Event generator to construct cross sections for the multiphonon excitation of a set of collective vibrational modes

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The construction of differential cross sections as a function of excitation energy for systems with a collection of low- and high-lying intrinsic vibrational modes has been attempted in the past. A prescription is proposed here that simplifies the implementation of such calculation schemes with a remarkable reduction in computational time.

I. INTRODUCTION

The possibility of exciting collective vibrational modes in heavy-ion reactions has captivated the attention of researchers in the field for several decades [1]. Of special interest has been the challenge of extending the set of participant states (i.e., open channels) well beyond the familiar low-lying surface modes and into the range of the nuclear giant resonances of the lowest multipolarities. Because of adiabatic considerations, these processes required, at the bombarding energies available during the 1970s and 1980s, very short effective collision times. Only a sharp exponential drop of the nuclear radial form factors was able to re-create these conditions at that time, a fact that was noted (and exploited) in the early studies of deep inelastic collisions [2,3].

Nowadays accelerators provide much higher beam energies and thus the population of giant resonances can also be mediated by the Coulomb excitation mechanism. In these circumstances the long range of the Coulomb-coupling matrix elements forces us to incorporate—in the theoretical analyses of the process—a considerably larger number of impact parameters or partial waves. This, naturally, increases the chances for exciting the high-energy part of the nuclear response.

Historically, the multiple-phonon excitation of high-lying modes was actively promoted as being the source of characteristic structures in the experimental distribution of cross sections as a function of excitation energy [4]. The study of these patterns became, in turn, a quite convenient source of information for learning about the actual features of giant resonances (energies of the modes, widths, strengths, anharmonicity, etc).

This link motivated an important body of recent theoretical work [5,6] where microscopic calculations for the structural aspects of the giant modes have been combined with standard reaction formalisms to yield concrete predictions for the shape of the distributions $d\sigma/dE$. Unfortunately, as stated earlier, a much larger number of impact parameters are now needed

to compute accurately the nuclear and Coulomb components of the excitation processes. This, together with the fact that there exists a distinct possibility of exciting a multiple number of collective phonons, has resulted in rather complicated and time-consuming coupled-channel schemes.

Upon close inspection of the results, however, one realizes that for practically the entire range of relevant partial waves, the excitation probabilities are very low. It is also possible to conclude that, in leading order, the different modes can be considered as being independent from each other. In other words, one can ignore terms in the Hamiltonian that involve simultaneously the coordinates of two or more collective variables $\alpha_{\lambda\mu}$. Note that this is *not* the same as claiming that one works within the perturbation limit; we have already stressed the relevance of multistep events. Multiphonon processes may occur as the excitation of the same mode (two phonons, three phonons) or as the simultaneous excitation of two or more different modes [5].

Considering the harmonic modes to be essentially uncoupled to each other has a significant practical advantage; it can be exploited to design an event generator that allows for a much simpler, yet accurate, method for constructing the differential cross sections $d\sigma/dE$. We shall describe this idea in detail in the following sections.

Such an approximation scheme is of course bound to fail for the very central impact parameters. However, these partial waves are unlikely to participate directly in the population of the inelastic channels explicitly taken into account. Within the semiclassical formalism their contribution is, in fact, strongly suppressed by the global absorption associated with the imaginary part of the optical potential. To take the grazing impact parameters into account properly may perhaps require a complete procedure such as the one exploited by the authors of Ref. [5]. Note that this would be necessary, at worst, for only a narrow window of impact parameters; for most of the relevant range (extending up to hundreds of fermis), the method we propose in this paper is appropriate and it is precisely here that the major savings of computational time can be achieved.

We elaborate further on these considerations in Sec. II, where we also recall a previous work that proves to be quite

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essential to the development of our proposal. In Sec. III we explain what is truly specific about the technique for generating the folding of probabilities that we need. In Sec. IV we take, as an example, the reaction $^{40}\text{Ar} + ^{208}\text{Pb}$ at 40 MeV per nucleon and include two low-lying and three high-lying resonances. The purpose is mainly to show that the function $d\sigma/dE(E)$ indeed reflects correctly the assumed input to the problem. We reserve Sec. V for a brief summary of the contribution and some closing remarks.

