Some Mathematical Methods and Tools for an Analysis of Harmony-Seeking Computations

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Summary. A general review of some topic concepts and methods of membrane computing [15], [19] which can be useful in an analysis of harmony-seeking computations [1] is presented. Then an application of a certain particular method of membrane computing in a discussion of mobility of some systems considered in city planning [2] is described in some details. A conclusion of the discussion states that systems of hierarchical organization in a form of a tree are mobile by means of massively parallel (local) moves of the parts of the systems, where the moves are related to the process capabilities of moves of ambients in [7].

1 Introduction

An idea of a harmony-seeking computation was introduced and discussed in [1] aiming, among others, to improve design and planning in architecture in order to achieve that internal coherence (harmony) between the designed and planned objects of various scales (from the rooms in buildings, through buildings themselves, the districts of cities, and to cities themselves) which natural living system possesses. Therefore the discussion of harmony-seeking computations in [1] expands far beyond the methods of design and planning in architecture and concerns also better understanding of the phenomena of life with a regard to a geometric adaptation.

The paper [1] contains general postulates for mathematical modeling of harmony-seeking computations.

The aim of the present paper is to propose and review some known mathematical methods and tools which may serve for modeling harmony-seeking computations according to those postulates.

The methods focus on modeling those evolution processes of natural (living) systems which may realize massively parallel computations themselves or inspire a planning of devices realizing these computations, where an aspect of geometric adaptation is respected (on topological level) by considering certain possible transformations of hierarchical organization of systems during their evolution processes. A hierarchical organization is meant here as determined by a nesting relation of the less complex parts of a system in the more complex parts of a system.

The proposed and reviewed in the paper (Section 2) mathematical methods and tools are mainly those already applied in membrane computing, a branch of natural computing initiated by Gh. Păun and described in [19] (see also the P systems page [15]), where the underlying structure of a system (called a membrane system) with respect to nesting of subsystems or its parts is a tree and this underlying structure may be transformed during an evolution process.

In Section 3 and Appendix we present in some details a concrete application of membrane computing methods for an analysis of mobility (with respect to massively parallel moves) of certain systems considered in city planning and discussed in [2]. Section 3 together with Appendix are self-contained.

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2 Membrane Systems and Their Evolution Rules; A General Review

In [1] a harmony-seeking computation is identified with its underlying process whose steps are wholeness-extending-transformations (briefly W-E-transformations), where each W-E-transformation operates on one wholeness to produce another wholeness which is illustrated as follows:

$$W_1 \xrightarrow{WE_1} W_2 \xrightarrow{WE_2} W_3 \xrightarrow{WE_3} W_4 \xrightarrow{WE_4} \ldots .$$

The character of W-E-transformations is established in [1] by five postulates A1–A5 about the structure of wholeness and three postulates B1–B3 about the definition of W-E-transformations themselves.

In Postulate A5 each wholeness is identified (defined as) with a system of configurations, where according to Postulate A2 the subconfigurations may be spatially nested, or overlapping, or disjoint.

One can see that membrane systems, the basic tools of membrane computing, meant as finite trees with nodes labeled by multisets are appropriate candidates to model (the structure of configurations identified with) a wholeness because the trees (and their Venn diagram presentation, cf. [19]) well describe the spatial nesting. Moreover, the description of evolution processes of membrane systems by using evolution rules in membrane computing also well suits to model the harmony-seeking computations. Namely, the evolution rules of membrane systems are similar to production rules generating languages and they can be simultaneously applied to membrane systems in a similar way like production rules for L-systems can be simultaneously applied to the processed expressions. We point out here that a simulation of harmony-seeking process of tree growth by using a context-free L-system is discussed in [1].

The known applications of membrane systems and their evolution rules for modeling processes in system biology presented among others in the recent papers contained in (Pre-)Proceedings of Workshops and Brainstorming Weeks on Membrane Computing (cf. [13], [14], [11], see the papers by D. Bezzossi, N. Busi and C. Zandron, L. Cardelli and Gh. Păun, V. Manca) show that it is worth to apply membrane computing methods and tools to model harmony-seeking computations.

The most remarkable are those applications which concern fractals generation by P systems presented in [12], because fractals represent geometry of dynamic systems with a regard to similarities in various scales which is an important aspect of fractals applications in architecture, see [20].

3 Semilattices of Subsets, Trees of Subsets, hereditarily finite sets, and Their Mobility

The paper [2] contains a discussion of a thesis that a city structure of a form of a semilattice of subsets is better (topologically) adapted or more fit to live in than a structure of a form of a tree of subsets.

In this section we introduce a representation of those semilattices and trees by certain hereditarily finite sets and then we show that this representation makes possible to transfer from [18] some results concerning mobile ambients and mobile membranes into the area of trees and semilattices of subsets and then into the realm of city planning.

We quote from [2] the definitions of semilattices and trees of subsets.

The semilattice axiom goes like this: *A collection of sets forms a semilattice if and only if, when two overlapping sets belong to the collection, the set of elements common to both also belongs to the collection.*

The tree axiom states: *A collection of sets forms a tree if and only if, for any two sets that belong to the collection, either one is wholly contained in the other, or else they are wholly disjoint.*

We use the following notion which is a generalization of the above defined concepts.

We define a [*finite*] *nesting structure* to be an ordered pair $\mathfrak{N} = (U_{\mathfrak{N}}, \mathcal{N}_{\mathfrak{N}})$ such that $U_{\mathfrak{N}}$ is a [finite] set, called the *underlying set of* $\mathfrak{N}$, and $\mathcal{N}_{\mathfrak{N}}$ is a collection of nonempty subsets of $U_{\mathfrak{N}}$ with $U_{\mathfrak{N}}$ belonging to $\mathcal{N}_{\mathfrak{N}}$. The elements of $\mathcal{N}_{\mathfrak{N}}$ are called the *parts in* $\mathfrak{N}$.

For two parts $\boldsymbol{n}, \boldsymbol{n}'$ in a nesting structure $\mathfrak{N}$ we define that $\boldsymbol{n}$ is *an immediate part of* $\boldsymbol{n}'$ *in* $\mathfrak{N}$ (and write $\boldsymbol{n} \prec \boldsymbol{n}'$) if $\boldsymbol{n} \subsetneq \boldsymbol{n}'$ and for every part $\boldsymbol{m}$ in $\mathfrak{N}$ if $\boldsymbol{n} \subseteq \boldsymbol{m} \subseteq \boldsymbol{n}'$, then $\boldsymbol{m} = \boldsymbol{n}$ or $\boldsymbol{m} = \boldsymbol{n}'$.

We recall now the notion of a hereditarily finite set used in [18].

For a potentially infinite set L of labels or names which are *urelements*, i.e., they are not (treated as) sets themselves, we define inductively a family of sets

HF_i for natural numbers $i \geq 0$ such that

$$\begin{aligned}\mathrm{HF}_0 &= \varnothing,\\ \mathrm{HF}_{i+1} &= \text{the set of nonempty finite subsets of } L \cup \mathrm{HF}_i.\end{aligned}$$

The elements of the union $\mathrm{HF} = \bigcup\{\mathrm{HF}_i \mid i \geq 0\} \cup \{\varnothing\}$ are called *hereditarily finite sets over* L or *hereditarily finite sets with urelements* in L, or simply *hereditarily finite sets* if there is no risk of confusion.

