Transparent expression of the A^2 condensate's renormalization

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We give a more transparent understanding of the vacuum expectation value of the renormalized local operator A^2 by relating it to the gluon propagator integrated over the momentum. The quadratically divergent perturbative contribution is subtracted and the remainder, dominantly due to the $O(1/p^2)$ correction to the perturbative propagator at large p^2 is logarithmically divergent. This provides a transparent derivation of the fact that this $O(1/p^2)$ term is related to the vacuum expectation value of the local A^2 operator and confirms a previous claim based on the operator product expansion (OPE) of the gluon propagator. At leading logarithms the agreement is quantitative, with a standard running factor, between the local A^2 condensate renormalized as described above and the one renormalized in the OPE context. This result supports the claim that the BRST invariant Landau-gauge A^2 condensate might play an important role in describing the QCD vacuum.

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I. INTRODUCTION

In a series of lattice studies [1–4] the gluon propagator in QCD has been computed at large momenta, and it was shown that its behavior was compatible with the perturbative expectation provided a rather large $1/p^2$ correction was considered. In an OPE approach this correction has been shown [2,3] to stem from an A^2 gluon condensate which does not vanish since the calculations are performed in the Landau gauge. It was also claimed [4] that this condensate might be related to instantons.

The role of such a condensate in the nonperturbative properties of QCD, in particular its relation to confinement, has been studied by several groups [5,6]. Of course any physics discussion about the A^2 condensate necessitates a clear definition of what we speak about, i.e., it needs a well defined renormalization procedure to define the renormalized local A^2 operator, since $A(0)^2$ is a quadratically divergent quantity as can easily be seen in perturbation theory. A renormalization of A^2 was defined in [2,3] within the OPE context which we now briefly summarize. It uses the notion of "normal order product" in a "perturbative vacuum" which is annihilated by the fields A. It implies that $\langle :A(0)^2: \rangle_{\mathrm{pert}} = 0$ in the perturbative vacuum.[1] The contribution to $\langle :A(0)^2: \rangle$ in the true QCD vacuum is then of nonperturbative origin. It has only logarithmic divergences and it is multiplicatively renormalized. Of course this notion of a perturbative vacuum in which Fock expansion could be performed has not a very transparent physical meaning, especially in a nonperturbative context such as the numerical Euclidean path integral method.

A^2 is not a gauge invariant operator but the bare A^2 condensate is a very special object since, by definition, it is a minimum of the gauge orbit [6]. In other words, some important physics seems to lie beneath the Becchi-Rouet-Stora-Tyutin (BRST) invariance of A^2 in Landau gauge. The authors of Ref. [5] discussed on the generalized[2] composite operator $A_\mu A^\mu + 2i(1-\xi)\overline{c}c$, which is BRST invariant in the manifestly Lorentz covariant gauge, and examined the survival of this invariance after renormalization. In this paper, although in a different context, we also examine the same point: the subtle relationship between the minimum of bare A^2 in the gauge orbit and any gauge-independant physical phenomenology associated to the renormalized condensate [6], emerging for instance from the OPE analysis [1–4]. To this aim, we will derive the renormalized A^2 vacuum expectation value without using the normal ordering but using only the OPE expansion of the gluon propagator.[3] It will provide a more transparent definition, related directly to a quantity which is actually measured.

We start from the observation that the non renormalized $\langle A(0)^2 \rangle$ is related to the integral of the gluon propagator over momentum. Hence it is expected that the nonperturbative contribution to A^2 has to do with the nonperturbative contribution to the gluon propagator. The latter contains precisely $1/p^2$ contributions due to the A^2 condensate at large momenta, and also strong deviations from perturbative QCD at small momenta, see Fig. 1 (taken from [1–4]). How does this fit together?

II. BARE, PERTURBATIVE AND NONPERTURBATIVE A^2

It is possible in principle from lattice calculations to define the nonperturbative gluon propagator in the Landau gauge. Lattice calculations provide the bare gluon propaga-

[1]The symbol ":⋯:" represents the normal ordered product in this perturbative vacuum.

[2]Landau gauge is recovered in the limit $\xi \to 1$.

[3]Of course the normal ordering has been used in [2,3] to compute the anomalous dimension of A^2 and the Wilson coefficient c_2.

