Analysis of stationary roving mass effect for damage detection in beams using wavelet analysis of mode shapes

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Abstract. One of the main challenges in damage detection techniques is sensitivity to damage. During the last years, a large number of papers have used wavelet analysis as a sensitive mathematical tool for identifying changes in mode shapes induced by damage. This paper analyzes the effect of adding a mass to the structure at different positions. Depending on the location and severity of damage, the presence of the mass affects the natural frequencies and mode shapes in a different way. The paper applies a damage detection methodology proposed by the authors, although it has been modified in order to consider the addition of the mas. This methodology is based on a wavelet analysis of the difference of mode shapes of a damaged and a reference state. The singular behavior of a normalized weighted addition of wavelet coefficients is used as an indicator of damage. The presence of damage is detected by combining all the information provided by mode shapes and natural frequencies for different positions of the roving mass. A continuous wavelet transform is used to detect the difference between the response of a healthy state and a damaged one. The paper shows the results obtained for a beam with different cracks. The paper analyzes the sensitivity to damage of the proposed methodology by considering some practical issues such as the size of the crack, the number of measuring points and the effect of experimental noise.

1. Introduction

It is well known that the presence of damage (cracks) in a beam implies a change in its dynamic properties. Thus, vibration based damage detection techniques try to detect the presence of damage by analyzing the change on natural frequencies, mode shapes and/or damping ratios. Some pioneering damage detection techniques (1) were based on the analysis of changes in natural frequencies, which are the most simple dynamic parameters to measure. However, natural frequencies lack of sensitivity to damage. Only a significant damage would induce a significant change in natural frequency. Moreover, the effect of damage may be masked by the effect of environmental changes, experimental noise and uncertainty, etc. On the other hand, natural frequencies are a global parameter of the structure, and therefore it can provide information only about the presence of damage but not about its location. In order to locate damage and to be able to detect more little damage, the mode shapes of the structure may be used. From a experimental point of view, the identification of mode shapes requires a larger amount of sensors (more complex and expensive experimental setups) as well as more sophisticated system identification methods. Despite of these experimental and mathematical efforts, the changes in mode shapes induced by damage are usually subtle (unless severe damage is present) so damage can not be identified from mode shapes in a straightforward way. There is a significant number of papers that propose different techniques and damage detection parameters to analyze the information provided by mode shapes (1).

The wavelet transform is a rather new mathematical tool that has been developed from the 90s for signal processing and information encoding. After some pioneering works that extended the use of wavelet transform to damage detection in structures (2), a number of authors have made different proposals for the same purpose by applying wavelet transform to mode shapes, time response, static deflection, etc (3; 4).

The wavelet transform is sensitive to local changes in the original signal. Wavelet coefficients show a singular behavior, ridges or peaks when some discontinuity or a sudden change occurs. Thus, they can be used as an indicator of damage when applied to mode shapes, assuming that damage may lead to some kind of discontinuity in mode shapes.

The authors have recently proposed a simple damage detection technique based on the wavelet analysis of difference in mode shapes from a healthy and a damaged state (5). The main idea is to combine all the information provided from all identified mode shapes and natural frequencies by a weighted addition of the wavelet coefficients according to changes in natural frequencies for each mode. The methodology has been successfully applied to cracked steel beams. This paper includes a new idea for making the proposed damage detection method more robust and sensitive to little damage. A non-structural mass is attached to the structure, and modal analysis is performed for different positions of the mass. The mass is at a fixed position for each experimental test (it is not a moving load) but it changes its position along the experimental campaign, so it is called a stationary roving mass.

The effect of the mass on the dynamic response of the structure will depend on the position and severity of the damage. As a result, additional information is available for damage detection and the methodology becomes more sensitive to damage. The information of all available mode shapes and frequencies is combined through the addition of wavelet coefficients of difference in mode shapes from a healthy and a damage state. The paper shows numerical results for a steel cantilever beam with different damage scenarios.

2. Combined modal-wavelet methodology for damage detection with a stationary roving mass

In this section, the proposed methodology for damage detection in beams is described step by step.