For $x \in \mathrm{HF}$ we define its weak transitive closure $\mathrm{WTC}(x)$ and support $\mathrm{supp}(x)$ by

$$\begin{aligned}\mathrm{WTC}(x) &= \bigcup\big\{\mathrm{WTC}(y) \mid y \in x \text{ and } y \in \mathrm{HF}\big\} \cup \{x\}\\ \mathrm{supp}(x) &= (x \cap L) \cup \bigcup\{\mathrm{supp}(y) \mid y \in x \text{ and } y \in \mathrm{HF}\},\end{aligned}$$

and the *depth of* x is defined to be the smallest natural number i for which $x \in \mathrm{HF}_i$.

The notion of a hereditarily finite set is applied in [10] to give a general characterization of physical computing devices. The characterization is improved in [4], [21], [22], and examples are given in [9]. Membrane computing applications of hereditarily finite sets are discussed in [16], [17], [18].

For a finite nesting structure $\mathfrak{N} = (U_{\mathfrak{N}}, \mathcal{N}_{\mathfrak{N}})$ we define *its hereditarily finite set* $\mathrm{hfs}(\mathfrak{N})$ by

$$\mathrm{hfs}(\mathfrak{N}) = \big(U_{\mathfrak{N}} - \bigcup\{\boldsymbol{n} \in \mathcal{N}_{\mathfrak{N}} \mid \boldsymbol{n} \prec U_{\mathfrak{N}}\}\big) \cup \{\mathrm{hfs}(\mathfrak{N}(\boldsymbol{n})) \mid \boldsymbol{n} \in \mathcal{N}_{\mathfrak{N}} \text{ and } \boldsymbol{n} \prec U_{\mathfrak{N}}\},$$

where for a part $\boldsymbol{n} \in \mathcal{N}_{\mathfrak{N}}$ we write $\mathfrak{N}(\boldsymbol{n})$ to denote a nesting structure whose underlying set $U_{\mathfrak{N}(\boldsymbol{n})}$ is $\boldsymbol{n}$ itself and the set $\mathcal{N}_{\mathfrak{N}(\boldsymbol{n})}$ of parts in $\mathfrak{N}(\boldsymbol{n})$ is the set $\{\boldsymbol{n}' \in \mathcal{N}_{\mathfrak{N}} \mid \boldsymbol{n}' \subseteq \boldsymbol{n}\}$.

For a hereditarily finite set x we define *its nesting structure* $\mathfrak{N}^x$ by

$$U_{\mathfrak{N}^x} = \mathrm{supp}(x), \quad \mathcal{N}_{\mathfrak{N}^x} = \{\mathrm{supp}(y) \mid y \in \mathrm{WTC}(x)\}.$$

A characterization of hereditarily finite sets of finite nesting structures is formulated in the following theorem which one can treat as a representation theorem of nesting structures by hereditarily finite sets.

Theorem 1. *For every finite nesting structure* $\mathfrak{N}$, *if* x *is the hereditarily finite set of* $\mathfrak{N}$, *i.e.* $x = \mathrm{hfs}(\mathfrak{N})$, *then the following conditions hold*:

0) $\mathfrak{N}^x = \mathfrak{N}$,
1) $\mathrm{supp}(y) = \mathrm{supp}(y')$ *implies* $y = y'$ *for all* y, y' *with* $\{y, y'\} \subseteq \mathrm{WTC}(x)$,
2) $y \in y'$ *if and only if* $\mathrm{supp}(y) \prec \mathrm{supp}(y')$ *for all* y, y' *with* $\{y, y'\} \subseteq \mathrm{WTC}(x)$,
3) $y \cap \bigcup\{\mathrm{supp}(y') \mid y' \in y\} = \varnothing$ *for every* $y \in \mathrm{WTC}(x)$.

For every hereditarily finite set x, *if it satisfies the conditions* 1)–3), *then* $\mathrm{hfs}(\mathfrak{N}^x) = x$.

Proof. One proves by induction on the number of elements of $\mathcal{N}_{\mathfrak{N}}$ that $x = \mathrm{hfs}(\mathfrak{N})$ implies that the conditions 0)–3) hold for x. One proves by induction on the depth of x that if the conditions 1)–3) hold for x, then $\mathrm{hfs}(\mathfrak{N}^x) = x$.

Corollary 1. *For a finite nesting structure* $\mathfrak{N}$ *the set* $\mathrm{WTC}(\mathrm{hfs}(\mathfrak{N}))$ *ordered by the membership relation* $\in$ *forms a structure which is isomorphic to that structure which is given by the set* $\mathcal{N}_{\mathfrak{N}}$ *of parts in* $\mathfrak{N}$ *ordered by* $\prec$.

Proof. The corollary is a consequence of Theorem 1. By 1) a mapping from $\mathrm{WTC}(\mathrm{hfs}(\mathfrak{N}))$ into $\mathcal{N}_{\mathfrak{N}}$ given by $y \mapsto \mathrm{supp}(y)$ is a bijection which preserves the ordering by 2), where 1) and 2) are the conditions from Theorem 1 which hold for $x = \mathrm{hfs}(\mathfrak{N})$.

Example 1. The set x of the form

$$\Bigg\{\Big\{\big\{1,\{2,\{3\}\}\big\},\big\{\{2,\{3\}\},\{\{3\},\{4\}\}\big\},\big\{\{\{3\},\{4\}\},\{5,\{4\}\}\big\}\Big\},\Big\{6,\big\{\{\{3\},\{4\}\},\{5,\{4\}\}\big\}\Big\}\Bigg\}$$

is a hereditarily finite set such that $\mathrm{hfs}(\mathfrak{N}^x) = x$, where $\mathfrak{N}^x$ is a semilattice illustrated in Fig. 0.

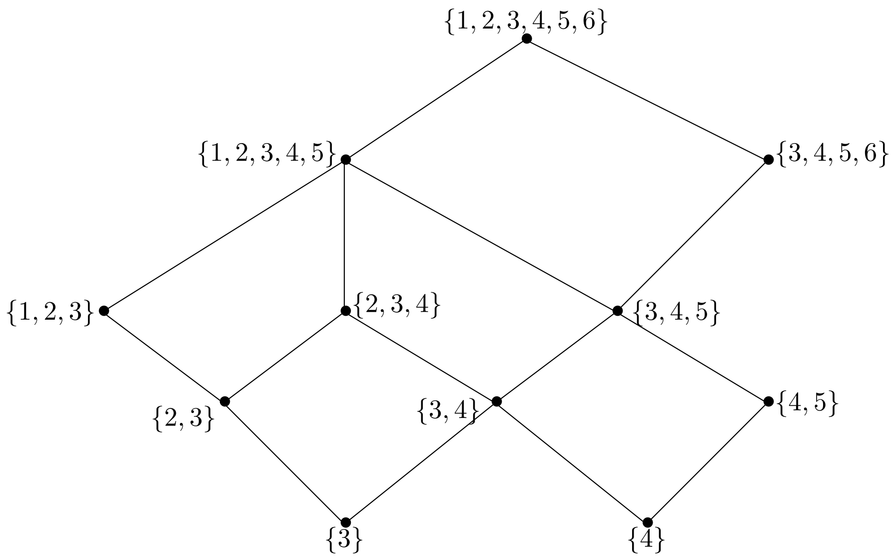


Fig. 0.

