Flattening P Systems with Active Membranes

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Summary. Given a P system Π with active membranes having several membranes, we construct a P system Π^f having only one membrane and rules involving catalysts, cooperation and priorities. The evolution of this "flat" P system Π^f simulates the evolution of initial P system with active membranes Π by replacing any rule that changes the configuration in Π by prioritized rules application in a configuration of Π^f.

1 Introduction

The family of membrane systems (also called P systems) is presented in the monograph [6] and in the handbook [7], while several applications of membrane computing are presented in [4]. Membrane systems are distributed, parallel and non-deterministic computing models inspired by biological entities. The structure of the cell is represented by a set of hierarchically embedded regions, each one delimited by a surrounding boundary (called membrane), and all of them contained inside a skin membrane. A membrane without any other membrane inside it is said to be elementary, while a membrane with other membranes inside is said to be composite. Multisets of objects are distributed inside these regions, and they can be modified or communicated between adjacent compartments. Objects represent the formal counterpart of the molecular species (ions, proteins, etc.) floating inside cellular compartments, and they are described by means of strings over a given alphabet. Evolution rules represent the formal counterpart of chemical reactions, and are given in the form of rewriting rules which operate on the objects, as well as on the membrane structure.

A membrane system can perform computations in the following way. Starting from an initial configuration that is defined by multisets of objects initially placed inside the compartmentalized structure and by sets of evolution rules, the system evolves by applying evolution rules in a non-deterministic and maximally parallel manner (every rule that is applicable inside a region *has to* be applied in that region). A rule is applicable when all the objects that appear on its left-hand side are available in the region where the rule is placed (there are not used by other

rules applied in the same step). Due to the competition for resources, some rules are applied non-deterministically. A halting configuration is reached when no rule is applicable.

A flat membrane system Π^f can be constructed to simulate a membrane system Π with multiple membranes and dissolution [1]. The central idea of this construction is to use pairs of objects and labels from Π as objects in Π^f. Each rule r of Π is translated into sets of rules r^f for Π^f. This simulation is at no extra cost: the flat-membrane system does not use more time or space than the membrane system with multiple membranes.

In order to simulate a P system Π with multiple active membranes by a flat membrane P system Π^f, a bigger amount of rules and more time is needed to perform the simulation. The increase in the number of rules derives from the fact that Π has a 'higher dimension' (the changes in the topology) in working with its symbols: for instance, it can change the polarization of one compartment affecting in this way all the symbols in it. The larger amount of time depends on the fact that in Π configuration changes are done in one step (one rule application), while in Π^f this cannot be achieved in one step.

A motivation of this study is this one: many researcher classify computational complexity classes in terms of the kinds of rules present in P systems with active membranes. For instance, NP problems can be solved with these kind of systems when rules of types (a), (b), (c) and (d) or when rules of kind (a), (c), (e) and polarities are there. These kinds of rules are rather powerful and they could hide some power in their definition. We aim to unify these rules in terms of simple rules, hoping to compare their power. For instance, we may prove that the simple rules needed to simulate rules of the kind (a), (b), (c) and (d) are the same as the simple rules needed to simulate rules of the (a), (c), (e) and polarities.

2 P Systems with Active Membranes

Biological membranes are not completely passive: when a molecule passes through a membrane, the molecule and the membrane itself can be modified. These ideas lead to the introduction of P systems with active membranes where the central role in computation is played by membranes. Each membrane is supposed to have an electrical polarization (also called charge), one of the three possible: positive $(+)$, negative $(-)$ and neutral (0).

Definition 1 ([6]). *A* P system with active membranes *is a construct*

$$\Pi = (V, T, H, \mu, w_1, \ldots, w_n, \alpha_1, \ldots, \alpha_n, R)$$

where:

1. $n \geq 1$ *represents the number of membranes;*
2. V *is an alphabet (the* total alphabet *of the system);*
3. $T \subseteq V$ *(the* terminal *alphabet);*
4. H *is a finite set of* labels *for membranes;*

