Estimating robust optimum parameters of tuned mass dampers using multi-objective genetic algorithms

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ABSTRACT

Tuned mass dampers (TMD) are a well-known control device widely used to control the vibratory problem originated by the pedestrian action on footbridges. The main purpose of this study is the robust multi-objective optimization design of a TMD using genetic algorithms to control the structural vibrations of a footbridge due to the pedestrian action. The performance of the TMD has been improved designing optimally its parameters, including, mass, stiffness and damping ratio using multi-objective genetic algorithms. Moreover, in order to take into account the uncertainties existing in the system, a robust design optimization procedure has been performed. As an example, a slender steel footbridge, modelled by 3-D frame elements, is used to assess numerically the performance and accuracy of the proposed method. The pedestrian action has been simulated by an equivalent harmonic force. The proposed approach is compared with the classical Den Hartog's proposal. This comparison shows that this approach is more effective than the classical reported method and more feasible due to the smaller TMD parameters.

Keywords: pedestrian, structural control, tuned mass damper, robust design optimization, multi-objective genetic algorithms.

1. INTRODUCTION

A tuned mass damper (TMD) is a passive control system formed by mechanical components such as mass, spring and viscous damper. This damping device has been installed in slender footbridges for controlling vibrations. Although active vibration control is more effective for control of civil engineering structures, its high cost and unreliability favour the practical use of passive control techniques. TMDs are commonly used for retrofit of footbridges experiencing vibratory problems, as these devices can be easily attached to the deck of the structure without any condition (**Figure 1.a**). Large advances have been performed in the design of this type of dampers to control the response of buildings under the earthquake action [**1**]. However, when it comes to control the response of a slender footbridge under a pedestrian flow, the current standards [**2**] still maintain the design principles proposed by Den Hartog [**3**] that considered only an undamped main system with a single degree of freedom.

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In this paper, the multi-objective genetic algorithms (MGA) are utilized to find the optimum parameters of a TMD which is implemented in the mid-span of a footbridge. A program has been developed for the optimization of the TMD parameters (mass, stiffness and damping coefficient). None of the TMD parameters have been preselected in order to obtain an economical result. As optimization criteria both the vertical acceleration at the mid-span of a steel footbridge under pedestrian flows and the mass of the TMD have been considered. In order to take into account the uncertainties existing in the system, a robust design optimization (RDO) procedure has been performed. A non-deterministic optimization approach has been implemented which probabilistic uncertainties have been considered for uncertain parameters and a stochastic optimal design has been applied. A numerical pedestrian footbridge, reported in the literature [4], under the crossing of different pedestrian densities has been used and the results have been compared with the classical Den Hartog's proposal [3] in order to show the efficacy of the proposed approach.

2. EQUATIONS OF MOTION

The implementation of the TMD-footbridge interaction model (**Figure 1.b**) has been performed from the following equations [5]

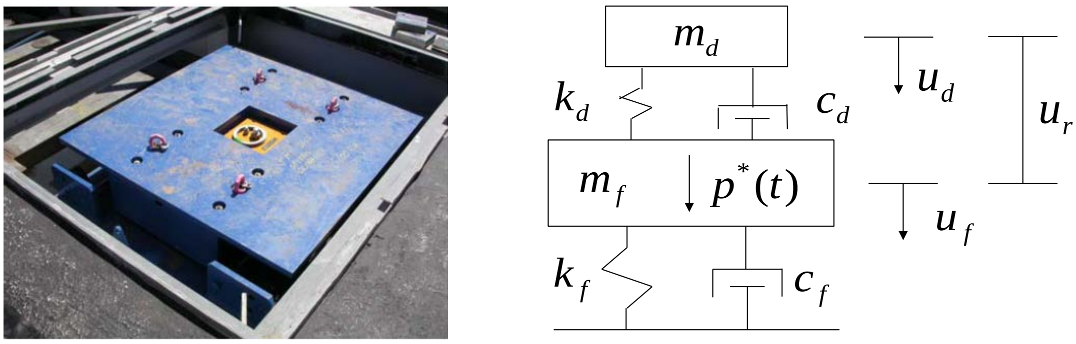

Figure 1. *a) TMD installed under the deck of a footbridge [2]. b) TMD-footbridge interaction model.*

$$m_f \cdot \ddot{u}_f + c_f \cdot \dot{u}_f + c_d \cdot \left(\dot{u}_f - \dot{u}_d\right) + k_f \cdot u_f + k_d \cdot \left(u_f - u_d\right) = p^*(t) \quad (1)$$

$$m_d \cdot \ddot{u}_d + c_d \cdot \left(\dot{u}_d - \dot{u}_f\right) + k_d \cdot \left(u_d - u_f\right) = 0 \quad (2)$$

$$p^*(t) = p(t) \cdot \phi(x) = G \cdot \cos(w_f \cdot t) \cdot n' \psi \cdot \phi(x) \quad (3)$$

where:

$\ddot{u}_f, \dot{u}_f, u_f$ is the acceleration, velocity and displacement of the footbridge [m/s^2, m/s, m].