2.1. Modal analysis

The first task of the proposed methodology is to obtain the mode shapes and natural frequencies of the structure. As any vibration based damage detection method, it requires modal information about the undamaged and damaged state. The modal parameters for each position of the added mass for the undamaged beam as well as for the damaged one must be obtained.

2.2. Extension of mode shapes

The next step of the methodology deals with a very important issue in wavelet analysis, especially when applied to space based damage detection. The wavelet transform is defined for an infinite integration interval, whereas the original signal is defined over a finite interval. When the wavelet transform is performed, there is a singular behavior at the beginning and at the end of the signal. The signal starts and finishes at those points, so there is a significant local change there, unless the signal trends softly in an asymptomatic way to a constant value, which will never be the case for a mode shape. This unstable behavior of the wavelet coefficients in the vicinity of the beginning and the end of the analyzed signal is known as the edge effect, and it is a serious drawback of the wavelet transform when the damage is close to the beginning or the end of the signal. Moreover, the high values of wavelet coefficients near those regions of the signal can mask the structural damage effect on the wavelet transform along the structure.

This paper applies a simple method to avoid edge effects. It consists of an anti-symmetric extension of the signal of the same length of the original signal at both ends (5; 6).

2.3. Smoothing, interpolation and noise reduction of mode shapes

Another important issue when analyzing mode shapes is the reduction of experimental noise effect. The experimental mode shapes will always be affected by noise, so they will always show some kind of irregularities. This undesirable behavior will affect the wavelet analysis and it could eventually mask the effect of damage. In order to reduce this effect, a smoothing technique may be introduced, so local peaks induced by experimental noise are eliminated without affecting any local trend that could have been induced by damage, since damage is not expected to produce only a local peak as it is the case of experimental noise. In this paper a softening technique already proposed by the authors is applied (5).It is based on a windowed quadratic regression.

2.4. Wavelet transform of difference of modes

Once the extended and smoothed mode shapes have been obtained, the wavelet analysis is applied. Firstly, the extended difference of each mode shapes (i) for each position of the mass (j) is obtained ($\Phi^{ij}_{diff,ext}$) by computing the difference between the smoothed extended damaged ($\Phi^{ij}_{s,ext,d}$) and undamaged ($\Phi^{ij}_{s,ext,u}$) mode shapes:

$$\Phi^{ij}_{diff,ext}(x) = \left(\Phi^{ij}_{s,ext,d}(x) - \Phi^{ij}_{s,ext,u}(x)\right) \tag{1}$$

Then, a CWT of each extended mode shape difference is done to give information about changes in mode shapes. The CWT for the ith mode shape and for the jth position of the mass can be written as:

$$CWT_{\Phi^{ij}_{diff,ext}}(u,s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} \Phi^{ij}_{diff,ext}(x)\Psi^*\left(\frac{x-u}{s}\right) dx \tag{2}$$

This paper uses the well-known Daubechies wavelet function with two vanishing moments. This function has already provided successful results in damage detection (5). From this point, only the CWT coefficients that corresponds to the original signal ($CWT_{\Phi^{ij}_{diff}}$), and therefore to the real structure, will be considered.

2.5. Normalized weighted addition of wavelet results based on frequency changes

In order to simplify the analysis of the CWT for each mode shape and to draw an overall result for damage detection in one single picture, the values of CWT coefficients of each mode shape and for each position of the mass are added up to obtain a global result for damage detection (Equation 3). On the other hand, this combination of the results mitigates the effect of experimental noise.

For a more precise and clear detection of singularities, the proposed methodology combines the absolute values of the wavelet coefficients. In addition, the coefficients for each mode shape are weighted according to its corresponding change in natural frequencies:

$$CWT_{sum}(u,s) = \sum_{j=1}^{M}\sum_{i=1}^{N} \left| CWT_{\Phi^{ij}_{diff}}(u,s) \right| \cdot \left(1 - \frac{\omega^{ij}_u}{\omega^{ij}_d}\right)^2 \tag{3}$$

where ω^{ij}_u and ω^{ij}_d stand for the natural frequencies of mode shape i for the position M of the added mass, for the undamaged and the damaged state, respectively.

