A GENERALIZATION OF THE PFÄHLER-LAMBERT DECOMPOSITION

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Abstract

The aim of this paper is to provide a generalization of the Pfähler (1990) and Lambert (1989, 2001) decomposition that allows us to overcome some limitations of the original methodology. In particular, our proposal allows avoiding the problem of sequentiality when the tax has several types of deductions or allowances, schedules or tax credits. In addition, our alternative decomposition is adapted to the dual income class of tax structures. Moreover, in order to adapt this methodology to real-world taxes, our alternative includes the re-ranking effects of real taxes, caused by the existence of differentiated treatments based on non-income attributes. This theoretical proposal is illustrated with an empirical analysis for the Spanish Personal Income Tax reform enforced in 2007.

Keywords: decomposition, redistributive effect, progressivity, personal income tax

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1. Introduction

In 1990, Wilhelm Pfähler published in *Bulletin of Economic Research* his well-known and widely used article "Redistributive effect of income taxation: Decomposing tax base and tax rates effects" (Pfähler, 1990). Since then, his methodology is used in almost every analysis that tries to determine how each piece of the structure of an income tax affects both progressivity and redistribution. The reinterpretation made by Lambert (1993, 2001) in his handbook *Distribution and Redistribution of Income* has widely contributed to the widespread use of this methodology of decomposition, and we can actually say that citations to Pfähler's article in academic papers and technical reports are almost boundless.

In our view, the explanation for this success is clear. On one hand, the methodology is based on the Gini index, certainly the measure of inequality most widely used in analyses of tax progressivity and redistribution. Moreover, Pfähler's proposal is directly linked to the fundamental Kakwani (1977) decomposition, which allows to explain the redistributive effect in terms of overall progressivity and average net tax rate. On the other hand, the results of Pfähler's decomposition are easy to interpret, because they provide partial measures which allow additively recomposing global indices.

However, the application of this methodology to real personal income taxes is not straightforward. The structures of real taxes are very complex, incorporating lots of differentiated treatments of very diverse nature (personal, family, disabilities, incentives for specific income sources, etc.) which are taken into account for the calculation of taxable income (exemptions, deductions, allowances) and tax liability (tax credits). According to the original methodology of Pfähler, the existence of several deductions and several tax credits faces us with the problem of establishing a sequential order to measure the contribution of the pieces corresponding to the same group. Nevertheless, as we show in this paper, to establish an order of application of these elements is an incorrect solution as an alternative choice leads to different values in each of the different sub-indices. A partial solution is adopted by Lambert (1993, 2001), using gross income as a fixed benchmark for every tax deduction (but not for tax credits, since he does not take them into account)

Besides the aforementioned problem, there are other two limitations that must also be tackled. On the one hand, Pfähler does not take into account the re-ranking effects of real taxes; Lambert does it only partially, as long as he uses concentration indices instead of Gini indices, but does not include either partial or global re-ranking terms in his formulae. On the other hand, the proliferation since the nineties of dual income taxes with (at least) two tax taxable bases and two tax schedules makes it necessary to adapt the methodology to a reality that was not foreseen by Pfähler or Lambert.

The aim of this paper is to provide a generalization of the Pfähler and Lambert decomposition that allows us to overcome the abovementioned limitations, being our main objective to improve the distributive analysis of the current personal income tax reforms. To illustrate the potential advantages of our proposal, we include an empirical illustration focused on the last reform of the Spanish personal income tax (PIT), using microdata from the Spanish Income Tax Return Panel. In particular we apply our alternative methodology of decomposition to the years 2006 and 2007, since the reform adopted in 2007 changed the treatment of personal and family circumstances (allowances were transformed into tax credits), and a dual income structure with two tax schedules was adopted (a flat rate for savings income and a progressive schedule for the remaining income).

The paper is structured as follows. After this introduction, the second section shows the main limitations of original Pfähler's and Lambert's methodologies, analysing in detail the differences between sequential and benchmark decomposition, both for redistribution and progressivity effects. In the third section we present our proposal for the generalization of the Pfähler-Lambert decomposition, adapted to a dual income tax structure with different tax base deductions and allowances and several tax credits. The fourth section presents the empirical illustration referred to the last reform of the Spanish PIT, comparing years 2006 and 2007. The section offers some concluding remarks.

