

Soil structure interaction effects on the resonant response of railway bridges under high-speed traffic

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In the present contribution the dynamic behaviour of beams traversed by moving loads including soil-structure interaction (SSI) is investigated. The main application of the study is to analyse the effects of SSI on the resonant response of bridges caused by railway traffic. As this phenomenon is highly influenced by the free vibration response of the deck, a numerical investigation is carried out analyzing the effects of the wave propagation problem on the transverse free vibration response of beams under moving loads in a wide range of velocities. To this end a coupled three-dimensional boundary element-finite element model formulated in the time domain is used to reproduce the soil and structural behaviour, respectively. A subset of bridges is defined considering span lengths ranging from 12.5 to 25 m and fundamental frequencies covering associated typologies. A single soil layer is considered with shear wave velocities ranging from 150 to 365 m/s. From the single load free vibration parametric analysis conclusions are derived regarding the conditions of maximum free vibration and cancellation of the response. These conclusions are used afterwards to justify how resonant amplitudes of the bridge under the circulation of railway convoys are affected by the soil properties, leading to substantially amplified responses or to almost cancelled ones, and numerical examples are included to show the aforementioned situations.

Keywords: railway bridges, soil-structure interaction, resonance, cancellation, moving loads, BEM-FEM coupled models.

1 Introduction

The dynamic response of beams under the circulation of moving systems has been a deeply investigated topic during the last decades [1]-[4], partly due its direct application to the problem of bridges subject to the action of travelling vehicles [5,6]. In this regard, railway bridges have received special attention, as the periodic nature of axle loads may induce important vibration levels in the structures, particularly under resonant conditions [7]. Especially critical in this regard are short-to-medium span bridges

composed by simply-supported decks with usually low associated masses (see Fig. 1), which may experience high levels of vertical accelerations at the deck level in these situations. This problem aggravates for low structural damping levels, typical in the aforementioned constructions [7]. Resonance in railway bridges may lead to adverse consequences such as ballast destabilization, general degradation of the track and a raise in the maintenance costs of the line.

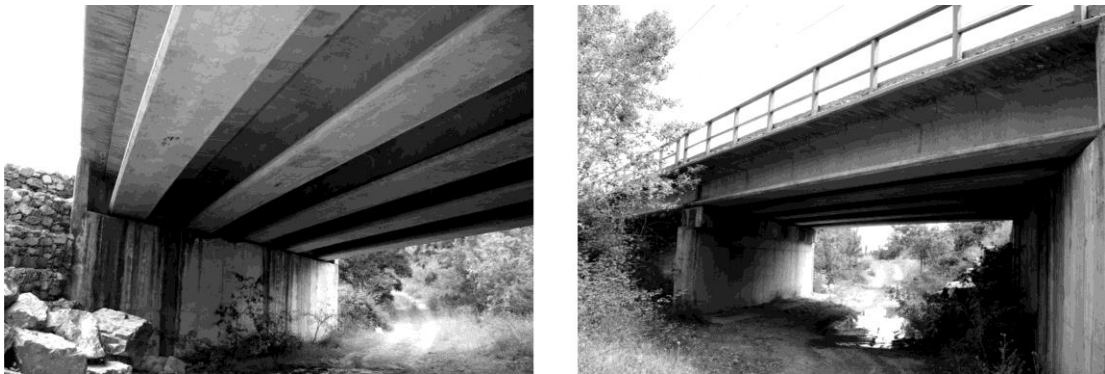


Figure 1. Railway bridge in High Speed line composed by short simply supported bays.

Resonance in simply-supported beams or bridges takes place when the excitation period of the axles, i.e. the ratio between the characteristic distance or distance causing resonance and the train speed, is a multiple of one of the structure's natural periods. When this occurs, the free vibration oscillations induced on the structure when each load abandons it accumulate, and the transverse response of the bridge progressively increases if the number of axles is sufficient. In short to medium span bridges with nowadays maximum train speeds, the characteristic distance associated with detrimental levels of transverse accelerations due to resonance usually corresponds to the length of the passengers' coaches. Therefore, the amplification of the transverse response of beams or bridges at resonance depends both on the periodicity of the loads and on the amplitude of the free vibrations left by every single load. When SSI is not taken into

account this level of free vibrations for a particular load depends on the travelling speed and on the structure natural frequencies according to literature. The aforementioned free vibration levels of beams or bridges under moving loads, and their effect on the amplification or cancellation of resonance have been evaluated in the past considering simple models for the bridge structure: generally simply supported (S-S) beams [4], [8], [9], elastically supported (E-S) beams [8], [10], and simply-supported or elastically supported plates [11] when the contribution of three-dimensional deformation modes of the deck needs to be considered. In all these studies, the soil-structure interaction (SSI) effect is neglected.

Only a few authors have investigated the resonant response of beams or bridges taking into account the effect of the waves transmitted through the soil from the substructure [12]-[14]. Some authors [12]-[13] suggest that the resonant response of a railway bridge could be considerably affected by the soil flexibility, leading to a reduction of the resonant speeds of circulation and of the transverse response amplitudes at the deck level due to the increase of damping. Others authors [14], nevertheless, indicate that for certain typologies the consideration of simply-supported conditions may provide non conservative results, when it comes to predicting the acceleration level. In the opinion of the authors of this contribution, there is a need to understand how the soil-structure interaction effects affects the free vibration response of beams, and the maximum free vibration and cancellation phenomena, which are the fundamental aspects governing resonance. Moreover, this study should be carried out considering different bridge lengths and deck typologies in order to be able to obtain general trends and conclusions. The present contribution shows the main parameters that govern the SSI effects in this regard and the fundamental trends in the evolution of the bridge resonant response with them.

