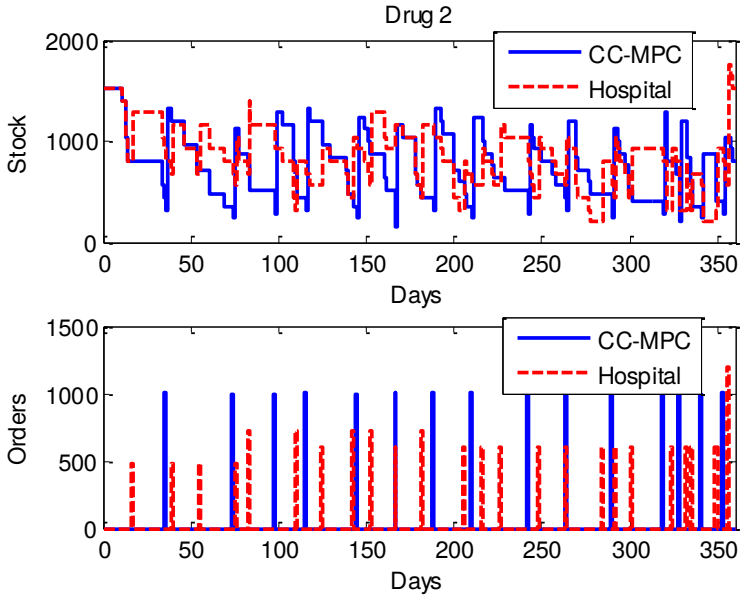


Figure 2.10: Real and simulated stock evolution and placed orders for drug 2.

Table 2.13: Comparison of the behavior of the drug 2 applying CC-MPC and hospital policy.

Approach	Orders	Stock-out	Mean	Desviation
CC-MPC	15	0	770	316
Hospital historical data	25	0	861	313

The optimization has been made taking into account the constraints (2.38)-(2.42). A problem is solved at every sampling time to compute a control sequence  $u$  that takes the system to the desired reference. For this simulation, the stock reference (security stock) has been set to 2. All optimization problems, solved for the exhaustive algorithm, were computed by using a linear programming routines (`linprog` in *Matlab*), on a machine with an Intel Core 2 Duo CPU with 3.33 GHz and 8 GB RAM. The time needed to calculate the optimal sequence of actions per drug and day was below 10 seconds. If we take into account that the orders are recomputed once a day, it is possible to

calculate optimal control actions for 8640 drugs a day with the current configuration, many more than those used in the hospitals.

These results have been published in [33].

## **Chapter 3**

# **Hierarchical Stochastic MPC**

This chapter will be submitted for publication as a journal article.



## **Chapter 4**

# **Stochastic MPC to Deal with Vulnerabilities in Distributed Schemes**

This chapter will be submitted for publication as a journal article.





## Chapter 5

# Conclusions and Future Researches

Several control approaches cope with the operational management of distribution systems as a hierarchical control given by layers of planning, management, and regulation of the nonlinear system. However, those techniques, in spite of the inherent robustness of optimization-based controllers, do not guarantee the proper disturbance rejection related to the uncertainties of the whole system.

The novelty of this work consists in the design and assessment of three stochastic controllers applied to the operational management of distribution systems, in this case, energy dispatch systems and water resource management. These stochastic controllers are designed to deal with internal and external uncertainties with different configurations, i.e., centralized, hierarchical, and distributed fashion. The common advantage of all the proposed approaches relies, apart of the robustness features, on the compromise between profits, reliability, and computational burden. Moreover, this thesis offers a deep discussion about the tractability and performance of the closed loops based on the proposed approaches.

During the last years, distributed model predictive control (DMPC) has become a very active optimal control field where many algorithms and strategies have been proposed to ease the coordination in multi-agent systems where there are coupled dynamics. This work raises an important issue in the field of DMPC. In particular, DMPC schemes rely on the assumption that the information shared across the network is trustworthy. A malicious controller could send false information to the rest of the controllers to steer the negotiation process, which may result in a loss of performance or

even in the instability of the closed loop system. To illustrate and raise awareness of this problem, this work has presented the vulnerabilities of a very popular distributed model predictive control scheme. More specifically, it has shown how a malicious agent in the network could exploit the information exchange to steer the negotiation process arbitrarily. Also, stochastic based MPCs and a heuristic technique have been proposed as alternatives to give some robustness the DMPC scheme against this problem.

## 5.1 Conclusions

Below, the main conclusions of each chapter of this thesis are provided.

