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Nuclear transparencies in relativistic $A(e, e'p)$ models

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Abstract

Relativistic and unfactorized calculations for the nuclear transparency extracted from exclusive $A(e, e'p)$ reactions for $0.3 \leqslant Q^2 \leqslant 10$ (GeV/c)^2 are presented for the target nuclei C, Si, Fe and Pb. For $Q^2 \geqslant 0.6$ (GeV/c)^2, the transparency results are computed within the framework of the recently developed relativistic multiple-scattering Glauber approximation (RMSGA). The target-mass and Q^2 dependence of the RMSGA predictions are compared with relativistic distorted-wave impulse approximation (RDWIA) calculations. Despite the very different model assumptions underlying the treatment of the final-state interactions in the RMSGA and RDWIA frameworks, they predict comparable nuclear transparencies for kinematic regimes where both models are applicable.

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The transparency of the nuclear medium to the propagation of protons is an issue of fundamental importance. The nuclear transparency provides a measure of the probability that a proton of a certain energy escapes from the nucleus without any further interaction. The nuclear transparency is a useful quantity for studying nuclear medium effects, and in particular, it is very well suited for investigations of the so-called color transparency (CT) phenomenon, which predicts a significant enhancement of the transmission of protons through nuclei [1,2] once QCD mechanisms start playing a role.

During the last decade, several investigations of the nuclear transparency have been carried out using the $A(e, e'p)$ reaction in the quasi-elastic (QE) regime. In this kinematic regime, the impulse approximation (IA), where a quasifree single-nucleon knockout reaction mechanism is assumed, has been proved to provide a good description of the reaction dynamics. Thanks to the electromagnetic character of the initial-state interactions in an $A(e, e'p)$ process, the entire nuclear volume can be probed.

Nuclear transparency measurements with the $A(e, e'p)$ reaction are available for a range of target nuclei. The first experiments were performed at Bates for $Q^2 \approx 0.3$ $(\text{GeV}/c)^2$ [3], and at SLAC for $1 \leqslant Q^2 \leqslant 7$ $(\text{GeV}/c)^2$ [4,5]. Recently, measurements at the Thomas Jefferson National Accelerator Facility (TJNAF) provided precise data for the target nuclei ^{2}D, ^{12}C and ^{56}Fe and $Q^2 = 3.3$, 6.1 and 8.1 $(\text{GeV}/c)^2$ [6]. The same facility provided an alternate set of data for the target nuclei ^{12}C, ^{56}Fe and ^{197}Au and $0.64 \leqslant Q^2 \leqslant 3.25$ $(\text{GeV}/c)^2$ [7,8].

The prediction of the nuclear transparency to protons poses a serious challenge for models dealing with the $A(e, e'p)$ reaction due to the wide range of proton energies which are probed in the present-day experiments. As a matter of fact, at present there is no uniform and realistic framework in which the proton-nucleus final-state interaction (FSI) effects can be computed for proton kinetic energies ranging from 0.3 to several GeV. For kinetic energies up to around 1 GeV, most theoretical $A(e, e'p)$ investigations are performed within the context of the distorted-wave impulse approximation (DWIA), where the effect of the scatterings on the emerging nucleon is estimated with the aid of proton–nucleus optical potentials. The parameterizations of these potentials are usually not available for proton kinetic energies T_p beyond 1 GeV. Beyond this energy, the Glauber model, which is a multiple-scattering extension of the eikonal approximation, offers a valid and economical alternative for describing FSI. In a Glauber framework, the effects of FSI on the $A(e, e'p)$ observables are computed directly from the elementary proton–nucleon scattering data through the introduction of a profile function. The Glauber method postulates linear trajectories and frozen spectator nucleons, and the lower limit of this treatment to $A(e, e'p)$ has not yet been established.

Numerous predictions for the nuclear transparencies within the context of non-relativistic Glauber theory have been reported in literature [9–17]. These results are typically obtained in a non-relativistic and factorized model for dealing with the $e + A \rightarrow e' + (A - 1) + p$ reaction dynamics. In this context, non-relativistic refers to the fact that the calculations use bound-state wave functions or nuclear densities from solutions to a Schrödinger equation and non-relativistic expressions for the electromag-netic photon–nucleus interaction Lagrangian. In the context of modeling $A(e, e'p)$ processes, factorization refers to the approximation of decoupling the electron–proton from the nuclear dynamics part in the calculations.