2 Numerical SSI formulation and implemented model

2.1 Fundamental hypotheses of the SSI formulation

In this work soil-structure interaction effects in beams and bridges traversed by moving loads are evaluated using a coupled three-dimensional Finite Element-Boundary Element (FEM-BEM) model integrated in the time domain implemented in the SSIFiBo toolbox developed by Galvín et al. [15]. The loads are considered constant in modulus, neglecting therefore vehicle-structure interaction effects, and are applied concentrated at their corresponding locations each time step. SSI analyses are carried out by domain decomposition in two subdomains. Soil behaviour is represented by the BEM, while structures are modelled with the FEM (see Fig. 2). Coupling of BEM and FEM equations is carried out by imposing equilibrium and compatibility conditions at the soil-structure interface. Both systems of equations are assembled into a single global system, together with the equilibrium and compatibility equations [12]. The BEM is based on a time marching procedure to obtain the time variation of the boundary unknowns, i.e. displacements and tractions. Piecewise constant time interpolation functions are used for tractions and piecewise linear functions for displacements. The fundamental solution for the displacement and traction response is evaluated analytically, and nine node rectangular quadratic elements are used for spatial discretization. Expressions of the fundamental solution for displacements and tractions due to an impulse point load in a three dimensional elastic full-space are included in [16]. An approach based on the idea of using a linear combination of equations for several time steps in order to advance one step is used to ensure that the stepping procedure is stable in time. After boundary unknowns are solved, the scattered wave field at any internal point is computed by means of the integral representation of Somigliana identity.

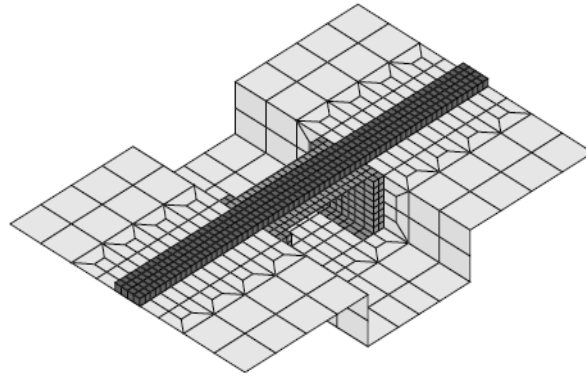


Figure 2. Discretization example of a coupled BEM-FEM for SSI analysis.

2.1 Implemented model for analysis

The objective of the investigation is to evaluate the SSI effects on the transverse response of beams traversed by moving loads at constant speeds. First the structure response will be analysed under the circulation of a single load in a wide range of velocities in order to determine the conditions for maximum response and cancellation of the response during the free vibration phase (once the load has left the structure). This relates with the amplification of resonance and cancellation of resonance that may occur when the beam is subjected to trains of equidistant loads at certain speeds. Museros et al. [8] investigated this phenomenon solving the analytical conditions for maximum free vibration response and cancellation in simply-supported and elastically-supported beams (Fig. 3b,c) and stated that these conditions, when coincide with resonant velocities, provoke very relevant resonant amplifications or almost inexistent resonant situations. Due to the importance of the free vibration amplitudes in the resonant response of beams and bridges, the model represented in Fig. 3a is investigated herein.

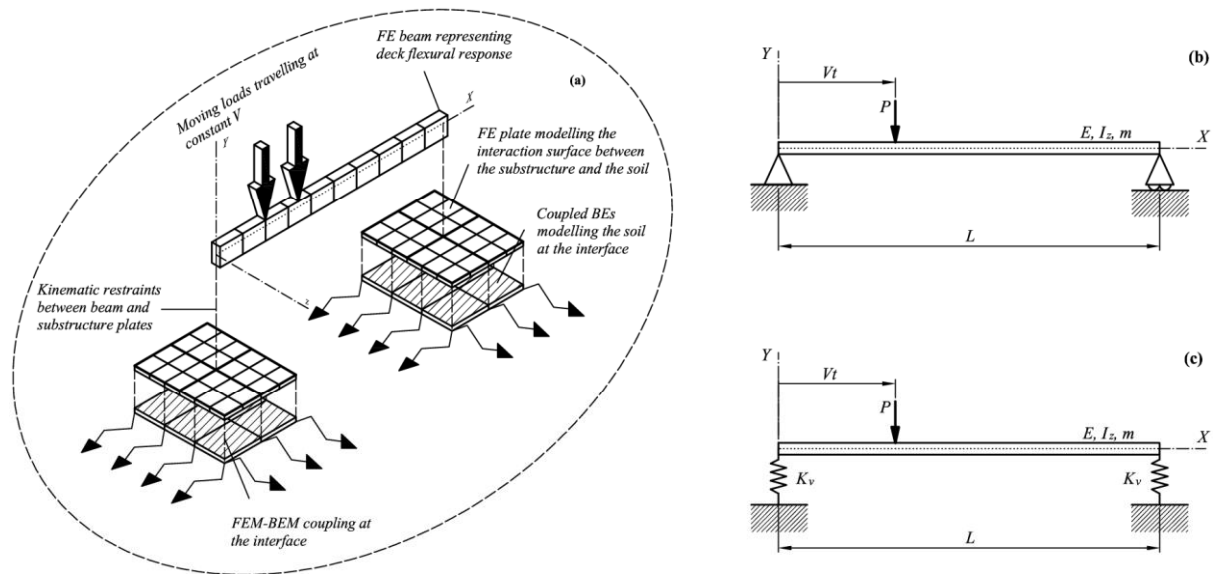


Figure 3. (a) Schematic representation of the model under analysis. (b) and (c) simply supported and elastically supported beam under travelling load.