$\ddot{u}_d, \dot{u}_d, u_d$ is the acceleration, velocity and displacement of the TMD [m/s^2, m/s, m].

$u_r = u_d - u_f$ is the relative displacement between m_d and m_f [m].

m_f is the modal mass of the footbridge [kg].

$c_f = 2 \cdot m_f \cdot 2 \cdot \pi \cdot f_f \cdot \zeta_f$ is the modal damping of the footbridge [Ns/m].

$k_f = m_f \cdot (2 \cdot \pi \cdot f_f)^2$ is the modal stiffness of the footbridge [N/m].

f_f is the natural frequency of the considered vibration mode of the footbridge [Hz].

ζ_f is the modal damping ratio of the footbridge [%].

m_d is the mass of the TMD [kg].

$c_d = 2 \cdot m_d \cdot 2 \cdot \pi \cdot f_d \cdot \zeta_d$ is the damping of the TMD [Ns/m].

$k_d = m_d \cdot (2 \cdot \pi \cdot f_d)^2$ is the stiffness of the TMD [N/m].

f_d is the natural frequency of the TMD [Hz].

ζ_d is the modal damping ratio of the TMD.

$p(t)$ is the equivalent harmonic pedestrian load [2].

G is the dynamic component of the pedestrian step load (280 N for vertical direction [2]).

n' is the equivalent number of the n pedestrians on the footbridge [2]. Its value may be determined from the following equation according to the pedestrian density, d , (P=Person/m[2]).

$$n' = \begin{matrix} 10.8 \cdot \sqrt{\zeta_f \cdot n} \\ 1.85 \cdot \sqrt{n} \end{matrix} \text{ if } \begin{matrix} d < 1.00 \ P/m^2 \\ d \geq 1.00 \ P/m^2 \end{matrix} \tag{4}$$

ψ is the reduction coefficient to take into account the probability that the footfall frequency approaches the natural frequency under consideration [2]. In the vertical direction, it may be estimated from the following equation according to the considered natural frequency of the footbridge, f_f

$$\psi = \begin{matrix} 0 \\ 2.22 \cdot \left(f_f - 1.25\right) \\ 1 \\ 1 - 5 \cdot \left(f_f - 2.10\right) \\ 0 \end{matrix} \text{ if } \begin{matrix} f_f \leq 1.25 & Hz \\ 1.25 < f_f < 1.70 & Hz \\ 1.70 \leq f_f \leq 2.10 & Hz \\ 2.10 < f_f < 2.30 & Hz \\ f_f \geq 2.30 & Hz \end{matrix} \tag{5}$$

$\phi(x)$ is the considered numerical vibration mode.

Substituting these relations in the overall dynamic equilibrium equation of the structure and organizing information in a matrix form, the following model of interaction is obtained.

$$\mathbf{M} \cdot \ddot{\mathbf{u}}(t) + \mathbf{C} \cdot \dot{\mathbf{u}}(t) + \mathbf{K} \cdot \mathbf{u}(t) = \mathbf{F}(t) \tag{6}$$

$$\mathbf{M} = \begin{bmatrix} m_f & 0 \\ 0 & m_d \end{bmatrix}; \ \mathbf{C} = \begin{bmatrix} c_f + c_d & -c_d \\ -c_d & c_d \end{bmatrix}; \ \mathbf{K} = \begin{bmatrix} k_f + k_d & -k_d \\ -k_d & k_d \end{bmatrix}; \ \mathbf{F}(t) = \begin{bmatrix} p^*(t) \\ 0 \end{bmatrix} \tag{7}$$

$$\ddot{\mathbf{u}}(t) = \begin{bmatrix} \ddot{u}_f(t) \\ \ddot{u}_d(t) \end{bmatrix}; \ \dot{\mathbf{u}}(t) = \begin{bmatrix} \dot{u}_f(t) \\ \dot{u}_d(t) \end{bmatrix}; \ \mathbf{u}(t) = \begin{bmatrix} u_f(t) \\ u_d(t) \end{bmatrix} \tag{8}$$

Considering the nature of the resulting system, the use of a β-Newmark integration method is proposed, with parameters β=1/4 and γ=1/2, thus ensuring an unconditionally stable system.

