

## Reply to "Comment on 'Dynamical polarization potential due to the excitation of collective states' "

M. V. Andrés,<sup>(1)</sup> F. Catara,<sup>(2,3)</sup> and E. G. Lanza<sup>(3)</sup>

<sup>(1)</sup>*Departamento de Física Atómica y Nuclear, Universidad de Sevilla, 41080 Sevilla, Spain*

<sup>(2)</sup>*Dipartimento di Fisica dell'Università, 95129 Catania, Italy*

<sup>(3)</sup>*Istituto Nazionale di Fisica Nucleare, Sezione di Catania, 95129 Catania, Italy*

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This is a reply to the preceding Comment. We confirm the results of our paper. The differences between our calculations and those of the preceding Comment come mainly from the form factors. Our random-phase-approximation-based form factors are in good agreement with those obtained by using experimental transition densities.

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In a recent paper [1] we calculated the heavy ion polarization potential due to the excitation of vibrational collective states which we describe in a microscopic way by using the random phase approximation (RPA). The aim of the work of Ref. [1] was to construct the optical potential starting from a microscopic description of nuclear properties. We restate our main results: (a) the contribution of the giant quadrupole resonance (GQR) to the real part of the polarization potential is very important at low and high incident energies; (b) the contribution of the GQR states increases with the incident energy until reaching a maximum around 650 MeV and then decreases very slowly to zero; and (c) the use of average excitation energies gives rise to a polarization potential with different energy dependence and magnitude from the one calculated with the proper energies.

In the preceding Comment, Vinh Mau *et al.* claim that our imaginary potential has a nonrealistic energy dependence and is abnormally weak. They argue that this is due to the neglecting of the low-lying  $2^+$ ,  $4^+$ , and  $5^-$  collective states and also to an underestimation of the low-lying  $3^-$  states.

We agree with the authors of the Comment that the inclusion of the  $(2^+, 4^+, 5^-)_{LL}$  states can give an appreciable contribution to the imaginary potential at least at low incident energies. We could have included the  $2^+$ ,  $4^+$  states losing in this way the consistency of our calculation, since these states are not RPA eigenstates, but we prefer to maintain the microscopic aspect of the model. The  $5^-$  state has been neglected because it exhausts only a few percent of the energy weighted sum rule. So its contribution to the polarization potential was not important and in any case it would not have changed the main results of Ref. [1]. On the other hand, as we will show later on, the contribution of the low-lying  $3^-$  state calculated within the RPA is very similar to that obtained by using experimental transition densities.

Even though we do not take into account the contribution of the  $(2^+, 4^+, 5^-)_{LL}$  states, it is true that the value of the imaginary potential due to the  $3^-$  and to the GQR states found by Vinh Mau *et al.* is higher than

ours. We believe the difference depends on the different form factors used in the two calculations. But, in order to exclude miscalculations in our code we checked it once again. To do so we calculated the imaginary part of the optical potential by using the analytical formula given by the same authors of the Comment and reported in Eq. (16) of Ref. [2]. This analytic formula is deduced in the case that the bare potential is real, and the off-diagonal potential is given by a Woods-Saxon derivative form, as the form factor defined in Eq. (4) of the Comment, which from hereafter we indicate with the symbol FF4. Since in their Comment the authors do not say exactly which parameters they have used, in order to make the check on our code we used the potential reported in the Broglia and Winther book [3], namely, the one corresponding to the Eqs. (40), (41), (44), and (45) of section III.1, as bare potential, and its derivative in the form factor FF4. The check has been done using the same energies and  $\beta$ 's reported in Table I of the Comment, while for the deformation lengths  $\delta_{J_i}$  we have used

$$\delta_{J_i} = \beta_{J_i} R_i = \beta_{J_i} 1.2A^{1/3}. \quad (1)$$

The result of the check is that the imaginary part of the polarization potential calculated by our code coincides with the one calculated by the analytic formula, Eq. (16) of Ref. [2]. But it does not coincide with the one reported in the Comment. In particular, the potential shown in their Fig. 2 seems to be larger by a factor of about 1.25 with respect to the one obtained with their analytic formula. We do not understand this discrepancy.