In a first approximation a beam FE model is used to represent the deck flexural behaviour under moving loads, assuming therefore that the deck vertical response is mainly governed by the contribution of longitudinal bending modes. The beam end sections are connected through kinematic constraints to two rigid FE plates representing in a first approach and simple approximation the lower surface of the shallow foundations at the abutments. These plates are coupled to the BEs simulating the interaction with the soil. In the proposed study the detailed geometry of the substructure has not been included for the following reasons: (i) the main objective is to detect the fundamental parameters that affect the SSI effects on the bridge deck resonant response and evaluate the main tendencies of these parameters; (ii) an exhaustive parametric analysis is performed in what follows considering a wide range of circulating velocities, structural and soil properties, entailing considerably high computational times; (iii) the fundamental effects of SSI on the resonant response of bridge decks has not been analysed before covering the proposed factors; and (iv) in the authors opinion the

investigation proposed herein will be very useful as starting point in the analysis of particular foundations geometries.

3 Parametric analysis: modal identification and free vibration response under single moving load

3.1 Design of the parametric analysis

In order to be able to derive general conclusions applicable to different bridge lengths, deck typologies, soil properties and circulating velocities, an extensive parametric numerical study is designed. Beams of lengths ranging from 12.5 to 25 m in increments of length of 2.5 m are considered. For each length, three theoretical fundamental frequencies, covering the Eurocode 1 frequency range for dynamic simplified analysis [17] of simply-supported railway bridges are selected (see Fig. 4). In what follows $f_{1,000}$, $f_{1,100}$ and $f_{1,050}$ stand for the Eurocode 1 fundamental frequency lower limit, upper limit and mean value for each length considered. Beam masses have been assigned in order to represent realistic deck typologies found in conventional and High-Speed lines structures, after the studies from [18]. In particular linear deck masses of $m_b=L(m) \cdot 1000 \text{ kg/m}^2$ are considered for each length. Regarding the soil properties, four single layer soil types are defined with flexibilities covering the AASHTO classification [19], in particular with s and p -wave velocities of $c_s=\{365, 220, 150, 80\} \text{ m/s}$ and $c_p=2c_s$. Soil density has been set equal to 1800 kg/m^3 . Regarding structural damping, in a first approach the study is performed without structural damping. No material damping is assigned to the soil either. Eliminating damping permits a better comparison of cancellation conditions with the analytical solution of the elastically-supported beam. In the numerical examples presented in section 4 Rayleigh damping is assigned to the bridge structure.

Two types of analyses are performed and presented in this section for all the bridge-soil combinations under study: (i) identification of fundamental frequencies and (ii) dynamic time-history analysis under the circulation of single axle load travelling at constant speed. The circulating velocities of the load are included in the following interval, expressed in terms of the non-dimensional speed parameter K_1 associated to the fundamental mode:

$$K_1 = \frac{V\pi}{L\omega_1} \in [0.1, 0.5] \quad (1)$$

where ω_1 is the fundamental frequency of the beam. The 0.5 limit is above the highest speeds that can be reached nowadays with existing rolling stock and railway infrastructures.

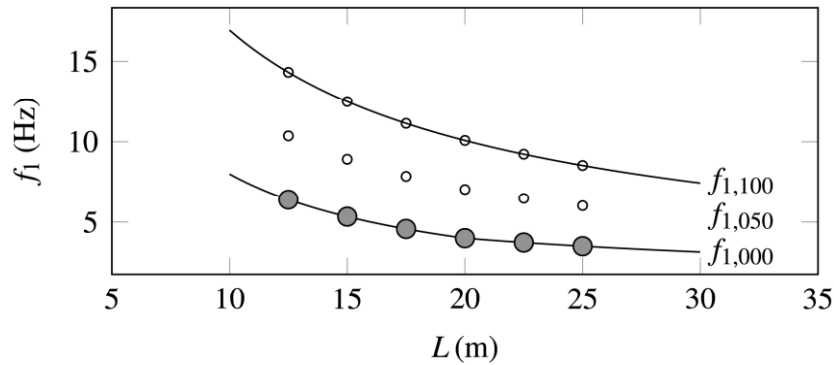


Figure 4. Eurocode 1 [17] lower and upper frequency limits for simplified dynamic analysis. Circles: reference bridges under study.

3.2 Modal identification of the bridges under study

First the fundamental natural frequencies of 72 bridges (6 lengths \times 3 frequencies \times 4 soil types) under study considering SSI have been identified from the response under impulse loading. Fig. 5 shows the evolution of the frequencies with the soil flexibility. In the vertical axis the fundamental frequency computed considering SSI has been divided by that of the infinitely rigid soil (S-S case). In the plot, three lengths

are included (12.5, 17.5 and 25 m) for the sake of clarity, as intermediate lengths show a comparable evolution.

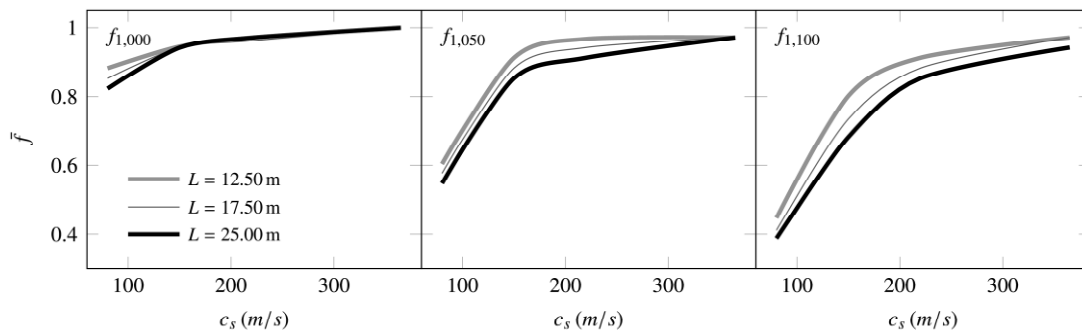


Figure 5. Evolution of the bridge fundamental frequencies with soil flexibility.

