# Reporting flock patterns on the GPU 

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#### Abstract

In this paper we study the problem of finding flock patterns in a set of trajectories of moving entities. A flock refers to a large enough subset of entities that move close to each other for a given time interval. We present a parallel approach, to be run on a Graphics Processing Unit, for reporting maximal flocks. We also provide experimental results that show the efficiency and scalability of our approach.


## 1 Introduction

A trajectory is a sequence of sampled time-stamped point locations describing the path of a moving entity over a period of time. Assuming that the movement of an entity between two consecutive positions is done at constant speed and without changing direction, its trajectory is modelled by the polygonal line, that can self-intersect, whose vertices are the trajectory points.

In this paper we study the flock pattern $[1,2,3]$, that identifies a group of entities moving close together during a given time interval (see Figure 1).


Figure 1: Example of two flocks.
The increasing programmability and high computational rates of Graphics Processing Units (GPUs), together with CUDA (Compute Unified Device Architecture) and some programming languages which use this architecture, make them attractive to solve problems which can be treated in parallel as an alternative to CPUs. GPUs are used in different computational tasks where a big amount of data or operations have to be done, whenever they can be processed or done in parallel. Some recent works show that demanding algorithms in different fields can take advantage of the GPU parallel processing $[4,5,6]$.

[^0]In this paper, we present an efficient parallel GPUbased algorithm, designed under CUDA architecture, for reporting maximal flocks. The experimental results obtained with the implementation of our algorithm show the significance of the presented approach according to its performance and scalability.

## 2 The flock pattern

Let $E=\left\{e_{0}, \ldots, e_{n-1}\right\}$ be a set of $n$ moving entities. The trajectory $T_{i}$ of the entity $e_{i}$ is a sequence of $\tau$ points in the plane $T_{i}: e_{0}^{i}, \ldots, e_{\tau-1}^{i}$, where $e_{j}^{i}$ denotes the position of $e_{i}$ at time $t_{j}$ with $0 \leq j<\tau$. We assume that the positions are sampled synchronously for all the entities, and that entity $e_{i}$ moves between two consecutive positions $e_{j}^{i}, e_{j+1}^{i}$ with constant speed and without changing direction. Consequently, the trajectory $T_{i}$ is described by the polygonal line, which may self-intersect, whose vertices are the trajectory points.

Flocks identify groups of entities whose trajectories are close together during a minimum period of time. Formally, according to [2], given a set $E$ of entities, a minimum number of entities $\mu \in \mathbb{N}$, a number of time-steps $\delta \in \mathbb{N}$, a time interval $I_{j}^{\delta}=\left[t_{j}, t_{j+\delta-1}\right]$, and a distance $\epsilon \in \mathbb{R}$ :
Definition 1 A flock $f(\delta, \mu, \epsilon)$ in a time interval $I_{j}^{\delta}$, consists of at least $\mu$ entities such that for every discrete time step $t_{\ell} \in I_{j}^{\delta}$, there is a disk of radius $\epsilon$ that contains all the $\mu$ entities.
The number of reported or determined flocks is a critical issue that can adversely affect the response time and the proper interpretation of the final results. For a time-step $t_{\ell} \in I_{j}^{\delta}$ there may be several circles with radius $\epsilon$ that yield a flock with the same subset of entities of $E$, so we consider that two flocks are different if they involve two different subsets. Since there can be $\Theta\left(n^{2}\right)$ combinatorially distinct ways to place a circle of radius $\epsilon$ among $n$ points in the plane, at each timestep there can be $\Theta\left(n^{2}\right)$ flock candidates involving different subsets of entities [9]. An algorithm having as output the position of the entities involved in a flock, would have an output size of $\Theta\left(n^{3}\right)$ per time-step. Moreover, it is easy to observe that a set of entities could be present in many flocks, and even one single entity can be involved in several flocks. For example, a flock of $\mu+1$ entities contains $\mu+1$ flocks of $\mu$ entities.

To overcome this problem, as in [3], instead of finding all the existent flocks, we are interested only in maximal flocks, so that the entities of one flock may not be a subset of the entities of another flock. Even in the case of finding maximal flocks, at every time-step there may still asymptotically be $\Theta\left(n^{2}\right)$ flock candidates, although in many real situations the number of candidates decreases considerably.