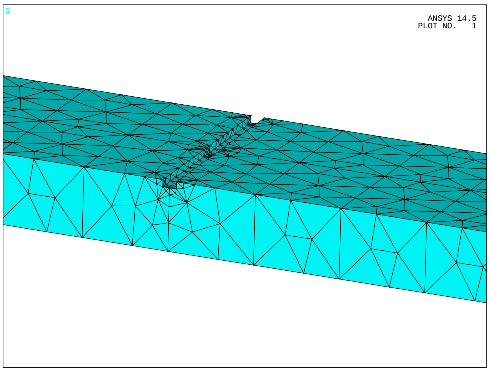

Figure 1. Detail of the finite element model around the notch (1mm depth).

The weighting of the difference of the damaged and undamaged mode shapes with their natural frequencies relations is used to emphasize the most sensitive mode shapes to damage. It is assumed that those modes that exhibit a higher frequency change are more sensitive to damage and therefore changes in those mode shapes are more significant. On the other hand, the mode shapes that do not change their natural frequencies are almost disregarded. They are likely to introduce mainly noise in the final result when all the mode shapes are combined.

Finally, the resulting weighted addition of CWT coefficients is normalized to unity for each scale:

$$CWT_{sum-mass-norm}(u,s) = \frac{CWT_{sum-mass}(u,s)}{max[CWT_{sum-mass}(u,s)]_s} \quad (4)$$

Normalized coefficients give a more clear final result since the information for all scales can be analyzed altogether. The idea of the addition and normalization of wavelet coefficients has also been applied by other authors (7).

2.6. Analysis of the results for damage detection and location

The normalized weighted addition of absolute values of CWT coefficients of mode shapes differences ($CWT_{sum-mass-norm}(u,s)$) is finally plotted so information for all scales is available along the beam in just one final figure. A singular behavior of wavelet coefficients may indicate the presence of damage. This singular behavior is typically shown as high values (unity values because of the normalization of coefficients) for all scales around damage location.

3. Numerical model

A finite element model was built to simulate the effect of a crack in a beam. The length (L) of the beam is 800mm and its cross section is 300mm wide and 10mm high. The beam was modeled with 4 node tetrahedral elements using Ansys software. Figure 1 shows a picture of the finite element model around the crack, which is modeled as a notch of 2mm width. The severity of the damage is defined by the depth of the crack and it will be referred as a percentage of the height of the beam. Thus, a 10% damage will mean a crack of 1mm depth. The mesh of the model is different depending on the depth of the crack since it has to be adapted to the irregular geometry around the crack. The added mass is modeled with a point mass element. The beam is made of steel, with an elastic modulus $E = 210GPa$, Poisson ratio $\nu = 0.3$ and density $7850kg/m^3$. The beam is fixed at one end and free at the other end. All displacements are constrained for all nodes at the fixed end. For damage detection purposes, only bending modes in the plane parallel to the shortest dimension of the beam (vertical plane) are considered. Vertical modal displacements of nodes along the axis on the upper side of the beam are considered as