3. TMD DESIGN BASED ON A CLASSICAL APPROACH

The design procedure proposed by international standards [2] for the optimum design of a TMD, based on the classical Den Hartog's proposal (DH) [3], may be summarized as follows:

(i) Choice of TMD mass, m_d, based on the mass ratio $\mu = m_d / m_f$, being typical values in the range from 0.01 to 0.05.

(ii) Calculation of the optimum TMD ratios: frequency, δ_{opt}, and damping ratio, ζ_{opt}.

$$\delta_{opt} = 1/(1+\mu) = f_d / f_f \tag{9}$$

$$\zeta_{opt} = \sqrt{3\mu/(8\cdot(1+\mu)^3)} \tag{10}$$

(iii) Calculation of the TMD constants: spring and damping constant.

$$k_d = m_d \cdot (2\cdot\pi\cdot\delta_{opt}\cdot f_f)^2 \tag{11}$$

$$c_d = 2\cdot m_d \cdot 2\cdot\pi\cdot\delta_{opt}\cdot f_f \cdot\zeta_{opt} \tag{12}$$

In this way, the design consists of selecting of the minimum mass ratio, μ, that reduces the acceleration of the structure under an allowable value, a_{adm}, according to the established comfort level of the structure [2]. Further, the relative displacement, u_r, between the TMD and the footbridge must check a physical limit established by the manufactured (20 mm for this study).

4. TMD DESIGN BASED ON MULTIOBJECTIVE GENETIC ALGORITHMS

In the multi-objective optimization process several objective functions are minimized, offering an optimal set of solutions rather than one optimal value and being difficult to find which set dominate the others. The optimal solutions are called Pareto front. The determination of the set optimal solution, inside the Pareto front, will be made considering additional information based on the physical knowledge of the problem. It is well-accepted that genetic algorithms work adequately to solve multi-objective optimization problems. In this study, the multi-objective optimization is performed by a program developed in Matlab software [6]. In multi-objective optimization, the purpose is to find a design vector $\mathbf{x} = \{x_1 \quad x_2 \quad \ldots \quad x_n\}^T$ which could optimize *k* objective functions $\mathbf{f} = \{f_1 \quad f_2 \quad \ldots \quad f_k\}^T$ .in a search domain, being *n* number of design variables.

5. ROBUST DESIGN OF THE TMD SYSTEM.

The dynamic response of a footbridge is conditioned by its natural frequencies, mode shapes and its damping. Therefore, in this study, it is assumed that the natural frequencies and the damping ratio of the footbridge may vary from the values considered originally in deterministic analyses, treating these parameters as uncertain by normal probabilistic distributions. These uncertainties can affect the design performance based on deterministic optimization, so they must be taken into account. A procedure for their consideration is the application of a robust design optimization [7]. In order to simulate this stochastic behaviour, the main procedure used in robust design method is Monte Carlo simulation. According to this method, random variables are generated assuming pre-defined probabilistic distributions for the uncertain parameters. The structure is then simulated considering each of these randomly generated variables, reflecting the percentage of cases situated in the failure region, defined by a limit state function, the probability of failure. Subsequently, the goal of the method is to minimize the mean and variance of each objective function simultaneously.

6. NUMERICAL EXAMPLE

We have selected an example, which exists in the literature [4], in order to present the method of optimization of the TMD parameters using MGA. The footbridge studied is a concrete-steel composite structure with an only span of 38.25 m. Its longitudinal profile is curved to a radius of 450 m. The framework is configured by two Warren-type lateral beams with a constant depth of 1.215 m. These beams are connected by floor beams located at the level of the lower member. A precast reinforced concrete slab, 10 cm thick, rest on these crossbeams. The distance between the two lateral beams (center lines) is about 2.90 m, giving a horizontal clearance of 2.50 m (**Figure 2**).