II. BRIEF BACKGROUND

A few years back we developed a general formalism for the excitation of a single collective vibrational mode [7]. We begin the presentation by mentioning this reference because it was essential in motivating the present contribution. Exploiting well-tested approximations, we were able in Ref. [7] to propose a semiclassical prescription to estimate the dynamical effects associated with a spreading width Γ of the mode. Modifications in the predicted cross sections owing to the presence of an eventual anharmonicity, manifested by an apparent ratio of state energies ε,

$$\nu = \frac{\varepsilon(\text{2-phonon state})}{\varepsilon(\text{1-phonon state})} \neq 2, \tag{1}$$

were also investigated. We should mention here that the development of this program—aiming mostly at the description of single- and double-phonon giant resonances—was done within the framework of perturbation theory.

Without going into detail about the implementation of Ref. [7], let us briefly recall the input of the calculation scheme and what is obtained as a result. Given a single collective vibrational mode of multipolarity λ, energy $\hbar\omega_\lambda$, width Γ_λ, and anharmonicity ν_λ, a run of the program densely sampling impact parameters ρ within the interval $[\rho_{\min}, \rho_{\max}]$ generated the total differential cross section $d\sigma/dE_\lambda(E)$. The energies of the transitions $0 \rightarrow 1$ phonon and $1 \rightarrow 2$ phonon, affected as they are by the anharmonicity factors and spreading widths, introduced interesting dynamical consequences that were the main object of investigation in Ref. [7].

To construct the differential cross sections $d\sigma/dE(E)$, it was necessary to define a procedure to distribute the probabilities of inelastic transitions for the one- and two-phonon vibrational states over the relevant excitation energy ranges. This prescription is quite analogous (except for its generalization from one mode to several modes) to the one we later use for our event generator. We can thus defer its presentation to the next section.

The effect of the absorption was taken into account in Ref. [7] by means of a multiplicative depletion factor that rapidly falls to zero as the overlap between the reacting nuclei increases for the lower impact parameters. This is—following standard practice—constructed from an integral along the trajectory $r(t)$,

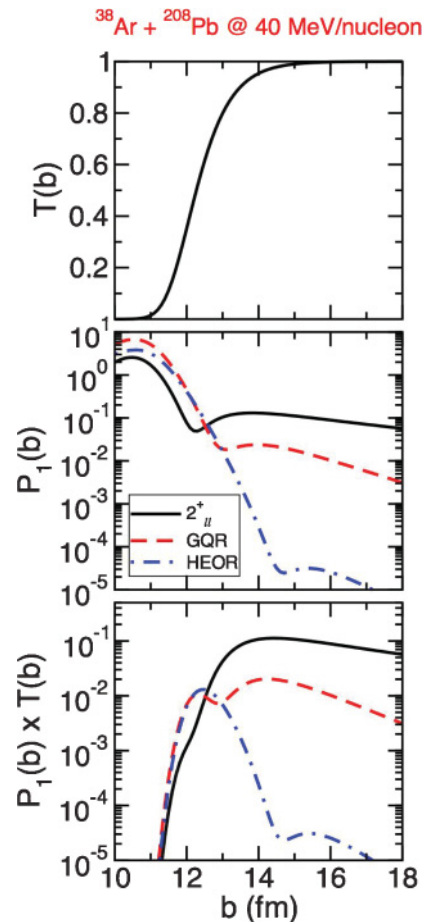$$T(b) = \exp\left\{-\frac{2}{\hbar} \int_{-\infty}^{+\infty} W(r(t'))dt'\right\}, \tag{2}$$

FIG. 1. (Color online) Treatment of absorption. Top: behavior of $T(b)$ as a function of the impact parameter b. Middle: excitation probability of a single phonon is plotted as function of b for the three states reported in the key. Bottom: functions obtained as a product of the two preceding quantities.

of the imaginary part of the optical potential W (see, e.g., Ref. [8] and references therein). For the current application the absorptive component was chosen following the prescription in Refs. [7] and [9].

The projection into the subspace of explicitly considered channels is now able to reverse the tendency of the second-order amplitudes to yield too large probabilities for impact parameters at or inside the grazing distance. Note that this uncomfortable situation would not become obvious when solving for coupled-channel amplitudes (even if the numerics may be equally absurd) because of a prescribed conservation of the norm by the integration algorithm.

The interplay between these contrasting effects can be appreciated in Fig. 1 for the reaction $^{38}\text{Ar} + ^{208}\text{Pb}$ at 40 MeV per nucleon. The figure shows the attenuation factor that defines what fraction of the contribution of a given impact parameter is actually retained in the inelastic channels. The top graph shows the transmission coefficient $T(b)$ as a function of the impact parameter and puts in evidence that the transition from $T(b) \approx 1$ to $T(b) \approx 0$ indeed occurs over an interval Δb that spans only a couple of fermis. The middle plot displays the excitation probabilities as a function of the

impact parameter in three situations. These are the low-lying quadrupole mode (solid curve), the giant quadrupole resonance (GQR; dashed curve), and a high-energy octupole resonance (HEOR; dash-dotted curve). The lower plot corresponds to the actual distribution $d\sigma/db$, constructed from the information displayed in the other two plots.