5. μ *is a* membrane structure*, consisting of* n *membranes, labelled in a one-to-one manner with elements of* H*;*
6. $w_1, \ldots, w_n$ *are strings over* V*, describing the* multisets of objects *placed in the* n *regions of* μ*;*
7. $\alpha_1, \ldots, \alpha_n$*, with* $\alpha_i \in \{+, -, 0\}$ *for* $i \in \{1, \ldots, n\}$*, are the initial polarizations of the membranes;*
8. R *is a finite set of* developmental rules*, of the following forms:*

   a) $[a \to v]_h^\alpha$*, for* $h \in H$*,* $\alpha \in \{+, -, 0\}$*,* $a \in V$*,* $v \in V^*$

   object evolution

   An object a *placed inside a membrane evolves into a multiset of objects* v*, depending on the label* h *and the charge* α *of the membrane. The membrane does not take part in the application of the rule and is not modified by it.*

   b) $a[\ ]_h^{\alpha_1} \to [b]_h^{\alpha_2}$*, for* $h \in H$*,* $\alpha_1, \alpha_2 \in \{+, -, 0\}$*,* $a, b \in V$

   communication

   An object a *is introduced in the membrane labelled* h *and with charge* α_1*. However, the object* a *may be modified to* b *and the polarization may be changed from* α_1 *to* α_2 *during the operation. The label of the membrane remains unchanged.*

   c) $[a]_h^{\alpha_1} \to [\ ]_h^{\alpha_2} b$*, for* $h \in H$*,* $\alpha_1, \alpha_2 \in \{+, -, 0\}$*,* $a, b \in V$

   communication

   An object a *is removed from the membrane labelled* h *and with charge* α_1*. However, the object* a *may be modified to* b *and the polarization may be changed from* α_1 *to* α_2 *during the operation. The label of the membrane remains unchanged.*

   d) $[a]_h^\alpha \to b$*, for* $h \in H$*,* $\alpha \in \{+, -, 0\}$*,* $a, b \in V$

   dissolving

   An object a *dissolves the surrounding membrane labelled* h *and with charge* α*. However, the object* a *may be modified to* b *during the operation.*

   e) $[a]_h^{\alpha_1} \to [b]_h^{\alpha_2} [c]_h^{\alpha_3}$*, for* $h \in H$*,* $\alpha_1, \alpha_2, \alpha_3 \in \{+, -, 0\}$*,* $a, b, c \in V$

   division of elementary membranes

   In reaction with an object a*, a membrane labelled* h *and with charge* α *is divided into two membranes with the same label, maybe of different polarizations. However, the object* a *may be replaced in the two new membranes by possibly new objects.*

   f) $[\ [\ ]_{h_1}^{\alpha_1} \ldots [\ ]_{h_k}^{\alpha_1} [\ ]_{h_{k+1}}^{\alpha_2} \ldots [\ ]_{h_n}^{\alpha_2}]_{h_0}^{\alpha_0} \to [\ [\ ]_{h_1}^{\alpha_3} \ldots [\ ]_{h_k}^{\alpha_3}]_{h_0}^{\alpha_5} [\ [\ ]_{h_{k+1}}^{\alpha_4} \ldots [\ ]_{h_n}^{\alpha_4}]_{h_0}^{\alpha_6}$ *for* $k \geq 1$*,* $n > k$ *,* $h_i \in H$*,* $0 \leq i \leq n$*, and* $\alpha_0, \ldots, \alpha_6 \in \{+, -, 0\}$ *with* $\{\alpha_1, \alpha_2\} = \{+, -\}$

   division of non-elementary membranes

   If a membrane labelled h_0 *contains other membranes than those with the labels* $h_1, \ldots, h_n$ *specified above, then they should have neutral charge for this rule* t *be applicable. The division of non-elementary membranes is possible only if a membrane contains two immediately lower membranes of opposite polarization,* $+$ *and* $-$*. The membranes of opposite polarizations are separated in the two new membranes, but the polarization can change. All*

membranes of opposite polarizations are separated always by applying this rule.

It can be noticed that the objects interact indirectly by means of membranes and their polarizations. The above rules are applied in the usual non-deterministic maximally parallel manner, with the following details:

- Any object can be subject of only one rule of any type and any membrane can be subject of only one rule of types (b)-(f);
- Rules of type (a) are not counted as applied to membranes, but only to objects;
- If a membrane is dissolved, then all the objects and membranes in its region are left free in the surrounding region;
- The rules are applied in a bottom-up manner;
- The skin membrane cannot be dissolved or divided, but it can be the subject of in/out operations.

