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Variants of global Carleman weights in one-measurement inverse problems and fluid-structure controllability problems

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Abstract

We review some recent results on variants of global Carleman weights and Carleman inequalities applied to singular controllability and inverse problems partially developed in collaboration with the authors in a series of papers. First of all, we explain how we can modify weights to study one measurement inverse problems for the heat and wave equations with discontinuous coefficients in the principal part, in a case of locally supported boundary observations for recovering coefficients in the wave equation and we mention also some recent results for the Schödinger equation. As another important application, we show how time-variable global Carleman weights are applied to study the null- controllability for a Navier-Stokes-rigid solid problem in moving domains.

1 Carleman weights

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain and P^* a second order adjoint operator in $Q = \Omega \times I$, where I is a time interval. We suppose that P^* depends on some stationary parameter $q \in L^\infty(\Omega)$. Given some regular weight function Φ defined in Q, we perform the following change of variables in the partial differential equation $P^* z = f$ called *conjugation*:

$$w = \rho\, z, \quad \rho = \exp(-s\Phi), \quad s > 0, \tag{1}$$

$$P^* z = f \quad \Leftrightarrow \quad \rho P^*(\rho^{-1} w) = \rho\, f. \tag{2}$$

We also introduce a function $\varphi(x,t)$ such that $\nabla\Phi = -\lambda \nabla\psi\, \varphi$.
Typical examples of weights Φ are:
HEAT EQUATION: $P^* = -\delta_t - \Delta + q$, $Q = \Omega \times (0,T)$

$$\Phi(x,t) = \frac{\exp(\lambda\alpha) - \exp(\lambda\psi(x))}{T-t} \tag{3}$$

for some $\lambda > 0$ and α large enough, where ψ is some suitable regular and bounded function defined in Ω (see for instance Table 1 for some conditions on ψ and Figure 1 for typical shapes of ψ).
WAVE EQUATION: $P^* = \delta_{tt} - \Delta + q$, $Q = \Omega \times (-T,T)$

$$\Phi(x,t) = -\exp(\lambda(\psi(x) - \beta\, t^2)),\ \psi(x) = |x - x_0|^2, \tag{4}$$

where x_0 is some given point outside $\overline{\Omega}$ and $\beta \in (0,1)$ is some suitably chosen parameter.
SCHRÖDINGER EQUATION: $P^* = i\partial_t + \Delta + q$, $Q = \Omega \times (-T,T)$

$$\Phi(x,t) = \frac{\exp(\lambda\alpha) - \exp(\lambda\psi(x))}{(T-t)(T+t)},\ \psi(x) = |x - x_0|^2, \tag{5}$$

for some $\lambda > 0$ and α large enough, where x_0 is some given point outside $\overline{\Omega}$.

Let $\omega \subset\subset \Omega$ be an *internal observational or control region* and let $\Gamma_0 \subset \partial\Omega$ be a *boundary observational or control region.* We will work with *global Carleman inequalities* of the form

$$\begin{aligned} p_1(s,\lambda)\|\varphi^{1/2}\rho\,\nabla z\|^2_{L^2(Q)} + p_0(s,\lambda)\|\varphi^{3/2}\rho\, z\|^2_{L^2(Q)} \leq \\ C\left(\|\rho\, f\|^2_{L^2(Q)} + p_1(s,\lambda)\|\varphi^{3/2}\rho\,\nabla z\cdot n\|^2_{L^2(\Gamma_0\times I)} + p_0(s,\lambda)\|\varphi^{1/2}\rho\, z\|^2_{L^2(\omega\times I)}\right), \end{aligned} \tag{6}$$

where n is the unit exterior normal to Ω, p_i are the polynomial weights given in Table 1. Notice that $\rho \to 0$ exponentially as $s\Phi \to +\infty$.

The internal observational or control region ω appearing at the right hand side of the global Carleman inequality is such that the pseudoconvexity of Φ with respect to P^* holds *outside* ω (for pseudoconvexity notion see [15], [26]), in particular, $|\nabla\psi(x)| > 0$ outside ω. On the other hand, the boundary observational or control region Γ_0 is such that a strong Lopatinskii condition holds *outside* Γ_0, that is $\nabla\psi(x)\cdot n < 0$ outside Γ_0 (see [26] for a more general statement of global Carleman inequalities in this cases).

