Two-dimensional discrete solitons in rotating optical lattices

Jesús Cuevas-Maraver

Grupo de Física No Lineal. Departamento de Física Aplicada I. EU Politécnica. Universidad de Sevilla, Spain

Boris A. Malomed

Department of Physical Electronics. School of Electrical Engineering. Faculty of Engineering. Tel Aviv University, Israel

P.G. Kevrekidis

Department of Mathematics and Statistics. University of Massachusetts. Amherst, Massachusetts, USA.

Rotating Optical Lattices

- Bose-Einstein condensates (BECs) trapped in an optical lattice (OL) can be described by the Gross-Pitaevskii equation (GPE).
- If the OL is strong enough, the underlying (GPE) for the wave function in the continuum may be approximated by its DNLS counterpart [1, 2, 3, 4].
- Discrete solitons of various kinds have been studied in detail theoretically in 1D, 2D, and 3D versions of the DNLS equation [5].
- All these localized states have their counterparts in continuum models with periodic potentials that emulate the lattices. 2D solitons of both the fundamental and vortex types, which are unstable in uniform continua, can be stabilized by the OL potential [6].
- 2D solitons obeying the GPE in the 2D continuum can also be supported by a *rotating* OL [7, 8].
- These solitons may be fully localized solutions to the equation with the self-focusing/attractive cubic nonlinearity, placed at some distance from the rotation pivot and revolving in sync with the holding 2D lattice.
- These *co-rotating* strongly localized solitons are stable provided that the rotation frequency does not exceed a critical value.

The model

- The starting point is the normalized 2D GPE including the potential in the form of an OL rotating at angular velocity Ω, and thus stirring a "pancake"-shaped BEC trapped in a narrow gap between two strongly repelling optical sheets.
- The GPE is written in the reference frame co-rotating with the lattice (and the rotation pivot located at the center of the lattice); hence the potential does not contain explicit time dependence:

$$i\frac{\partial\psi}{\partial t}=\sigma|\psi|^2\psi-\left(\frac{1}{2}\nabla^2+\Omega\hat{L}_z\right)\psi-\epsilon\left[\cos\left(kx\right)+\cos(ky)\right]\psi \qquad (1)$$

- $\hat{L}_z=i(x\partial_y-y\partial_x)$ is the operator of the z-component of the orbital momentum; σ determines the sign of the interaction, attractive ($\sigma=-1$) or repulsive ($\sigma=+1$).
- In the limit of a very deep OL, a discrete model can be derived from the underlying GPE in the *tight-binding approximation* [2]. Eventually, it amounts to a straightforward discretization of the GPE.
- Thus, the discrete counterpart of Eq. (1) is

$$\begin{aligned} i\frac{d\psi_{m,n}}{dt} &= \sigma|\psi_{m,n}|^2\psi_{m,n}-\frac{C}{2}\left\{(\psi_{m+1,n}+\psi_{m-1,n}+\psi_{m,n+1}+\psi_{m,n-1}-4\psi_{m,n})\right. \\ &\left. -i\Omega\left[m\left(\psi_{m,n+1}-\psi_{m,n-1}\right)-n\left(\psi_{m+1,n}-\psi_{m-1,n}\right)\right]\right\} \qquad (2)\end{aligned}$$

- (m,n) are discrete coordinates; $C>0$ is the corresponding coupling constant, which accounts for the linear tunneling of atoms between BEC droplets trapped in deep nodes of the lattice; Ω, which takes positive values, is the rotation frequency.
- As in the usual 2D DNLS equation (with $\Omega=0$), values $\sigma=\pm1$ in Eq. (2) may be transformed into each other by the *staggering transformation* ($\psi_{m,n}\rightarrow(-1)^{m+n}\psi_{m,n}$). Therefore we fix $\sigma\equiv-1$ (self-attraction).
- Our first objective is to find stationary localized solutions to (2) in the form of FSs (fundamental solitons) and VSs (vortex solitons).
- We substitute the standing wave ansatz, $\psi_{m,n}=e^{-i\mu t}\phi_{m,n}$ (where μ is the normalized chemical potential); then, the stationary lattice field $\phi_{m,n}$ obeys the equation

