XX Congreso de Ecuaciones Diferenciales y Aplicaciones
X Congreso de Matemática Aplicada
Sevilla, 24-28 septiembre 2007
(pp. 1–8)

Existence and Uniqueness of Strong Solutions for the Incompressible Micropolar Fluid Equations in Domains of $\mathbb{R}^3$

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Palabras clave: Micropolar fluids, unbounded domains, hydrodynamics. existence of solutions

Resumen

We consider the initial boundary value problem for the system of equations describing the nonstationary flow of an incompressible micropolar fluid in a domain Ω of $\mathbb{R}^3$. Under hypotheses that are similar to the Navier-Stokes equations ones, by using an iterative scheme, we prove the existence and uniqueness of strong solution in $L_p(\Omega)$, for $p > 3$.

1. Introduction

The objective of the present work is to study the existence of strong solutions of the evolution equations for the motion of incompressible micropolar (asymmetric) fluids in a bounded or unbounded domain $\Omega \subset \mathbb{R}^3$ having a compact C^2-boundary. That is, the domains we are considering include the the so called exterior domains. To describe these equations, let $T > 0$ and $Q_T \equiv \Omega \times (0, T)$; then the system we will study is the following:

$$\left\{\begin{array}{l} \dfrac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} - (\mu+\mu_r)\Delta\mathbf{u} + \nabla\eta = 2\mu_r \operatorname{rot}\mathbf{w} + \mathbf{f} \qquad \text{in } Q_T\,, \\ \operatorname{div}\,\mathbf{u} = 0 \qquad \text{in } Q_T\,, \\ \dfrac{\partial \mathbf{w}}{\partial t} + (\mathbf{u}\cdot\nabla)\,\mathbf{w} - (c_a+c_d)\Delta\mathbf{w} + 4\mu_r\mathbf{w} \\ \qquad\qquad\qquad - (c_0+c_d-c_a)\,\nabla\operatorname{div}\,\mathbf{w} = 2\mu_r \operatorname{rot}\,\mathbf{u} + \mathbf{g} \qquad \text{in } Q_T\,, \end{array}\right. \tag{1}$$

together with the following boundary and initial conditions

$$\begin{cases} \mathbf{u} = \mathbf{0} \qquad \text{on } S_T\,, \\ \mathbf{w} = \mathbf{0} \qquad \text{on } S_T\,, \\ \mathbf{u}(x,0) = \mathbf{u}_0(x) \qquad \text{in } \Omega\,, \\ \mathbf{w}(x,0) = \mathbf{w}_0(x) \qquad \text{in } \Omega\,, \end{cases} \tag{2}$$

where $S_T \equiv \partial\Omega \times (0,T)$. The vector-valued functions $\mathbf{u} = (u_1, u_2, u_3)$, $\mathbf{w} = (w_1, w_2, w_3)$ and the scalar function η denote respectively the velocity, the angular velocity of rotation of particles and the pressure of the fluid. The vector-valued functions $\mathbf{f}$ and $\mathbf{g}$ denote respectively the external sources of linear and angular momentum. The positive constants μ, μ_r, c_0, c_a and c_d are viscosities-type coefficients satisfying the following inequality $c_0 + c_d > c_a$.

For the derivation and physical discussion of equations (1)-(2) see Petrosyan [15], Condiff and Dalher [1], Eringen [4], [5] and Lukaszewicz [9]. We observe that this model of fluid include as particular case the classical Navier-Stokes equations, which has been widely studied (see for instance the books by Ladyzhenskaya [6] or Temam [24], and the references therein). In this case, since $\mu_r = 0$, equations (1) and (2) decouple.

It is appropriate to recall earlier works on the initial-value problems closely related to (1)-(2) in order to clarify the intended contribution of the present work.