Definition 2 A maximal flock $m f(\delta, \mu, \epsilon)$ in the time interval $I_{j}^{\delta}$, is a maximal flock $f(\delta, \mu, \epsilon)$ in $I_{j}^{\delta}$ with respect to the subset inclusion.

We will denote $\mathcal{M}_{j}^{\delta}$ the family of maximal flocks in the time interval $I_{j}^{\delta}$, and $\mathcal{M}^{\delta}$ the family of all maximal flocks, that is, the flocks of $\mathcal{M}_{j}^{\delta}$, for $j=0, \ldots, \tau-1$.

### 2.1 Characterization

Vieira et al. [3] prove that, for each pair of entity locations, there exists two such disks. Thus, $2 n^{2}$ disks per time step must be tested. Since we are interested in those flocks that are maximal, we prove that keeping just one of the two disks is enough. Consequently, only $n^{2}$ disks per time step must be tested, thus the number of tests is reduced by half.

The following lemma, similar to one presented in [3], allows us to bound the number of disks to be tested in each time step.
Lemma 3 Let $P$ be a set of n points and $D^{\prime}$ be a disk of radius $r$ that covers a subset $P^{\prime} \subseteq P,\left|P^{\prime}\right|=k \leq n$. There exist another disk $D^{\prime \prime}$ of radius $r$ such that: a) has at least two points $p_{i}, p_{j} \in P^{\prime}$ on its boundary; b) covers a superset $P^{\prime \prime}$ of $P^{\prime}, P^{\prime} \subseteq P^{\prime \prime} \subseteq P$; c) it is located at the right side of points $p_{i}, p_{j}$.

## 3 Reporting flock patterns

In this paper study the problem of reporting the family $\mathcal{M}^{\delta}$ of all maximal flocks. To report $\mathcal{M}^{\delta}$, we must find the family $\mathcal{M}_{j}^{\delta}$ in the time interval $I_{j}^{\delta}$, for $j=0, \ldots, \tau-1$.

### 3.1 Computing the family $\mathcal{M}_{j}^{\delta}$

Next, we describe the two main steps of the algorithm used to compute the family $\mathcal{M}_{j}^{\delta}$ in the time interval $I_{j}^{\delta}$.

First step. At each time step $t_{i}$ in the time interval $I_{j}^{\delta}$, we compute the family $\mathcal{F}_{i}$ of potential flocks containing at least $\mu$ entities. Each family $\mathcal{F}_{i}$ is obtained by using the characterization given in Lemma 3. Next, we find the maximal sets of each family $\mathcal{F}_{i}$. Abusing of notation, we still denote $\mathcal{F}_{i}$ the obtained subfamily of potential maximal flocks. At the end of the step, we have the array of families of potential maximal flocks $\mathcal{F}_{j}^{\delta}=\left[\mathcal{F}_{j}, \ldots, \mathcal{F}_{j+\delta-1}\right]$ in the time interval $I_{j}^{\delta}$.

Second step. We first compute the family $\mathcal{I}_{j}^{\delta}$ obtained intersecting the $\delta$ families of potential maximal flocks in $\mathcal{F}_{j}^{\delta}$. That is, $\mathcal{I}_{j}^{\delta}$ contains the sets which are the intersection of $\delta$ sets, one of each family $\mathcal{F}_{i}$ in $\mathcal{F}_{j}^{\delta}$. Finally, we find the subfamily of maximal sets of $\mathcal{I}_{j}^{\delta}$ that contain at least $\mu$ elements. The obtained subfamily is the desired $\mathcal{M}_{j}^{\delta}$.

## 4 GPU implementation

In this Section, we describe how to implement the different steps of the algorithm that computes $\mathcal{M}_{j}^{\delta}$, when GPU parallel techniques are used.