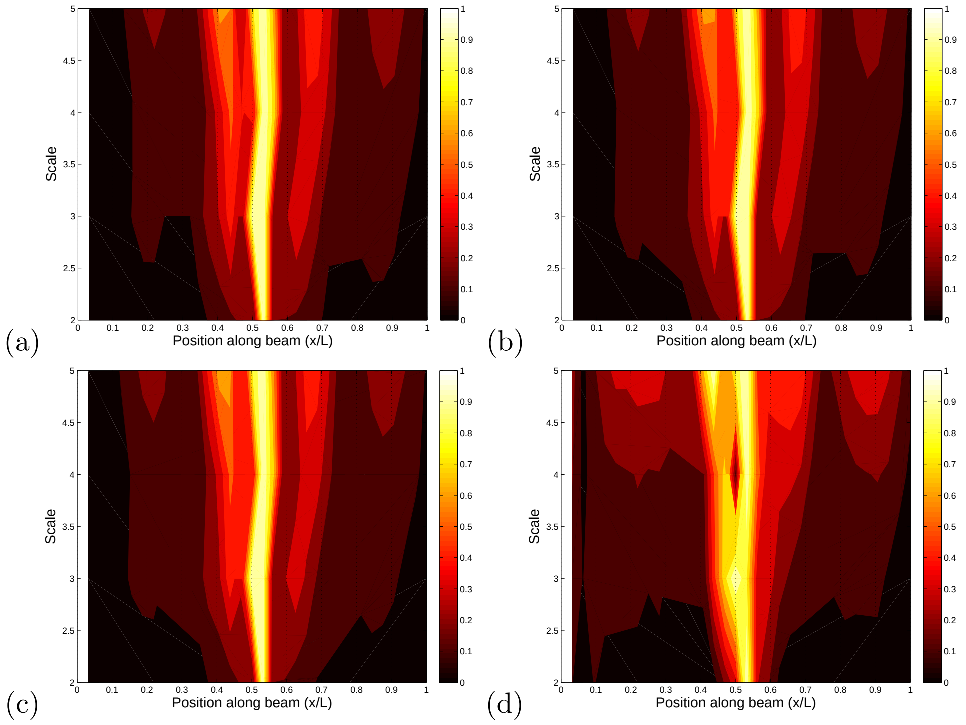

Figure 2. Normalized weighted addition of wavelet coefficients using 32 measuring points, no added noise and crack depth 20% (a), 10% (b), 5% (c) and 1% (d).

virtual measuring points, simulating the modal information that could be obtained by attaching accelerometers to that side of the beam. The influence of the amount of sensors along the beam on the final results is analyzed by considering different number of nodes along the beam. These nodes must be uniformly spaced, since the wavelet transform assumed that the samples of the signal to be transformed are equally spaced. In real applications, if the accelerometers are not uniformly distributed then some kind of interpolation is required (5).

4. Results

This section analyzes the sensitivity to damage of the proposed methodology considering the effect of the number of measuring points, the presence of experimental noise and the use of the added roving mass. The damage is located at the middle section of the beam ($0.5L$). The result for each case is presented by a coloured 2D picture of the normalized weighted addition of wavelet coefficients of mode shapes differences. The vertical axis is the scale of the wavelet transform whereas the horizontal axis is the position along the beam. The values of the wavelet coefficient are represented by a color map. The results are obtained considering the first five bending modes of the beam.

Figure 2 shows the results for several depths of the crack, considering 32 measuring points along the beam. The added mass is 20% of the total mass of the beam and it is located at 10 different positions along the beam (from the position at $0.1L$ from the fixed end of the beam to its free end, in steps of $0.1L$). It can be observed that the damage can be clearly identified for all the damage scenarios presented: high values (about unity) of normalized wavelet coefficients are located at damage location for every scale. The methodology is able to identify the presence of a damage even when the notch is only 1% deep ($0.1mm$).

The results of Figure 2 shows that the methodology is very sensitive to damage from a