As c_s increases, and therefore the soil becomes stiffer, the fundamental frequency of the beams tends to that of the S-S case. The structures that are less affected by the soil flexibility are those with lower natural frequency for all the lengths ($f_{1,000}$ stands for the lower frequency limit in figure 4). These beams fundamental frequency is reduced around 20% for the most flexible soils and the longest spans. Bridges with highest natural frequencies ($f_{1,100}$, upper limit in Fig. 4) are most affected by the SSI effects, experiencing maximum reductions in the fundamental frequency that reach 50% in the softer soils. It must be clarified that $c_s = 80$ m/s, most flexible soil under consideration in the modal identification, is a considerably soft soil, but it has been included in this section in order to point out the interaction effect. These results are consistent with the frequency evolution included in [8] for the elastically-supported beam. In this contribution it was shown that natural frequencies were more affected as the ratio between the supports flexibility and the structure flexural flexibility increased. As all the beams with the same length present the same mass, lower frequencies entail more flexible structures as well.

3.3 Maximum free vibration response under a moving load

In this section the maximum response of the beams in the free vibration phase left by the circulation of a single load is evaluated. In figures 6 and 7 the maximum transverse displacement at mid-span, non-dimensionalized by the static deflection, R , computed in the free vibration phase (once the load has left the beam) is represented for bridges with the lowest natural frequencies (those marked as $f_{1,000}$ in Fig. 4) in terms of the circulating velocity. Figs. 6a and 7a show the analytical solution for the elastically-supported beam (Fig. 3c), included in [8]. In particular R_1 stands for the maximum transverse response associated to the fundamental mode of the E-S beam divided by the static solution; and κ is the ratio between the supports vertical flexibility and the beam flexural flexibility ($\kappa=0$ corresponds to the S-S case). In figures 6b to 6d and 7b to 7d the dynamic response of the BEM-FEM bridge model has been represented for values of $L=12.5, 15$ and 17.5 m and $L=20, 22.5$ and 25 m, respectively. Both the analytical E-S response and the numerical one have been computed in the absence of damping, in order to be able to visualize more clearly the evolution of the cancellation conditions.

From the analysis of figures 6 and 7 several aspects should be pointed out: (i) when the SSI is taken into account velocities leading to maximum free vibration response and to cancellation sequentially take place, in the same way that occurs for the E-S beam; (ii) as c_s and c_p decrease, going from stiffer to softer soils, the cancellation non-dimensional velocities increase as in the E-S case. This is related with the alteration in the beams natural frequencies only due to the soil effect. In fact cancellation linear velocities remain unmodified with the flexibility of the soil; (iii) in the plot, depending on the non-dimensional speed interval, the maximum free vibration response may be associated to stiffer or softer soils; (iv) even though for the SSI problem the response is obtained with the full model, and it is not limited to the fundamental modal response as

in 6a and 7a, cancellation takes place at certain speeds and the response in free vibration practically vanishes; (v) when the beam transverse response is represented as a nondimensional magnitude (R) in terms of the nondimensional velocity (K_1), the maximum free vibration and cancellation conditions are practically not affected by the beam length.