In this communication we present a collection of Carleman weights whose applications illustrate the extent of Carleman inequalities when they are applied to the study of some inverse and controllability problems.

	p_1	p_0
Heat	$s\lambda^2$	$s^3\lambda^4$
Wave	$s\lambda$	$s^3\lambda^3$
Schrödinger	$s\lambda$	$s^3\lambda^4$

Table 1: Polynomial weights in global Carleman inequalities

2 Inverse source problem for heat transmission

We consider a heat operator with discontinuous coefficients in the principal part. In this case, the function ψ has to be well adapted to this new situation and then specific global Carleman estimates can be derived. As an application of the Carleman inequality is the study of one measurement inverse problems using the general Bukhgeim-Klibanov approach [9]. The results of this section have been collected from [11], [6], [7] and [2].

Given $\Omega \subset \mathbb{R}^n$ be a bounded and regular subset. Take $\overline{\Omega}_1 \subset \Omega$ and let set $\Omega_0 = \Omega \setminus \overline{\Omega}_1$. Define S as the interface between Ω_0 and Ω_1 with unit normal n exterior to Ω_1 and let S^+ and S^- be its outer and inner sides with respect to n and $\Sigma^+ = S^+ \times (0,T)$, $\Sigma^- = S^- \times (0,T)$.

Let us consider the transmission problem

$$\left\{ \begin{array}{ll} y_t - \mathrm{div}\,(a_0(x)\nabla y) = f(x)g(x,t) & \text{in } \Omega_0 \times (0,T) \\ y_t - \mathrm{div}\,(a_1(x)\nabla y) = f(x)g(x,t) & \text{in } \Omega_1 \times (0,T) \\ y|_{\Sigma^+} = y|_{\Sigma^-}, \quad a_0 \frac{\partial y}{\partial n}|_{\Sigma^+} = a_1 \frac{\partial y}{\partial n}|_{\Sigma^-}, & y = 0 \text{ on } \partial\Omega \times (0,T)\ (*) \end{array} \right. \tag{7}$$

with $a_i \geq c_0 > 0$ a.e. in Ω and let us introduce the space $V = \{y \in C^2(\overline{\Omega}_i \times [0,T]),\ i = 0,1,\ y \text{ satisfies } (*)\}$. The inverse source problem consists in retrieving the source $f(x)$ from the knowledge of $g(x,t)$, the local trace of the solution y in $\omega_0 \times (0,T)$, where $\overline{\omega}_0 \subset \Omega_0$ and from a time slice $y(\cdot,T_0)$ for some $T_0 \in (0,T)$, but without any knowledge of the initial condition $y(\cdot,0)$. We have to assume also that some technical isotopy type condition is satisfied, see details in [11]. The inverse stability result that is obtained using a Carleman estimate for the heat operator with discontinuous coefficients is

Theorem 2.1 ([6], [7]) *Let $T_0 \in (0,T)$ and $\omega_0 \subset \Omega_0$ and let us assume that Ω_1 and Ω_0 satisfy the isotopy type conditions of [11]. Assume that y solution of (7) is such that $y, y_t \in V$. Assume that $a_1|_{S^-} - a_0|_{S^+} \geq 0$ and that $g \in C^2(\overline{\Omega} \times [0,T])$, $|g(\cdot,T_0)| \geq r_0 > 0$ a.e. in Ω. Then there exists a constant $C = C(g,\omega_0,T_0)$ such that for all $f \in L^2(\Omega)$*

$$\|f\|_{L^2(\Omega)} \leq C \left(\|y(\cdot,T_0)\|_{H^2(\Omega_0)} + \|y(\cdot,T_0)\|_{L^2(\Omega_1)} + \|y\|_{H^1(0,T;L^2(\omega_0))} \right). \tag{8}$$