2. Sequential decomposition vs. benchmark decomposition

2.1. Redistribution

The original paper by Pfähler (1990) decomposes the redistributive effect of an income tax (measured by the Reynolds-Smolensky index, RS) in two parts, indirect (corresponding to exemptions, allowances and deductions subtracted from the tax base) and direct (corresponding to the effect of tax rates and tax credits). In the former part he separates the effect of exemptions and allowances from the effect of deductions, while in the latter part he differentiates the effect of the tax rates and the effect of the tax credits. The partial indices are calculated sequentially; this means that, within each effect (indirect and direct), the redistributive effect of a particular "piece" of the tax is measured once the previous "pieces" have been applied.

Equation [1] shows the Pfähler decomposition into direct effect (first term) and indirect effect (second term), presenting also each effect split into its components[1]:

$$\Pi^{RS}_{\text{Pfähler}} = G_{NI} - G_{I} = \frac{\overline{R_f}}{\overline{NI}}\left[(G_R - G_{TI}) + \left(G_{R_f} - G_R\right)\right] - \frac{\overline{T_f}}{\overline{NI}}\left[(G_{AI} - G_I) + (G_{TI} - G_{AI})\right] \quad [1]$$

where

G_Z is the Gini index of variable Z;
$\overline{Z}$ is the average of variable Z;
I is gross income;
T_f is final tax liability (gross tax liability minus tax credits);
NI is net income (gross income minus final tax liability);
TI is taxable income (gross income minus exemptions, allowances and deductions);
R is residual income (taxable income minus gross tax liability);
R_f is the final residual income (taxable income minus final tax liability);
AI is adjusted income (gross income minus exemptions and allowances)

The Pfähler decomposition was later reinterpreted by Lambert (2001) as follows[2]:

$$\Pi^{RS}_{\text{Lambert}} = \frac{1}{1-t}\Big[(1-\alpha-\delta-t)(C_{X-A-D} - C_{Y-T}) - \frac{t(1-\alpha)}{1-\alpha-\delta}(G_X - C_{X-A}) - \frac{t(1-\delta)}{1-\alpha-\delta}(G_X - CX-D \quad [2]$$

where

[1] This equation is the result of replacing equations (6) and (7) in equation (5) (Pfähler, 1990: 125 and 127).
[2] This equation is the result of replacing equation 8.40 in 8.42, and then 8.42, 8.45 and 8.48 in equation 8.53 (Lambert, 2001: 214-216).

G_Z is the Gini index of variable Z;
C_Z is the concentration index of variable Z;
X is gross income;
Y is taxable income;
A are allowances (equivalent to Pfähler's exemptions and allowances);
D are deductions (as in Pfhäler's decomposition);
A is residual income (as in Pfhäler's decomposition);
α is the average allowance divided by average gross income
δ is the average deduction divided by average gross income
t is the average tax rate (average final tax liability divided by average gross income)

In order to make the equations clearer and allow an easier comparison, we homogenize equations [1] and [2] by changing the notation. We choose to use intuitive letters for each concept, and also reduce the number of them, not naming all the derived variables but showing some of them as a sum or difference between two or more variables.

Equation [3] shows the Pfähler decomposition using our notation. We have also changed the signs so that positive Π^{RS} is interpreted as redistributive (Pfähler takes as a starting point the original Reynolds-Smolensky equations, where negative means positive redistribution):

$$\Pi^{\mathrm{RS}}_{\mathrm{Pfähler}} = \frac{\overline{\mathrm{B-T}}}{\overline{\mathrm{Y-T}}}\left[(\mathrm{G_B} - \mathrm{G_{B-S}}) + (\mathrm{G_{B-S}} - \mathrm{G_{B-T}})\right] - \frac{\overline{\mathrm{T}}}{\overline{\mathrm{Y-T}}}\left[(\mathrm{G_Y} - \mathrm{G_{Y-A}}) + (\mathrm{G_{Y-A}} - \mathrm{G_{Y-A-D}})\right] \quad [3]$$

where

Y is gross income;
A are allowances and exemptions;
D are deductions;
B is the tax base or taxable income, i.e. gross income minus allowances, exemptions and deductions ($\mathrm{B} = \mathrm{Y} - \mathrm{A} - \mathrm{D}$);
S is the gross tax liability, i.e. the result of applying the tax schedule to the tax base;
C are tax credits;
T is the final tax liability, i.e. gross tax liability minus tax credits ($\mathrm{T} = \mathrm{S} - \mathrm{C}$);

Every other variable is not shown explicitly, but can be easily understood. For example, final residual income is expressed as $\mathrm{B} - \mathrm{T}$ (taxable income minus final tax liability) and net income as $\mathrm{Y} - \mathrm{T}$ (gross income minus final tax liability).