### 4.1 The multi-grid structure $\mathcal{G}$

We denote by $E_{i}=\left\{e_{i}^{0}, \ldots, e_{i}^{n-1}\right\}$ the set of locations of the entities of $E$ at time step $t_{i}, i \in\{0, \ldots, \tau-$ $1\}$. In [3], a single grid is used to perform quick disk range searching queries over each set $E_{i}$. Instead of using a grid for each time step, we build a multi-grid structure, denoted $\mathcal{G}$, in parallel in the GPU, that allows us to search simultaneously in each one of the $\delta$ sets of $E_{j}^{\delta}=\left[E_{j}, \ldots, E_{j+\delta-1}\right]$.

The multi-grid structure contains $\delta$ regular grids, where the edges of the grid cells have length $\epsilon$. Each grid $j$ contains the set of points corresponding to $E_{j}$. In order to maximize the parallel performance, the top-right corner is defined so that the grid is a square, that is, it has the same number of rows and columns (see Figure 2).

The multi-grid structure $\mathcal{G}$ is composed of 7 arrays allocated in the GPU global memory. In Figure 2 we can see an example for $\delta=3$. The bottom-left and the top-right corners of each grid are stored in the array $b_{\delta}$ of size $\delta$, where $b_{\delta}[j]$ stores the two corner points of the grid containing $E_{j}$. The number of cells per row (or per column) of each grid is stored in $\widetilde{c_{\delta}}$ and its prefix sum is stored in $\widetilde{p_{\delta}}$. Both arrays are integer arrays of size $\delta$.

To represent the $\delta$ grids, we use the array of integers $c_{\delta}$, whose size depends on the size of the different grids which directly depend on the distribution of the points. For a given grid cell, we store in the corresponding $c_{\delta}$ position the number of points contained on that cell. The array $p_{\delta}$ of the same size of $c_{\delta}$ is obtained as the prefix sum of $c_{\delta}$. Note that, for a better understanding, in Figure 2 the arrays $c_{\delta}$ and $p_{\delta}$ are showed as several $2 D$ grids, but in the GPU memory they are stored one $1 D$ array each, by storing one grid after the other starting from left to right and from top to bottom, thus, the first grid corresponds to the time step $t_{i}$ and the last one of the time step $t_{i+\delta}$. With this structure we can easily access, in a parallel way, to any element of the $\delta$ grids, by using the fact that the set of points of a cell $w$ starts at position $p_{\delta}[w]$ of $e_{\delta}$, and is stored in $c_{\delta}[w]$ consecutive positions of $e_{\delta}$. In each $e_{\delta}$ position we store the two floats of the
corresponding location. Finally, $\widetilde{e}_{\delta}$ is an integer array of size $n \delta$ containing at position $j$ the entity id of the $e_{\delta}[j]$ location.


Figure 2: Example of multi-grid $\mathcal{G}$ for $\delta=3$.
We construct the $\mathcal{G}$ structure in parallel as follows.
We allocate CPU arrays $\widetilde{c}_{\delta}^{\prime}, \widetilde{p}_{\delta}^{\prime}, b_{\delta}^{\prime}$ of size $\delta$, and $p^{\prime}$ and $\widetilde{e}_{\delta}^{\prime}$ of size $n \delta$. While we load the input $P$ into $p^{\prime}$ the arrays $\widetilde{c_{\delta}}{ }^{\prime}, \widetilde{p}_{\delta}^{\prime}$ and $b_{\delta}^{\prime}$ are accordingly filled. Then $\widetilde{c_{\delta}}, \widetilde{p_{\delta}}, b_{\delta}$ and $p$ are allocated in GPU memory and the ${\widetilde{c_{\delta}}}^{\prime}, \widetilde{p}_{\delta}^{\prime}, b_{\delta}^{\prime}$ and $p^{\prime}$ values are transferred to GPU memory to $\widetilde{c_{\delta}}, \widetilde{p_{\delta}}, b_{\delta}$ and $p$ respectively.

Then, $c_{\delta}$ and $p_{\delta}$ are allocated in the GPU memory according to $\widetilde{c_{\delta}}$ and $\widetilde{p_{\delta}}$ values. Additionally, the arrays $e_{\delta}$ and $\widetilde{e_{\delta}}$ are allocated with size $n \delta$ also in GPU memory. Because GPU has no dynamic memory we have to construct $\mathcal{G}$ in two steps.