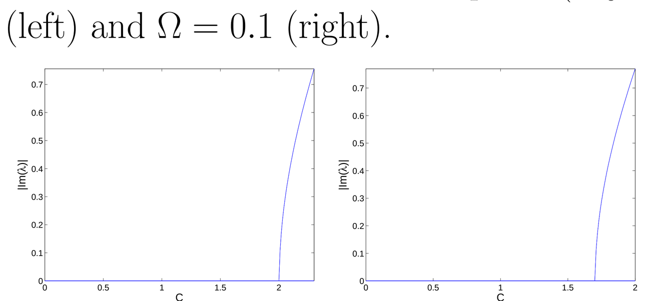
$$\begin{aligned} -\mu\phi_{m,n} &= |\phi_{m,n}|^2\phi_{m,n}+\frac{C}{2}\left(\phi_{m+1,n}+\phi_{m-1,n}+\phi_{m,n+1}+\phi_{m,n-1}-4\phi_{m,n}\right) \\ &+i\frac{C}{2}\Omega\left[m\left(\phi_{m,n+1}-\phi_{m,n-1}\right)-n\left(\phi_{m+1,n}-\phi_{m-1,n}\right)\right] \qquad (3)\end{aligned}$$

- The second objective is to examine the stability of the discrete solitons, assuming small perturbations in the form of $\delta\psi_{m,n}\sim\exp\left(-i\mu t+i\lambda t\right)$. the onset of instability indicated by the emergence of $\mathrm{Im}(\lambda)\neq0$.
- To parameterize the soliton families, the scales are fixed by setting $\mu\equiv-1$, while C is varied.

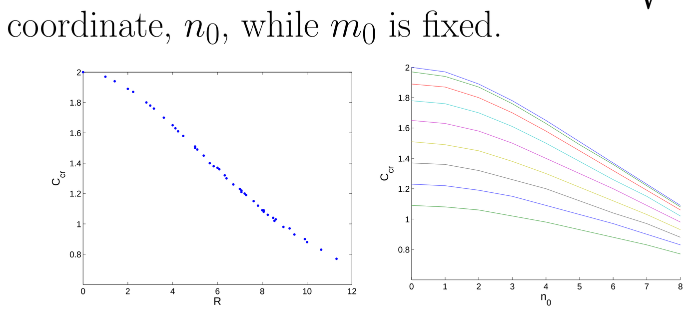
Fundamental solitons

- Rotation makes the discrete lattice inhomogeneous, hence properties of solitons strongly depend of the location of their centers.
- Without the rotation, FSs are stable at $C\leq C_{\mathrm{cr}}=-2\mu\equiv2$ (see Ref. [5]).
- The onset of their instability is accounted for by a pair of eigenfrequencies of small perturbations with finite imaginary and zero real parts, i.e., the instability leads to the exponential growth of perturbations.
- Numerical simulations of the instability development show spontaneous transformation of unstable FSs into lattice breathers [9].
- Our analysis aims to determine the stability border for the FSs, C_{cr}, for each set of values of the discrete coordinates of the soliton's center, $\{m_0,n_0\}$.
- We present results for angular velocity $\Omega=0.1$. This choice makes it possible to explore the existence and stability of FSs in a clear form; larger values of Ω give rise to a resonance with linear lattice modes, leading to Wannier-Stark ladders and hybrid solitons [13] and making the continuation in C and identification of C_{cr} difficult.
- We carried out the calculations on the lattice of size 21×21. To avoid effects of the boundaries, the range of the soliton-center coordinates was restricted to $|m_0|,|n_0|\leq8$.