3 Flattening P Systems with Active Membranes

We may reduce an initial P system Π with active membranes, as described in Section 2, to the following P system Π^f with only one membrane, priorities, catalysts and cooperative rules as described below.

$$\Pi^f = (V^f, T^f, [\,], w^f, R^f)$$

where

1. $V^f = \{a_h \mid a \in V, h \in H \cup H_{\mathrm{N}}\}$
   $\cup\, \{\alpha_h, \alpha_h^{di}, \alpha_h^m \mid \alpha \in \{+,-,0\}, h, i \in H \cup H_{\mathrm{N}}\}$
   $\cup\, \{p_{ij}, p'_{ij} \mid i, j \in H \cup H_{\mathrm{N}}\}$
   - a_h - models an object a from membrane h;
   - α_h - models the polarization α of membrane h;
   - p_{ij} - represents the fact that membrane i is included in membrane j; this object is used to model the membrane structure of the initial system;
   - $H_{\mathrm{N}} = \{h_i \mid h \in H, i \in \mathrm{N}\}$ is used to uniquely track the copies of the membranes from H created by division;
2. $T^f = \{a_h \mid a \in T, h \in H \cup H_{\mathrm{N}}\}$
   - the terminal alphabet is obtained by considering all combination of terminal objects and locations from initial system;
3. $w^f = \{a_h, a'_h \mid a \in w_h, h \in H\}$
   $\cup\, \{\alpha_h \mid h \in H\} \cup \{p_{ij} \mid \text{membrane } i \text{ is included in membrane } j \text{ in } \mu\}$
   - a_h - models an object a from the initial multiset w_h;
   - α_h - models the initial polarization α of membrane h;
- R^f is a finite set of evolution rules:
  a) a rule $[a \to v]_h^\alpha$ is simulated with the rule:

i. $\alpha_h a_h \to \alpha_h v'_h$;
v'_h denotes the fact that to all objects from the multiset v a label h is added and that the newly created objects cannot be used by any other rules in the current evolution step;

b) a rule $a[\ ]_h^{\alpha_1} \to [b]_h^{\alpha_2}$ is simulated with the rule:
   i. $p_{hi} a_i \alpha_{1h} \to p_{hi} b'_h \alpha_{2h}$;
   a_i is an object from the membrane i surrounding membrane h; this relation between membranes is captured by the object p_{hi}.

c) a rule $[a]_h^{\alpha_1} \to [\ ]_h^{\alpha_2} b$ is simulated with the rule:
   i. $p_{hi} a_h \alpha_{1h} \to p_{hi} b'_i \alpha_{2h}$;
   b_i is an object from the membrane i surrounding membrane h; this relation between membranes is captured by the object p_{hi}.

d) a rule $[a]_h^{\alpha} \to b$ is simulated with the rules:
   i. $p_{hi} a_h \alpha_h \to p'_{hi} \alpha_h^{di} b'_i$
   α_h^{di} represents a special object that models the fact that the membrane labelled h in the initial membrane structure is marked to be dissolved (d - symbolizes dissolution, i - the parent membrane of dissolved membrane h), and to signal that objects that in the initial membrane structure were in membrane h should move to membrane i. When a membrane is marked dissolution, some objects p_{ij} need to be modified in order to keep track with the modification of the initial structure. To do this the object p_{hi} is replaced with the object p'_{hi} in order to announce any children of membrane h, if any, to change the parent to i.
   ii. $p'_{hi} p_{jh} \to p'_{hi} p_{ji}$
   In the presence of the object p'_{hi} any membrane contained in the dissolved membrane h, if any, changes its parent from h to i, namely the object p_{jh} is replaced with the object p_{ji}.
   iii. $p'_{hi} \to \varepsilon$
   If there are no more membranes with parent h, the intermediate object p'_{hi} is removed.
   iv. $\alpha_h^{di} a_h \to \alpha_h^{di} a'_i$
   In the presence of the object α_h^{di} any object from the initial membrane h is moved to membrane i, namely an object a_h is replaced with an object a'_i.
   v. $\alpha_h^{di} \to \varepsilon$
   If there are no more objects in the dissolving membrane h, then intermediate object α_h^{di} is removed.