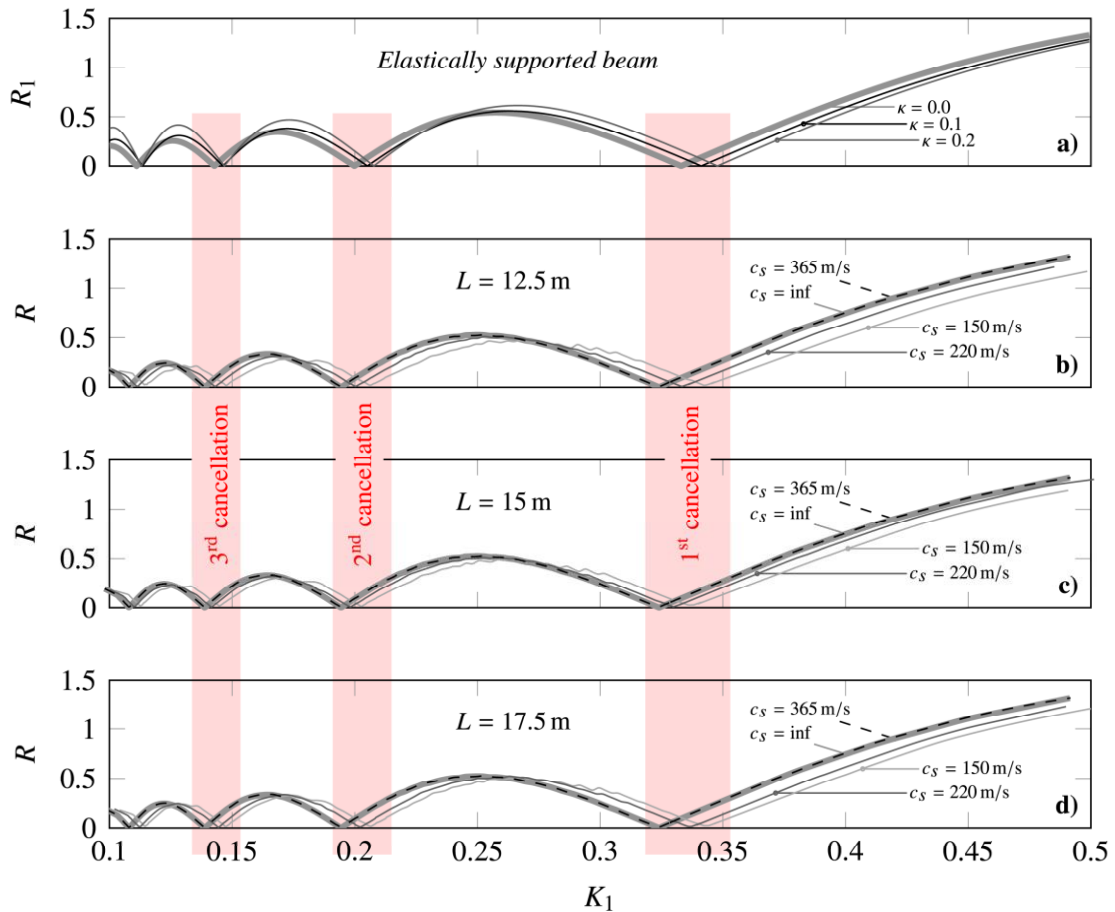


Figure 6. Maximum free vibration displacement response of (a) elastically-supported beam; and (b)-(d) BEM-FEM model with $L=12.5-17.5$ m and $f_{1,000}$ under constant moving force including SSI ($\zeta_b=0$, $\zeta_s=0$).

The practical application of these results is that conclusions regarding the type of resonant response to be expected when the same structure is subjected to the circulation of a train of loads (instead of a single axle) may be drawn. In particular if a resonant velocity is close to a cancellation speed the resonant amplitude will drastically

reduce and may practically be imperceptible. On the other hand, if the resonant velocity takes place at close to a maximum free vibration condition the amplification should be substantial. In the following section a few cases of particular bridges subjected to resonance are presented to show the aforementioned situations.

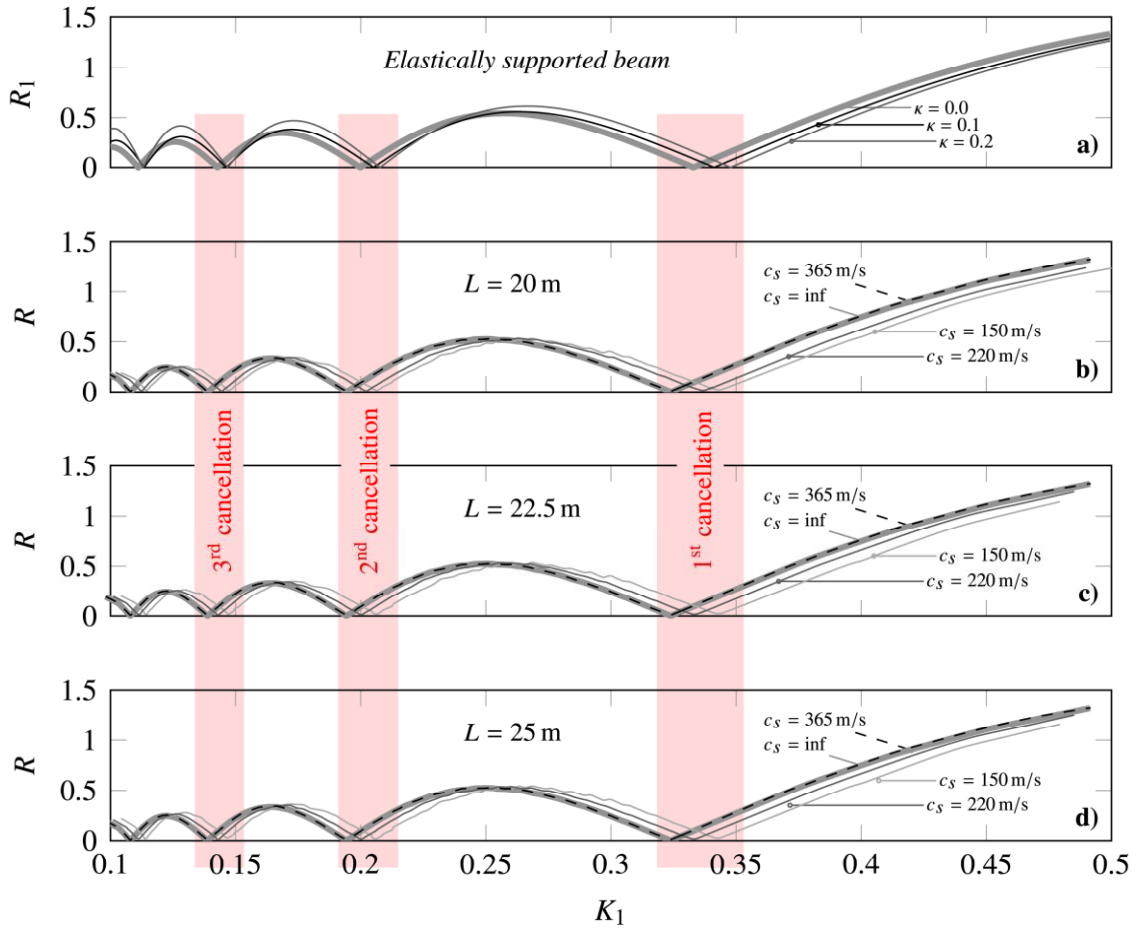


Figure 7. Maximum free vibration displacement response of (a) elastically-supported beam; and (b)-(d) BEM-FEM model with $L=20-25$ m and $f_{1,000}$ under constant moving force including SSI ($\zeta_b=0$, $\zeta_s=0$).