III. FORMALISM

We consider a pair of reacting heavy ions that accumulate a number N of intrinsic surface vibrational modes. Each one is characterized by its excitation energy $\hbar\omega_i$, multipolarity λ_i, strength β_i, and width Γ_i. That is,

$$\hbar\omega_i, \lambda_i, \beta_i, \Gamma_i, \quad \text{with} \quad 1 \leqslant i \leqslant N. \quad (3)$$

We proceed immediately to define two separate groups of these modes: high- and low-lying modes. The main reason for establishing the subdivision has to do with the way people in the field have traditionally dealt with their widths. The differentiation is actually an old story that dates back to the 1970s and the use of surface vibrational models to describe specific dynamical features of deep inelastic reactions [2,3]. It aims to reflect two clear experimental facts.

(i) Low-lying modes (excitation energies $\hbar\omega_\lambda < 5$–6 MeV). At the zero- and one-phonon level, low-lying modes are sharp and display no width. Clearly the energy range quoted previously is only qualitative. The modes we have in mind are those, for example, known—in the harmonic oscillator terminology for even multipolarities—as $\Delta N = 0$ excitations. At the two-phonon level they show a spread that is mainly associated with the anharmonicity of the mode. Let us be a bit more explicit. Suppose we have a low-lying quadrupole mode with $\hbar\omega_{\lambda=2} \approx 4$ MeV. At about double that excitation energy a multiplet of states 0^+, 2^+, and 4^+ is found, spanning an interval of energy that we call $\Delta E_{\lambda=2}$. Typically this quantity has an order of magnitude of about 0.5 MeV. Finally, the last piece of experimental evidence to be incorporated in the formalism is that there are practically no known three-phonon states associated with low-lying modes. This can be formally done by assuming that, at the three-phonon level, the mode assumes a width that equals the separation energy $\hbar\omega_\lambda$. All of these features are best implemented by ascribing an energy-dependent width to the mode (see the following).

(ii) High-lying modes (i.e., giant resonances). In this situation the zero-phonon state is taken to be sharp, while at the one-phonon level the state displays the well-known spreading width Γ_λ. The distribution of the excitation amplitudes to higher levels is achieved by a straightforward folding (see below).

The prescription that emerges from the two items listed may appear, at first, difficult to grasp. However, it has led to practical conclusions in the treatment of deep inelastic collisions that are in very good agreement with the experimental evidence. Obviously one could come up with different but somewhat

TABLE I. Width prescription as it is used in the construction of the cross section shown in Figs. 2 and 3.

n_λ	0	1	2	3
High-lying	0	Γ	$\sqrt{2}\Gamma$	$\sqrt{3}\Gamma$
Low-lying	0	0	ΔE	E

equivalent operating procedures. It is easier and reasonable, however, to adhere to this established practice, as the details of its implementation have already been described and tested in the literature (see, e.g., Ref. [3]). A practical reminder of the spreading prescription as it is applied for low- and high-lying modes is summarized in Table I.

The event generator works in practice just like any other similar device in a wide variety of physics subjects. We take a given impact parameter and consider a very large number of possible "events", N_{ev}. In every instance one generates the probability of occurrence of each independent mode by "throwing a Poisson dice" N times, in agreement with an average number of phonons $\langle n_i \rangle$. This means that the random number generator is designed to return a number of phonons n_i for each of the independent modes consistent with the law

$$P(n_i) = \frac{\langle n_i \rangle^{n_i}}{\langle n_i \rangle !} \exp\{-\langle n_i \rangle\}. \quad (4)$$

The differential cross section we search for is to be constructed for a large range of relevant impact parameters $[\rho_1, \rho_2]$, which are sampled at a uniform interval $\Delta\rho$. The average number of phonons for the ith independent collective
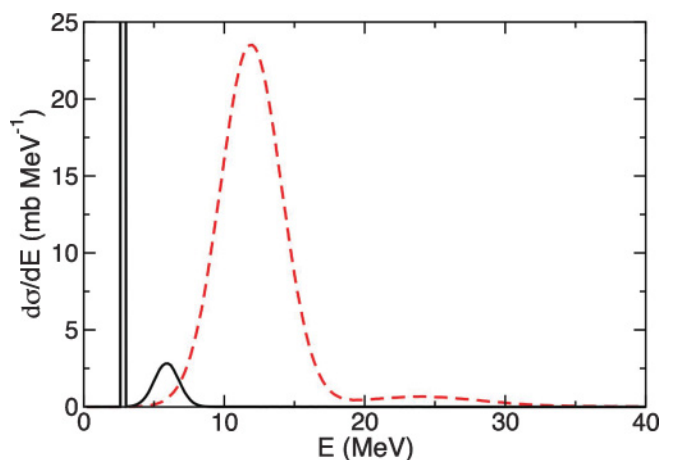