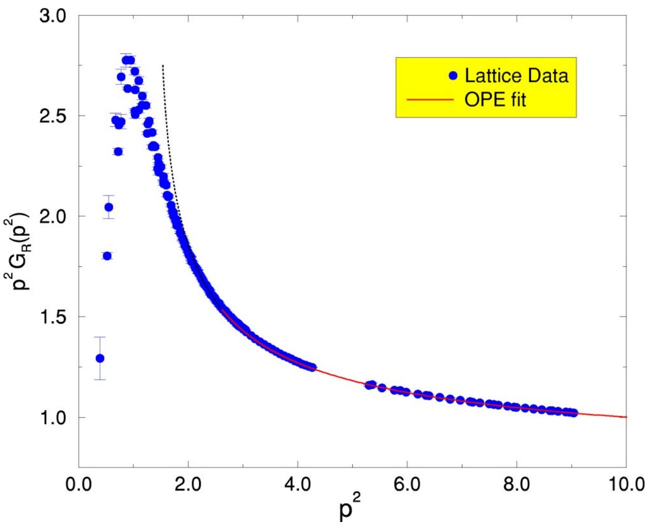

FIG. 1. Gluon propagator extracted from lattice calculations renormalized at $\mu = 10$ GeV and plotted between 0 and 9 GeV. The curve corresponds to the fit written in Eq. (7). It results that the infrared cutoff p_{min} can be safely taken around 2.6–3.0 GeV.

tor. From the gluon propagator computed with a series of different values of the lattice spacing one can in principle compute the renormalized gluon propagator from zero momentum up to as large a momentum as one wishes. An example of such a nonperturbative propagator is shown in Fig. 1. We can choose, for example, the momentum subtraction (MOM) renormalization scheme,[4] such that

$$G_{\mathrm{R}}^{(2)}(p^2 = \mu^2) = \frac{1}{\mu^2}. \tag{1}$$

This implies a renormalization of the gluon fields

$$A_{\nu\mathrm{R}} = Z_3(\mu)^{-1/2} A_{\nu\,\mathrm{bare}}, \quad Z_3(\mu) \equiv \mu^2 G_{\mathrm{bare}}^{(2)}(\mu^2). \tag{2}$$

The renormalization constant Z_3 has to be understood as related to any regularization method and any value of the UV regulator provided that the latter is larger than the momenta carried by the gluons. The coupling constant is also renormalized in a MOM scheme. Initially, the particular kinematics of the three-gluon vertex leading to the definition of the coupling should be specified. In fact, the perturbative + OPE analysis we show in Fig. 1 is performed with the choice of the asymmetric $\widetilde{\mathrm{MOM}}$ scheme, one of the incoming gluon momenta being zero. Concerning the conclusions of the present paper, they shall be derived from leading-logarithm computations which are not affected by the kinematics of the renormalization point for α, then we do not specify the kinematics any more on the following. The Yang-Mills theory is thus fully renormalized and from now on we will consider only renormalized gauge fields and propagators.

The propagator is defined in Euclidean space by

$$\int d^4x e^{ip\cdot x} \langle A^a_{\mu\,\mathrm{R}}(0) A^b_{\nu\,\mathrm{R}}(x)\rangle = \delta_{a,b}\left[\delta_{\mu,\nu} - \frac{p_\mu p_\nu}{p^2}\right] G_{\mathrm{R}}(p^2). \tag{3}$$

Inverting the Fourier transform,

$$\sum_{a,\mu} \langle A^a_{\mu\,\mathrm{R}}(0) A_a^{\mu\,\mathrm{R}}(0)\rangle = \frac{3(N_c^2-1)}{(2\pi)^4}\int d^4p\, G_{\mathrm{R}}^{(2)}(p^2) = \frac{3(N_c^2-1)}{16\pi^2}\int p^2 dp^2 G_{\mathrm{R}}^{(2)}(p^2). \tag{4}$$