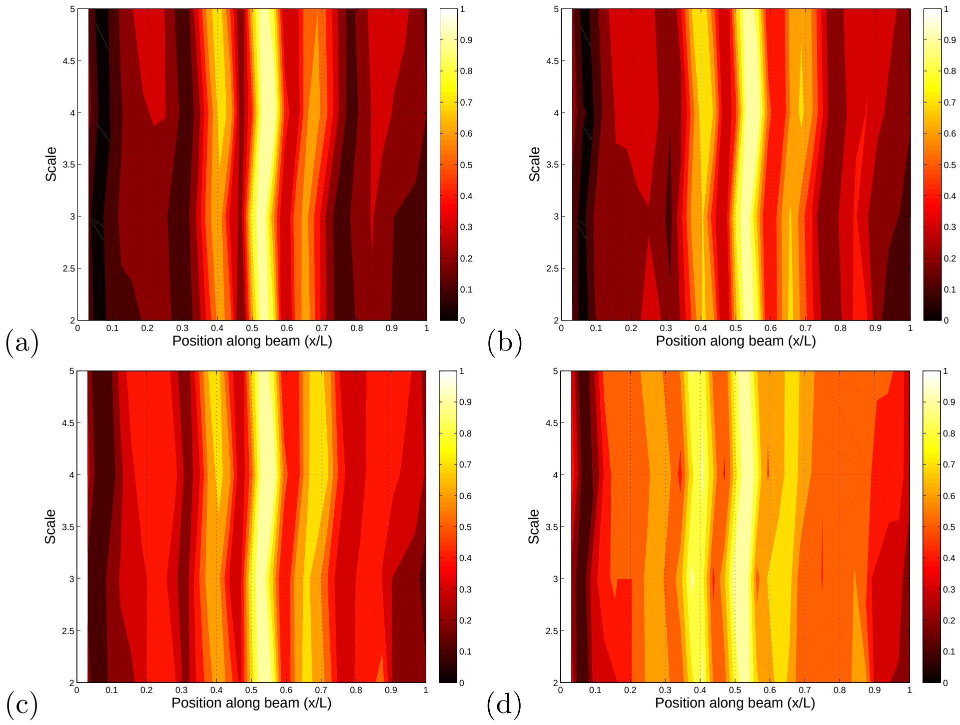

Figure 3. Normalized weighted addition of wavelet coefficients using 32 measuring points, 40db gaussian noise and crack depth 50% (a), 40% (b), 30% (c) and 20% (d).

mathematical point of view, when noise-free mode shapes are obtained from the numerical model of the beam. However, experimental noise will always be present in real applications. Experimental errors will affect the modal displacements values and noisy mode shapes will be obtained. In order to analyze the effect of noise in the numerical simulations, a white gaussian noise is added to the mode shapes using the 'awgn' built-in function of Matlab software. The intensity of this synthetic noise is defined by the Signal to Noise Ratio expressed in terms of the average power of the original signal (mode shape vector) and the added noise.

Figure 3 shows the results obtained when the mode shapes are contaminated with high level of noise (40db). The sensitivity to damage is significantly affected by the presence of noise. The effect of the presence of the crack is present for a crack depth higher than 20%, but it can not be not clearly identified, even for crack depths up to 50%.

Figure 4 shows the results obtained when 64 measuring points are considered. It is clear from figures 3 amd 4 that the undesirable effect of noise is reduced by increasing the number of measuring points. When using 64 measuring points the results are satisfactory from crack depth values of 20%, even with a 40db noise level.

It should be noted here that, according to the changes in natural frequencies, all the considered damage scenarios are very demanding for damage detection purposes. Even when the crack is 50% deep, the drift in natural frequencies from the healthy state is less than 4%. When the crack depth is 20%, the drift is less than 0.6%, and for a crack of 10% the drift is less than 0.2%. The use of the added roving mass enhance the sensitivity to damage, as it can be seen by comparing figures 4 and 5. When no mass is added, the results are much more affected by noise. If the level of noise is reduced to 60db, then up to a 10% deep crack can be clearly identified with 32 measuring points, whereas even a very small crack of 5% depth can be identified with 64 measuring points (Figure 6).