Let us firstly consider the situation when Ω is a bounded regular domain. In this case, Lukaszewicz [9] established for a restricted class of initial data, the existence of weak and strong global solutions using, in both cases, an iterative linearized scheme together with a fixed point result. For initial data similar to the case of the classical Navier-Stokes equations, by applying the spectral Galerkin method, Rojas-Medar & Boldrini [17] proved the global existence and uniqueness of weak solutions in the two-dimensional case; existence of the local and global in time strong solution was obtained respectively by Rojas-Medar in [18] and by Ortega-Torres and Rojas-Medar in [13]. Ortega-Torres and Rojas-Medar, following the arguments given by Serrin in [21], also considered the uniqueness of weak solution in [14]. In [19], Rojas-Medar obtained the convergence rates associated to the approximate solutions constructed by the Galerkin method. The existence of reproductive solution (so called periodic weak solution) for the previous system was proved in [17]. Recently, Reséndiz and Rojas-Medar [16] have proved the existence of weak solution in a smooth time dependent domain. By using and interactive approach Rojas-Medar and Ortega-Torres [20] show the existence and uniqueness of the strong solutions in bounded domains in the L_2-context. The existence and uniqueness of periodic strong solutions was done in [10] using the Galerkin method. Yamaguchi [25] also studied the problem (1)-(2)in bounded domains using the semigroup approach in L_p, $1 < p < \infty$; he shows the existence of global strong solutions for small data.

The case of unbounded domains Ω is less studied. When Ω is an exterior domain, for the related model of the magneto-micropolar fluid, existence of a stationary weak solution was studied by Durán *et al.* in [2], while the existence of reproductive solution was established in [3]. For two-dimensional unbounded domains, one can look at the word by Lukaszewicz an Sadowski [12].

In the present work, as we said previously, we are interested in the flow of micropolar fluids in bounded ou unbounded domains of $\mathbb{R}^3$ with compact C^2-boundaries. By using an iterative procedure we will prove the existence and uniqueness of strong solutions in

$L_p(\Omega)$, for any $p > 3$. Specifically, we will prove the following (local) existence result of strong solutions.

Theorem 1.1 *Let $\Omega \subset \mathbb{R}^3$ have a non-void regular boundary $\partial\Omega$ in the sense of Solonnikov and let $p > 3$. Assume that $\mathbf{u}_0(x) \in W^{2-\frac{2}{p}}(\Omega)$, $\mathbf{u}_0|_{S_T} = 0$, $\operatorname{div} \mathbf{u}_0 = 0$, $\mathbf{w}_0(x) \in W_p^{2-\frac{2}{p}}(\Omega)$, $\mathbf{w}_0|_{S_T} = 0$, $\mathbf{f}, \mathbf{g} \in L_p(Q_T)$.*

Then there exists $T_1 \in (0, T]$ such that problem (1)-(2) has a unique solution $(\mathbf{u}, \mathbf{w}, \eta)$ satisfying $\mathbf{u} \in W_p^{2,1}(Q_{T_1})$, $\nabla\eta \in L_p(Q_{T_1})$, $\mathbf{w} \in W_p^{2,1}(Q_{T_1})$.

In this statement, we used the classical notations for the Sobolev-type spaces $W_p^k(\Omega)$ and $W_p^{2,1}(Q_T)$.

The present work is organized as follows: in Section 2 we fix the notations, and state preliminaries results that will be useful in the rest of the paper. More precisely, we state the existence, the uniqueness and regularity (a priori estimates) for two linear problems closely related to (1)-(2). We also describe in this section the iterative scheme that construct the approximate solutions. In Section 3, we obtain estimates in several norms for such approximate solutions. Finally, in Section 4, we show that the approximate the solutions converge to a strong solution of our original problem.

We remark that, as it is usual in this kind of context to simplify the notations, we will denote by c, C_0, M_0 and so on generic finite positive constants depending only on Ω and the other fixed parameters of the problem (like the initial data). That is, they may have different values in different expressions. In a few points to emphasize the fact that the constants are in fact different, we use $C_1, C_2, ..., M_1, M_2. \cdots$ and so on.