The representation of finite nesting structures by hereditarily finite sets described in Theorem 1 and Corollary 1 provides that already defined constructions and proved properties of hereditarily finite sets x satisfying $\mathrm{hfs}(\mathfrak{N}^x) = x$ can be transferred or interpreted in the class of finite nesting structures. In particular, the basic concepts and constructions describing mobile systems[1] modeled by hereditarily finite sets, see [18] and Appendix in the present paper, can be transferred to the class of finite nesting structures via the representation. The following theorem is a starting point of this transfer.

Theorem 2. *Let x be a hereditarily finite set such that $\mathrm{hfs}(\mathfrak{N}^x) = x$. Then the following conditions hold*:

$\mathbb{C}_1$) *if $y \in x$ with $y \in \mathrm{HF}$, then for $u_1 = (x - \{y\}) \cup y$ the condition $\mathcal{N}_{\mathfrak{N}^{u_1}} = \mathcal{N}_{\mathfrak{N}^x} - \{\mathrm{supp}(y)\}$ holds,*
$\mathbb{C}_2$) *if $\{y, z\} \subseteq x \cap \mathrm{HF}$ with $y \neq z$, then for $u_2 = (x - \{y, z\}) \cup \{z \cup \{y\}\}$ the condition $\mathcal{N}_{\mathfrak{N}^{u_2}} = (\mathcal{N}_{\mathfrak{N}^x} - \{\mathrm{supp}(y)\}) \cup \{\mathrm{supp}(y) \cup \mathrm{supp}(z)\}$ holds,*
$\mathbb{C}_3$) *if $z \in y \in x$ with $z \in \mathrm{HF}$, then for $u_3 = (x - \{y\}) \cup \{y - \{z\}, z\}$ the condition $\mathcal{N}_{\mathfrak{N}^{u_3}} = (\mathcal{N}_{\mathfrak{N}^x} - \{\mathrm{supp}(y)\}) \cup \{\mathrm{supp}(y - \{z\})\}$ holds.*

Moreover, if $\mathfrak{N}^x$ is such that $\mathcal{N}_{\mathfrak{N}^x}$ is a tree, then for every $i \in \{1, 2, 3\}$ the set $\mathcal{N}_{\mathfrak{N}^{u_i}}$ is also a tree and $\mathrm{hfs}(\mathfrak{N}^{u_i}) = u_i$.

Proof. We prove the theorem by induction on the depth of x.

We interpret Theorem 2 in the following way. The conditions $\mathbb{C}_1$), $\mathbb{C}_2$), $\mathbb{C}_3$) correspond to the following process capabilities discussed in [7]:

- condition $\mathbb{C}_1$) corresponds to the capability “can open an ambient”,
- condition $\mathbb{C}_2$) corresponds to the capability “can enter an ambient”,
- condition $\mathbb{C}_3$) corresponds to the capability “can exit out an ambient”.

The above capabilities are capabilities of some “spatial” moves of parts of systems with respect to hierarchical organization of systems determined by nesting relation of parts.

A mathematical description of the mentioned capabilities for systems modeled by hereditarily finite sets x (with $\mathrm{WTC}(x)$ meant as a collection of parts of x) is contained in conditions $\mathbb{C}_1$), $\mathbb{C}_2$), $\mathbb{C}_3$), where for every $i \in \{1, 2, 3\}$ the conditions written between “if” and “then” in $\mathbb{C}_i$) are (pre)conditions which provide a realization of a move and the equation defining u_i written after “then” in $\mathbb{C}_i$) is a (post)condition describing the result u_i of the move.

For a hereditarily finite set x modeling a system with parts represented by elements of $\mathrm{WTC}(x)$ the capabilities of moves can be applied (or referred) to the elements of $\mathrm{WTC}(x)$ and these applications are called *local moves in x*. The local moves are described in terms of Gh. Păun’s evolution rules and their applications in [18], see also Appendix of the present paper, where a local action is a mathematical description of a local move.

[1] Related to mobile ambient systems in [7].

A collisionless set of simultaneous local moves in a given x (more than one local move in a unit of time) is described in [18] and Appendix as a proper set of local actions over x.

An inductive formula for assembly of a whole system from the results of local moves belonging to a collisionless set of simultaneous local moves in a hereditarily finite set x is given in [18], see also the inductive definition of $\mathrm{Ap}(\mathcal{A}, x)$ in Appendix, where $\mathrm{Ap}(\mathcal{A}, x)$ is the result of assembly for a proper set $\mathcal{A}$ of local actions over x. Theorem 3 in Appendix is the final and concluding step of the discussed transfer of the basic concepts and constructions describing mobile systems modeled by hereditarily finite sets into the area of nesting structures.

Remark. For x given in Example,

$$y = \Big\{\Big\{1, \{2, \{3\}\}\Big\}, \Big\{\{2, \{3\}\}, \{\{3\}, \{4\}\}\Big\}, \Big\{\{\{3\}, \{4\}\}, \{5, \{4\}\}\Big\}\Big\},$$

and

$$z = \Big\{\{2, \{3\}\}, \{\{3\}, \{4\}\}\Big\}$$

we have that $z \in y \in x$ which means that preconditions in $\mathbb{C}_3$) hold for these x, y, z. Then for $u_3 = (x - \{y\}) \cup \{y - \{z\}, z\}$ we have that $\mathfrak{N}^{u_3} = \mathfrak{N}^x$ (because $\mathrm{supp}(y - \{z\}) = \mathrm{supp}(y)$ in this case) which means that capability "can exit out an ambient" does not lead to any real move leaving $\mathfrak{N}^x$ unchanged.

Conclusion

By virtue of Theorem 2 and Theorem 3 in Appendix every finite nesting structure $\mathfrak{N}$ with $\mathcal{N}_{\mathfrak{N}}$ being a tree is mobile with respect to simultaneous (massively parallel) local moves determined by process capabilities "can open an ambient", "can enter an ambient", and "can exit out an ambient". The case discussed in Remark shows that mobility of some nesting structures $\mathfrak{N}$ with $\mathcal{N}_{\mathfrak{N}}$ being a semilattice is problematic.

Appendix

We consider those evolutive transformations of hereditarily finite sets into hereditarily finite sets which are determined by evolution rules written in Păun's manner as the parenthesis expressions, cf. [19]:

R_1) [ ] $\rightarrow$ (*dissolution rule*),
R_2) [ ][ ] $\rightarrow$ [[ ]] (*in-rule*),
R_3) [[ ]] $\rightarrow$ [ ][ ] (*out-rule*),

The single applications from the top of the above rules to hereditarily finite sets are described in the following way:

- if $y \in x \cap \mathrm{HF}$, then the dissolution rule $[\,] \rightarrow$ *can be applied to* x and the result of its application is a new hereditarily finite set of the form

$$(x - \{y\}) \cup y,$$

- if $\{y, z\} \subseteq x \cap \mathrm{HF}$, and $z \neq y$, then the in-rule $[\,][\,] \rightarrow [[\,]]$ *can be applied to* x and the result of its application is a new hereditarily finite set of the form

$$(x - \{y, z\}) \cup \{z \cup \{y\}\},$$

- if $z \in y \in x \in \mathrm{HF}$, $z \in \mathrm{HF}$, and $y - \{z\} \neq \varnothing$, then the out-rule $[[\,]] \rightarrow [\,][\,]$ *can be applied to* x and the result of its application is a new hereditarily finite set of the form

$$(x - \{y\}) \cup (\{y - \{z\}, z\} - \{\varnothing\}).$$

The above described single applications of evolution rules R_1), R_2), R_3) from the top determine evolutive transformations of hereditarily finite sets into new hereditarily finite sets from the top. One sees that these rules are related to process capabilities "can open an ambient", "can enter an ambient", "can exit out an ambient", introduced in [6]. The evolution rules may describe forces in patterns, cf. [3]. We describe by using $\cup$, $-$, and $\{?\}$ a more complicated case of evolutive transformations of hereditarily finite sets, where these transformations are determined by simultaneous applications of different rules to many different elements of $\mathrm{WTC}(x)$ for a hereditarily finite set x to be transformed.