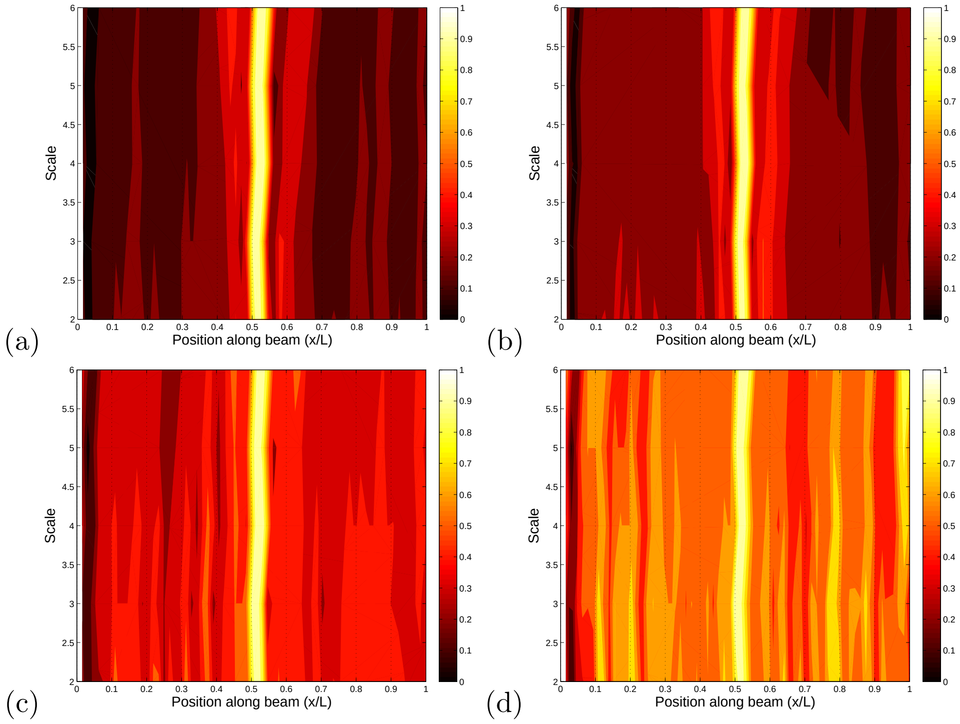

Figure 4. Normalized weighted addition of wavelet coefficients using 64 measuring points, 40db gaussian noise and crack depth 50% (a), 40% (b), 30% (c) and 20% (d).

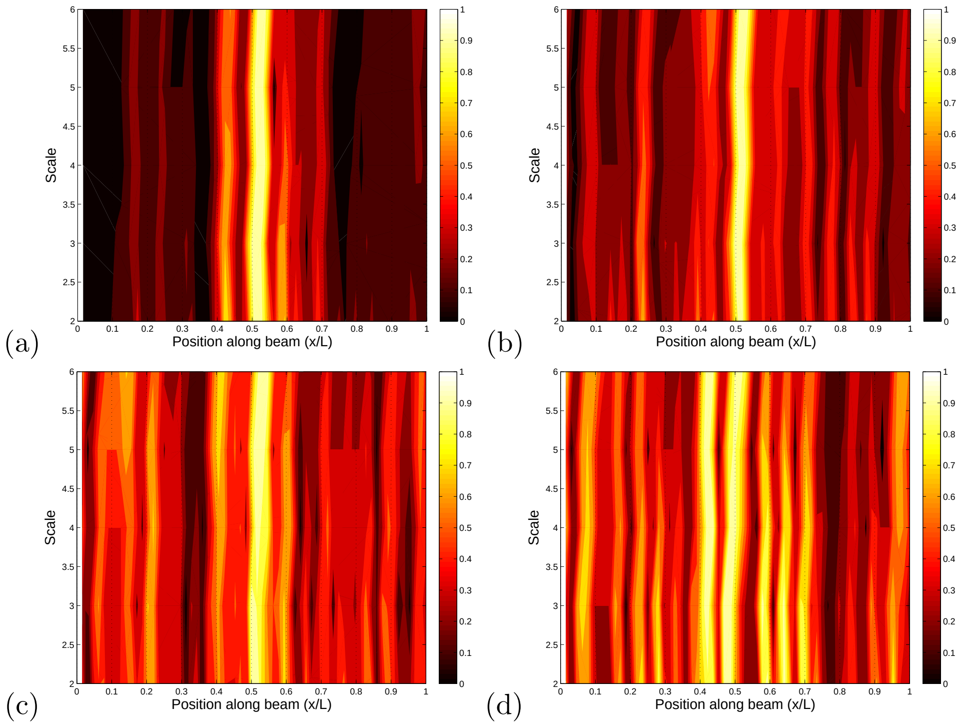

Figure 5. Normalized weighted addition of wavelet coefficients with no added mass,using 64 measuring points, 40db gaussian noise and crack depth 50% (a), 40% (b), 30% (c) and 20% (d).