2. Preliminaries and iterative scheme

For any $t \in (0, T]$, we will denote $Q_t = \Omega \times (0, t)$. As previously said, we will use classical notations for the Sobolev-type spaces; we will also use freely the standard results for such spaces. Here we just recall that the restriction of a function in $W_p^{2,1}(Q_T)$ on the hyperplane $t =$ constant belongs for $\forall\ t \in [0, T]$ to the Slobodetskii-Besov space $W_p^{2-\frac{2}{p}}(\Omega)$ and depend continuously on t in the norm of $W_p^{2-\frac{2}{p}}(\Omega)$. Moreover, it holds that

$$\|\mathbf{u}(\cdot, t)\|_{W_p^{2-\frac{2}{p}}(\Omega)} \leq \|\mathbf{u}(\cdot, 0)\|_{W_p^{2-\frac{2}{p}}(\Omega)} + \widehat{c}\|\mathbf{u}\|_{W_p^{2,1}(Q_T)}, \tag{3}$$

where the constant $\widehat{c}$ does not depend on $t \in [0, T]$. For more details of the Slobodetskii-Besov space see [8], for instance.

Next, we recall some results associated to two linear problems closely related to (1)-(2). The first result is proved in Solonnikov [23] and is the following:

Lemma 2.1 *Let $F(x, t) \in L_p(Q_T)$ and $\mathbf{u}_0(x) \in W_p^{2-\frac{2}{p}}(\Omega)$ with $\mathbf{u}_0|_{S_T} = 0$ and $\operatorname{div} \mathbf{u}_0 = 0$, then the following problem*

$$\begin{aligned}
&\mathbf{u}_t - (\mu + \mu_r)\Delta\mathbf{u} + \nabla\eta = F, \\
&\operatorname{div} \mathbf{u} = 0, \\
&\mathbf{u}|_{S_T} = 0, \\
&\mathbf{u}(0) = \mathbf{u}_0(x)
\end{aligned}$$

has a unique solution $\mathbf{u} \in W_p^{2,1}(Q_T)$*,* $\eta \in W_p^{1,0}(Q_T)$ *(*η *is unique up to a constant,) satisfying*

$$\|\mathbf{u}\|_{W_p^{2,1}(Q_{T_1})} + \|\nabla\eta\|_{L_p(Q_{T_1})} \le K_1(T_1)(\|\mathbf{u}_0\|_{W_p^{2-\frac{2}{p}}(\Omega)} + \|F\|_{L_p(Q_{T_1})}),$$

where $K_1(\cdot)$ *is an increasing function of* $T_1 \in (0,T]$

The following result is a special case of the result for parabolic system given in [22].

Lemma 2.2 *Let* $G(x,t) \in L_p(Q_T)$ *and* $\mathbf{w}_0(x) \in W_p^{2-\frac{2}{p}}(\Omega)$ *with* $\mathbf{w}_0|_{S_T} = 0$*, then the following problem*

$$\begin{aligned}
&\mathbf{w}_t - (c_a + c_d)\Delta\mathbf{w} - (c_0 + c_d - c_a)\nabla \mathrm{div}\ \mathbf{w} + 4\mu_r\mathbf{w} = G\\
&\mathbf{w}|_{S_T} = 0,\\
&\mathbf{w}(0) = \mathbf{w}_0(x)
\end{aligned}$$

has a unique solution $\mathbf{w} \in W_p^{2,1}(Q_T)$*, satisfying*

$$\|\mathbf{w}\|_{W_p^{2,1}(Q_{T_1})} \le K_2(T_1)(\|\mathbf{w}_0\|_{W_p^{2-\frac{2}{p}}(\Omega)} + \|G\|_{L_p(Q_{T_1})}),$$

where $K_2(\cdot)$ *is an increasing function of* $T_1 \in (0,T]$*.*

Iterative Scheme:

Next, we describe the iteration scheme used to construct approximate solutions of our problem.