   The rules are applied according to the following sequence of priorities:
   $$(d).i > (d).ii > (d).iii > (d).iv > (d).v$$

e) a rule $[a]_h^{\alpha_1} \to [b]_h^{\alpha_2} [c]_h^{\alpha_3}$ is simulated with the rules:
   i. $p_{hi} a_h \alpha_{1h} \to p_{h_1 i} b'_{h_1} p_{h_2 i} c'_{h_2} \alpha_h^m$
   α_h^m represents a special object that models the fact that the membrane labelled h in the initial membrane structure is multiplied (m - symbolizes multiplication). In order to keep track which objects belong to

which membrane in the initial system, we consider that the new copies of the membrane and its inner objects have labels that uniquely identify them, namely h_1 and h_2. To be able to apply similar rules as for the initial membrane h also to the newly created membranes labelled by h_1 and h_2 we duplicate the rules from h in the newly created membranes by replacing the objects a_h by the objects a_{h_1} or a_{h_2}, respectively. As an example, for the rule $\alpha_h a_h \to \alpha_h v_h$ of membrane h, an instance of it is created for each membrane h_i, $i \in \{1, 2\}$, namely $\alpha_{h_i} a_{h_i} \to \alpha_{h_i} v_{h_i}$.

ii. $\alpha_h^m a_h \to \alpha_h^m a'_{h_1} a'_{h_2}$
In the presence of the object α_h^m any object from the initial membrane h is duplicated in the two new copies of membrane h, namely the object a_h is replaced with the objects a'_{h_1} and a'_{h_2}.

iii. $\alpha_h^m \to \alpha_{h_1} \alpha_{h_2}$
After all objects of membrane h are replicated into the new copies of membrane h, namely h_1 and h_2, the object α_h^m is replaced with the objects α_{h_1} and α_{h_2}, representing the polarizations of the newly created membranes.

The rules are applied according to the following sequence of priorities:

$$(e).i > (e).ii > (e).iii$$

f) a rule $[\,[\,]_{h_1}^{\alpha_1} \dots [\,]_{h_k}^{\alpha_1} [\,]_{h_{k+1}}^{\alpha_2} \dots [\,]_{h_n}^{\alpha_2}]_{h_0}^{\alpha_0} \to [\,[\,]_{h_1}^{\alpha_3} \dots [\,]_{h_k}^{\alpha_3}]_{h_0}^{\alpha_5} [\,[\,]_{h_{k+1}}^{\alpha_4} \dots [\,]_{h_n}^{\alpha_4}]_{h_0}^{\alpha_6}$ is simulated with the rules:

i. $p_{h_i h_0} +_{h_i} p_{h_j h_0} -_{h_j} \to \alpha_{0h_0}^m p_{h_i h_{01}} +'_{h_i} p_{h_j h_{02}} -'_{h_j}$
The division of the non-elementary membrane h_0 is possible only if it contains two immediately lower membranes h_i, h_j of opposite polarization, $+$ and $-$. In this case, the membranes of opposite polarizations are separated in the two new membranes h_{01} and h_{02} together with all the membranes of similar charge, but for which the polarizations can change. The membranes with neutral charge are copied in both obtained membranes. The newly created object $\alpha_{h_0}^m$ indicates that the membrane h_0 will multiply. The labels h_{01} and h_{02} are used to indicate uniquely the two instances of membrane h_0. We shall use the subscript 1 for the membrane containing the membranes initially charged with $+$, and the subscript 2 for the membrane containing the membranes initially charged with $-$.

ii. $\alpha_{h_0}^m p_{h_i h_0} +_{h_i} \to \alpha_{h_0}^m p_{h_{i1} h_{01}} +'_{h_i}$

iii. $\alpha_{h_0}^m p_{h_i h_0} -_{h_i} \to \alpha_{h_0}^m p_{h_{i2} h_{02}} -'_{h_i}$

iv. $\alpha_{h_0}^m p_{h_i h_0} 0_{h_i} \to \alpha_{h_0}^m p_{h_{i1} h_{01}} p_{h_{i2} h_{02}} 0'_{h_i}$
Using these rules, is modelled the change of the membrane structure in the initial membrane system such that the membranes with charge $+$ are included in membrane h_{01} and the label is changed from h_i to h_{i1}, while the membranes with charge $-$ are included in membrane h_{02} and the label is changed from h_i to h_{i2}. For the membranes with charge 0 a fresh copy is included both in membranes h_{01} and h_{02} and the label

h_i is changed to h_{i1} and h_{i2}. The charge is primed so no other rule can be used for the obtained membranes until the division ends.