To simplify the equation, let's define $\Pi^{\mathrm{RS}}_{\mathrm{X,Y}}$ as the difference between the Gini indices for variables X and Y ($\Pi^{\mathrm{RS}}_{\mathrm{X,Y}} = \mathrm{G_X} - \mathrm{G_Y}$), so equation [3] can be expressed as:

$$\Pi^{\mathrm{RS}}_{\mathrm{Pfähler}} = \frac{\overline{\mathrm{B-T}}}{\overline{\mathrm{Y-T}}}\left(\Pi^{\mathrm{RS}}_{\mathrm{B,B-S}} + \Pi^{\mathrm{RS}}_{\mathrm{B-S,B-T}}\right) - \frac{\overline{\mathrm{T}}}{\overline{\mathrm{Y-T}}}\left(\Pi^{\mathrm{RS}}_{\mathrm{Y,Y-A}} + \Pi^{\mathrm{RS}}_{\mathrm{Y-A,B}}\right) \quad [4]$$

Equation [4] shows the Lambert decomposition using our notation. The original equation has been also multiplied and divided by $\overline{\mathrm{Y}}$ to express it in terms of average values (as Pfähler does), and not in terms of average "rates" (as Lambert does).

$$\Pi^{\mathrm{RS}}_{\mathrm{Lambert}} = \frac{\overline{\mathrm{B-T}}}{\overline{\mathrm{Y-T}}}(\mathrm{C_B} - \mathrm{C_{B-T}}) - \frac{\overline{\mathrm{T}}}{\overline{\mathrm{Y-T}}}\left[\frac{\overline{\mathrm{Y-A}}}{\overline{\mathrm{B}}}(\mathrm{G_Y} - \mathrm{C_{Y-A}}) + \frac{\overline{\mathrm{Y-D}}}{\overline{\mathrm{B}}}(\mathrm{G_Y} - \mathrm{C_{Y-D}})\right] \quad [5]$$

Defining now $\Pi^{RS}_{X,Y}$ as the difference between the concentration indices for variables X and Y ($\Pi^{RS}_{X,Y} = C_X - C_Y$) (except for gross income, where Gini and concentration indices are the same), equation [4] can be expressed as follows:

$$\Pi^{RS}_{\text{Lambert}} = \frac{\overline{B-T}}{\overline{Y-T}} \Pi^{RS}_{B,B-T} - \frac{\bar{T}}{\overline{Y-T}} \left(\frac{\overline{Y-A}}{\bar{B}} \Pi^{RS}_{Y,Y-A} + \frac{\overline{Y-D}}{\bar{B}} \Pi^{RS}_{Y,Y-D} \right) \quad [6]$$

Comparing equations [4] and [6] it can be easily seen that Lambert's computations represent actually a significant departure from Pfähler proposal, even though Lambert does not mention it explicitly. This important difference can be seen observing the indirect effect, where Lambert calculates the redistributive effect of each "piece" in relation to the same benchmark, instead of calculating the sequential effect of each of them; this means that the effect of tax deductions is calculated against gross income, and not against income minus allowances and exemptions that had been applied before, as Pfähler did. Therefore not only there is a weight for the indirect effect, but each piece within that effect has also its own weight; in contrast, in Pfähler decomposition the pieces of the indirect effect are simply summed up.

Although less relevant, there are two additional differences between both approaches. On one hand, Lambert uses concentration indices instead of Gini indices (as Pfähler does), thus keeping the order of the observations derived of their gross income. On the other hand, Lambert does not take tax credits into account, as can be seen comparing the first terms of the two equations; the direct effect can be then interpreted either as the effect of a tax with no tax credits either as the joint effect of tax rates and tax credits (however this is not a conceptual difference, but only an instrumental one).