- *Centralized Stochastic Model Predictive Control*

In Chapter 2, stochastic MPC schemes have been designed and applied to ensure robustness against external disturbances in the context of distribution systems, more specifically, a real microgrid, the drinking water network of Barcelona, and the stock management of a hospital pharmacy. In this context, it is possible to conclude that MS-MPC controller is over-conservative because it does not consider the controller capacity to adapt. It calculates a control series valid to all possible scenarios by means an open-loop formulation. However, it is possible to solve the optimization problem by using a control tree and increasing the number of optimization variables and the computational time. Regarding the control point of view, TB-MPC controller works in a closed-loop fashion to adapt the control actions to the expected evolution of the disturbances. Finally, CC-MPC controller formulates the optimization problem by taking into account the statistical features of the uncertainty without increasing the number of variables. The results obtained with the three presented versions of stochastic MPC controllers show their effectiveness in energy management and drinking water network under economic and optimal criteria. According to the results obtained and the evaluation of the case studies, it can be said that CC-MPC controller relaxes the constraints of the optimization problem by assuming a risk to offer better performance, resulting in a lower cost, less energy exchange with the network and a lower number of constraints violation for energy management and drinking water networks compared MS-MPC and TB-MPC controllers. This is also the approach with the lowest computational burden. The downside of this approach is that it requires a statistical characterization of the disturbances.

Also, a solution for the problem of stock management in a hospital pharmacy has been proposed. A control methodology has been described to deal with the different and contradictory objectives of the problem. The proposed control strategy

is based on MPC, which allows the fulfillment of the management objectives while imposing different operational constraints. In that way, it has been possible to guarantee, with a high probability, that the drugs will be available for the patients, knowing explicitly the allowed risk level. In addition, we have shown that several hospitals could collaborate to reduce their stock levels. Finally, some simulations have been carried out to show the performance of the proposed management approach. It has been seen how the average level of stocked drugs has been reduced, which reduces the economical costs for the hospital, and how the work burden in the pharmacy was also reduced while guaranteeing the needs of the patients.

- *Hierarchical Stochastic MPC*

In Chapter 3, this work has shown a risk assessment methodology applied to microgrids. Although many risks have been identified, only a reduced set of them has been used in the example for illustrating the method. Two different model predictive controllers have been used. One for the external loop in order to evaluate risks and determine the optimal mitigation actions and another for the control of the plant. Results show that the benefits that can be obtained are very positive.

By another site, it has been considered a water resource management system, which is subject to dynamical uncertainty. The system is distributed into subsystems at the bottom, but the overall performance is checked for the whole system at the top layer. The final goal was to decompose the overall problem into different regions and under different hydrological conditions. In this sense, a scenario based Hierarchical and Distributed Model Predictive Control have been used to address the disturbances and uncertainties, which commonly affect this kind of systems. For the uncertainty, the tree based approach based on scenarios has been widely applied principally in the field of centralized water systems due to its adaptability at the moment to generate the disturbances tree. A drawback that presents this approach is its higher computational burden, this problem was addressed by the hierarchical controller, which collects the whole information and sends only the most likely scenarios to be taken into account by the distributed controllers at the time to solve the local optimization problem. Results show the effectiveness of this method to ensure the water level into the desired reference despite the presence of uncertainties for a large scale system.

- *Stochastic MPC to Deal with Vulnerabilities in Distributed Schemes*

In Chapter 4, cyber-security issues in DMPC have been considered. An analysis of the vulnerability of a popular Lagrange-based DMPC scheme has been

presented. By using a relevant case study involving a power network, we have illustrated the potential of this mechanism, and how a controller can attack the system to obtain benefits. It has been addressed the problem of providing robustness to DMPC for defending it from a malicious controller by carrying out scenario-based mechanisms. We have also proposed a heuristic mechanism to defend the attacked agents, that is, to identify false information and perform the negotiation process among agents regardless of the actions of the attacker. A highly relevant case study involving a power network illustrates the potential of this mechanism.

## 5.2 Future Researches

The research of stochastic MPC techniques applied to distribution systems has been a major issue in recent years. However, some important issues fall out of the scope of this thesis and can be studied in the future. Next, several research lines are pointed out for their study and analysis.