First, we fill $c_{\delta}$ by counting the number of points contained in each cell. This is done in parallel by launching a kernel with $n \delta$ threads, that is, a thread per point and per time step. Each thread $i d x$ reads its $p[i d x]$ value and determines its position in the grid by using its position on the plane, the grid corresponding to the time step which the point belongs to, and the $\widetilde{c_{\delta}}$ and $\widetilde{p_{\delta}}$ arrays. The corresponding $c_{\delta}$ position is incremented in 1. Because many threads may correspond to the same $c_{\delta}$ position we use atomic operations to ensure no thread interferences.

Once $c_{\delta}$ is computed, $p_{\delta}$ is obtained as the prefix sum of $c_{\delta}$ and we allocate an auxiliary array $v$ with the same size of $c_{\delta}$, initialized to zeros. Now, we launch a parallel kernel with $n \delta$ threads where each thread $i d x$ reads its $p[i d x]$ value, determines its position $w$ in the grid and stores $p[i d x]$ at $e_{\delta}\left[p_{\delta}[w]+v[w]\right]$. Every time a thread stores its associated point into $e_{\delta}$ the corresponding $v$ value is incremented in one by using an atomic operation.

### 4.2 Finding potential flocks

The computation of the array of potential flocks $\mathcal{F}_{j}^{\delta}$ of a given interval $I_{j}^{\delta}$ is done in two main steps. First, we
need to find the center of the disks which are obtained applying the characterization given in Lemma 3, and second, we need to report the entities contained in those disks. Additionally, because GPUs do not have dynamic memory, we split the process in several steps.

First, we allocate an array of integers $D_{c}$ of size $n \delta$ to count how many disks there exist. This is done in parallel by launching a kernel with $n \delta$ threads, that is, a thread per point within $I_{j}^{\delta}$. Using the structure $\mathcal{G}$ each thread $i d x$ goes through its neighbors and counts the number of points which are at distance $\leq 2 \epsilon$. The results are stored in $D_{c}[i d x]$. Because this is computed in parallel, each disk will be reported twice, once for each of the two points defining the disk. In order to avoid this, we force threads to check those points whose entity id is bigger than itself. Then, $D_{p}$ of size $n \delta$, is computed as the prefix sum of $D_{c}$, and the array $D$ is allocated with size $D_{p}[n \delta-1]+D_{c}[n \delta-1]$, where we will store the centers of disks. The same process is repeated, but, instead of counting, each thread reports the center of the disk. Let us recall that only the right disk per paired point has to be reported.

In the second step, we actually report the potential flocks. To do this we use $\mathcal{G}$ to perform the range searching queries, using the values of the array $D$ as the center of the queries with radius $\epsilon$. Note that, because in order that the regular grid structure works for range searching queries, we must be able to locate any point within the grid. It may happen that some disk centers were placed outside the grid for a given time step. Thus, before we perform the range searching queries we have to reconstruct $\mathcal{G}$ so that the center of the disks were contained in $\mathcal{G}$. We could modify $\mathcal{G}$, but it is faster to rebuild it than to modify it.

To count the number of points contained on each disk we first allocate $P_{c}$, the array of integers of size equal to the total number of disks reported which is $D_{p}[n \delta-1]+D_{c}[n \delta-1]$, initialize it to zeros and launch a kernel with one thread per disk. Each thread $i d x$ reads the center of its disk $D[i d x]$, locates $D[i d x]$ on the grid and checks the distance between the center of the disk and the points of its cell neighbors. Each thread counts how many points are at distance $\leq \epsilon$ to its disk center, and if there are at least $\mu$ the number is stored in $P_{c}[i d x]$. Finally, we compute $P_{p}$ as the prefix sum of $P_{c}$.

If there is at least one disk per time step with at least $\mu$ entities, we continue to report the potential flocks. The last position of $P_{p}$ plus the last position of $P_{c}$ gives us $\omega$, the size of the array used to store the potential flocks. Thus, the array $P$ is allocated as an integer array with size $\omega$ and is filled in parallel using a thread per disk. The process is the same as before but this time, instead of counting, each thread stores in $P$ the entities contained inside each disk.