4 Analysis of resonant conditions under load trains

4.1 Description of the dynamic analyses

In what follows, the bridge under study is evaluated under the circulation of trains of constant loads, therefore neglecting vehicle-structure interaction effects. Two types of train models are considered: the HSLM-A model from Eurocode 1 [17], which is a train

composed by equidistant pairs of loads, and a hypothetical equidistant load train. Both models are shown in Fig. 8. In Table 1 the particular parameters that define the four trains that are used in the following examples are included, where N stand for the number of passenger coaches, d for the characteristic distance of the train (or distance causing resonance) and P for the load value per axle.

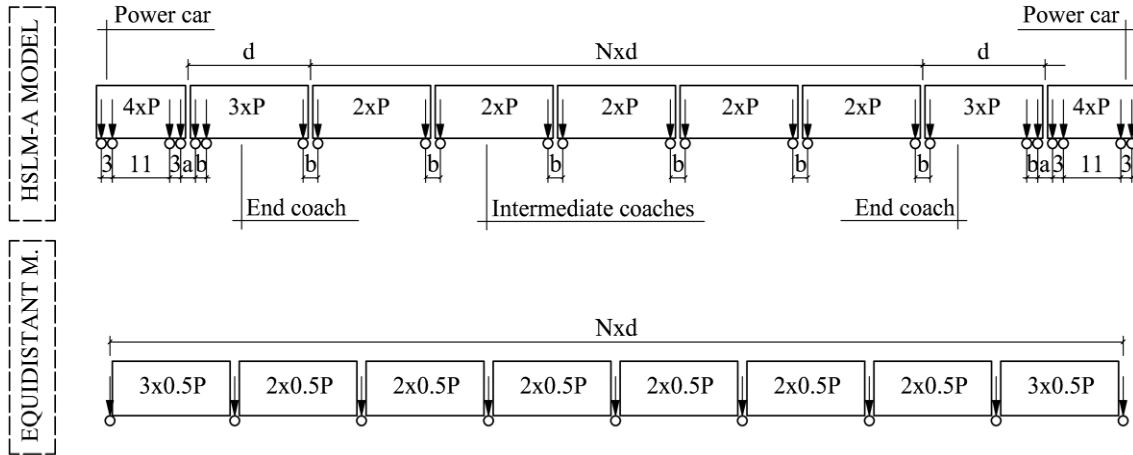


Figure 8. HSLM-A and equidistant train load schemes.

Train name	Type name	N	$a(m)$	$b(m)$	$d(m)$	$P(kN)$
A5	HSLM-A	14	3.525	2	22	170
A7	HSLM-A	13	3.525	2	24	190
A30	Equidistant	49	---	---	16.675	170
A31	Equidistant	49	---	---	16.200	170

Table 1. Train load models definition.

4.2 Cancellation of resonance

In the following example the 12.5 m length bridge with the lowest natural frequency, $f_{1,000}$, is considered in the S-S case (neglecting SSI effects) and including SSI with $c_s=220$ m/s. In both cases a second resonance of the bridge fundamental mode is forced in two scenarios: (a) the resonant velocity coincides with a cancellation condition; and (b) the second resonance does not coincide with a cancellation condition. A suitable

train is selected to force these two situations. The condition for a second resonance to be cancelled occurs when:

$$V_{2nd, res} = \frac{d \cdot f_1}{2} = \frac{K_{1, canc}^i L \omega_1}{\pi} \rightarrow d = 4 K_{1, canc}^i L \quad (2)$$

where i is the cancellation order. The first cancellation for the second resonance of this particular structure takes place for $K_{1, canc}^{i=1} = 0.3335$ when $c_s=220$ m/s and for

$K_{1, canc}^{i=1} = 0.324$ for infinitely rigid soil (see Fig. 6b). For these values the characteristic

distances of the trains leading to cancellation of the beam second resonance are computed, along with the resonant velocity (same in both cases due to the alteration in both the cancellation condition and the bridge natural frequency considering the soil effect). In Table 2 these values are included.

c_s (m/s)	f_1 (Hz)	$d(m)_{canc, 2nd, res}$	Train	$V_{2nd, res}$ (km/h)	K_1	Cancel. of 2 nd res.?
220	6.3328	16.675	A30	190.10	0.3335	Yes
∞	6.5247	16.200	A31	190.10	0.3240	Yes
220	6.3328	22	HSLM-A5	250.78	0.44	No
∞	6.5247	22	HSLM-A5	258.37	0.44	No

Table 2. Cancellation of second resonance of 12.5 m $f_{1,000}$ case for $c_s=\infty$ and $c_s=220$ m/s.

In Fig. 9 the maximum acceleration at mid-span is represented in terms of the quotient V/d for circulating velocities in the interval 144-360 km/h. Fig. 9 shows that the cancellation of the second resonance indeed takes place for this bridge when the SSI effects are included, in the same way that it happens for rigid boundary conditions.

In the same figure 9, the response of the bridge has been obtained for a second resonance caused by a different train such that the resonant velocity is not close to a cancellation condition and, therefore should not be cancelled. That is the case of the HSLM-A5 train with characteristic distance $d=22$ m. This train excites a second

resonance of the bridge fundamental mode when travelling close to 70 m/s (252 km/h) (see Table 2). This corresponds with a non-dimensional velocity of $K_1=0.44$ which is far from the first cancellation situation (as it can be observed in figure 6b). Moreover, the resonant amplitude reached in the absence of soil is considerably higher than when SSI is included. As the soil has not been assigned any damping this should be related with (i) the radiation capacity of the soil and (ii) the higher level of free vibrations associated to the S-S model for a $K_1=0.44$ value.