This integral is quadratically divergent in the ultraviolet. Indeed, if the gauge fields and the coupling constant have been renormalized, the local A^2 operator has not yet. Let us introduce an ultraviolet cutoff Λ and define

$$\langle (A_{\mathrm{R}}(\mu))^2\rangle_\Lambda = \frac{3(N_c^2-1)}{16\pi^2}\int_0^{\Lambda^2} p^2 dp^2 G_{\mathrm{R}}^{(2)}(p^2), \tag{5}$$

where $(A_{\mathrm{R}}(\mu))^2$ refers to the square of the gauge fields renormalized at the scale μ, but where A^2 has *not been renormalized as a local product of operators*. The symbol "$\langle \cdots \rangle$" represents the vacuum expectation value (VEV). $(A_{\mathrm{R}})^2$ is clearly an UV divergent quantity. The index Λ refers to the ultraviolet cutoff and μ to the renormalization point for the gauge fields and the coupling constant. The cutoff Λ has nothing to do with the lattice cutoff a^{-1}. The renormalization in Eqs. (1) and (2) has eliminated any dependence in the different lattice spacings which have been used to produce the renormalized propagator. Λ is introduced simply to control the quadratic and logarithmic divergences we encounter here.

The dominant contribution to this integral is the perturbative one. To separate the perturbative contribution from the nonperturbative we will now use the results of [3],

$$p^2 G_{\mathrm{R}}^{(2)}(p^2,\mu^2) \equiv \frac{p^2 G^{(2)}(p^2)}{\mu^2 G^{(2)}(\mu^2)} = c_0\left(\frac{p^2}{\mu^2},\alpha(\mu)\right) + c_2\left(\frac{p^2}{\mu^2},\alpha(\mu)\right) \times \frac{\langle (A^2)_{\mathrm{R}}(\mu)\rangle}{4(N_c^2-1)}\,\frac{1}{p^2}, \tag{6}$$

where $G^{(2)}(p^2)$ is the bare propagator. This expansion does not exactly separate the perturbative from the nonperturbative contribution because of the denominator $\mu^2 G^{(2)}(\mu^2)$

[4]Notice that the chosen renormalization scheme is not relevant in our argument in this paper, but we clearly need a scheme in which nonperturbative quantities coming from lattice simulations can be accommodated. MOM is one of the simplest. On the contrary the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme does not satisfy this condition.

which contains a nonperturbative contribution. It is therefore convenient to introduce a slightly different renormalization R':

$$p^2 G^{(2)}_{\mathrm{R}'}(p^2,\mu^2) \equiv \frac{p^2 G^{(2)}(p^2)}{\mu^2 G^{(2)}_{\mathrm{pert}}(\mu^2)} = \frac{c_0\left(\frac{p^2}{\mu^2},\alpha(\mu)\right)}{c_0(1,\alpha(\mu))} + \frac{c_2\left(\frac{p^2}{\mu^2},\alpha(\mu)\right)}{c_0(1,\alpha(\mu))} \times \frac{\langle (A^2)_{\mathrm{R}}(\mu)\rangle}{4(N_c^2-1)}\frac{1}{p^2}, \tag{7}$$

where $(A^2)_{\mathrm{R}}(\mu)$ represents the A^2 operator renormalized as a local operator at the scale μ. Here the denominator is only the perturbative contribution to the Green function whence the first term in Eq. (7) is purely perturbative: it runs perturbatively with a perturbative MOM renormalization condition at $p^2=\mu^2$. Let us define for simplicity the constant

$$z_0 \equiv \frac{1}{c_0(1,\alpha(\mu))} = \frac{G^{(2)}(\mu^2)}{G^{(2)}_{\mathrm{pert}}(\mu^2)} = 1 + O\left(\frac{1}{\mu^2}\right). \tag{8}$$

We know from [3] that the first term in Eq. (7), $z_0 c_0(p^2/\mu^2,\alpha(\mu))$, represents the three loop perturbative contribution. The second, $z_0 c_2(p^2/\mu^2,\alpha(\mu))$, appears as the leading logarithm Wilson coefficient of the first nonperturbative correction $(O(1/p^2))$, which we attributed [3] to the vacuum expectation value of the renormalized local operator $\langle (A^2)_{\mathrm{R}}(\mu)\rangle$.