In this Letter we focus on relativistic and un-factorized descriptions of nuclear transparencies extracted from quasi-elastic $A(e, e'p)$ processes. In the past, relativistic distorted-wave impulse approximation (RDWIA) $A(e, e'p)$ calculations for the nuclear transparency have been presented by Kelly [18], Meucci [19] and Greenberg [20]. Kelly used an effective current operator containing the Dirac potentials, two-component bound states and distorted waves obtained as solutions to relativized Schrödinger equations. Meucci used bound-state wave functions from a relativistic mean-field approach, while the effective Pauli reduction was adopted to construct the ejectile's wave function.

For $T_p \geqslant 1$ GeV many partial waves need to be computed, thus a description of the FSI mechanisms in terms of phenomenological optical potentials may not be the most economical way at higher energies. In addition, the global optical potentials we use in this work were fitted to data with a limited range in T_p below 1 GeV, hence the RDWIA studies of the nuclear transparency do not cover the kinematic ranges beyond $T_p = 1$ GeV.

Recently, a relativized and unfactorized version of the Glauber model has been proposed [21–23]. In this Letter, transparencies obtained within this so-called relativistic multiple-scattering Glauber approximation (RMSGA) framework will be compared with those obtained in the RDWIA framework as it has been implemented by the Madrid–Sevilla group [24–27]. The comparison is made in a consistent way. For the transparency results which will be presented below, this implies that the two frameworks only differ in the way they treat the final-state interactions. All the remaining ingredients are kept identical.

In what follows we will first sketch the basic ingredients entering the RDWIA and the RMSGA frameworks, thereby indicating the similarities and differences between the two. Then, the energy dependence, expressed in terms of the four-momentum transfer $Q^2 = q^2 - \omega^2$, and target-mass dependence of the nuclear transparencies obtained in the two approaches will be compared and confronted with the data.

We adopt the relativistic IA to $A(e, e'p)$ which translates the outstanding success of the Dirac phenomenology in describing elastic proton scattering [28] to the electron scattering case. Within the relativistic IA approach to $A(e, e'p)$, the central quantity to be computed is the current matrix element [29]

$$\langle J^\mu \rangle = \int d\vec{r}\, \bar{\phi}_F(\vec{r})\hat{J}^\mu(\vec{r})e^{i\vec{q}\cdot\vec{r}}\phi_B(\vec{r}). \tag{1}$$

In the RDWIA framework, the target nucleons interact with each other through static mean fields of Lorentz scalar and vector character [30]. We consider, as usual, only spherically symmetric nuclei with closed shells. The ground-state nuclear wave function is computed in the Dirac–Hartree approximation [31]. For the ejected nucleon, ϕ_F is a scattering solution to a Dirac-like equation, which includes scalar (S) and vector (V) global optical potentials obtained by fitting elastic *proton–nucleus* scattering data. The scattering wave function, expressed in terms of a partial-wave expansion in configuration space, reads

$$\phi_F(\vec{r}) = 4\pi \sqrt{\frac{E_F + M_p}{2E_F}} \sum_{\kappa\mu m} e^{-i\delta_\kappa^*} i^\ell \left\langle \ell m \frac{1}{2} s_F \,\middle|\, j\mu \right\rangle$$
$$\times Y_\ell^{m*}(\Omega_{p_F})\Psi_\kappa^\mu(\vec{r}), \tag{2}$$

where $\Psi_\kappa^\mu(\vec{r})$ are four-spinors of the same form as the ground-state wave function, except for the fact that they have complex phase-shifts and radial functions. The outgoing nucleon momentum, energy and spin are denoted as $\vec{p}_F$, E_F and s_F, respectively.

In the RMSGA framework, the scattering wave function takes on the following form

$$\phi_F(\vec{r}) \equiv \phi_{p_F s_F}(\vec{r})\mathcal{G}(\vec{b}, z), \tag{3}$$

where $\phi_{p_F s_F}$ is a relativistic plane wave. The entire effect of the FSI mechanisms is contained in the Dirac–Glauber phase $\mathcal{G}(\vec{b}, z)$, which is an A-body operator and reads

$$\mathcal{G}(\vec{b}, z)$$
$$= \prod_{\alpha \neq B}\left[1 - \int d\vec{r}' |\phi_\alpha(\vec{r}')|^2 \theta(z' - z)\Gamma(\vec{b} - \vec{b}') \right]. \tag{4}$$