- On the one hand, centralized stochastic MPCs have been analyzed and implemented via simulation and experimental setup, describing their advantages and drawbacks. However, in this work the bounds for violating constraints were assumed in the same way for MS-MPC and TB-MPC, this assumption may be modified, and a further study for TB-MPC may be developed. Moreover, some algorithms for obtaining three disturbances deserve particular attention for improving the computational burden. Regarding CC-MPC, the formulation of deterministic constraints to replace the stochastic ones by considering other well-known probability distribution functions instead of normal distribution functions or historical data, as been discussed, may be an accurate bullet for being developed.
- On the other hand, regarding hierarchical and distributed MPC, it is important to remark the possibility to design, implement, and compare the assessment and performance of other stochastic MPC at the lower level to deal with the uncertainty that the distribution system present. Furthermore, some results have been carried out by simulation and show the benefit of these approaches described in this document; however, an aggregated value will apport an experimental setup in real case studies.
- Cyber-security issues is a very relevant and critical topic that has not been explicitly considered in the DMPC literature in a structured way. In this thesis,

one of the most popular distributed MPC schemes has been considered. However, other kinds of attacks will be discussed in different distributed algorithms. In this sense, there are many important issues in the context of distribution systems related to the reliable information exchange that can be exploited, such as uncertainty in the demand patterns will be considered as a manner to robustify the system against possible internal attackers. Moreover, the use of the quality of service (QoS) as an index of trustworthiness of each agent will be investigated. It will also address cyber security issues in DMPC from other points of view, e.g., by considering other stochastic MPC methods or by developing new approaches to identify and isolate the attackers from the whole distributed system. In this manner, it is possible to mitigate losses and the impact from malicious agents in the performance of the system.

### 5.3 Publications from this work

Several publications have taken place as peer-reviewed articles and conference papers. The list of scientific articles is enumerated as follows:

- Jurado, I., Maestre, J. M., Velarde, P., Ocampo-Martinez, C., Fernandez, I., Tejera, B. I., del Prado, J. R. (2016). Stock management in hospital pharmacy using chance-constrained model predictive control. *Computers in biology and medicine*, 72, 248-255.
- Grosso, J. M., Velarde, P., Ocampo-Martinez, C., Maestre, J. M., Puig, V. (2016). Stochastic model predictive control approaches applied to drinking water networks. *Optimal Control Applications and Methods*. DOI:10.1002/oca.2269.
- Velarde, P., Valverde, L., Maestre, J. M., Ocampo-Martinez, C., Bordons, C. (2017). On the comparison of stochastic model predictive control strategies applied to a hydrogen-based microgrid, *Journal of Power Sources*, 343, 161-173.
- Velarde, P., Maestre, J. M., Ishii, H., Negenborn, R. R., (2017). Vulnerabilities in Lagrange-based Distributed Model Predictive Control. Submitted to *Optimal Control Applications and Methods*.
- Maestre, J. M., Velarde, P., Jurado, I., Ocampo-Martinez, C., Fernandez, I., Tejera, B. I., del Prado, J. R. (2014, December). An application of chance-constrained model predictive control to inventory management in hospitalary pharmacy. In 53rd IEEE Conference on Decision and Control (pp. 5901-5906). IEEE.

- Velarde, P., Maestre, J. M., Jurado, I., Fernandez, I., Tejera, B. I., del Prado, J. R. (2014, September). Application of robust model predictive control to inventory management in hospitalary pharmacy. In Proceedings of the 2014 IEEE Emerging Technology and Factory Automation (ETFA) (pp. 1-6). IEEE.
- Velarde, P., Maestre, J. M., Ocampo-Martinez, C., Bordons, C. (2016, June). Application of robust model predictive control to a renewable hydrogen-based microgrid. In Proceedings of the 2016 European Control Conference (ECC), Aalborg, Denmark, 2016, pp. 1209-1214.
- Velarde, P., Maestre, J. M., Ishii, H., Negenborn, R. R., Vulnerabilities in Lagrange-based DMPC in the Context of Cyber-Security. Submitted to The 20th World Congress of the International Federation of Automatic Control (IFAC).
- Velarde, P., Maestre, J. M., Ishii, H., Negenborn, R. R., Scenario Based Defense Mechanism for Distributed Model Predictive Control. Submitted to The 20th World Congress of the International Federation of Automatic Control (IFAC).
- Zafra-Cabeza, A., Velarde, P., Maestre, J. M., Multicriteria Optimal Operation of a Microgrid considering Risk Analysis, Renewable Resources and MPC. Submitted to The 20th World Congress of the International Federation of Automatic Control (IFAC).
- Maestre, J. M., Velarde, P., Muros, F. J., An Application of the Logarithmic Mean Divisia Index Method to Distributed and Coalitional Control. Submitted to The 20th World Congress of the International Federation of Automatic Control (IFAC).

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