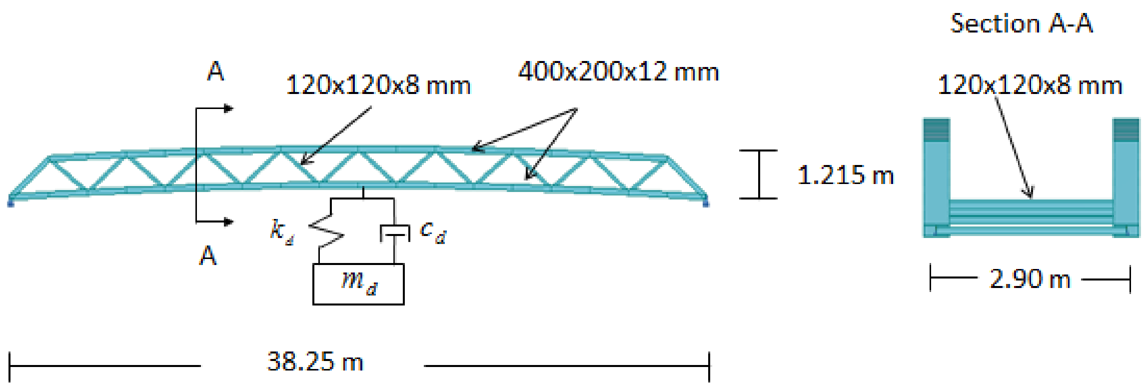

Figure 2. *Elevation of the considered footbridge [3] and location of the TMD.*

A finite element model of the footbridge has been developed by the software Ansys [8]. A numerical modal analysis of the footbridge has been performed under five load cases (L.C,) according different values of the pedestrian density, d , estimated on the footbridge. A medium pedestrian mass of 70 kg has been considered. The change of modal mass, m_f , and the first natural frequency, f_f , of the footbridge is shown in **Table 1**.

Table 1. Change of the dynamic properties of the footbridge versus the load scenario.

L.C.	d [P/m^2]	a_{adm} [m/s^2]	n [P]	m_f [kg]	f_f [Hz]	a_{max} [m/s^2]
I	<0.20	0.50	15	34706	2.14	1.05
II	0.20	0.50	19	35984	2.10	1.16
III	0.50	0.50	48	37710	2.05	1.74
IV	0.80	0.50	76	39500	2.00	2.08
V	1.00	0.50	96	40750	1.97	4.96

Subsequently, the maximum vertical acceleration under the five load case has been determined, considering, as it is recommended by different authors [2, 9], a variable damping modal ratio according to the displacement of the footbridge $\zeta_f = \min(0.3 + 0.3/0.005 \cdot u_f, 0.6)\,\%$. The demanded comfort level, a_{adm}, is not checked (Table 1).

6.1. Deterministic TMD design

In order to reduce the vibration level of the footbridge a TMD device has been added at the maximum deflection location of the first vertical vibration mode (Figure 2). The parameters of the TMD have been obtained numerically from two deterministic approaches, the classical Den Hartog's proposal (DH) and the proposed multi-objective optimization using genetic algorithms (MGA). In the DH proposal the minimum value of the mass ratio, μ, has been obtained by its iterative variation in order to check that the maximum vertical acceleration, $a_{\max}$, under the five load conditions was lower than the allowable acceleration, a_{adm}. For the genetic algorithm optimization, the multi-objective function is defined as follows:

$$\mathbf{f} = \left\{ \mu \quad \max\left(a_{\max}^{I} / a_{\lim} \quad \ldots \quad a_{\max}^{V} / a_{\lim} \right) \right\} \tag{13}$$

being $a_{\max}^{i}$ the maximum vertical acceleration at mid-span under the load conditions I, and $a_{\lim}$ the allowable acceleration. Only one parameter (μ) has been considered for the evaluation of the Den Hartog's proposal while three parameters (μ, δ_{opt}, ζ_{opt}) have been determined in the multi-objective optimization. To avoid ill-conditioning problems, a search domain has been developed, constraining the problem, $\mu \in [0.01 - 0.10]$, $\delta_{opt} \in [0.85 - 1.15]$ and $\zeta_{opt} \in [0.05 - 0.20]$. The diagram of Pareto front of this optimization process is shown in Figure 3.a. From Figure 3.a it may be determined the first value of the function f_1 that produces on the function f_2 a value less than the unity. This point allows establishing the set optimal solutions (Table 2).

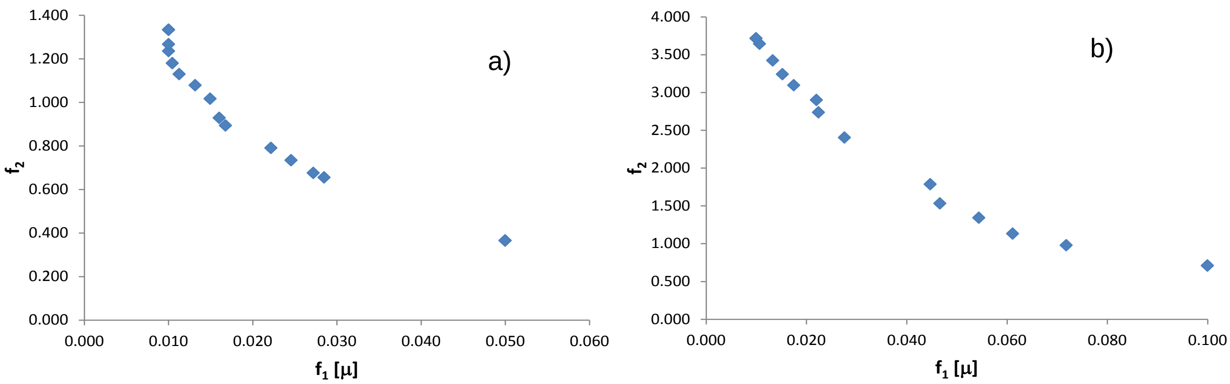

Figure 3. *Diagram of Pareto front of the TMD. a) Deterministic design and b) Robust design.*