Here, the product over α extends over all occupied single-particle levels, but for the one from which the detected nucleon was emitted. Further, the profile

function for pN scattering is defined as

$$\Gamma(\vec{b}) = \frac{\sigma_{pN}^{\text{tot}}(1 - i\epsilon_{pN})}{4\pi\beta_{pN}^2} \exp\left(\frac{-b^2}{2\beta_{pN}^2}\right). \tag{5}$$

The parameters σ_{pN}^{tot}, β_{pN} and ϵ_{pN} depend on the proton kinetic energy and are obtained through interpolation of the data base available from the Particle Data Group [32]. We wish to stress that Glauber-based models come in very different flavours and that the RMSGA formulation used here borrows a lot of ingredients from the RDWIA approach to $A(e, e'p)$ processes, except for the way of computing the effect of FSI mechanisms on the hit proton, which is through-and-through different.

In order to make the comparisons between the RDWIA and RMSGA transparency predictions as meaningful as possible, all the ingredients in the $A(e, e'p)$ calculations not related to FSI, as those concerning the implementation of relativistic dynamics and nuclear recoil effects, are kept identical. In particular, both pictures use the relativistic bound-state wave functions from a Hartree calculation with the W1 parameterization for the different field strengths [33]. Further, all the results presented in this work are obtained within the Coulomb gauge using the so-called CC2 current operator [34]. For the description of nuclear transparencies, the effect of Coulomb distortions has been recognized as negligible [18]. Therefore, no attempt has been made to correct for the Coulomb-distortion effect.

We wish to stress that the RDWIA, as implemented by the Madrid–Sevilla group, and RMSGA codes adopt very different numerical techniques to compute the scattering wave functions and the corresponding matrix elements of Eq. (1). The Madrid RDWIA code employs a partial-wave expansion to solve the Dirac equation for the ejectile. The cylindrical symmetry of the Glauber phase of Eq. (4) prohibits any meaningful use of this technique in the RMSGA calculations. Instead, the multi-dimensional integrals are computed numerically. In the limit of vanishing FSI mechanisms, however, the two codes should predict identical results. In the Glauber approach this limit is reached by putting the Glauber phase of Eq. (4) equal to unity. In the RDWIA picture, the effect of FSI can be made vanishing by nullifying the optical potentials. Then, the computed partial waves sum to a relativistic plane

wave. To guarantee convergence, the total amount of partial waves grows with increasing proton kinetic energy. Convergence of the partial wave expansion was tested against the analytical plane-wave result [35]. In this so-called relativistic plane-wave impulse approximation (RPWIA) limit and assuming identical input options, the RMSGA-Gent and RDWIA-Madrid codes produce differential cross sections with an agreement to better than 5% for the whole range of kinetic energies considered in this work. The remaining can be partially attributed to the numerical evaluation of the multi-dimensional integrals in RMSGA. This comparison gives us confidence about the consistency of the calculations.

The nuclear transparency provides a measure of the likelihood that a struck nucleon with kinetic energy T_p escapes from the nucleus. The nuclear transparency is extracted from the measured $A(e, e'p)$ differential cross sections $d^5\sigma^{\exp}(e, e'p)$ on the basis of the following ratio

$$
\begin{aligned}
&T_{\exp}(Q^2) \\
&= \frac{\int_{\Delta^3 p_m} d\vec{p}_m \int_{\Delta E_m} dE_m \, S_{\exp}(\vec{p}_m, E_m, \vec{p}_F)}{c_A \int_{\Delta^3 p_m} d\vec{p}_m \int_{\Delta E_m} dE_m \, S_{\text{PWIA}}(\vec{p}_m, E_m)}.
\end{aligned}
\tag{6}
$$

Here, $S_{\exp}$ is the experimentally determined reduced cross section

$$
S_{\exp}(\vec{p}_m, E_m, \vec{p}_F) = \frac{\frac{d^5\sigma^{\exp}}{d\Omega_p \, d\epsilon' \, d\Omega_{\epsilon'}}(e, e'p)}{K\sigma_{ep}},
\tag{7}
$$

where K is a kinematical factor and σ_{ep} is the off-shell electron–proton cross section, which is usually evaluated with the CC1 prescription of de Forest [34]. The quantities $\Delta^3 p_m$ and ΔE_m specify the phase-space volume in the missing momentum and energy and are commonly defined by the cuts $|p_m| \leqslant 300 \, \text{MeV}/c$ and $E_m \leqslant 80 \, \text{MeV}$. These kinematic cuts, in combination with the requirement that the Bjorken variable $x = \frac{Q^2}{2M_p\omega} \approx 1$, guarantee that the electro-induced proton-emission process is predominantly quasi-elastic. For example, the effects of two-body meson-exchange and isobar currents, which are neglected within the IA, have been shown to be at the percent level for quasi-elastic kinematics [36,37].