The global Carleman estimate for (7) stated in [11] was firstly used in order to prove the exact controllability to trajectories for a semilinear system similar to (7) that is controlled in $\omega_0 \times (0,T)$. In the general case when Ω_1 is not simply connected, and in order to construct the weight functions, an isotopy type condition between S and the boundary of two disjoint open subsets O_i, $i = 1,2$ of Ω_1 is used. Two weights similar to (3) are then constructed of the form

$$\Phi_i(x,t) = \frac{\exp(\lambda\alpha) - \exp(\lambda\psi_i(x))}{T-t},\ i = 1,2, \tag{9}$$

where $\psi_i \in V$ and $\nabla\psi = 0$ only in O_i (see Figure 1 left). Notice that you can also consider the opposite case when $\overline{\Omega}_0 \subset \Omega$ and $\Omega_1 = \Omega \setminus \overline{\Omega}_0$, and always $\overline{\omega}_0 \subset \Omega_0$. In this case, an isotopy type condition between $\partial\Omega$ and S is a sufficient condition. See Figure 1 right).

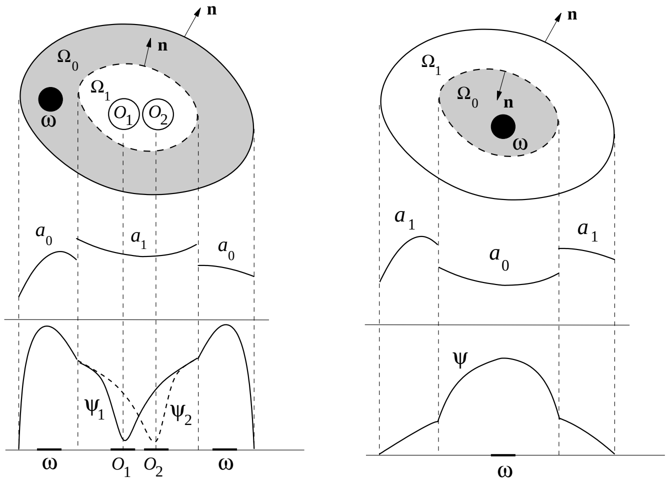

Figure 1: Construction of the global Carleman weight (bottom curves) for the heat equation with discontinuous coefficients such that $a_1(S^-) - a_0(S^+) > 0$ (middle curves). In the case $\overline{\Omega}_0 \subset \Omega$ (left) two combined weights are used and in the case $\overline{\Omega}_1 \subset \Omega$ (right) one weight suffices. In both cases the observation zone ω is represented by a black dot.

3 Inverse problem for waves from partial boundary data

Here we focus on one measurement inverse problems for the wave equation from *local boundary observations*. In this case, the function ψ is modified in order to obtain some strong Lopatinskii condition of the form $(x - x_0) \cdot Tn < 0$, where T is some linear transformation of the normal field. For further details we refer to [12]. The main idea is to modify the weight function Φ given in (4) in such a way that its gradient $\nabla\Phi$ is a rotation of the original field $(x - x_0)$ with a radially dependent magnitude. This concept come up from multipliers techniques commonly used in controllability [18].

Let Ω be a domain in $\mathbb{R}^n$, $n = 2, 3$. In order to solve the Dirichlet to Neumann one measurement inverse problem, it suffices to measure on a *rotated exit part* of the boundary Γ_r which corresponds in fact to a particular case of the geometrical optics condition BLR [1], [19]. If $n = 2$, this region depends on a point $x_0 \in \mathbb{R}^n$ and on a rotation T_θ in an angle $\theta \in (-\pi/2, \pi/2)$. If $n = 3$, it depends also on a unit direction $\alpha \in \mathbb{R}^3$ and the rotation T_θ is considered on the orthogonal plane to α denoted here by $\alpha^\perp$. More precisely, f we use the notation $v^\perp = v - (v \cdot \alpha)\alpha$ for the projection of the field v on $\alpha^\perp$

$$\Gamma_r = \{x \in \partial\Omega \mid (x - x_0) \cdot T_\theta n > 0\} \quad \text{if } n = 2 \tag{10}$$

$$\Gamma_r = \{x \in \partial\Omega \mid (x - x_0) \cdot (\cos\theta(n - n^\perp) + T_\theta n^\perp) > 0\} \quad \text{if } n = 3. \tag{11}$$