Let us now introduce some notations:

$$(A^2)_{\mathrm{R}}(\mu) = Z^{-1}_{A^2}(\mu) : A^2_{\mathrm{bare}} : = [Z_{A^2}(\mu) Z_3^{-1}(\mu)]^{-1} : (A_{\mathrm{R}}(\mu))^2 :, \tag{9}$$

where the symbol $:\cdots:$ represents the normal ordered product in the perturbative vacuum.[5] We define $\hat{Z}(\mu) \equiv Z_3^{-1}(\mu) Z_{A^2}(\mu)$, the anomalous dimension for these renormalization constants given by

$$\gamma_{A^2}(\alpha(\mu)) \equiv \frac{d}{d\ln\mu^2}\ln Z_{A^2}(\mu) = -\frac{35 N_C}{12}\frac{\alpha(\mu)}{4\pi} + \cdots,$$

$$\hat{\gamma}(\alpha(\mu)) \equiv \frac{d}{d\ln\mu^2}\ln \hat{Z}(\mu) = -\hat{\gamma}_0 \frac{\alpha(\mu)}{4\pi} + \cdots = -\frac{3N_C}{4}\frac{\alpha(\mu)}{4\pi} + \cdots. \tag{10}$$

Our main goal in this paper is to understand better the connection between $\langle (A^2)_{\mathrm{R}}(\mu)\rangle$ defined in [3] and the $(A_{\mathrm{R}}(\mu))^2$ object considered here.

[5] The $:\cdots:$ symbols have been erroneously omitted in [3].

The expansion in Eq. (7) is only valid above some momentum $p \geqslant p_{\min}$. Typically we have taken $p_{\min} = 2.6$ GeV for our fits reported in [1–4].

From Eqs. (5), (7) we decompose

$$\langle (A_{\mathrm{R}}(\mu))^2\rangle_\Lambda = \langle (A_{\mathrm{R}}(\mu))^2\rangle_\Lambda^{\mathrm{pert}} + \langle (A_{\mathrm{R}}(\mu))^2\rangle_\Lambda^{\mathrm{OPE}} + \langle (A_{\mathrm{R}}(\mu))^2\rangle_\Lambda^{\mathrm{IR}} \tag{11}$$

where

$$\langle (A_{\mathrm{R}}(\mu))^2\rangle_\Lambda^{\mathrm{pert}} = \frac{3(N_c^2-1)z_0}{16\pi^2}\int_{p^2_{\min}}^{\Lambda^2} dp^2 c_0\left(\frac{p^2}{\mu^2},\alpha(\mu)\right), \tag{12}$$

$$\langle (A_{\mathrm{R}}(\mu))^2\rangle_\Lambda^{\mathrm{OPE}} = \frac{3(N_c^2-1)z_0}{16\pi^2}\int_{p^2_{\min}}^{\Lambda^2} \frac{dp^2}{p^2} c_2\left(\frac{p^2}{\mu^2},\alpha(\mu)\right) \times \frac{\langle (A^2)_{\mathrm{R}}(\mu)\rangle}{4(N_c^2-1)}, \tag{13}$$

$$\langle (A_{\mathrm{R}}(\mu))^2\rangle_\Lambda^{\mathrm{IR}} = \frac{3(N_c^2-1)}{16\pi^2}\int_0^{p^2_{\min}} p^2 dp^2 G^{(2)}_{\mathrm{R}'}(p^2). \tag{14}$$

A few comments are in order here. $\langle (A_{\mathrm{R}}(\mu))^2\rangle_\Lambda^{\mathrm{pert}}$ corresponds indeed to the perturbative computation of the vacuum expectation value of A^2, i.e., to the connected diagrams with no external legs and with one A^2 inserted. However, the coupling constant and the gluon fields in the diagrams have been consistently renormalized at the scale μ. To leading order Eq. (12) leads to

$$\langle (A_{\mathrm{R}}(\mu))^2\rangle_\Lambda^{\mathrm{pert}} \underset{\Lambda\to\infty}{\rightarrow} \frac{3(N_c^2-1)}{16\pi^2}\Lambda^2 \left\{ \left(\frac{\ln\left(\frac{\Lambda}{\Lambda_{\mathrm{QCD}}}\right)}{\ln\left(\frac{\mu}{\Lambda_{\mathrm{QCD}}}\right)} \right)^{\gamma_0/\beta_0} \times \left[1 + O\left(\frac{1}{\ln\left(\frac{\Lambda}{\Lambda_{\mathrm{QCD}}}\right)} \right) \right] + O\left(\frac{p^2_{\min}}{\Lambda^2}\right) \right\}, \tag{15}$$

which diverges more than quadratically. Note that the dependence in $p^2_{\min}$ is subdominant.