The parameters of the TMD, according the two considered deterministic approaches, are shown in Table 2. The implementation of the MGA allows achieving a clear reduction of the parameters of the TMD (mass 54.14 %, stiffness 55.11 % and damping ratio 76.55 %).

Table 2. Design of TMD according to deterministic approaches.

Method	μ [%]	δ_{opt}	ζ_{opt} [%]	m_d [kg]	k_d [N/m]	c_d [Ns/m]	$a_{\max}$ [m/s^2]	a_{adm} [m/s^2]
DH	3.49	0.96	10.87	1211	204465	3420	0.49	0.50
MGA	1.60	0.95	5.61	555.30	91780	802	0.47	0.50

The maximum vertical acceleration at mid-span of the footbridge corresponding to the fifth load case is shown (**Table 2**). For both methods (DH and MGA), the comfort limit is checked with the demanded maximum level ($a_{adm} = 0.50$ m/s^2).

6.2. Robust TMD design

Subsequently, in order to take account in the design the uncertain of the system, a stochastic robust design optimization procedure has been performed. 50 possible variations of the original footbridge have been determined considering a variation of 20 % of the deterministic first vertical natural frequency of the structure and its damping ratio. In the DH method the minimum value of the mean mass ratio, μ, has been estimated in this case.

On the other hand, for the stochastic robust design optimization, the multi-objective function is defined as follows:

$$\mathbf{f} = \left\{ \mu \quad E(\max(a^I_{\max}/a_{\lim} \quad \dots \quad a^V_{\max}/a_{\lim})) + 2\cdot\sigma(\max(a^I_{\max}/a_{\lim} \quad \dots \quad a^V_{\max}/a_{\lim})) \right\} \tag{14}$$

being $E()$ the mean value and $\sigma()$ the standard deviation.

The design variables and the search domain are the same one defined in the deterministic procedure. The diagram of Pareto front of this robust MGA process is shown in **Figure 3.b**. From **Figure 3.b** it may be determined the first value of the function f_1 that produces on the function f_2 a value less than the unity. This point allows establishing the set optimal solutions (**Table 3**).

Table 3. Robust design of TMD.

Method	μ [%]	δ_{opt}	ζ_{opt} [%]	m_d [kg]	k_d [N/m]	c_d [Ns/m]	$a_{\max}$ [m/s^2]	a_{adm} [m/s^2]
DH	9.60	0.91	16.54	3332	501466	13518	0.48	0.50
MGA	7.20	0.99	14.90	2498	442786	9912	0.49	0.50

The parameters of the TMD, according the two considered non-deterministic approaches, are shown in **Table 3**. Again, the implementation of the robust MGA allows achieving a clear reduction of the parameters of the TMD (mass 25.00 %, stiffness 11.70 % and damping ratio 26.67 %).

The maximum vertical acceleration at mid-span of the footbridge corresponding to the fifth load case is shown (**Table 3**). In both methods (robust DH and MGA), the comfort limit is checked with the demanded maximum level ($a_{adm} = 0.50$ m/s^2).

7. CONCLUSIONS

The object of this paper is to determine the optimum parameters of TMD in order to reduce the responses of footbridge under pedestrian walking action. Optimum parameters must be minimum physical, practical and economical values. Multi-objective genetic algorithms are used to optimize the parameters of TMD (mass, stiffness and damping). The response of a footbridge under a pedestrian flow is checked during the optimization process being the pedestrian action simulated as a harmonic load. The uncertainties existing in the footbridge, variation of its modal parameters from its deterministic values has been considered by the implementation of a robust design optimization procedure in the optimization design process. The efficiency of the method is validated comparing the results between the proposed method and the classical Den Hartog's proposal. The TMD parameters obtained from the proposed methodology are smaller than the values obtained from the compared method which is beneficial for both the footbridge, the force applied by the damper is less, and the damping device also because its cost is also less. However, further studies are needed in order to apply the methodology under different optimization methods and asses experimentally the efficiency of the tuned mass dampers on real footbridges.

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