In any case there is a difference between our potential and the one calculated using the form factor FF4 and this difference can be mainly attributed to the different form factors used.

We agree with Vinh Mau *et al.* that the main reason of the discrepancy comes from the low-lying  $3^-$  states, in particular the  $3^-$  of  $^{16}\text{O}$ . Indeed, as it is shown in Fig. 1, our RPA form factor (solid line), calculated as described in Ref. [1], is quite different, also in the important peripheral region, from the FF4 one (short-dashed line). In the same figure there are also shown the form factors of the

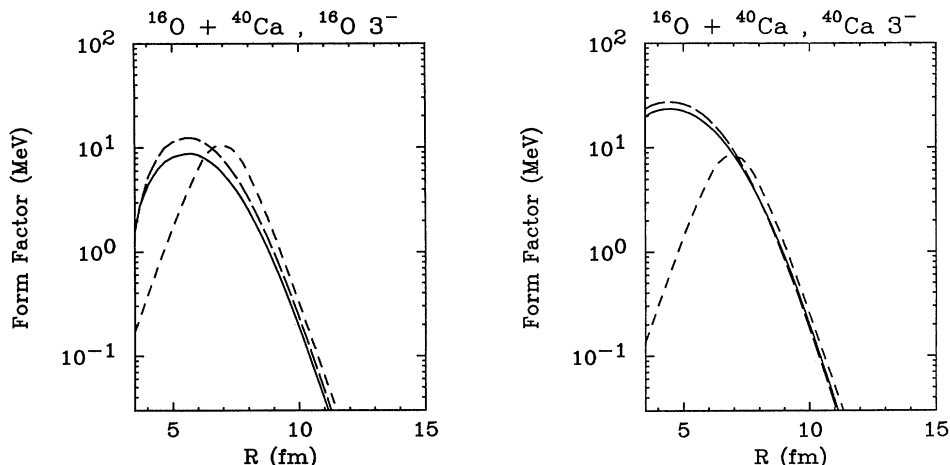


FIG. 1. Form factors for the low-lying  $3^-$  state of the nuclei  $^{16}\text{O}$  (left) and  $^{40}\text{Ca}$  (right). The three curves correspond to RPA form factor (solid line), FF4 form factor (short dashed line), and FFE form factor (long dashed line) (see text).

$3^-_{LL}$  of  $^{40}\text{Ca}$ , which are again different but not so much as in the  $^{16}\text{O}$  case.

These differences are due to the fact that our microscopic form factor has been calculated by double folding the M3Y effective interaction with the transition density of the specific state and with the ground state density of the nucleus which is generating the excitation. On the contrary, the macroscopic form factor FF4 has a fixed geometry which is determined by the bare potential used and is mainly concentrated on the surface. Then the strength of the form factor FF4 is determined by the value of the  $\beta$  [or the reduced transition probability  $B(E\lambda)$ ] used. The importance of the strength of the form factors in the peripheral region for this kind of calculations is evidenced by the fact that the scaling of the FF4, in order to get the same values of our form factors in the external region, produces a potential very similar to ours. These scaling factors are not simply the ratio between the  $\beta$ 's values.

In order to have a further check on our form factors we have calculated them by double folding the M3Y effective interaction with the experimental density of a nucleus and with the experimental transition density of the other. In Ref. [4] the transition charge densities for the low-lying excited states of  $^{16}\text{O}$  are represented by a Fourier-Bessel expansion. Then, by using the method illustrated in Ref. [5], we have constructed the proton point transition density by making a deconvolution with the proton form factor [6]. For the low-lying  $3^-$  and  $2^+$  states of  $^{40}\text{Ca}$  we have used the charge transition density reported in Ref. [7], deconvoluted with the same proton form factor by the Fourier transform method. The same method has been used to extract the proton-point ground-state density from the charge density written as a parabolic Fermi distribution of the form