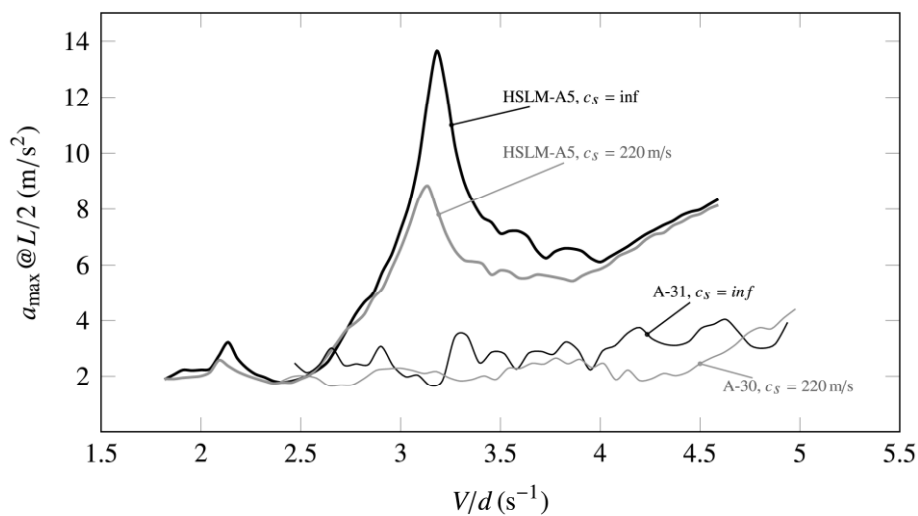


Figure 9. a_{max} vs. V/d at beam mid-span section for case $L=12.5$ m and $f_{1,000}$. Cancellation of second resonance of the bridge fundamental mode.

In Fig. 10 the acceleration time history at the bridge mid-span under the HSLM-A5 circulating at 253.44 km/h train has been represented for infinitely rigid soil conditions and including SSI for the particular soil with $c_s=220$ m/s. From the figure it can be detected how the bridge experiences two cycles of oscillation between the passage of two pair of axles leading to a progressive increase of the resonant response. When SSI is included in the model resonance still takes place reaching lower amplitudes.

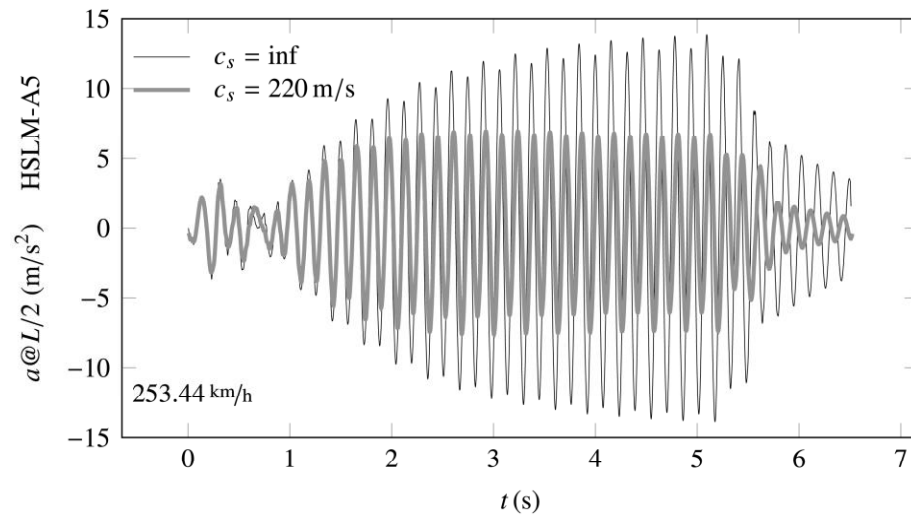


Figure 10. Acceleration time-history at beam mid-span section for HSLM-A5 at 253.44 km/h without SSI case and including SSI for $c_s=220$ m/s ($L=12.5$ m and $f_{1,000}$). Second resonance of bridge fundamental mode.

In Fig. 11 the acceleration time history at the bridge mid-span under the equidistant trains A30 and A31 circulating at 190 km/h has been represented again for infinitely rigid soil conditions and including SSI. This velocity corresponds to the velocity for cancellation of this second resonance and that explains the considerably low levels of vibration experienced by the structure.

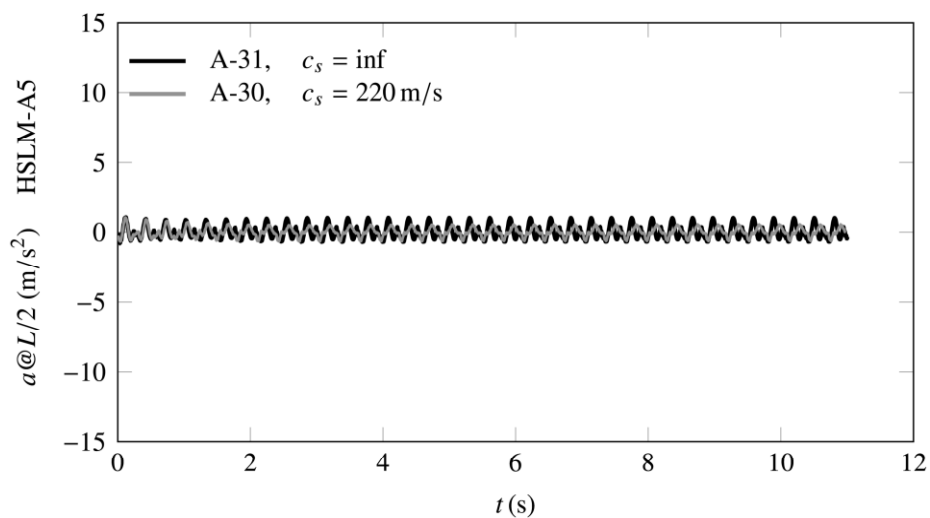


Figure 11. Acceleration time-history at beam mid-span section for HSLM-A5 at 190 km/h without SSI case and including SSI for $c_s=220$ m/s ($L=12.5$ m and $f_{1,000}$). Cancellation of second resonance of bridge fundamental mode.