In the above equation, S_{PWIA} is the reduced cross section within the plane-wave impulse approximation (PWIA) in the non-relativistic limit. The factor c_A in the denominator of Eq. (6) has been introduced to correct in a phenomenological way for short-range mechanisms and is assumed to be moderately target-mass dependent. It accounts for the fact that short-range correlations move a fraction of the single-particle strength to higher missing energies and momenta and, hence, beyond the ranges covered in the integrations $\int d\vec{p}_m \int dE_m$ of Eq. (6). The values for c_A which are adopted to extract the transparency from the $A(e, e'p)$ measurements are 0.9 (^{12}C), 0.88 (^{28}Si), 0.82 (^{56}Fe) and 0.77 (^{208}Pb).

Theoretically, the nuclear transparencies are extracted from the computed relativistic $A(e, e'p)$ angular cross sections for the individual single-particle states, according to

$$
T_{\text{theo}}(Q^2) = \frac{\sum_\alpha \int_{\Delta^3 p_m} d\vec{p}_m \, S^\alpha(\vec{p}_m, E_m, \vec{p}_F)}{c_A \sum_\alpha \int_{\Delta^3 p_m} d\vec{p}_m \, S^\alpha_{\text{PWIA}}(\vec{p}_m, E_m)}.
\tag{8}
$$

This expression reflects the one used to determine $T_{\exp}$. Indeed, in our approach, we obtain the "theoretical" transparencies by adopting identical expressions and cuts as in the experiments. Essentially, we replace the measured $A(e, e'p)$ angular cross sections by the computed ones. As our relativistic models to compute the S^α's adopt the IA and a mean-field approach, it may be argued that the factor c_A in the denominator of Eq. (8) needs to be let out. This would, however, necessarily introduce an additional model dependence in our calculations. Indeed, one would need to assume that the effect of correlations is Q^2 independent and that it can be quantified by the aforementioned values of c_A. As neither of these two assumptions can be considered realistic, we prefer to stick with a theoretical definition of the transparency that matches as closely as possible the one adopted in the analysis of the data. We wish to stress that the factors c_A have no impact on the comparisons between the two relativistic approaches RDWIA and RMSGA. In addition, the integration over the missing energy $\int_{\Delta E_m} dE_m$ has been substituted by a sum over all occupied shells ($\sum_\alpha$) in the ground state of the target nucleus. Indeed, the relativistic Hartree approximation does predict bound-state eigenfunctions with a fixed energy-eigenvalue and zero width. When determining the denominator in Eq. (8), in our calculations the PWIA limit is accomplished by nullifying all sources of FSI mechanisms and neglecting those contributions introduced by the

presence of negative-energy components in the relativistic bound nucleon wave function [35].

Transparencies have been calculated for the nuclei ^{12}C, ^{28}Si, ^{56}Fe and ^{208}Pb. All numerical calculations are performed in planar and constant (q, ω) kinematics. The adopted values for q and ω are the central values of the kinematics in the $A(e, e'p)$ transparency experiments reported in Refs. [3,4,7,8]. For each shell α, the kinetic energy of the outgoing nucleon is calculated by means of the relationship $T_p = \omega + \epsilon_\alpha$, where ϵ_α is the energy eigenvalue of the corresponding single-particle state. Due to the internal motion of the confined protons, the ejected protons emerge in a cone about the transferred momentum. The boundaries of the cone are restricted by the requirement that the "initial" proton momentum $|p_m| \leqslant$ 300 MeV/c.

In the RDWIA calculations, we have employed the global $S - V$ parameterizations of Cooper et al. [38], which provide the best phenomenological optical potentials to date. As the highest kinetic energy in these parametrizations is 1 GeV, RDWIA transparencies are obtained up to four-momentum transfers of $Q^2 \approx$ 1.8 (GeV/c)^2. Due to its use of the eikonal approximation, the validity of RMSGA becomes questionable when approaching low values of Q^2. For this reason the RMSGA model is not used for calculating transparencies below $Q^2 \approx 0.6$ (GeV/c)^2. Hence, the kinematic range $0.6 \leqslant Q^2 \leqslant 1.8$ (GeV/c)^2 will be covered in both the RMSGA and the RDWIA frameworks.