- The figure displays the dependence of the imaginary part of eigenfrequencies of small perturbations $\mathrm{Im}(\lambda)$ on the lattice coupling constant, for the FS with its center set at point ($m_0=3,n_0=2$) and $\Omega=0$ (left) and $\Omega=0.1$ (right).

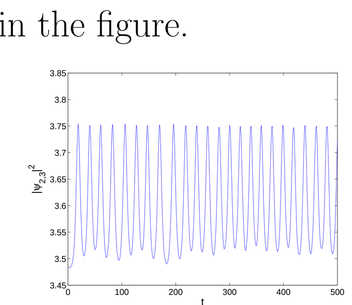


- The instability sets in at $C=C_{\mathrm{cr}}=1.70$, which is smaller than the critical value in the ordinary model, $C_{\mathrm{cr}}^{(\Omega=0)}=2$.
- Next figure summarizes the results obtained for the FSs placed at different positions, in the form of dependences of C_{cr} on the distance of the FS's center from the pivot, $R\equiv\sqrt{m_0^2+n_0^2}$, and on one coordinate, n_0, while m_0 is fixed.



- It is observed that C_{cr} *monotonously decreases* with R. Note that there are different pairs (m_0,n_0) which have equal values of R and give slightly different C_{cr}.

- Direct simulations of the dynamical evolution of unstable FSs demonstrate that the instability does not destroy the solitary wave. Instead, it transforms the waveform into a persistent breathing structure, as shown in the figure.

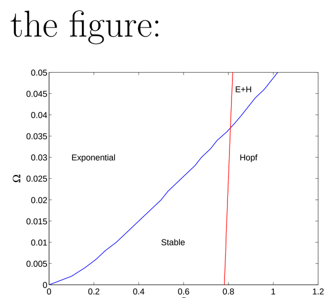


- A quasi-continuum approximation predicts an estimated dependence $C_{\mathrm{cr}}\sim1/R^2$ [10], which is qualitatively consistent with the numerical findings showing the decrease of C_{cr} with R.
- That quasi-continuum approximation makes it also possible to estimate the order of magnitude of the rotation frequency $\Omega=0.1$ in physical units. Assuming a lattice spacing ~1 μm and the condensate of ^{7}Li or ^{85}Rb [11, 12], corresponds, respectively, to ~100 Hz or 10 Hz.

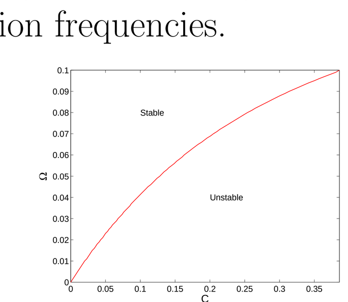
Vortex solitons

- Discrete vortex solitons (VSs) of the 2D DNLS equation are stationary solutions which feature a phase circulation of $2\pi S$ around the central point, at which the amplitude vanishes, with integer S identified as the vorticity [14].

- VS crosses with $S=1$ are stable for $C<C_{\mathrm{cr}}^{(S=1)}=0.781$; the instability above this points transforms the VS into an ordinary FS, with $S=0$. All VSs with $S=2$ are unstable in the ordinary 2D DNLS equation. In all cases, the instability sets in via a Hamiltonian Hopf bifurcation.
- We consider here only *on-axis* VSs, whose centers coincide with the rotation pivot ($R=0$). On the contrary, Ω is varied.
- The stability region of VSs with $S=1$, for given Ω, features not only the upper bound, $C_{\mathrm{cr}}^{(S=1)}$ but also a *lower* one, $\tilde{C}_{\mathrm{cr}}^{(S=1)}$.
- At a given value of Ω, VSs are exponentially unstable for $C<\tilde{C}_{\mathrm{cr}}^{(S=1)}$, and they feature oscillatory (Hopf) instabilities for $C>C_{\mathrm{cr}}^{(S=1)}$. This situation takes place up at $\Omega<\Omega_{\mathrm{cr}}^{(S=1)}=0.037$.
- $C_{\mathrm{cr}}^{(S=1)}$ slightly increases with Ω, while the growth of $\tilde{C}_{\mathrm{cr}}^{(S=1)}$ with Ω is fast. As a result, at $\Omega>\Omega_{\mathrm{cr}}^{(S=1)}$ the stability region does not exist, coexisting exponential and Hopf instabilities.
- The overall stability region for the VSs with $S=1$ in the plane (C,Ω) is presented in the figure:



- VSs at $C>C_{\mathrm{cr}}^{(S=1)}$ are transformed into persistent breathers, which loses the vortical structure. For $C<\tilde{C}_{\mathrm{cr}}^{(S=1)}$ the nonlinear development eventually leads to a persistently pulsating localized state with zero vorticity. The transformation to a FS state is also observed in the region of the coexistence of exponential and oscillatory instabilities.
- At $\Omega=0$, VSs with $S=2$ are unstable due to an imaginary eigenfrequency. In the rotating lattice, the solitons with $S=2$ acquire a finite *stability region*. Only an upper stability border exists for the $S=2$ solitons, i.e., the respective stability interval is $0<C<C_{\mathrm{cr}}^{(S=2)}$.
- The stability diagram for the VSs with $S=2$ in the (C,Ω) plane is presented in the figure, indicating the increasing stabilization effect of larger rotation frequencies.



- Instabilities of the VSs with $S=2$ transform them into persistent breathers without the vortical structure. In fact, a similar qualitative conclusion was made in the ordinary model, with $\Omega=0$, where all VSs with $S=2$ are unstable.

References

[1] A. Trombettoni and A. Smerzi, Phys. Rev. Lett. **86**, 2353 (2001); F. Kh. Abdullaev, B. B. Baizakov, S. A. Darmanyan, V. V. Konotop, and M. Salerno, Phys. Rev. **A64**, 043606 (2001).

[2] G. L. Alfimov, P. G. Kevrekidis, V. V. Konotop, and M. Salerno, Phys. Rev. E **66**, 046608 (2002).

[3] F. S. Cataliotti, S. Burger, C. Fort, P. Maddaloni, F. Minardi, A. Trombettoni, A. Smerzi, and M. Inguscio, Science **293** , 843 (2001).

[4] M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature **415**, 39 (2002).

[5] P. G. Kevrekidis, K. Ø. Rasmussen, and A. R. Bishop, Int. J. Mod. Phys. B **15**, 2833 (2001).

[6] B. B. Baizakov, B. A. Malomed, and M. Salerno, Europhys. Lett. **63**, 642 (2003); J. K. Yang and Z. H. Musslimani, Opt. Lett. **28**, 2094 (2003).

[7] H. Sakaguchi and B. A. Malomed, Phys. Rev. A **75**, 013609 (2007).

[8] Y. V. Kartashov, B. A. Malomed, and L. Torner, Phys. Rev. A **75**, 061602(R) (2007)

[9] J. Gómez-Gardeñes, L. M. Floría, and A. R. Bishop, Physica D **216**, 31 (2006).

[10] J. Cuevas, B.A. Malomed, and P.G. Kevrekidis. Phys. Rev. E **76**, 046608 (2007).

[11] K. E. Strecker, G. B. Partridge, A. G. Truscott and R. G. Hulet, Nature **417**, 150 (2002); L. Khaykovich, F. Schreck, G. Ferrari, T. Bourdel, J. Cubizolles, L. D. Carr, Y. Castin, and C. Salomon, Science **256**, 1290 (2002).

[12] S. L. Cornish, S. T. Thompson, and C. E. Wieman, Phys. Rev. Lett. **96**, 170401 (2006).

[13] T. Pertsch, U. Peschel, and F. Lederer, Phys. Rev. E **66**, 066604 (2002).

[14] B. A. Malomed and P. G. Kevrekidis. Phys. Rev. E **64**, 026601 (2001).