FIG. 2. (Color online) Width prescription as applied to low- and high-lying modes. The distribution of the total differential cross section $d\sigma/dE(E)$ is constructed as an example for two separate quadrupole modes with $\hbar\omega = 3$ and 12 MeV. The different prescriptions for low- and high-lying modes summarized in Table I can be appreciated here. The giant quadrupole state [$\Gamma(n = 1) = 5$ MeV] clearly shows the second phonon, with a wide and increasing width. The low-lying mode ($\Delta E = 1$ MeV) displays, on the contrary, a much narrower structure at the two-phonon level and has the three-phonon structure practically washed out (not visible in the figure). A large number of events and an impact parameter range of 100 fm were used for this illustration.

surface mode is previously calculated, for each impact parameter ρ_k, using the formalism developed in Ref. [7]. These figures are collected in an auxiliary data file that has the structure

$$
\begin{aligned}
\rho_1, & \quad \langle n_{i=1} \rangle, \ldots, \langle n_{i=N} \rangle, \\
& \quad \ldots \\
\rho_k, & \quad \langle n_{i=1} \rangle, \ldots, \langle n_{i=N} \rangle, \qquad (5) \\
& \quad \ldots \\
\rho_2, & \quad \langle n_{i=1} \rangle, \ldots, \langle n_{i=N} \rangle.
\end{aligned}
$$

The smaller impact parameter ρ_1 should be such that any possible contribution from it is definitely eliminated by absorption. Similarly, one should verify that contributions from $\rho > \rho_2$ can also be neglected.

Adding to this prepared data set the characteristic information that specifies the different vibrational modes i, one is ready to run the event generator and construct $d\sigma(E)/dE$. For each impact parameter in the file, Eqs. (5), the probability assigned to the current event is, naturally,

$$
P = \prod_{i=1}^{N} P(n_i). \qquad (6)
$$

It is in the energy scale that we have to be careful with the character of the low-lying vibrational state or giant resonance of the particular mode i. Following the prescription summarized in Table I, we assign a spread Γ_i to the contribution to the excitation energy ϵ_i of this mode. This quantity is thus defined as

$$
\epsilon_i = n_i \hbar \omega_i + \mathcal{G}(0, \Sigma_i), \qquad (7)
$$

where $\mathcal{G}(0, \Sigma_i)$ is a random number obtained from a normal distribution with zero centroid and standard deviation $\Sigma_i \approx \Gamma_i/2.3$. Slightly different prescriptions could once more be obtained by replacing $\mathcal{G}$ with a similar type of random number generator, but these choices are not of much consequence at the level of approximation we have chosen to maintain.

The total excitation energy for the collection of N independent modes is then simply given by

$$
E = \sum_{i=1}^{N} \epsilon_i. \qquad (8)
$$

In the corresponding abscissa—in a properly designed histogram—we accumulate a weighted version of the probability previously given in Eq. (6). The proper units for the differential cross section as a function of energy are obtained by multiplying that number by $(20\pi \rho_k \Delta\rho)/(\Delta E N_{\text{ev}})$, where ρ_k is the impact parameter sampled at this moment, and $\Delta\rho$ and ΔE are the extents of the impact parameter mesh and the energy mesh, respectively. The final result is then given as millibarns per megaelectronvolt, and a drawing of this histogram represents the predicted distribution of cross section $d\sigma(E)/dE$.

To test the validity of our method, a comparison with a more sophisticated method such as that in Ref. [5] is in order. In this approach one starts with a Hartree-Fock plus random phase approximation calculation in order to identify the most collective one-phonon states. For each of these chosen states

TABLE II. Total excitation cross section (mb) calculated with the events generator method and with the method from Ref. [5] for the states listed in the first column.

State	Events generator	Ref. [5] calculations
3^-	1.10	1.05
GQR	58.7	58.6
$3^- \times 3^-$	0.11×10^{-3}	0.10×10^{-3}
GQR $\times$ GQR	0.14×10^{-1}	0.14×10^{-3}

one calculates the transition densities and the corresponding form factors. These are used in a semiclassical coupled-channel scheme to determine the excitation probabilities for all possible one-, two-, and three-phonon states that one can construct.