In Eq. (13) the left-hand side has been defined from the decomposition of the integral (5) according to Eq. (7). The right-hand side contains $\langle (A^2)_{\mathrm{R}}(\mu)\rangle$ already discussed. The latter is just a number which factorizes out of the integral in Eq. (13). We thus see that $\langle (A_{\mathrm{R}}(\mu))^2\rangle_\Lambda^{\mathrm{OPE}}$ and $\langle (A^2)_{\mathrm{R}}(\mu)\rangle$ are proportional.

Our next task is to compute the proportionality coefficient and to compare $\langle (A_{\rm R}(\mu))^2\rangle_\Lambda^{\rm OPE}$ with the other subleading term, $\langle (A_{\rm R}(\mu))^2\rangle_\Lambda^{\rm IR}$. From [3] and Eq. (8) we know that $z_0 = 1 + O(1/\mu^2)$. In our calculation of the integral in Eq. (13), being performed to the leading logarithm, we will take $z_0 c_2 = c_2$ in the following. From (23) in [3],

$$c_2\left(\frac{p^2}{\mu^2},\alpha(\mu)\right) = 12\pi\alpha(p)\left(\frac{\alpha(p)}{\alpha(\mu)}\right)^{-\hat{\gamma}_0/\beta_0}. \tag{16}$$

Let us also recall

$$Z_3(\mu) \propto (\alpha(\mu))^{\gamma_0/\beta_0}, \quad \langle (A^2)_{\rm R}(\mu)\rangle \propto (\alpha(\mu))^{-\gamma_{A^2}/\beta_0} \tag{17}$$

with

$$\beta_0 = 11, \quad \gamma_0 = 13/2, \quad \gamma_{A^2} = \frac{35}{4}, \quad \hat{\gamma}_0 = \gamma_{A^2} - \gamma_0 = \frac{9}{4}. \tag{18}$$

From Eq. (13) and the leading logarithm relation

$$dp^2/p^2 = d\log(p^2) \simeq -\frac{4\pi}{\beta_0}\frac{d\alpha}{\alpha^2} \tag{19}$$

we find

$$\begin{aligned}\langle (A_{\rm R}(\mu))^2\rangle_\Lambda^{\rm OPE} &= \frac{3(N_c^2-1)}{16\pi^2}\frac{(12\pi)}{(\alpha(\mu))^{-\hat{\gamma}_0/\beta_0}} \\ &\quad \times \frac{\langle (A^2)_{\rm R}(\mu)\rangle}{4(N_c^2-1)}\frac{4\pi}{\beta_0}\int_{\alpha(\Lambda)}^{\alpha(p_{\rm min})} d\alpha\,\alpha^{-1-(\hat{\gamma}_0/\beta_0)} \\ &= \langle (A^2)_{\rm R}(\mu)\rangle \\ &\quad \times\left[\left(\frac{\alpha(\Lambda)}{\alpha(\mu)}\right)^{-\hat{\gamma}_0/\beta_0} - \left(\frac{\alpha(p_{\rm min})}{\alpha(\mu)}\right)^{-\hat{\gamma}_0/\beta_0}\right].\end{aligned} \tag{20}$$

It is interesting to notice that the coefficient $\beta_0/\hat{\gamma}_0$ stemming from the integration over α is exactly compensated by the prefactors outside the integral, the origin of which does not appear at first sight to be related to the anomalous dimension of A^2. Had we taken any other anomalous dimension instead of $\hat{\gamma}_0$, say some γ', we would have ended with a constant $9/(4\gamma')$ in front of the RHS of Eq. (20).

In the large Λ limit, $\alpha(p_{\rm min}) \gg \alpha(\Lambda)$ whence, since $\hat{\gamma}_0$ is positive, the main result of this paper comes from

$$\langle (A_{\rm R}(\mu))^2\rangle_\Lambda^{\rm OPE} \underset{\Lambda\to\infty}{\simeq} \langle (A^2)_{\rm R}(\mu)\rangle\left(\frac{\alpha(\Lambda)}{\alpha(\mu)}\right)^{-\hat{\gamma}_0/\beta_0}. \tag{21}$$

To leading logarithms and keeping μ fixed,

$$\langle (A_{\rm R}(\mu))^2\rangle_\Lambda^{\rm OPE} \propto \alpha(\Lambda)^{-\hat{\gamma}_0/\beta_0} \underset{\Lambda\to\infty}{\to} \infty. \tag{22}$$