The main stability result is the following in the simplest case $x_0 \notin \overline{\Omega}$ (see [12])

Theorem 3.1 ([12]) *Let $P^* = \partial_{tt} - \Delta + q$ and let $u(q)$ and $u(\overline{q})$ be the respective solutions of $P^*u = 0$ with Dirichlet boundary conditions associated to $q, \overline{q} \in L^\infty(\Omega)$ and with Neumann measurements ξ and $\overline{\xi}$ on $\Gamma_r \times (0,T)$ respectively. There exists a time $\overline{T} > 0$ such that if $T > \overline{T}$, if $u(\overline{q}) \in H^1(0,T; L^\infty(\Omega))$ and if $|u(0)| \geq \alpha_0 > 0$ a.e. in Ω, then there exists a positive constant C_M depending on $M = \|q\|_{L^\infty(\Omega)}$ such that*

$$||\overline{q} - q||_{L^2(\Omega)} \leq C_M \left\|\overline{\xi} - \xi\right\|_{H^1(0,T;L^2(\Gamma_r))} \quad \forall q \text{ with } \|q\|_{L^\infty(\Omega)} \leq M\,. \tag{12}$$

The proof of this Theorem is based on a global Carleman estimate using (compare with (4))

$$\Phi(x,t) = -\lambda \exp\left(\cos\theta|x - x_0|^2 \exp\left(2\tan\theta \arg(x - x_0)^\perp\right) - \beta\, t^2\right) \tag{13}$$

for some suitable constant $\beta \in (0,1)$. The main steps in the deduction of such inequality are taken from [22] following a well known technique due to Bukhgeim and Klibanov [9].

There are also geometrical exit type conditions for the analogous inverse problem in the case of the Scrhödinger equation (see [3]). The present method should also work in this case because the spatial part used in Carleman weights for wave and Schödinger equations are the same (compare (4) with (5)) but this is an open problem. Nevertheless, in the case of the Scrhödinger operator, it is certain that the geometrical optics condition is not necessary to solve the one measurement inverse problem (see [20]). Other completely different problem is the case when you have Dirichlet to Newmann map measurements. In this case, a suitable arbitrarily small boundary of measurements is enough to solve the inverse problem both for Schödinger and wave equations (see [17]). Finally, it has been shown [5] that you can solve the one measurement problem for the wave equation with an arbitrarily boundary measurement region (in the case of Neumann boundary conditions and Dirichlet measurements), but the corresponding inequality analogous to (12) is logarithmic.

4 Inverse coefficient problem for wave transmission

Notice that recently, Global Carleman estimates and applications to one measurement inverse problems for the wave equation were obtained in the case of variable but still regular coefficients [4], [16]. The inverse problem of retrieving coefficients from a wave equation with discontinuous coefficients from single boundary measurements arise naturally in geophysics and in seismic prospection [27].

Let Ω and $\Omega_1 \subset \Omega$ be two open subsets of $\mathbb{R}^2$ with smooth boundaries Γ and Γ_1 respectively and let $\Omega_2 = \Omega \setminus \overline{\Omega}_1$. To fix ideas we assume that Ω_1 is simply connected. We set: $a(x) = a_1$ in Ω_1 and a_2 in Ω_2 with $a_j > 0$ for $j = 1, 2$, for each $q \in L^\infty(\Omega)$, we consider $u(q)$ as the solution of the following wave transmission equation

$$\left\{\begin{array}{rcll} u_{tt} - \operatorname{div}(\bar{a}(x)\nabla u) + q(x)u &=& 0 & \text{in } Q = \Omega \times (0,T) \\ u &=& 0 & \text{on } \Sigma = \Gamma \times (0,T) \\ u(0) &=& u_0 & \text{in } \Omega \\ u_t(0) &=& u_1 & \text{in } \Omega. \end{array}\right. \tag{14}$$

The following inverse stability result holds (see the preprint [2]):