First, we investigate the sensitivity of the computed transparencies to the adopted parameterizations for the optical potentials. In Fig. 1 results for ^{12}C and ^{208}Pb are displayed as a function of Q^2 for different optical-
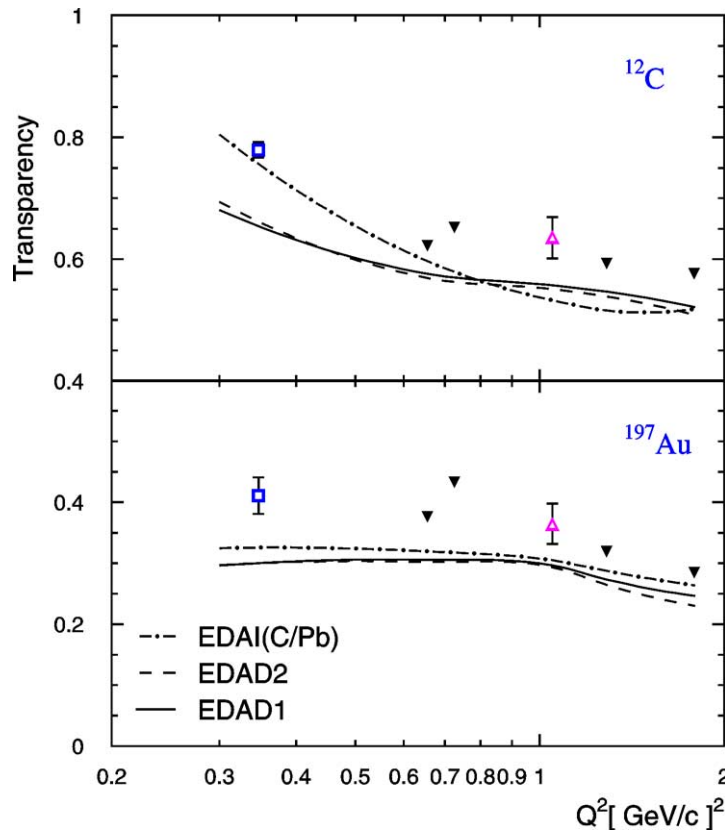

Fig. 1. The sensitivity of the computed nuclear transparencies in ^{12}C and ^{197}Au to the adopted choice for the parameterization of the relativistic optical potentials. Results of RDWIA calculations with the EDAD1 (solid curve), EDAD2 (dashed curve) and EDAIC/EDAIPb (dot-dashed curve) are shown. Data points are from Refs. [3] (open squares), [4,5] (open triangles), and [7,8] (solid triangles).

potential parameterizations contained in Ref. [38]. For ^{12}C, both the predicted Q^2 dependence and the value of the transparency depend on whether A-dependent (EDAD1/EDAD2) or A-independent (EDAIC) fits for the potentials are selected. For ^{208}Pb, the noted differences between the different types of optical-potential sets are less pronounced. Within the class of A-dependent parameterizations, the versions EDAD1 and EDAD2 give rise to comparable nuclear transparencies. In the remainder of the Letter, the EDAD1 version will be used. There are various arguments to motivate this choice. First, the A-independent parameterization is only available for a very limited number of nuclei, and extrapolation to other nuclei has been discouraged [38]. Second, all energy-dependent A-dependent parameterizations in Ref. [38] produce similar transparency predictions. Finally, the relativistic transparency calculations by Kelly [18] and

Meucci [19] employed the EDAD1 parametrization. Adopting the same choice makes easier the comparison between these predictions and ours.

In Fig. 2, the transparencies predicted by the RMSGA and RDWIA models are displayed as a function of Q^2 and compared to the world data. The ^{197}Au data are compared to ^{208}Pb calculations. The RDWIA approach systematically underestimates the data by roughly 5–10%. The presented RDWIA transparency results for ^{56}Fe and ^{208}Pb are in better agreement with the data than those reported in [7]. The RDWIA transparencies obtained in Ref. [19], on the other hand, are rather comparable to ours for low Q^2, the differences increasing for higher values.