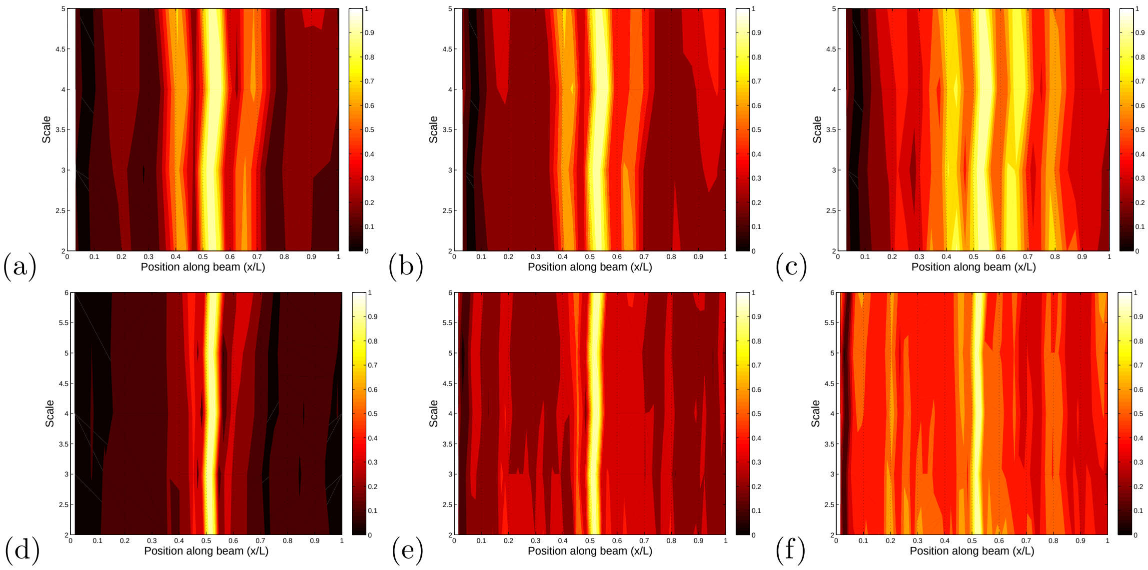

Figure 6. Normalized weighted addition of wavelet coefficients with 60db gaussian noise, using 32 measuring points (a-c) and 64 measuring points (d-f), for crack depth 20% (a,d), 10% (b,e) and 5% (c,f).

5. Conclusions

This paper has presented a novel damage detection methodology based on the wavelet analysis of changes in mode shapes due to the presence of damage. A stationary roving mass is added in order to emphasize the effect of damage as well as to reduce the effect of experimental noise. The influence of experimental noise, size of the crack and number of measuring points has been addressed in the paper. The obtained results show that the proposed methodology is a promising tool for detecting small cracks in beams. The actual sensitivity in real applications will depend on the real experimental noise and the available number of sensors.

6. Acknowledgments

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References

[1] Fan W and Qiao P 2011 *Structural Health Monitoring* **10** 83–111

[2] Surace C and Ruotolo R 1994 *Proceedings of the 12th International Modal Analysis Conference* pp 1141–1147

[3] Taha M M R, Noureldin A, Lucero J L and Baca T J 2006 *Structural Health Monitoring* **5** 267–295

[4] Katunin A 2013 *International Journal of Composite Materials* **3** 1–9

[5] Solís M, Algaba M and Galvín P 2013 *Mechanical Systems and Signal Processing* **40** 645–666

[6] Strang G and Nguyen T 1996 *Wavelets and Filter Banks* (Wellesley- Cambridge Press)

[7] Katunin A 2015 *Archives of Civil and Mechanical Engineering* **15** 251–261