$$\rho_c(r) = \rho_0(1 + \omega r^2/r_0^2) \left[ 1 + \exp\left(\frac{r-r_0}{a}\right) \right]^{-1} \quad (2)$$

with  $r_0 = 3.7984$  fm,  $a = 0.5795$  fm, and  $\omega = -0.1779$  for  $^{40}\text{Ca}$  [7] and  $r_0 = 2.608$  fm,  $a = 0.513$  fm, and  $\omega = -0.051$  for  $^{16}\text{O}$  [8]. Then, assuming that the ground state and the transition densities of the neutron part is the same as that of the protons, we can calculate the form factor. This is done by double folding the effective interaction M3Y with the experimental transition density of a nucleus and with the experimental ground-state density of the other. The results are reported in Fig. 1 as long-dashed lines. One can see that in the case of the  $3^-$  of  $^{16}\text{O}$  the double folding form factor obtained from experimental densities (which we will indicate as FFE) is much closer to ours than to the one used by Vinh Mau *et al.* In any case we will see that this difference in the form factor will induce a difference in the calculations of the polarization potential. For the  $3^-$  of  $^{40}\text{Ca}$  case the agreement between the form factor FFE and ours is excellent.

In Fig. 2 we show the contribution of the low-lying  $3^-$  states to the imaginary part of the optical potential for the system  $^{16}\text{O} + ^{40}\text{Ca}$  at a fixed distance  $R = 9$  fm. The solid lines correspond to the calculation of Ref. [1]. Here we have separated the contribution of the  $3^-$  from the one due to the GQR (explicitly indicated in the figure). The dashed line corresponds to the calculation done with the form factor FF4 (as said before, the corresponding result by Vinh Mau *et al.*, is a factor of 1.25 higher). The results of the calculations done with the form factors FFE are also shown in Fig. 2 (long-short dashed line).

The first striking evidence is that the imaginary potential calculated with the form factors FFE and the experimental energies is very close to ours [1] and so in disagreement with the one calculated by Vinh Mau *et al.* Furthermore, our potential is even closer to the one calculated with the FFE form factor if, instead of the RPA energies, we use the experimental ones. The second observation regards the relative contribution of the GQR states with respect to the low-lying  $3^-$  ones, as a function

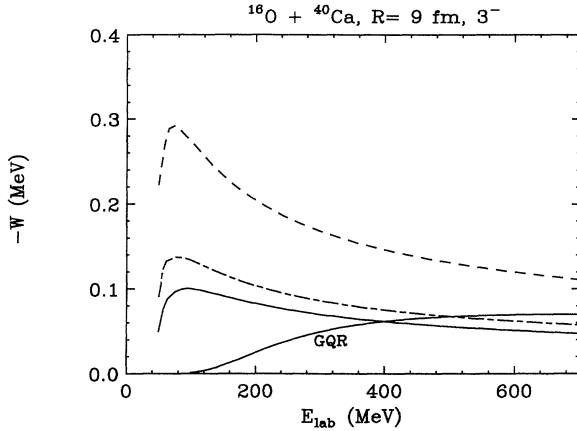


FIG. 2. Contribution of the low-lying  $3^-$  states to the imaginary part of the polarization potential for  $^{16}\text{O} + ^{40}\text{Ca}$  as functions of the incident energy for a fixed value of  $R = 9$  fm. The solid lines are the result of the calculation done with the RPA form factors, the short dashed line corresponds to the use of the FF4 form factors, while the long-short dashed line refers to the calculation where the FFE form factors were used. In the figure is also shown the contribution due to the GQR states (solid line with the GQR label).

of the incident energy. The two solid lines of Fig. 2 cross each other at  $E_{\text{lab}} \sim 400$  MeV, hence for incident energies higher than this, the contribution of the GQR is higher than that of the low-lying  $3^-$ . If we consider as contribution to  $W$  the FFE one (short-long dashed line), the cross point moves to a higher energy,  $E_{\text{lab}} \sim 500$  MeV, in contradiction with Figs. 1 and 2 of the Comment. The