The evolutive transformations of hereditarily finite sets considered above can be "transferred" to nesting structures by using the construction of $\mathfrak{N}^x$ (see Theorem 1 and Corollary 1) to define evolutive transformations of nesting structures themselves.

We restrict our considerations to *tree-like hereditarily finite sets* which are defined to be such that $\mathrm{hfs}(\mathfrak{N}^x) = x$ and $\mathfrak{N}^x$ is a tree.

Let x be a tree-like hereditarily finite set. By a *local action over* x we mean an ordered pair $\mathfrak{a} = (P^{\mathfrak{a}}, R^{\mathfrak{a}})$, where $P^{\mathfrak{a}}$ is a bijection from $\mathrm{dom}(\mathfrak{a})$ into $\mathrm{scope}(\mathfrak{a})$ with $\mathrm{scope}(\mathfrak{a}) \subset \mathrm{WTC}(x)$ and $R^{\mathfrak{a}}$ is an evolution rule such that

a_1) if $R^{\mathfrak{a}}$ is a dissolution rule $[\,] \rightarrow$, then $\mathrm{dom}(\mathfrak{a}) = \{0, 1\}$ and $P^{\mathfrak{a}}(1) \in P^{\mathfrak{a}}(0)$,
a_2) if $R^{\mathfrak{a}}$ is an in-rule $[\,][\,] \rightarrow [[\,]]$, then $\mathrm{dom}(\mathfrak{a}) = \{0, 1, 2\}$ and $\{P^{\mathfrak{a}}(1), P^{\mathfrak{a}}(2)\} \subset P^{\mathfrak{a}}(0)$,
a_3) if $R^{\mathfrak{a}}$ is an out-rule $[[\,]] \rightarrow [\,][\,]$, then $\mathrm{dom}(\mathfrak{a}) = \{0, 1, 2\}$ and $P^{\mathfrak{a}}(2) \in P^{\mathfrak{a}}(1) \in P^{\mathfrak{a}}(0)$.

For a local action $\mathfrak{a}$ over x the bijection $P^{\mathfrak{a}}$ is meant as a *place of application* of the rule $R^{\mathfrak{a}}$, where it will be seen later than one can interpret $\mathrm{scope}(\mathfrak{a})$ as the scope of the local transformation of x according to the rule $R^{\mathfrak{a}}$.

Let $\mathcal{A}$ be a set of local actions over x. For a set $y \in \mathrm{WTC}(x)$ and a set $z \subseteq y$ we write $\mathcal{A} \restriction (y - z)$ to denote the set of local actions $\mathfrak{a}$ over $y - z$ such that $\mathfrak{a} \in \mathcal{A}$ or $P^{\mathfrak{a}}(0) = y - z$ with $\mathfrak{a}^* = (P^{\mathfrak{a}^*}, R^{\mathfrak{a}}) \in \mathcal{A}$ for $P^{\mathfrak{a}^*} : \mathrm{dom}(\mathfrak{a}) \rightarrow (\mathrm{scope}(\mathfrak{a}) - \{y - z\}) \cup \{y\}$ with $P^{\mathfrak{a}^*}(i) = P^{\mathfrak{a}}(i)$ for all $i \in \mathrm{dom}(\mathfrak{a}) - \{0\}$. If $z = \varnothing$, then $\mathcal{A} \upharpoonright (y - z) = \mathcal{A} \upharpoonright y$ is simply the set of those local actions over y which belong to $\mathcal{A}$. If $z = y$, then $\mathcal{A} \upharpoonright (y - z) = \mathcal{A} \upharpoonright \varnothing = \varnothing$.

For a set $\mathcal{A}$ of local actions over x we adopt the following notation

$$\mathcal{A}_\alpha = \{\mathfrak{a} \in \mathcal{A} \,|\, R^{\mathfrak{a}} \text{ is an } \alpha\text{-rule}\} \quad \text{for} \quad \alpha \in \{\text{in}, \text{out}\},$$
$$\mathcal{A}_{\text{diss}} = \{\mathfrak{a} \in \mathcal{A} \,|\, R^{\mathfrak{a}} \text{ is a dissolution rule}\}.$$

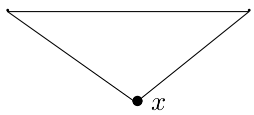
We define now a property of sets $\mathcal{A}$ of local actions over tree-like hereditarily finite sets x such that if $\mathcal{A}$ has this property, then one can construct the result of transformation of x with respect to $\mathcal{A}$ in a consistent (unambiguous) way, where x is transformed according to simultaneous application of the rules $R^{\mathfrak{a}}$ in places $P^{\mathfrak{a}}$, respectively for all $a \in \mathcal{A}$.

A set $\mathcal{A}$ of local actions over x is called a *proper set of local actions over x* if for all local actions $\mathfrak{a}, \mathfrak{a}'$ in $\mathcal{A}$ if $\mathfrak{a} \neq \mathfrak{a}'$, then $\text{scope}(\mathfrak{a}) \cap \text{scope}(\mathfrak{a}') = \varnothing$ or the disjunction of the following conditions holds:

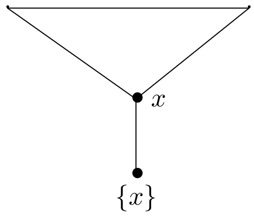
(C_1) $P^{\mathfrak{a}}(0) = P^{\mathfrak{a}'}(0)$ and $(\text{scope}(\mathfrak{a}) - \{P^{\mathfrak{a}}(0)\}) \cap (\text{scope}(\mathfrak{a}') - \{P^{\mathfrak{a}'}(0)\}) = \varnothing$,
(C_2) if $\{\mathfrak{a}, \mathfrak{a}'\} \subseteq \mathcal{A}_{\text{diss}}$, then $P^{\mathfrak{a}}(0) = P^{\mathfrak{a}'}(1)$,
(C_3) if $\{\mathfrak{a}, \mathfrak{a}'\} \subseteq \mathcal{A}_{\text{in}}$, then $P^{\mathfrak{a}}(1) = P^{\mathfrak{a}'}(1)$ or $P^{\mathfrak{a}'}(0) \in \{P^{\mathfrak{a}}(1), P^{\mathfrak{a}}(2)\}$,
(C_4) if $\{\mathfrak{a}, \mathfrak{a}'\} \subseteq \mathcal{A}_{\text{out}}$, then $P^{\mathfrak{a}}(0) = P^{\mathfrak{a}'}(2)$
or $\{P^{\mathfrak{a}}(1), P^{\mathfrak{a}}(2)\} \cap \{P^{\mathfrak{a}'}(0), P^{\mathfrak{a}'}(1)\} = \{P^{\mathfrak{a}}(1)\}$,
(C_5) if $\mathfrak{a} \in \mathcal{A}_{\text{diss}}$ and $\mathfrak{a}' \in \mathcal{A}_{\text{in}}$, then $P^{\mathfrak{a}}(1) = P^{\mathfrak{a}'}(0)$
or $P^{\mathfrak{a}}(0) \in \{P^{\mathfrak{a}'}(1), P^{\mathfrak{a}'}(2)\}$,
(C_6) if $\mathfrak{a} \in \mathcal{A}_{\text{diss}}$ and $\mathfrak{a}' \in \mathcal{A}_{\text{out}}$, then $P^{\mathfrak{a}}(1) = P^{\mathfrak{a}'}(0)$ or $\{P^{\mathfrak{a}}(0), P^{\mathfrak{a}}(1)\} \cap \{P^{\mathfrak{a}'}(1), P^{\mathfrak{a}'}(2)\} = \{P^{\mathfrak{a}}(0)\}$,
(C_7) if $\mathfrak{a} \in \mathcal{A}_{\text{in}}$ and $\mathfrak{a}' \in \mathcal{A}_{\text{out}}$, then $P^{\mathfrak{a}}(1) = P^{\mathfrak{a}'}(1)$ or $P^{\mathfrak{a}'}(0) \in \{P^{\mathfrak{a}}(1), P^{\mathfrak{a}}(2)\}$
or $\text{scope}(\mathfrak{a}) \cap \{P^{\mathfrak{a}'}(1), P^{\mathfrak{a}'}(2)\} = \{P^{\mathfrak{a}}(0)\}$.

We adopt the following conventions to explain and illustrate the notion of a proper set of local actions.

For a tree-like non-empty hereditarily finite set x whose content is not specified (or is not important for considerations) we illustrate x by a drawing given by a triangle below whose bottom vertex is labeled by x.

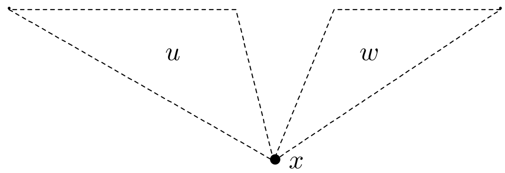


For a tree-like non-empty hereditarily finite set x whose content is not specified we illustrate one-element set $\{x\}$ by a drawing given by a triangle with an arrow glued to the bottom vertex of the triangle as below



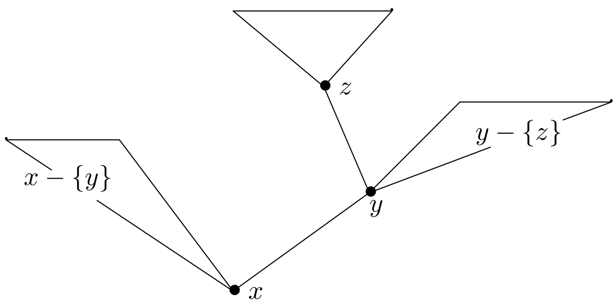
where the bottom vertex of the drawing is that vertex which is labeled by $\{x\}$.

If a tree-like hereditarily finite set x is such that $x = u \cup w$ for hereditarily finite sets u, w with $(\mathrm{HF} \cap u) \cap (\mathrm{HF} \cap w) = \varnothing$ such that there are given the drawings used for illustrations of u and w, respectively, then we illustrate x by a drawing below



where the meta-triangles labeled by u and w contain the drawing used to illustrate u and the drawing used to illustrate w, respectively. In the above drawing which illustrates $x = u \cup w$ the bottom vertex labeled by x is the result of gluing of the bottom vertex of the drawing used to illustrate u and the bottom vertex of the drawing used to illustrate w. Here the intersection of the set of vertices of the drawing for u and the set of vertices of the drawing for w is the one-element set containing the result of gluing described above, which is the vertex labeled by x.

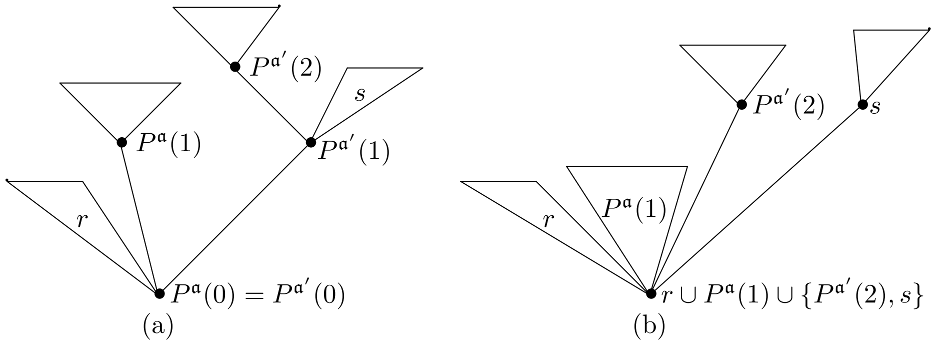
Thus for tree-like hereditarily finite sets x, y, z such that $z \in y \in x$ one can illustrate x by the drawing



where the contents of $x - \{y\}$, $y - \{z\}$, and z are not specified.

We explain and illustrate the conditions (C_1)–(C_7).

Ad (C_1). For two different local actions $\mathfrak{a} \in \mathcal{A}_{\mathrm{diss}}$ and $\mathfrak{a}' \in \mathcal{A}_{\mathrm{out}}$ satisfying (C_1) the places $P^{\mathfrak{a}}$ and $P^{\mathfrak{a}'}$ are illustrated in Fig. 1(a). The result of simultaneous application of the rules $R^{\mathfrak{a}}$ and $R^{\mathfrak{a}'}$ in places $P^{\mathfrak{a}}$ and $P^{\mathfrak{a}'}$, respectively, is illustrated in Fig. 1(b), where $P^{\mathfrak{a}}(1)$ is "dissolved" in $P^{\mathfrak{a}}(0)$ and $P^{\mathfrak{a}'}(2)$ is "sent out" of $P^{\mathfrak{a}'}(1)$

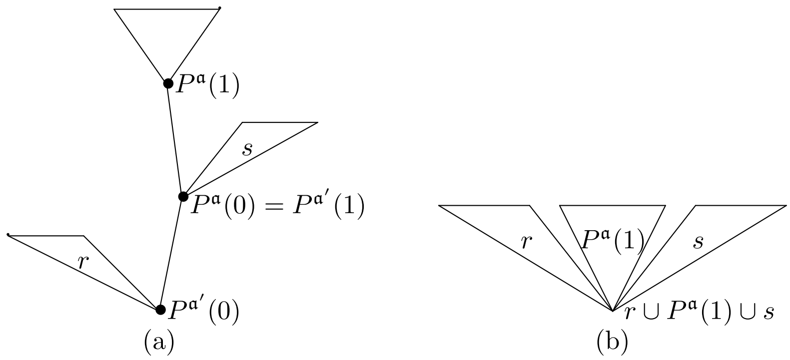


where $r = P^{\mathfrak{a}}(0) - \{P^{\mathfrak{a}}(1), P^{\mathfrak{a}'}(1)\}$ and $s = P^{\mathfrak{a}'}(1) - \{P^{\mathfrak{a}'}(2)\}$

Fig. 1.

into $P^{\mathfrak{a}}(0) = P^{\mathfrak{a}'}(0)$. The remaining cases of $\mathfrak{a}$ and $\mathfrak{a}'$ satisfying (C_1) are explained and illustrated in a similar way.