Theorem 4.1 ([2]) *Assume* Ω_1 *is* strictly convex *and* $a_1 > a_2 > 0$. *Let* $\mathcal{U}$ *be a bounded subset of* $L^\infty(\Omega)$, $\overline{q} \in L^\infty(\Omega)$ *and* $r > 0$. *If* $|u_0(x)| \geq r > 0$ *a. e. in* Ω *and* $u(\overline{q}) \in H^1(0,T;L^\infty(\Omega))$, *then there exists* $C = C(\Omega, T, \|q\|_{L^\infty(\Omega)}, \mathcal{U}) > 0$ *such that:*

$$\|\overline{q} - q\|_{L^2(\Omega)} \leq C \left\|\partial_n u(\overline{q}) - \partial_n u(q)\right\|_{H^1(0,T;L^2(\Gamma))}$$

for all $u_0 \in H^1_0(\Omega)$ *and* $q \in \mathcal{U}$.

This Theorem is proved by combining the Carleman inequality for the wave equation with discontinuous coefficients proved in [2] and the method of Bukhgeim-Klibanov explained in section 3. To this end, system (14) is viewed as two wave equations with constant coefficients coupled with transmission conditions (see [18]). Then, a global Carleman inequality is found out for this transmission problem by working with variants of Carleman weights of the form $\Phi = -\exp(\lambda g)$ where (compare with (4))

$$g(x,t) = \begin{cases} \eta(x)\frac{a_2}{r(x)^2}|x - x_0|^2 - \beta t^2 + M_1 & \text{in } \Omega_1 \times (-T,T) \\ \frac{a_1}{r(x)^2}|x - x_0|^2 - \beta t^2 + M_2 & \text{in } \Omega_2 \times (-T,T) \end{cases} \tag{15}$$

where M_1 and M_2 are constants such that $M_1 - M_2 = a_1 - a_2$, $r(x) = |x_0 - y(x)|$, $y(x) = \Gamma_1 \cap [x_0, x]$ and η is some cut-off function with support in Ω_1 centered at x_0. We also combine the Carleman inequalities obtained from two different interior points as we did in section 2, see also Figure 1, left. The convexity hypothesis on Ω_1 comes from the fact that the positiveness of the Hessian of the weight Φ is related with the curvature of Γ_1 with respect to x_0.

There are a lot of important works concerning this inverse problem in the case that a wide class of measurements are available. In these cases, microlocal analysis has been used and it gives positive answer to the problem of retrieving coefficients and discontinuity interfaces without restrictive hypothesis of convexity of the interfaces or monotonicity of the speed of waves. These kinds of results are fundamental for seismic prospection. For an overview on this subject see [27] and the references therein.

5 Controllability problems in fluid-structure interaction

Here we consider the case of *mobile domains* in fluid-structure problems, when studying the boundary null controllability of an immersed solid into a viscous Navier-Stokes fluid. In this case, the weight function ψ depends also on time, and the global Carleman inequality is more complicated than (6) due to the incompressibility in Navier-Stokes and the presence of the structure. The results we present in this communication were adapted from the article [8]. The first result of this kind using global Carleman estimates were obtained in [10] for a one-dimensional Burgers-particle system studied in [28]. Also, similar results to the one presented here has been simultaneously and independently obtained in [25].

Let $\Omega \subset \mathbb{R}^2$ be a fixed bounded connected open subset with regular boundary. Let $\Omega_S(t)$ and $\Omega_F(t) = \Omega \setminus \overline{\Omega_S(t)}$ be the domains occupied by the structure and by the fluid respectively and let n be the unit exterior normal to $\partial\Omega_S(t)$. The fluid is descibed in velocity-pressure (u,p) with $\sigma(u,p) = \nu(\nabla u + \nabla u^t) - p\,\mathrm{Id}$ for $\nu > 0$. The solid of mass

$m > 0$ and inertia $J > 0$ is described by the velocity of its center of mass $a(t) \in \mathbb{R}^2$ and by its angular velocity $r(t) \in \mathbb{R}$. The system is

$$\begin{cases} \partial_t u + (u \cdot \nabla)u - \operatorname{div} \sigma(u,p) = f 1_\omega, \ \operatorname{div} u = 0 \text{ in } \Omega_F(t) \\ m\ddot{a} = \displaystyle\int_{\partial\Omega_S(t)} \sigma(u,p) n d\sigma, \ J\dot{r} = \int_{\partial\Omega_S(t)} (\sigma(u,p)n) \cdot (x-a)^\perp d\sigma, \\ u = \dot{a} + r(x-a)^\perp \text{ on } \partial\Omega_S(t), \ u = 0 \text{ on } \partial\Omega, \\ u(0,\cdot) = u_0 \text{ in } \Omega_F(0), \ a(0) = a_0, \ \dot{a}(0) = a_1, \ r(0) = r_0, \end{cases} \tag{16}$$