Take

$$\mathbf{u}^{(0)} = \mathbf{0},\ \mathbf{w}^{(0)} = \mathbf{0}$$

and for $k = 1,2,3,\ldots$ recursively take $\{\mathbf{u}^{(k)}, \eta^{(k)}\}$ and $\{\mathbf{w}^{(k)}\}$ respectively as the solutions of problems

$$\begin{aligned}
&\mathbf{u}_t^{(k)} - (\mu + \mu_r)\triangle\mathbf{u}^{(k)} + \nabla\eta^{(k)} = \mathbf{f} + 2\mu_r \ \mathrm{rot}\ \ \mathbf{w}^{(k-1)} - (\mathbf{u}^{(k-1)} \cdot \nabla)\mathbf{u}^{(k-1)},\\
&\mathrm{div}\ \mathbf{u}^{(k)} = 0,\\
&\mathbf{u}^{(k)}|_{S_T} = 0,\\
&\mathbf{u}^{(k)}(0) = \mathbf{u}_0(x)
\end{aligned}$$

and

$$\begin{aligned}
&\mathbf{w}_t^{(k)} - (c_a + c_d)\triangle\mathbf{w}^{(k)} - (c_0 + c_d - c_a)\nabla\ \mathrm{div}\ \mathbf{w}^{(k)} + 4\mu_r\mathbf{w}^{(k)}\\
&\qquad\qquad = \mathbf{g}+2\mu_r\ \mathrm{rot}\ \mathbf{u}^{(k-1)} - (\mathbf{u}^{(k-1)} \cdot \nabla)\mathbf{w}^{(k-1)},\\
&\mathbf{w}^{(k)}|_{S_T} = 0,\\
&\mathbf{w}^{(k)}(0) = \mathbf{w}_0(x).
\end{aligned}$$

3. Estimates of the approximate solutions

To obtain the required estimates for the sequence $(\mathbf{u}^k, \eta^k, \mathbf{w}^k)$, we start by defining:

$$\Phi^{(k)}(T_1) = \|\mathbf{u}^{(k)}\|_{W_p^{2,1}(Q_{T_1})} + \|\mathbf{w}^{(k)}\|_{W_p^{2,1}(Q_{T_1})} + \|\nabla\eta^{(k)}\|_{L_p(Q_{T_1})}, \tag{4}$$

for $0 < T_1 \le T$

Then, we can prove the following two lemmas.

Lemma 3.1 *The elements of the sequence $\{\mathbf{w}^{(k)}\}$ satisfy for any $T_1 \in (0,T]$ the following estimate:*

$$\|\nabla \mathbf{w}^{(k-1)}\|_{L_p(Q_{T_1})} \le C(\|\mathbf{w}_0\|_{W_p^{2-\frac{2}{p}}(\Omega)} + aT_1^{\frac{1-a}{ap}}\Phi^{(k-1)}(T_1) + T^{\delta_1}\Phi^{(k-1)}(T_1)),$$

where C is independent of $T_1 \in (0,T]$ and

$$a = \frac{p-3}{2p-3} \quad \textit{and} \quad \delta_1 = (1-\frac{1}{p})(1-\frac{3}{p})(1-a) + \frac{1-a}{p}.$$

Remark 3.2 *Analogous result is valid for $\{\mathbf{u}^{(k)}\}$.*

Lemma 3.3 *Let $0 < T_1 \le 1$. Then, there is a constant $\alpha > 0$ such that*

$$\|(\mathbf{u}^{(k-1)} \cdot \nabla)\mathbf{w}^{(k-1)}\|_{L_p(Q_{T_1})} \le C[\|\mathbf{u}_0\|^2_{W_p^{2-\frac{2}{p}2}(\Omega)} + \|\mathbf{w}_0\|^2_{W_p^{2-\frac{2}{p}2}(\Omega)} + T^{\alpha}(\Phi^{(k-1)}(T_1))^2].$$

where C is independent of $T_1 \in (0,T]$.

Next, we prove the boundness of the sequence $\{\mathbf{u}^{(k)}, \eta^{(k)}, \mathbf{w}^{(k)}\}$.