Ad (C_2). For two different local actions $\mathfrak{a}, \mathfrak{a}'$ belonging to $\mathcal{A}_{\text{diss}}$ with $P^{\mathfrak{a}}(0) = P^{\mathfrak{a}'}(1)$ the places $P^{\mathfrak{a}}$ and $P^{\mathfrak{a}'}$ are illustrated in Fig. 2(a). The result of simultaneous application of the rules $R^{\mathfrak{a}}$ and $R^{\mathfrak{a}'}$ in places $P^{\mathfrak{a}}$ and $P^{\mathfrak{a}'}$, respectively, is illustrated in Fig. 2(b), where both $P^{\mathfrak{a}}(1)$ and $P^{\mathfrak{a}'}(1) - \{P^{\mathfrak{a}}(1)\}$ are "dissolved" simultaneously in $P^{\mathfrak{a}'}(0)$.

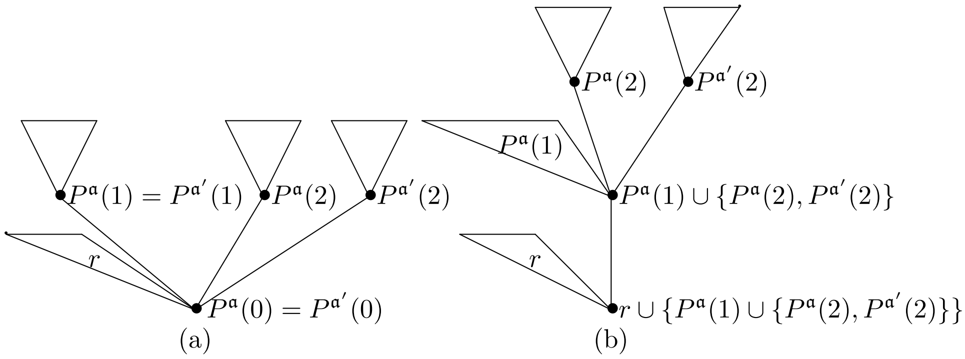


where $r = P^{\mathfrak{a}'}(0) - \{P^{\mathfrak{a}}(0)\}$ and $s = P^{\mathfrak{a}}(0) - \{P^{\mathfrak{a}}(1)\}$

Fig. 2.

Ad (C_3). For two different local actions $\mathfrak{a}, \mathfrak{a}'$ belonging to $\mathcal{A}_{\text{in}}$ with $P^{\mathfrak{a}}(1) = P^{\mathfrak{a}'}(1)$ the places $P^{\mathfrak{a}}$ and $P^{\mathfrak{a}'}$ are illustrated in Fig. 3(a). The result of simultaneous application of the rules $R^{\mathfrak{a}}$ and $R^{\mathfrak{a}'}$ in these places $P^{\mathfrak{a}}$ and $P^{\mathfrak{a}'}$, respectively, is illustrated in Fig. 3(b), where both $P^{\mathfrak{a}}(2)$ and $P^{\mathfrak{a}'}(2)$ are "sent into" $P^{\mathfrak{a}}(1) = P^{\mathfrak{a}'}(1)$ simultaneously. We point out that for all two different local actions $\mathfrak{a}$ and $\mathfrak{a}'$

with scope($\mathfrak{a}$) $\cap$ scope($\mathfrak{a}'$) $\neq \varnothing$ the condition (C_3) implies $P^{\mathfrak{a}}(1) \neq P^{\mathfrak{a}'}(2)$, which excludes the case such that simultaneous application of $R^{\mathfrak{a}}$ and $R^{\mathfrak{a}'}$ in places $P^{\mathfrak{a}}$ and $P^{\mathfrak{a}'}$ is ambiguous. The remaining cases of $\mathfrak{a}$ and $\mathfrak{a}'$ satisfying (C_3) are explained and illustrated in a similar way.



where $r = P^{\mathfrak{a}}(0) - \{P^{\mathfrak{a}}(1), P^{\mathfrak{a}}(2), P^{\mathfrak{a}'}(2)\}$

Fig. 3.

Ad (C_4). For two different local actions $\mathfrak{a}, \mathfrak{a}'$ belonging to $\mathcal{A}_{\text{out}}$ we explain the case of $\{P^{\mathfrak{a}}(1), P^{\mathfrak{a}}(2)\} \cap \{P^{\mathfrak{a}'}(0), P^{\mathfrak{a}'}(1)\} = \{P^{\mathfrak{a}}(1)\}$ which is equivalent to the disjunction of the following two conditions:

i) $P^{\mathfrak{a}'}(0) = P^{\mathfrak{a}}(1)$ and $P^{\mathfrak{a}'}(1) \neq P^{\mathfrak{a}}(2)$,
ii) $P^{\mathfrak{a}}(1) = P^{\mathfrak{a}'}(1)$.

The places $P^{\mathfrak{a}}$ and $P^{\mathfrak{a}'}$ for the case i) are illustrated in Fig. 4(a). The result of simultaneous application of $R^{\mathfrak{a}}$ and $R^{\mathfrak{a}'}$ in these places $P^{\mathfrak{a}}$ and $P^{\mathfrak{a}'}$, respectively, is illustrated in Fig. 4(b), where $P^{\mathfrak{a}}(2)$ and $P^{\mathfrak{a}'}(2)$ are simultaneously "sent out" of $P^{\mathfrak{a}}(1)$ into $P^{\mathfrak{a}}(0)$ and of $P^{\mathfrak{a}'}(2)$ into $P^{\mathfrak{a}'}(0) = P^{\mathfrak{a}}(1)$, respectively. The condition $P^{\mathfrak{a}'}(1) \neq P^{\mathfrak{a}}(2)$ in i) excludes the case such that simultaneous application of $R^{\mathfrak{a}}$ and $R^{\mathfrak{a}'}$ in places $P^{\mathfrak{a}}$ and $P^{\mathfrak{a}'}$ is ambiguous. The case ii) and the remaining cases in (C_4) are explained and illustrated in a similar way.

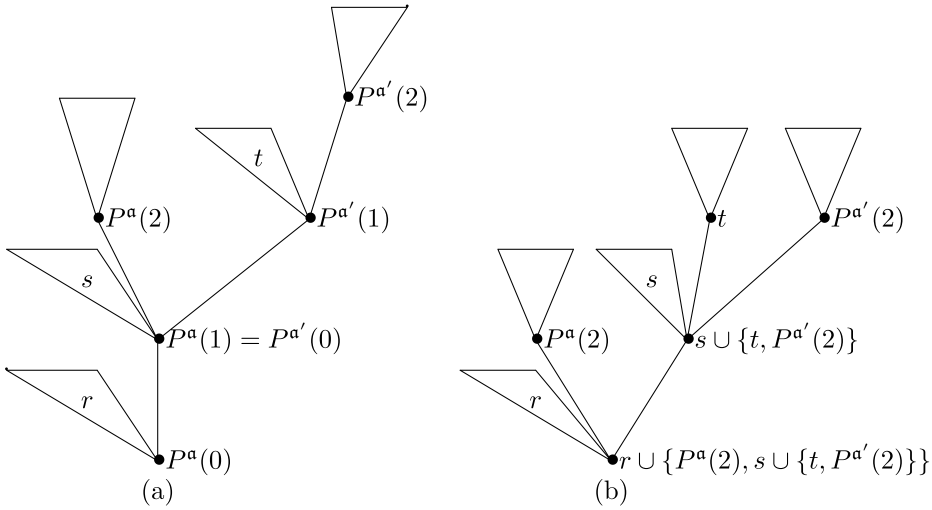
Ad (C_5). One explains and illustrates this condition in a way similar to (C_1) and (C_3).