In Table II we compare some results for the two approaches. The calculations are performed for the system $^{40}\text{Ca} + ^{208}\text{Pb}$ at 50 MeV per nucleon. We take a simple example where only two one-phonon states are considered as input: the low-lying 3^- state ($E = 4.9$ MeV) and the GQR ($E = 16.9$ MeV). The range of impact parameters used in the calculations (15–100 fm) corresponds to the peripheral region where the Coulomb interaction yields the most important contribution. This is also the region where the excitation probability distribution is of a Poisson type. The results of the two methods are very close.

So, one can envisage a calculation performed in two steps: Make use of the method in Ref. [5] in the inner region, where the nuclear interaction plays an important role, and then use our novel approach in the peripheral region for large impact parameters, which is the most time-consuming part.

Finally, we stress that in the case where one wants to take into account the contribution of anharmonicities, our method does not apply.

IV. APPLICATION

We now proceed to illustrate the possibilities of the event generator with an application to the specific reaction $^{38}\text{Ar} + ^{208}\text{Pb}$ at 40 MeV per nucleon. We take two low-lying modes (one quadrupole, one octupole) with energies $\hbar\omega = 3$ and 5 MeV, respectively, and a common value $\Delta E = 1$ MeV. The deformation parameter assumes, for the low-lying modes, a value $\beta = 0.1$. Three giant resonances are then added; a dipole, a quadrupole, and an octupole mode, with energies $\hbar\omega = 18$, 17, and 31 MeV, and widths $\Gamma = 6$, 6, and 8 MeV, respectively. The effective spread of all these modes is found in all cases following the prescriptions described in Table I. Accumulating the cross section for a range of impact parameters from $\rho_{\text{min}} = 12$ fm to $\rho_{\text{max}} = 100$ fm, in steps of 0.5 fm, we obtain the distribution shown in Fig. 3. The characteristics of the curve reflect the assumptions made and, in addition, are reminiscent of what is obtained by the time-demanding method of Ref. [5] in similar circumstances. Just to stress the advantage of the proposed method, let us note that one can save about two orders of magnitude in computing time by using the event generator.
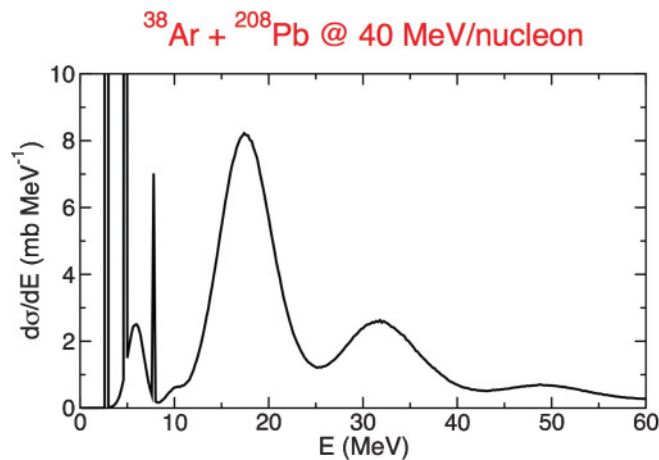
FIG. 3. (Color online) Constructed distributions of the total differential cross section $d\sigma/dE(E)$ for the reaction ^{38}Ar $+ ^{208}$Pb at 40 MeV per nucleon. Some 10^8 events per impact parameter have been generated and their contributions accumulated to construct this figure. The calculation time is however a few minutes CPU. The example is described in detail and the relevant parameters given in the text of Sect. IV.

V. CONCLUDING REMARKS

The developments described and the results presented in this report were motivated by sheer necessity. In fact, while the complete calculations performed by Lanza *et al.* are very important, the absolute times involved in the computation of cross sections by the procedure described in Ref. [5] are quite long. This becomes even more critical when one takes into account that the practical implementation of such a prescription involves a considerable amount of leeway, which can only be sorted out by trying alternative calculations with equally acceptable sets of parameters. This appears to be the only sensible way to learn, by gaining experience, how the various input numbers do indeed affect the calculated distributions of cross sections. Even if a final presentation with the full procedure of Ref. [5] is contemplated, a number of previous calculations exploiting the event generator would be—no doubt—very convenient to prepare the ground.

We consider that the use of an event generator like the one described in these pages is highly advisable for the class of problems that can use suggestive results to judge the soundness of the answers they provide. Actually this should be done even before embarking on sophisticated schemes without the proper benefit of an educated intuition.

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