A global feature of the RDWIA and RMSGA calculations presented here, is that they tend to underestimate the measured transparencies. Further, the RMSGA predictions for the nuclear transparencies are
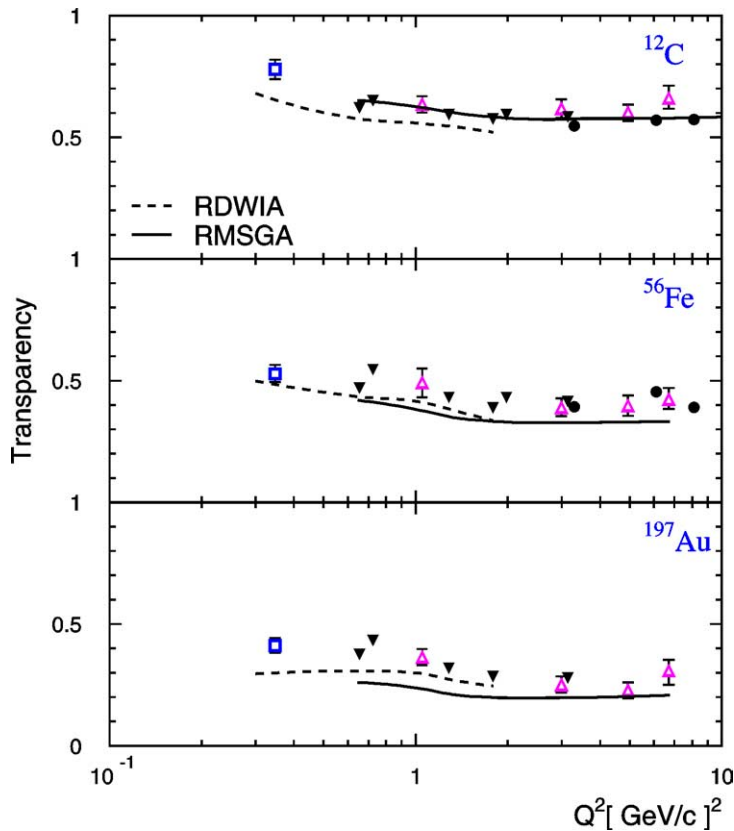

Fig. 2. Nuclear transparencies versus Q^2 for $A(e, e'p)$ reactions in quasi-elastic kinematics. The RMSGA (solid lines) are compared to the RDWIA (dashed lines) results. Data are from Refs. [3] (open squares), [4,5] (open triangles), [6] (solid circles) and [7,8] (solid triangles).

in reasonable agreement with those of the typical non-relativistic Glauber approaches, as for example reported in Ref. [16]. In Ref. [23] it was pointed out that genuine relativistic effects play a minor role in the predicted FSI effects as computed in the RMSGA model. The relativistic effects in the electromagnetic couplings tend to become visible in some well identified structure functions, like the transverse-longitudinal interference, and at high missing proton momenta. Neither of these is highly important for a phase-space averaged quantity like the nuclear transparency. This makes the nuclear transparency to exhibit modest sensitivity to the details of the relativistic dynamics, once the relativistic kinematics is properly taken into account. We insist, however, on the necessity of including relativistic ingredients in transparency calculations, in particular when the kinetic energies of the ejected protons are of the order of the nucleon mass.