Ad (C_6). One explains and illustrates this condition in a way similar to (C_4). We point out here that for two different local actions $\mathfrak{a} \in \mathcal{A}_{\text{diss}}$ and $\mathfrak{a}' \in \mathcal{A}_{\text{out}}$ with scope($\mathfrak{a}$) $\cap$ scope($\mathfrak{a}'$) $\neq \varnothing$ the condition

$$\{P^{\mathfrak{a}}(0), P^{\mathfrak{a}}(1)\} \cap \{P^{\mathfrak{a}'}(1), P^{\mathfrak{a}'}(2\} = \{P^{\mathfrak{a}}(0)\}$$

is equivalent to the disjunction of the following two conditions:

iii) $P^{\mathfrak{a}}(0) = P^{\mathfrak{a}'}(1)$ and $P^{\mathfrak{a}}(1) \neq P^{\mathfrak{a}'}(2)$,



where $r = P^{\mathfrak{a}}(0) - \{P^{\mathfrak{a}}(1)\},\ s = P^{\mathfrak{a}}(1) - \{P^{\mathfrak{a}}(2), P^{\mathfrak{a}'}(1)\}$
and $t = P^{\mathfrak{a}'}(1) - \{P^{\mathfrak{a}'}(2)\}$

Fig. 4.

iv) $P^{\mathfrak{a}}(0) = P^{\mathfrak{a}'}(2)$.

The condition $P^{\mathfrak{a}}(1) \neq P^{\mathfrak{a}'}(2)$ in iii) excludes the case such that simultaneous application of $R^{\mathfrak{a}}$ and $R^{\mathfrak{a}'}$ in the places $P^{\mathfrak{a}}$ and $P^{\mathfrak{a}'}$ is ambiguous.

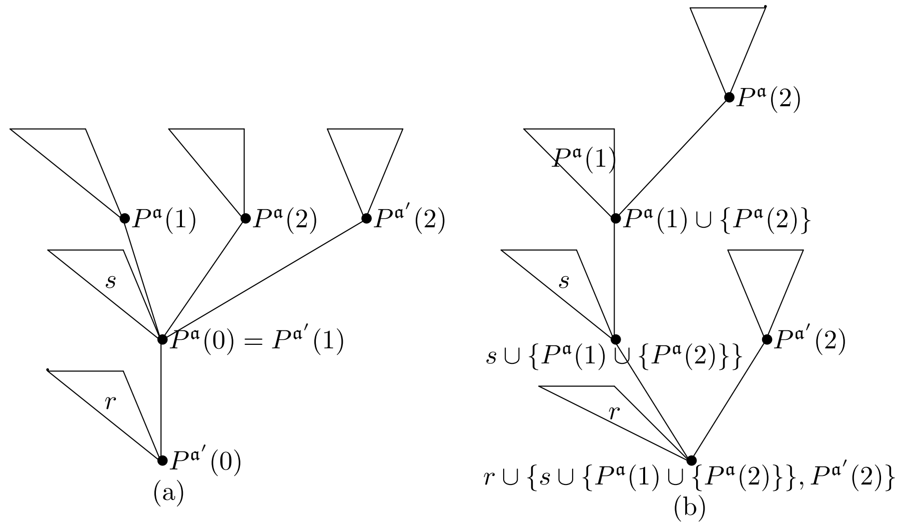
Ad (C_7). For two different local actions $\mathfrak{a} \in \mathcal{A}_{\text{in}}$ and $\mathfrak{a}' \in \mathcal{A}_{\text{out}}$ we explain the case of $\text{scope}(\mathfrak{a}) \cap \{P^{\mathfrak{a}'}(1), P^{\mathfrak{a}'}(2)\} = \{P^{\mathfrak{a}}(0)\}$ which is equivalent to the disjunction of the following two conditions:

v) $P^{\mathfrak{a}}(0) = P^{\mathfrak{a}'}(1)$ and $P^{\mathfrak{a}'}(2) \notin \{P^{\mathfrak{a}}(1), P^{\mathfrak{a}}(2)\}$,
vi) $P^{\mathfrak{a}}(0) = P^{\mathfrak{a}'}(2)$.

The places $P^{\mathfrak{a}}$ and $P^{\mathfrak{a}'}$ in the case v) are illustrated in Fig. 5(a). The result of simultaneous application of $R^{\mathfrak{a}}$ and $R^{\mathfrak{a}'}$ in these places $P^{\mathfrak{a}}$ and $P^{\mathfrak{a}'}$, respectively, is illustrated in Fig. 5(b), where $P^{\mathfrak{a}}(2)$ is "sent into" $P^{\mathfrak{a}}(1)$ and $P^{\mathfrak{a}'}(2)$ is "sent out" of $P^{\mathfrak{a}'}(1) = P^{\mathfrak{a}}(0)$ into $P^{\mathfrak{a}'}(0)$ simultaneously. The condition $P^{\mathfrak{a}'}(2) \notin \{P^{\mathfrak{a}}(1), P^{\mathfrak{a}}(2)\}$ in v) excludes the case such that simultaneous application of $R^{\mathfrak{a}}$ and $R^{\mathfrak{a}'}$ in the places $P^{\mathfrak{a}}$ and $P^{\mathfrak{a}'}$ is ambiguous. The case vi) and the remaining cases in (C_7) are explained and illustrated in a similar way.

Let $\mathcal{A}$ be a proper set of local actions over a tree-like hereditarily finite set x. By the *result of evolutive transformation of x with respect to $\mathcal{A}$* we mean a set, denoted by $\text{Ap}(\mathcal{A}, x)$, which is defined inductively (with respect to the number of elements of $\mathcal{A}$ and the depth of x) by the following equations:

1) $\text{Ap}(\varnothing, x) = x$ and $\text{Ap}(\varnothing, \varnothing) = \varnothing$,



where $r = P^{\mathfrak{a}'}(0) - \{P^{\mathfrak{a}'}(1)\}$ and $s = P^{\mathfrak{a}}(0) - \{P^{\mathfrak{a}}(1), P^{\mathfrak{a}}(2), P^{\mathfrak{a}'}(2)\}$

Fig. 5.