Note that, $P_{c}, P_{p}$ and $P$ denote a structure contain-
ing $\delta$ families of sets. This is the array of potential flocks $\mathcal{F}_{j}^{\delta}$, that is, $\delta$ families of sets containing the potential flocks of $\delta$ consecutive time steps starting at time step $t_{j}$. We denote $\mathcal{F}_{j}$ the family of potential flocks of the time step $t_{j}$.

The last step is to reduce each $\mathcal{F}_{i} \in \mathcal{F}_{j}^{\delta}$ so that it only contains the sets which are maximal, with respect to the partial order induced by the subset relation in each family. To this end, we use the parallel algorithm of Fort et al. [7]. When the process finishes, we have in $\mathcal{F}_{j}^{\delta}$ the maximal potential flocks with $\mu$ or more entities at each time step of the interval $I_{j}^{\delta}$.

### 4.3 Reporting $\mathcal{M}^{\delta}$ flocks

We want to report all the flocks $M_{j}^{\delta}$ for all $I_{j}^{\delta}, j=$ $[0, \ldots, \tau-\delta]$. Thus, we could perform the same algorithm over each $I_{j}^{\delta}$, but many information computed for a given time step within a time interval can be reused for the following time intervals. The idea is to compute the potential flocks for a given interval and compute their intersections in a way that we can reuse the information for the next time interval. To this aim we proceed as follows.

We start at the interval $I_{0}^{\delta}$ and we compute $\mathcal{F}_{0}^{\delta}$. Then, in order to obtain $\mathcal{M}_{0}^{\delta}=\mathcal{F}_{\delta-1} \cap \mathcal{F}_{\delta-2} \cap \cdots \cap \mathcal{F}_{0}$ we start at $\mathcal{F}_{\delta-1}$ and we intersect $F_{\delta-2} \cap \mathcal{F}_{\delta-1}$. Next, we intersect $\mathcal{F}_{\delta-3} \cap \mathcal{F}_{\delta-2} \cap \mathcal{F}_{\delta-1}$ and so on. After each intersection, we compute the maximal sets for each resulting intersection discarding those sets which are non maximal or containing less than $\mu$ entities. When we are done, we add, to $\mathcal{M}^{\delta}$, the family $\mathcal{M}_{0}^{\delta}$, which contains the flocks of $I_{0}^{\delta}$.

For the following time steps, instead of recomputing all the potential flocks and all the intersections, we reuse the previous computations. If for a given time step $t_{j}$, a family of potential flocks $\mathcal{F}_{k}, j \leq k<j+\delta$ is not inside the array $\mathcal{F}_{j}^{\delta}$, we compute $\mathcal{F}_{j+\delta-1}^{\delta}$. That is, instead of computing the potential flocks of one single time step we compute them in $\delta$ intervals. Then, we perform the corresponding intersections. This technique does not decrease the number of intersection we have to do, but, in practice, the $\delta$ families we intersect in each step are smaller than the ones containing the potential flocks, leading to faster results. We repeat the process until $j=\tau-\delta$.

The intersection process is also performed in parallel using the GPU algorithm presented by Fort et al in. [8].

## 5 Experimental results

The experimental results are obtained using an Intel Core2 CPU 6400 with a Nvidia GTX 480. We split the running time into 2 main steps, the potential flocks reporting process and the intersection process. The accumulated value, corresponding to the columns height, gives the total running time of the algorithm.

The algorithm has been tested with a data set extracted from [10], that consists of 145 trajectories of 2 school buses around Athens metropolitan area in Greece with a total of 66,096 time steps.

The results (Figure 3) shows that when we vary $\epsilon$ while we maintain $\mu=5$ and $\delta=10$, the number of flocks reported increases as long as we increase $\epsilon$. The most affected part is the intersection process. This is because the more flocks we report the more intersections we have to compute. When the parameter $\mu$ is varied, while $\epsilon=1200$ and $\delta=10$, the condition to form flock is more restrictive. Thus, the number of flocks decreases and, consequently, the running times too.


Figure 3: Running times varying $\epsilon$ and $\mu$.

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