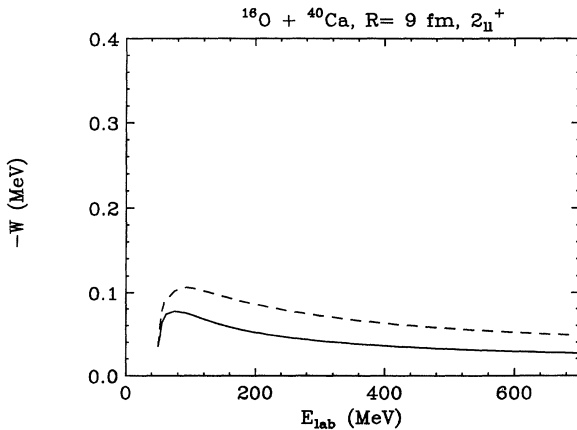


FIG. 3. Contribution of the low-lying  $2^+$  states to the imaginary part of the polarization potential for  $^{16}\text{O} + ^{40}\text{Ca}$  as function of the incident energy for a fixed value of  $R = 9$  fm. The solid lines are the result of the calculation done with the FFE form factors, while the dashed line corresponds to the use of the FF4 form factors.

same trend has been found at  $R = 8$  fm.

We have used the same procedure to extract the form factors FFE, for the low-lying  $2^+$  states. In Fig. 3 we compare their contribution to the optical potential (solid line) with the one obtained by using the form factor FF4. We can see that the latter contribution is bigger than the one given by the form factors FFE.

Finally we want to make a remark about Fig. 4 of the Comment. In this figure Vinh Mau *et al.* show two curves labeled with  $W_A$  and  $\bar{W}_A$  which should be equivalent to our calculations. They found that they are very different from the ones shown in Ref. [1]. Since they have used very different form factors this is hardly surprising.

In any case, looking only to their results,  $W$  and  $\bar{W}$ , we notice that the use of low average energies increases the value of  $W$  of about 60% at  $E_{\text{lab}} \sim 100$  MeV. We think that this is a big and unphysical effect and it does not have anything to do with the transfer channels, since in this case one is not using the closure approximation, so there is no summation over all possible states but rather only very few and specific open channels are taken into account.

The different enhancement factor between their calculations and ours (Fig. 10 of Ref. [1]) is due partly to the fact that they include the  $(2^+, 4^+, 5^-)_{LL}$  states which have already low energies. On the other hand, they do not have the high-lying  $3^-$  states present in our calculation [1], which give a big contribution when, by using the low average energy approximation, they are decreased from  $\sim 30$  to  $\sim 5$  MeV. Then in their case only the GQR states contribute to the enhancement, while in our case both GQR and high-lying  $3^-$  states contribute.

In summary, we have excluded miscalculations by checking our numerical code by using the analytical formula for the imaginary potential [Eq. (16) of Ref. [2]]. Then we have shown that the main difference between the results presented in the preceding Comment and our paper [1] is due to the different form factors used in the calculations. By using experimental transition densities and ground-state densities we have calculated the form factors FFE, which are very much closer to our RPA form factors than to the ones used by Vinh Mau *et al.* As a consequence the polarization potential calculated with the form factors FFE is very similar to the one obtained in Ref. [1] rather than to the one presented in the Comment. In particular, at high incident energies the contribution of the GQR states is higher than the one corresponding to the low-lying  $3^-$  states, confirming in this way the energy dependence of the optical potential we have found in Ref. [1].

Finally, the use of the approximation of low average energy produces an unphysical enhancement on the polarization potential also in the calculation presented in the Comment, although it is less pronounced than in our case (Fig. 10 of Ref. [1]). This difference can be explained by the fact that Vinh Mau *et al.* use more low-lying states and less high-lying states than us.

In conclusion, the calculation of the polarization potential is very sensitive to the form factor used, so one should not be surprised if, giving as input different form factors, one gets different results.

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