On the other hand, from Eq. (14)

$$\langle (A_{\rm R}(\mu))^2\rangle_\Lambda^{\rm IR} = \text{const.} \tag{23}$$

since it does not depend on Λ. It results that $\langle (A_{\rm R}(\mu))^2\rangle_\Lambda^{\rm OPE}$ is dominant over $\langle (A_{\rm R}(\mu))^2\rangle_\Lambda^{\rm IR}$ in the decomposition (11). This dominance will lead to the coming interpretation (next section) of the decomposition (11), which is indeed the main result of this note. Of course, we perform our analysis in the Landau gauge because of its conceptual [6] and numerical (lattice Green functions [3]) particular interest. However, the survival after renormalization of BRST invariance in covariant gauges for the generalized composite operator claimed in Ref. [5] seems to point out that an analogous analysis, with similar results, for these gauges might be performed. However, this is hard to do because of the renormalization mixing of both local operators A^2 and $\bar{c}c$ (the renormalization of the generalized composite operator not being diagonal except for very particular cases [5]).

As an interesting special case, if $\mu = \Lambda$

$$\langle (A_{\rm R}(\Lambda))^2\rangle_\Lambda^{\rm OPE} \underset{\Lambda\to\infty}{\to} \langle (A^2)_{\rm R}(\Lambda)\rangle \propto (\alpha(\Lambda))^{-\gamma_{A^2}/\beta_0}. \tag{24}$$

III. CONCLUSION AND DISCUSSION

Our conclusion is summarized in

$$\begin{aligned}\langle (A^2)_{\rm R}(\mu)\rangle &\simeq \langle (A_{\rm R}(\mu))^2\rangle_\Lambda^{\rm OPE}\left(\frac{\alpha(\Lambda)}{\alpha(\mu)}\right)^{\hat{\gamma}_0/\beta_0} \\ &\simeq [\langle (A_{\rm R}(\mu))^2\rangle_\Lambda - \langle (A_{\rm R}(\mu))^2\rangle_\Lambda^{\rm pert}]\left(\frac{\alpha(\Lambda)}{\alpha(\mu)}\right)^{\hat{\gamma}_0/\beta_0}.\end{aligned} \tag{25}$$

Since notations are not conventional let us recall that the $\langle\cdots\rangle_\Lambda$'s in the RHS represent the gluon propagator integrated over momentum up to an UV cutoff, Λ, see Eqs. (5) and (12). The gluon fields and coupling constants are renormalized in all the terms appearing in these equations. Thus we learn that the further renormalization of the local operator A^2 proceeds by substracting to the plain vacuum expectation value of A^2 the same object computed in perturbation. This logarithmically divergent difference is then renormalized by the powers of α in the RHS of Eq. (25). Not unexpectedly, we retrieve in essence the initial expression of the renormalization of the A^2 operator through normal ordering (i.e., subtraction of the perturbative VEV), followed by the multiplicative, logarithmic renormalization Z_{A^2}. But apart from a nontrivial consistency check, involving in particular the detailed expression of the Wilson coefficient, we obtain an expression which is more transparent, since it only involves a measurable quantity, the integral over the renormalized propagator.

Equation (25) presents a separation between perturbative and nonperturbative contributions to the integrated propagator, i.e., to A^2. Of course, such a separation depends on the renormalization scheme, and on the order in perturbation theory in which the Green functions are computed. It is also well known that summing to infinity the perturbative series may generate renormalons which behave like nonperturbative condensates. To avoid any such problem we stick to a finite order in the perturbative series. Furthermore, if the quantitative separation between perturbative and nonperturbative contributions depends on these prescriptions, the results summarized in Eq. (25) *do not depend on them* provided that we use *the same* scheme and order when computing both sides of Eq. (25). Of course the anomalous dimensions to leading logarithms do not either depend on them.