4.3 SSI effect on resonant amplitudes

Finally the effect of different soil properties is shown on the resonant amplitude of the bridge. In Fig. 12 the maximum acceleration at mid-span is represented vs. the ratio V/d for the same bridge under study ($L=12.5$ m and $f_{1,000}$) subjected to the circulation of the HSLM-A7 train with characteristic distance $d=24$ m. This train excites on the structure a second resonance when travelling at 282 km/h (condition for second resonance in the absence of SSI). As the flexibility of the soil increases, the critical velocity slightly reduces along with the structure fundamental frequency. This velocity corresponds to a value of $K_1 \approx 0.48$, associated with considerably high levels of free vibration. From Fig. 6b it should be expected that the model leading to the maximum resonant response would be the one without SSI, and that the maximum acceleration response would reduce with the soil flexibility. Fig. 12 shows that the bridge response aligns with this prediction and the resonant amplitude monotonically reduces with the soil flexibility.

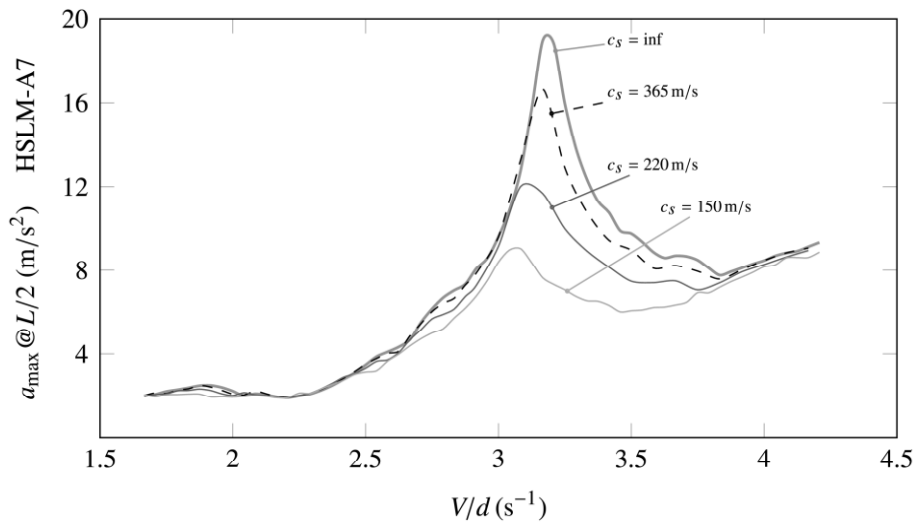


Figure 12. a_{max} vs. V/d at beam mid-span section for case $L=12.5$ m $f_{1,000}$. Second resonant amplitude for different soil conditions.

5 Conclusions

In the present contribution, the dynamic response of beams travelled by moving loads is analyzed taking into account soil-structure interaction effects using a three-dimensional BEM-FEM coupled numerical model integrated in the time domain. The main practical application of the study is the analysis of the transverse vibrations of simply-supported railway bridges considering short to medium span lengths.

In a first approach, the fundamental frequencies of all the bridges under study are identified from the response under impulse loading. Secondly, the maximum response of the beams is obtained in the free vibration phase right after a single travelling load has crossed the structure. A wide range of circulating velocities is defined and envelopes of maximum response are obtained and analysed.

From the preliminary results it is concluded that the fundamental frequency of the structures tends to the S-S one as the soil stiffness increases. The structures that are most affected by the soil flexibility are those with highest natural frequency for all the lengths. These results are consistent with the frequency evolution included in [8] for the elastically supported beam. Regarding the analysis of maximum free vibration under the circulation of single loads, it is concluded that:

- When the SSI is taken into account velocities leading to maximum free vibration response and to cancellation sequentially take place, in the same way that occurs for the E-S beam analytical case.

- As c_s and c_p decrease, going from stiffer to softer soils, the cancellation non-dimensional velocities increase as in the E-S case. This is related with the alteration in the beams natural frequencies due to the soil effect, and cancellation linear velocities remain unmodified with the flexibility of the soil.
- Depending on the non-dimensional speed interval, the maximum free vibration response may be associated to stiffer or softer soils.
- Cancellation takes place at certain speeds and the response in free vibration practically vanishes.
- When the beam transverse response is represented as a nondimensional magnitude (R) in terms of the nondimensional velocity (K_1), the maximum free vibration and cancellation conditions are practically unaffected by the beam length.

Finally the response of the bridges under study is evaluated under trains of several moving loads exciting resonant situations of the structure fundamental frequency. Through a few case studies it is shown that when resonant velocities take place close to cancellation conditions, the structural response drastically reduces and the resonant peak responses become almost imperceptible. In the same way, the amplitude of the structure at resonance varies with the soil properties following the trends observed in the free vibration analysis.

Acknowledgements

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