As can be inferred from Fig. 2, the RMSGA framework predicts less absorption than RDWIA for a light nucleus like ^{12}C. With increasing target mass the opposite holds true and when approaching the heaviest target nuclei considered here, the Glauber framework predicts 5 to 10 percent more absorption. The measured Q^2 dependence is reasonably well reproduced by both relativistic calculations. For low Q^2 the models reproduce the trend of decreasing transparencies. For $Q^2 \geqslant 2$ (GeV/c)^2, the RMSGA transparencies are close to constant, in line with the measured ones and those predicted in typical non-relativistic Glauber models. In fact, the modest energy variation of the transparency in the RMSGA model is a reflection of the fact that the total and elastic proton–nucleon cross sections remain fairly constant once $T_p \geqslant 1.7$ GeV.
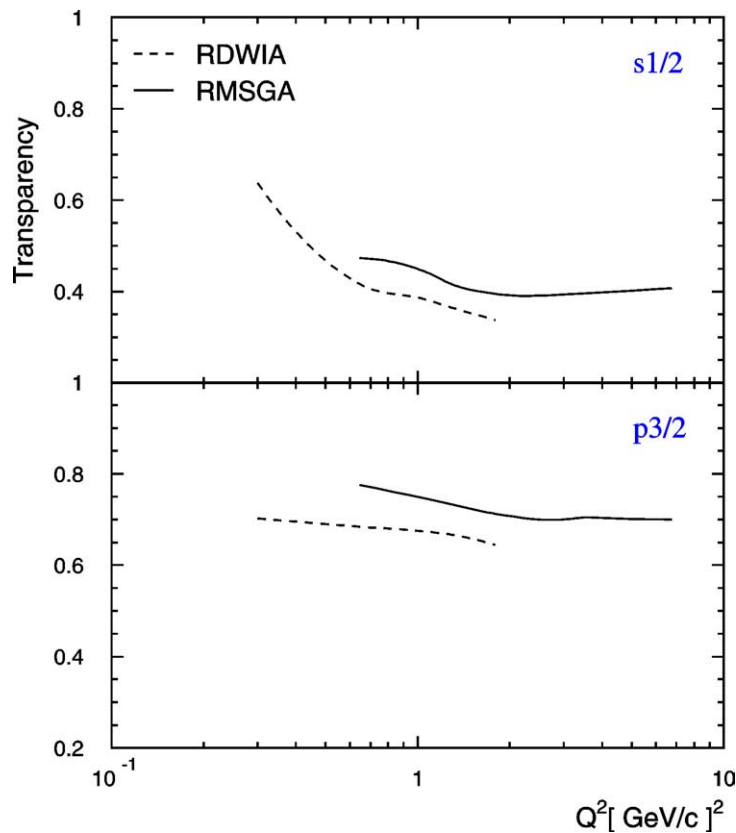

Fig. 3. The Q^2 dependence of the computed nuclear transparency for the two single-particle orbits in ^{12}C as obtained in the RDWIA and RMSGA approach.

In Ref. [18] large discrepancies were observed between the DWIA $A(e, e'p)$ transparencies and the ones from Glauber calculations of Nikolaev [10,11]. In contradistinction, Fig. 2 indicates reasonably good agreement between our RDWIA and RMSGA model predictions for the medium-heavy nucleus ^{56}Fe and modest variations in opposite directions when moving to a lighter or heavier nucleus. In Ref. [18] the noted differences between the transparencies obtained from DWIA and those from the particular Glauber

approach of Refs. [10,11], are attributed to the fact that the latter adopts a closure property in deriving the expression for the attenuation factor. We wish to stress that this approximation is NOT used in the RMSGA formulation of Glauber theory. In computing the effect of FSI mechanisms on the $A(e, e'p)$ cross sections, the sum extending over the occupied states α in Eq. (8) is carried out in a similar fashion in RMSGA and RDWIA.
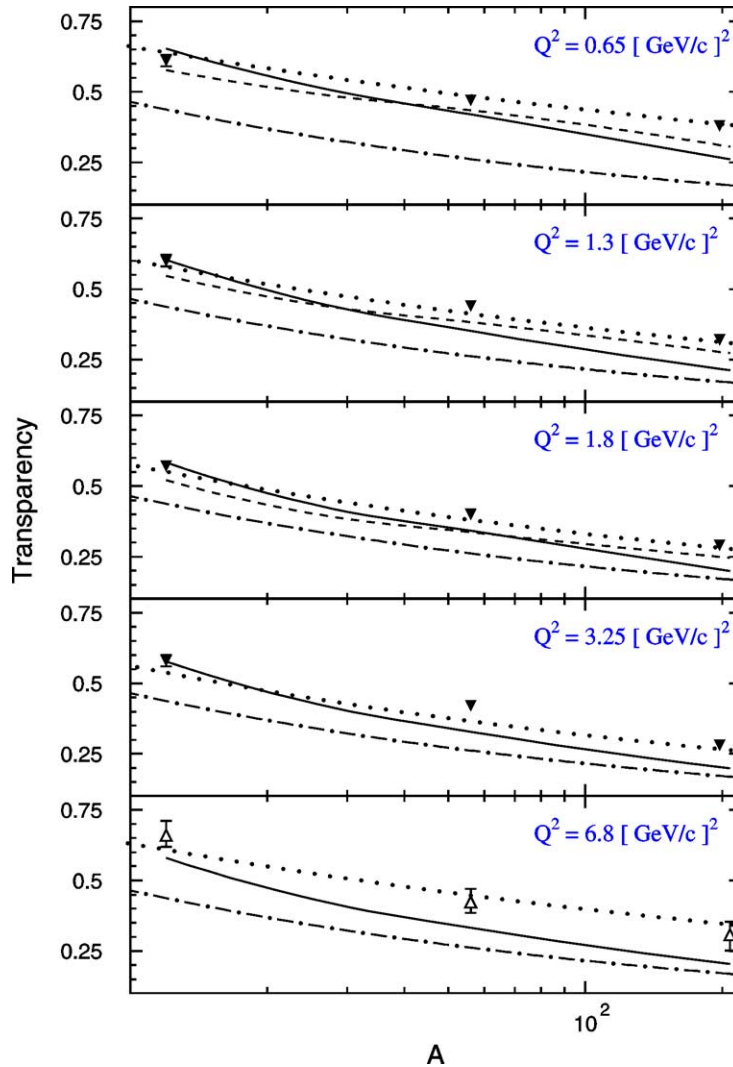