Lemma 3.4 *For sufficiently small $T_1 \in (0,T]$, the sequence $\{\mathbf{u}^{(k)}, \eta^{(k)}, \mathbf{w}^{(k)}\}$ is bounded in $W_p^{2,1}(Q_{T_1}) \times L_p(Q_T) \times W_p^{2,1}(Q_{T_1})$.*

4. Proof of Theorem 1.1

Setting $\mathbf{u}^{(n,s)}(t) = \mathbf{u}^{(n+s)}(t) - \mathbf{u}^{(n)}(t)$, $\eta^{(n,s)} = \eta^{(n+s)} - \eta^{(n)}$ and $\mathbf{w}^{(n,s)} = \mathbf{w}^{(n+s)} - \mathbf{w}^{(n)}$, we have

$$\begin{aligned} &\mathbf{u}^{(n,s)}t - (\mu + \mu_r)\triangle \mathbf{u}^{(n,s)} + \nabla \eta^{(n,s)} = F^{(n,s)}, \\ &\operatorname{div} \mathbf{u}^{(n,s)} = 0, \\ &\mathbf{u}^{(n,s)}|_{S_T} = 0, \\ &\mathbf{u}^{(n,s)}(0) = 0, \end{aligned} \tag{5}$$

where

$$F^{(n,s)} = 2\mu_r \text{ rot } \mathbf{w}^{(n-1,s)} - (\mathbf{u}^{(n-1,s)} \cdot \nabla)\mathbf{u}^{(n+s-1)} - (\mathbf{u}^{(n-1)} \cdot \nabla)\mathbf{u}^{(n-1,s)}. \tag{6}$$

Also

$$\begin{aligned} &\mathbf{w}_t^{(n,s)} - (c_a + c_d)\triangle \mathbf{w}^{(n,s)} - (c_0 + c_d - c_a)\nabla \operatorname{div} \mathbf{w}^{(n,s)} + 4\mu_r \mathbf{w}^{(n,s)} = G^{(n,s)}, \\ &\mathbf{w}^{(n,s)}|_{S_T} = 0, \\ &\mathbf{w}^{(n,s)}(0) = 0, \end{aligned} \tag{7}$$

where

$$G^{(n,s)} = 2\mu_r \text{ rot } \mathbf{u}^{(n-1,s)} - (\mathbf{u}^{(n+s-1)} \cdot \nabla)\mathbf{w}^{(n-1,s)} - (\mathbf{u}^{(n-1,s)} \cdot \nabla)\mathbf{w}^{(n-1)}. \tag{8}$$

We then are able to prove that

$$\begin{aligned}
\|F^{(n,s)}\|^p_{L_p(Q_t)} \;\leq\; & c\int_0^t \|\mathbf{u}^{(n-1,s)}\|^p_{W_p^{2,1}(Q_\tau)}d\tau + (\|\mathbf{u}_0\|_{W_p^{2-\frac{2}{p}}(\Omega)} \\
& +\widehat{c}\|\mathbf{u}^{(n-1+s)}(\tau)\|_{W_p^{2,1}(Q_t)})^p \int_0^t \widehat{c}^p \|\mathbf{u}^{(n-1,s)}\|^p_{W_p^{2,1}(Q_\tau)}d\tau \\
& +(\|\mathbf{u}_0\|_{W_p^{2-\frac{2}{p}}(\Omega)} \\
& +\widehat{c}\|\mathbf{u}^{(n-1+s)}(\tau)\|_{W_p^{2,1}(Q_t)})^p \int_0^t \widehat{c}^p \|\mathbf{u}^{(n-1,s)}\|^p_{W_p^{2,1}(Q_\tau)}d\tau.
\end{aligned} \tag{9}$$

$$\begin{aligned}
\|G^{(n,s)}\|^p_{L_p(Q_t)} \;\leq\; & c(\|\nabla \mathbf{u}^{(n-1,s)}\|^p_{L_p(Q_t)} + \|(\mathbf{u}^{(n-1,s)}\cdot\nabla)\mathbf{w}^{(n-1)}\|^p_{L_p(Q_t)} \\
& +\|(\mathbf{u}^{(n+s-1)}\cdot\nabla)\mathbf{w}^{(n-1,s)}\|^p_{L_p(Q_t)} \\
\leq\; & c\int_0^t \|\mathbf{u}^{(n-1,s)}\|^p_{W_p^{2,1}(Q_\tau)}d\tau + c(\|\mathbf{w}_0\|_{W_p^{2-\frac{2}{p}}(\Omega)} \\
& +\widehat{c}\|\mathbf{w}^{(n-1)}(\tau)\|_{W_p^{2,1}(Q_t)})^p \int_0^t \|\mathbf{u}^{(n-1,s)}\|^p_{W_p^{2,1}(Q_\tau)}d\tau \\
& +c(\|\mathbf{u}_0\|_{W_p^{2-\frac{2}{p}}(\Omega)} \\
& +\widehat{c}\|\mathbf{u}^{(n+s-1)}\|_{W_p^{2,1}(Q_t)})^p \int_0^t \|\mathbf{w}^{(n-1,s)}\|^p_{W_p^{2,1}(Q_\tau)}d\tau.
\end{aligned} \tag{10}$$