Here the function f is the *control* function which acts over a fixed small nonempty open subset ω (with characteristic function 1_ω). We have used the notation $x^\perp = (x_1, x_2)^\perp = (-x_2, x_1)$. The total angle θ associated to the angular velocity r is defined by $\theta(t) = \theta_0 + \int_0^t r(s)\, ds$, where $\theta_0 \in \mathbb{R}$ complements the initial data. The existence of solutions and regularity for this system has been recently studied in several papers (see [24] and the references therein).

The controllability result is the following, saying that it is possible to drive the structure and the fluid at rest and the immersed solid up to its reference position in arbitrarily small time with a localized control f, provided the initial conditions are sufficiently small.

Theorem 5.1 ([8]) *: Suppose that: i) the initial body solid shape satisfies $\Omega_S(0) \subset \Omega \setminus \omega$, $d(\Omega_S(0), \partial(\Omega\setminus\omega)) > 0$, $\int_{\partial\Omega_S(0)} (y - a_0)\, d\sigma = 0$, ii) $u_0 \in H^3(\Omega_F(0))^2$, $a_0 \in \mathbb{R}^2$, $a_1 \in \mathbb{R}^2$, $\theta_0 \in \mathbb{R}$ and $r_0 \in \mathbb{R}$ satisfy $\operatorname{div} u_0 = 0$ in $\Omega_F(0)$, $u_0 = a_1 + r_0(x-a_0)^\perp$ on $\partial\Omega_S(0)$ and $u_0 = 0$ on $\partial\Omega$ iii) the accelerations u_1 of the fluid and a_2 and r_1 of the structure at $t = 0$ satisfy $u_1 = 0$ on $\partial\Omega$, $u_1 = a_2 + r_1(x-a_0)^\perp - r_0^2(x-a_0) - \nabla u_0\big(a_1 + r_0(x-a_0)^\perp\big)$ on $\partial\Omega_S(0)$. Then for all $T > 0$ there exists $\varepsilon > 0$ and $f \in L^2((0,T)\times\omega)^2$ such that if $\|u_0\|_{H^3(\Omega_F(0))^2} + |a_0| + |a_1| + |\theta_0| + |r_0| \le \varepsilon$ then $u(T,\cdot) = 0$ in $\Omega_F(T)$, $a(T) = 0$, $\dot{a}(T) = 0$, $\theta(T) = 0$ and $r(T) = 0$.*

The first condition above is a symmetry restriction over the shape of the solid. The result only holds for small initial data because we want to keep the non-collision condition on the whole interval $(0,T)$ $\inf_{t\in(0,T)} \mathrm{d}(\Omega_S(t), \partial(\Omega\setminus\omega)) > 0$.

The proof follows ideas from [13] used to study the local exact controllability to trajectories of the Navier-Stokes equation and the ideas of [10] for a Burgers-mass model. We consider a linearized problem with operator and then a fixed point strategy. The Carleman inequality for the corresponding adjoint operator is expressed in the *moving* domains $\widetilde{\Omega}_S(t)$ and $\widetilde{\Omega}_F(t)$ and the transport theorem is used on its deduction. The weight Φ is of the form (3) and the weight function $\psi(x,t)$ is chosen as the standard weight for the heat equation but here it follows the shape of $\widetilde{\Omega}_S(t)$. More precisely, $\psi(x,t)$ is a regular function such that $\frac{\partial\psi}{\partial n} < 0$ on Σ, $|\nabla\psi| > 0$ outside ω and it satisfies the time dependent conditions $\frac{\partial\psi}{\partial n} > 0$ on $\partial\widetilde{\Omega}_S(t)$, ψ constant on $\partial\widetilde{\Omega}_S(t)$.

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