Fig. 4. The A-dependence of the nuclear transparency at five values of the four-momentum transfer Q^2. The solid (dashed) curves are RMSGA (RDWIA) calculations. The dotted curves represent the $A^{-\alpha(Q^2)}$ parametrization, while the dot-dashed curve gives $A^{-1/3}$. Data are from [7,8] (solid triangles) and [4,5] (open triangles).

Investigating the attenuation for each individual shell in the target nucleus allows one to study the radial dependence of the FSI mechanisms. In the ^{12}C case, for example, the $1s_{1/2}$ has spatial characteristics which are very different from the $1p_{3/2}$ orbit. The attenuation for the individual states represents also a more stringent test of the (non-)similarity of the optical-potential and Glauber-based models for describing proton propagation through nuclei. In Fig. 3, the RMSGA and RDWIA predictions for the attenuation for the individual shells in ^{12}C are compared. These numbers are computed according to the definition of Eq. (8) without performing the sum over the states α. Obviously, the optical-potential approach predicts more absorption for both shells. As expected, both models predict a stronger attenuation for proton emission from a level which has a larger fraction of its density in the nuclear interior. Again, the results of Fig. 3 illustrate that the proton–nucleus (RDWIA) picture and the proton–nucleon picture (RMSGA) are not dramatically different in their predictions. These findings provide us additional confidence that the "low-energy" and "high-energy" regime can be bridged in a relatively smooth manner. Note further that the observed tendency of increasing ^{12}C transparencies at low Q^2, can almost be entirely attributed to the $1s_{1/2}$ orbital.

The A-dependence of the nuclear transparencies at various values of the four-momentum transfer is studied in Fig. 4. The RDWIA framework reproduces the measured A-dependence, while RMSGA slightly overestimates it. Under the assumption that the attenuation effect is proportional to the radius of the target nucleus one would naively expect that the A-dependence of the nuclear transparency can be parameterized as

$$T(Q^2) = c(Q^2)A^{-\alpha(Q^2)}, \tag{9}$$

with $\alpha = 1/3$. In the work of Ref. [7] it was shown that the dependence of $T_{\exp}(Q^2)$ on the mass number could be nicely fitted with $c(Q^2) \equiv 1$ and $\alpha \equiv 0.17 \pm 0.04(Q^2 = 0.65)$, $0.22 \pm 0.05(Q^2 = 1.3)$, $0.24 \pm 0.04(Q^2 = 1.8)$, $0.25 \pm 0.04(Q^2 = 3.3)$, $0.20 \pm 0.02(Q^2 = 6.8)$. To guide the eyes these curves are also displayed in Fig. 4.

In conclusion, we have presented for the first time a relativistic calculation for the nuclear transparency for the process $e + A \to e' + (A - 1) + p$ covering the wide range of quasi-elastic kinematics in which experiments have been performed. An optical-potential approach has been used up to the highest kinetic energy ($T_p \approx 1$ GeV) for which potentials are readily available. Beyond that region we gathered our results within the context of a relativized and unfactorized Glauber framework. In a medium-Q^2 range, both models have been applied and their predictions compared. Both frameworks accommodate relativistic effects in the bound-state and scattering wave functions, as well as in the electromagnetic current operator. Despite the very different assumptions underlying the description of FSI effects in an optical-potential and Glauber based approach to $A(e, e'p)$, their predictions for the nuclear transparency and, in general, the effect of attenuation for different single-particle levels, are comparable.

Acknowledgements

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