From estimates (9)-(10) and Lemma 3.4, we conclude that for $t \in [0,T_1]$ and $p > 3$, if we call

$$\Psi^{(n,s)}(t) = \|\mathbf{u}^{(n,s)}\|_{W_p^{2,1}(Q_t)} + \|\mathbf{w}^{(n,s)}\|_{W_p^{2,1}(Q_t)} + \|\nabla\eta^{(n,s)}\|_{L_p(Q_t)}, \tag{11}$$

we then have

$$\Psi^{(n,s)}(t) \leq c\left(\int_0^t \Psi^{(n-1,s)}(\tau)^p\right)^{\frac{1}{p}}.$$

Therefore,

$$\left[\Psi^{(n,s)}(t)\right]^p \leq c^p \int_0^t \left[\Psi^{(n-1,s)}(\tau)\right]^p d\tau, \tag{12}$$

and consequently $\Psi^{(n,s)}(t) \to 0$ as $n \to \infty$, $\forall\, t \in [0,T_1]$.

In particular, since $W_p^{2,1}(Q_{T_1})$ and $L_p(Q_{T_1})$ are Banach spaces, there exist $\mathbf{u},\mathbf{w} \in W_p^{2,1}(Q_{T_1})$ and $\eta \in L_p(Q_{T_1})$ such that

$$\begin{aligned}
\mathbf{u}^n &\to \mathbf{u} \text{ strongly in } W_p^{2,1}(Q_{T_1}),\\
\mathbf{w}^n &\to \mathbf{w} \text{ strongly in } W_p^{2,1}(Q_{T_1}),\\
\eta^n &\to \eta \text{ strongly in } L_p(Q_{T_1}).
\end{aligned}$$

The next step is to take the limit as $n \to +\infty$ in the approximate equations in the iterative scheme. However, once the above convergences have been established, this is standard and we obtain that $\mathbf{u},\mathbf{w},\eta$ is a strong solution of the problem (1)-(2).

We need only to consider the uniqueness of the solution in order to complete the proof of Theorem. For this, suppose that there exists another solution $\mathbf{u}_1, \mathbf{w}_1, \eta_1$ of (1) and (2) with the same regularity as stated in the theorem. Then, define

$$U = \mathbf{u}_1 - \mathbf{u},\, W = \mathbf{w}_1 - \mathbf{w},\, P = \eta_1 - \eta,$$

and observe that these auxiliary functions verify a set of equations similar to (5)-(7). Repeating the arguments used to obtain (12), using the known regularity of the solutions, we get for $\theta(t) = \|U\|^p_{W^{2,1}_p(Q_t)} + \|W\|^p_{W^{2,1}_p(Q_t)} + \|P\|^p_{L_p(Q_t)}$ an inequality of the following type

$$\theta(t) \leq c \int_0^t \theta(\tau) d\tau$$

which by Gronwall's inequality implies that $U = 0$, $W = 0$, $P = 0$ and thus the uniqueness of our strong solutions.

Acknowledgments

During this research J.L. Boldrini was partially supported by CGCI MECD-DGU Brazil/Spain Grant 2137-05-4; M.A. Rojas-Medar was partially supported by D.G.E.S. and M.C. y T. (Spain) Grant BFM2003-06446-C02-01 and CGCI MECD-DGU Brazil/Spain Grant 2137-05-4. These authors are grateful for such support.

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