2) if $\mathcal{A} \neq \varnothing$, then $\mathrm{Ap}(\mathcal{A}, x) = (L \cap x) \cup \mathrm{Ap}^{\bullet}(\mathcal{A}, x)$ for

$$\mathrm{Ap}^{\bullet}(\mathcal{A}, x) = \bigcup_{1 \leq i \leq 4} \mathrm{Ap}_i(\mathcal{A}, x),$$

where

- $\mathrm{Ap}_1(\mathcal{A}, x) = \{\mathrm{Ap}(\mathcal{A} \restriction y, y) \,|\, y \in x \cap \mathrm{HF}$ and $y \notin \bigcup\{\mathrm{scope}(\mathfrak{a}) \,|\, P^{\mathfrak{a}}(0) = x$ and $\mathfrak{a} \in \mathcal{A}\}\}$,
- $\mathrm{Ap}_2(\mathcal{A}, x) = \bigcup\{\mathrm{Ap}^{\bullet}(\mathcal{A} \restriction P^{\mathfrak{a}}(1), P^{\mathfrak{a}}(1)) \,|\, P^{\mathfrak{a}}(0) = x$ and $\mathfrak{a} \in \mathcal{A}_{\mathrm{diss}}\}$,
- $\mathrm{Ap}_3(\mathcal{A}, x) = \{\mathrm{Ap}^{\bullet}(\mathcal{A} \restriction P^{\mathfrak{a}}(2), P^{\mathfrak{a}}(2)) \,|\, P^{\mathfrak{a}}(0) = x$ and $\mathfrak{a} \in \mathcal{A}_{\mathrm{out}}\}$,
- $\mathrm{Ap}_4(\mathcal{A}, x) = \{\mathrm{Ap}^{\bullet}(\mathcal{A} \restriction (y - \boldsymbol{P}^y), y - \boldsymbol{P}^y) \cup \boldsymbol{Q}^y \,|\, y \in \mathrm{INOUT}^x_{\mathcal{A}}\}$ for

$$\begin{aligned} \mathrm{INOUT}^x_{\mathcal{A}} &= \{P^{\mathfrak{a}}(1) \,|\, P^{\mathfrak{a}}(0) = x \text{ and } \mathfrak{a} \in \mathcal{A}_{\mathrm{in}} \cup \mathcal{A}_{\mathrm{out}}\}, \\ \boldsymbol{P}^y &= \{P^{\mathfrak{a}}(2) \,|\, P^{\mathfrak{a}}(1) = y \text{ and } \mathcal{A} \in \mathcal{A}_{\mathrm{out}}\}, \\ \boldsymbol{Q}^y &= \{\mathrm{Ap}^{\bullet}(\mathcal{A} \restriction P^{\mathfrak{a}}(2), P^{\mathfrak{a}}(2)) \,|\, P^{\mathfrak{a}}(1) = y \text{ and } \mathfrak{a} \in \mathcal{A}_{\mathrm{in}}\}. \end{aligned}$$

The result $\mathrm{Ap}(\mathcal{A}, x)$ of evolutive transformation of x with respect to $\mathcal{A}$ is the result of simultaneous application of the rules $R^{\mathfrak{a}}$ in places $P^{\mathfrak{a}}$, respectively for $\mathfrak{a} \in \mathcal{A}$, such that $\mathrm{Ap}(\mathcal{A}, x)$ inherits some basic properties of x which are described in the following theorem.

Theorem 3. *Let x be a tree-like hereditarily finite set and let $\mathcal{A}$ be a proper set of local action over x. Then $\mathrm{Ap}(\mathcal{A}, x)$ is a tree-like hereditarily finite set.*

Proof. One proves the theorem by induction on the number of elements of $\mathcal{A}$ and the depth of x. Theorem 2 in Section 3 provides the first inductive step.

References

1. Ch. Alexander: Harmony-Seeking Computations. *International Journal of Unconventional Computation*, to appear; see also `http://www.livingneighborhoods.org`.
2. Ch. Alexander: A city is not a tree. *Architectural Forum*, 122, 1 (1965), 58–61; 2 (1965), 58–62; `http://www2.rudi.net/bookshelf/classics/city`.
3. Ch. Alexander: *Pattern Languages*. Oxford Univ. Press, New York, 1977.
4. J. Byrnes, W. Sieg: An abstract model for parallel computations: Gandy's Thesis. *The Monist*, 82, 1 (1999), 150–164.
5. L. Cardelli: Brane calculi. Interactions of biological membranes. In *Proc. Computational Methods in System Biology*, 2004, Springer, Berlin.
6. L. Cardelli, A.D. Gordon: Mobile ambients. In *Foundations of Software Science and Computation Structures*, Lecture Notes in Comput. Sci. 1378, Springer, Berlin, 1998, 140–155.
7. L. Cardelli, A.D. Gordon: Mobile ambients. Coordination. *Theoret. Comput. Sci.*, 240 (2000), 177–213.
8. L. Cardelli, Gh. Păun: An universality result for (mem)brane calculus based on mate/drip operations. In *Proc. Cellular Computing (Complexity Aspects)*, ESF PESC Exploratory Workshop, January 31–February 2, 2005 (M.A. Gutiérrez-Naranjo et al., eds.), 75–94.
9. N. De Pisapia: *Gandy Machines: an Abstract Model for Parallel Computations, for Turing Machines, the Game of Life, and Artificial Neural Networks*. M.S. Thesis, Carnegie Mellon Univ., Pittsburgh 2000, `http://artsci.wustl.edu/~ndepisap`.
10. R. Gandy: Church's thesis and principles for mechanisms. In *The Kleene Symposium* (J. Barwise et al., eds.), North-Holland, Amsterdam, 1980, 123–148.
11. C. Graciani Diaz et al., eds.: *Proceedings of Fourth Brainstorming Week on Membrane Computing*, Sevilla, January 30–February 3, 2006, vol. I and II, Sevilla, 2006.
12. M.A. Gutiérrez-Naranjo, M.J. Pérez-Jiménez: Fractals and P systems. In *Proceedings of Fourth Brainstorming Week on Membrane Computing*, Sevilla, January 30–February 3, 2006 (C. Graciani Diaz et al., eds.), Sevilla 2006, vol. II, 65–86.
13. M.A. Gutiérrez-Naranjo et al., eds.: *Proc. Cellular Computing* (*Complexity Aspects*). ESF PESC Exploratory Workshop, January 31–February 2, 2005, Sevilla, 2005.
14. H.J. Hoogeboom et al., eds.: *Pre-proceedings of the 7th Workshop on Membrane Computing WMC7*. 17–21 July 2006, Lorenz Center, Leiden, Leiden, 2006.
15. Membrane computing web page `http://psystems.disco.unimib.it`.
16. A. Obtułowicz: Mathematical (denotational) semantics of some reducts of Ambient Calculus and Brane Calculi. *Romanian Journal of Information Science and Technology*, to appear.
17. A. Obtułowicz: Gandy's principles for mechanisms and membrane computing. In *Proc. Cellular Computing (Complexity Aspects)*, ESF PESC Exploratory Workshop, January 31–February 2, 2005 (M. A. Gutiérrez-Naranjo et al., eds.), Sevilla, 2005, 267–276.
18. A. Obtułowicz: Relational membrane systems. In *Membrane Computing, 6th International Workshop, WMC 2005,* Vienna, Austria, July 18–21, 2005 (R. Freund et al., eds.), Lecture Notes in Comput. Sci. 3850, Springer, Berlin, 2006, 342–355.
19. Gh. Păun: *Membrane Computing. An Introduction*. Springer-Verlag, Berlin, 2002.
20. N.A. Salingaros: *Fractals in New Architecture*. Katarxis 3, September 2004, `http://www.katarxis3.com/Salingaros-Fractals.htm`.

21. W. Sieg: *Computability Theory*. Seminar Lectures, University of Bologna, November 2004,
http://www.phil.cmu.edu/summerschool/2006/Sieg/computability_theory.pdf.
22. W. Sieg: Calculations by man and machine: conceptual analysis. *Lecture Notes in Logic*, 15 (2002), 390–409.