This simple result has several interesting consequences. First, it has been advocated [4] that the A^2 condensate could be dominantly due to the contribution to the path integral of semiclassical gauge field configurations such as instantons liquids. It is useful to consider this hypothesis through a background field picture, i.e., factorizing the path integral into an integral over semiclassical gauge field configurations, and for each value of these an integral over quantum fluctuations around this background configuration. It means that the Hermitian matrix A_μ is decomposed into

$$A_\mu = B_\mu + Q_\mu(B), \quad A^2 = B^2 + \{B.Q\}_+ + Q^2(B), \tag{26}$$

B_μ being the background, assumed to be nonperturbative, and Q_μ the quantum fluctuations assumed to be perturbative. $\{B\cdot Q\}_+ \equiv B\cdot Q + Q\cdot B$. In principle, Q_μ depends on B_μ and differs from the quantum fluctuations around the trivial vacuum $B_\mu = 0$ which is what perturbative QCD computes. The hypothesis that $\langle (A^2)_{\mathrm{R}} \rangle$ is due[6] to these semiclassical gauge configurations is translated into $(A^2)_{\mathrm{R}} \simeq B^2$. From Eqs. (25) and (26)

$$\langle (A^2)_{\mathrm{R}} \rangle \simeq \langle B^2 \rangle \simeq \langle A^2 \rangle - \langle Q^2(B=0) \rangle \tag{27}$$

i.e., that $[Q^2(B) - Q^2(B=0)]$ is subleading.[7] In other words, the dependence of Q_μ on B_μ is subleading. The hard quantum fluctuations are not sensitive to the soft background field.

A most interesting consequence of our result is related to some discussions in [6]. These authors extend to QCD some remarks stemming from compact $U(1)$. They attribute a special role to the A^2 condensate, even if a gauge dependent quantity, by arguing that A^2 in the Landau gauge is the minimum of A^2 on the gauge orbit. One difficulty in this argument is the following: Fixing the Landau gauge amounts to minimizing the $\langle A^2_{\mathrm{bare}} \rangle$ while the condensate refers to some renormalized quantity free of the quadratic and logarithmic divergences. In compact $U(1)$ life is simpler:

$$\langle A^2_{\mathrm{bare}} \rangle = \langle A^2_{\mathrm{pert}} \rangle + \langle A^2_{\mathrm{nonpert}} \rangle, \tag{28}$$

the perturbative theory is trivial and the nonperturbative contribution is due, roughtly speaking, to the topology. A phase transition when the coupling constant varies allows us to measure directly the non perturbative contribution. We refer to [6] for more details. Our result Eq. (25), exhibits in QCD, up to subleading contributions, a linear decomposition similar to Eq. (28), although such a similarity is not at all obvious at first sight. The next question could be whether in some sense the $\langle A^2 \rangle^{\mathrm{OPE}}$ computed in the Landau gauge is the minimum of some quantity on the gauge orbit.

Last but not least, let us simply say that the result in Eq. (25) provides a fairly simple understanding of what the A^2 condensate is. *It confirms that indeed the $O(1/p^2)$ correction to perturbative QCD at large momenta has to do with the A^2 condensate.* Indeed, if one starts with some doubt about the relation of the RHS of Eq. (13) with an A^2 condensate, just considering it as an unidentified $1/p^2$ contribution, we end-up with the conclusion that it yields a nonperturbative contribution to the A^2 VEV. The fact that in our derivation this term has precisely the anomalous dimension of an A^2 condensate comes form the fact that c_2 in the RHS of Eq. (13) has been computed under the assumption that it is due to an A^2 condensate, an assumption which has been shown to fit fairly well the lattice data. Had we used another scale dependence for c_2 we would have ended with a wrong scale dependence for the resulting nonperturbative contribution to the A^2 VEV. We would have also ended with a constant different from 1 in front of the RHS of Eq. (25); see the discussion following Eq. (20). In fact, the necessity of this factor 1 in front of the RHS of Eq. (25) could be thought to introduce, at least up to one loop, a bound for A^2 and gluon anomalous dimensions and β function. A very recent work [7], which has appeared while this note was under consideration, confirms such a bound up to all the orders in pure Yang-Mills theory in the Landau gauge within the algebraic renormalization. Thus the picture is fully consistent. On the other hand, this result strongly supports the existence of some underlying Slavnov-Taylor identity at the origin of such a bound and opens the possibility to extend the results of this paper, in particular Eq. (25), beyond the leading logarithm approximation. How to do it, within the MOM renormalization scheme, is a work in progress.

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[6]This discussion is qualitative and we do not know how to define rigorously the corresponding scale μ. We therefore prefer to omit writing μ here.

[7]If B is a classical solution of the field equations, the term linear in Q will vanish. B should be close to such a solution and we therefore neglect $\{B\cdot Q\}_+$.

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