v. $+'_{h_i} a_{h_i} \rightarrow +'_{h_i} a'_{h_{i1}}$
vi. $-'_{h_i} a_{h_i} \rightarrow -'_{h_i} a'_{h_{i2}}$
vii. $0'_{h_i} a_{h_i} \rightarrow 0'_{h_i} a'_{h_{i1}} a'_{h_{i2}}$
The objects are relocated to the new obtained membranes, and when necessary duplicated.
viii. $+'_{h_i} \rightarrow \alpha_{h_{i1}}$
ix. $-'_{h_i} \rightarrow \alpha_{h_{i2}}$
x. $0'_{h_i} \rightarrow \alpha_{h_{i1}} \alpha_{h_{i2}}$
xi. $\alpha^m_{h_0} \rightarrow \alpha_{h_{01}} \alpha_{h_{02}}$
This rules have a lower priority than the one above. When there are no more objects to be transferred to the newly obtained membranes, the primed polarizations are changed to new polarizations. If there are no primed polarizations remained, then modelling of the division process is finished by changing the label $\alpha^m_{h_0}$ to two new membranes $\alpha_{h_{01}}$ and $\alpha_{h_{02}}$. To be able to apply similar rules as for the initial membranes $h_0, h_1, \ldots, h_n$ also to the newly created membranes the rules for h_i are duplicated for the newly created membranes using instead of objects a_{hi}, objects $a_{h_{i1}}$ or $a_{h_{i2}}$, respectively. As an example, for the rule $\alpha_{h_i} a_{h_i} \rightarrow \alpha_{h_i} v_{h_i}$ of membrane h_i, an instance of it is created for each membrane h_{ij}, $j \in \{1, 2\}$ namely $\alpha_{h_{ij}} a_{h_{ij}} \rightarrow \alpha_{h_{ij}} v_{h_{ij}}$.

The rules are applied according to the following sequence of priorities:

$$(f).ii > (f).i > (f).xi$$
$$(f).iii > (f).i > (f).xi$$
$$(f).iv > (f).i > (f).xi$$
$$(f).v > (f).viii$$
$$(f).vi > (f).ix$$
$$(f).vii > (f).x$$

g) $a'_h \rightarrow a_h$
After applying all possible rules in a step, in order to mve the next step of the evolution, the primed objects (e.g., a'_h) are transformed into simple objects (e.g., a_h).

The rules are simulated using the application details for P systems with active membranes, except for rule (g) that has the lowest priority among all rules and is applied last in order to prepare the system for a new evolution step.

Remark 1. We end by emphasizing the size of the P system Π^f with respect to that of Π. The cardinality depends on the size of a halting configuration of the initial membrane system. Consider that the largest configuration has m membranes. Thus, the cardinality of the alphabet V^f is,

$$card(V^f) = m \times (card(V) + 10 \times m)$$

while the cardinality of the rule set R^f is,:

$$card(R^f) = m \times (card(R(a)) + card(R(a)) + card(R(c)) +$$

$+5 \times card(R(d)) + 3 \times card(R(e)) + 11 \times card(R(f)) + card(V^f))$,
where $R(a)$ denotes the number of (a) rules from the set of rules R.

4 Conclusion

The idea of using a flat membrane system to simulate P systems with multiple membranes has previously appeared in several papers; a formal presentation can be found in [5]. However, the P systems with multiple membranes had a static structure, not involving dissolution. The dissolution aspects where tackled in [1].

In this paper we presented a general approach for P systems with active membranes, based on the use of catalysts, cooperation and priorities.

The translation presented here can be used to simplify proofs of statements involving general P systems with active membranes by using only flat P systems. However, concerns may appear regarding the increasing number of objects and rules in the P system Π^f, according to Remark 1.

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