2.2. Progressivity

Both Pfähler and Lambert use progressivity measures in their papers. Following Kakwani (1977), they express the redistributive effect of each "piece" of the tax structure as the product of the progressivity effect of that "piece" (measured by the Kakwani index of progressivity) and a level effect (measured by the monetary weight that the corresponding piece has on the whole tax).

There is no conceptual difference here between Pfähler and Lambert approaches, so we can express this relationship in a single equation:

$$\Pi^{RS}_{X,Y} = \frac{\overline{X-Y}}{\bar{Y}} \Pi^{K}_{X,Y} \quad [7]$$

where $\Pi^{K}_{X,Y}$ is the Kakwani index, that expresses the progressivity effect of changing from variable X to variable Y, being $\Pi^{K}_{X,Y} = C_{X-Y} - C_X$ ($\Pi^{K}_{X,Y} = G_{X-Y} - G_X$ in Pfähler's methodology).

Replacing each term in [4] and [6] by the corresponding expression of [7] we have the redistributive effect as a weighted sum of progressivity effects, instead of redistributive effects:

$$\Pi^{RS}_{\text{Pfähler}} = \frac{\overline{B-T}}{\overline{Y-T}} \left(\frac{\bar{S}}{\overline{B-S}} \Pi^{K}_{B,B-S} - \frac{\bar{C}}{\overline{B-T}} \Pi^{K}_{B-S,B-T} \right) - \frac{T}{\overline{Y-T}} \left(\frac{\bar{A}}{\overline{Y-A}} \Pi^{K}_{Y,Y-A} + \frac{\bar{D}}{\overline{Y-A-D}} \Pi^{K}_{Y-A,Y-A-D} \right) \quad [8]$$

$$\Pi^{RS}_{Lambert} = \frac{\overline{B-T}}{\overline{Y-T}}\frac{\overline{T}}{\overline{B-T}}\Pi^{K}_{B,B-T} - \frac{\overline{T}}{\overline{Y-T}}\left(\frac{\overline{Y-A}}{\overline{B}}\frac{\overline{A}}{\overline{Y-A}}\Pi^{K}_{Y,Y-A} + \frac{\overline{Y-D}}{\overline{B}}\frac{\overline{D}}{\overline{Y-D}}\Pi^{K}_{Y,Y-D}\right) = \frac{\overline{T}}{\overline{Y-T}}\Pi^{K}_{B,B-T} - \frac{\overline{T}}{\overline{Y-T}}\left(\frac{\overline{A}}{\overline{B}}\Pi^{K}_{Y,Y-A} + \frac{\overline{D}}{\overline{B}}\Pi^{K}_{Y,Y-D}\right) \quad [9]$$

C are tax credits in equation [8] ($C = S - T$).

Finally, we can also express total progressivity as a weighted sum of partial progressivity effects. Using expression [6], we can express total Π^{RS} in equations [8] and [9] as the product of the progressivity and level effects. Isolating the progressivity effect afterwards we obtain:

$$\Pi^{K}_{Pfähler} = \frac{\overline{T}}{\overline{Y-T}}\left(\frac{\overline{S}}{\overline{B-S}}\Pi^{K}_{B,B-T} - \frac{\overline{C}}{\overline{B-T}}\Pi^{K}_{B-S,B-T}\right) - \left(\frac{\overline{A}}{\overline{Y-A}}\Pi^{K}_{Y,Y-A} + \frac{\overline{D}}{\overline{Y-A-D}}\Pi^{K}_{Y-A,Y-A-D}\right) \quad [10]$$

$$\Pi^{K}_{Lambert} = \Pi^{K}_{B,B-T} - \left(\frac{\overline{A}}{\overline{B}}\Pi^{K}_{Y,Y-A} + \frac{\overline{D}}{\overline{B}}\Pi^{K}_{Y,Y-D}\right) \quad [11]$$

The differences between [10] and [11] are exactly the same as between [4] and [6]: the reference for the partial decompositions (sequential approach vs. benchmark approach), the decomposition of the tax base (Pfähler does it and Labert does not) and the re-ranking effects (Pfähler uses Gini indices and Lambert concentration indices, although in expressions [10] and [11] they are implicit in the Kakwani indices).

3. Generalization of the Pfähler-Lambert decomposition

Lambert's approach seems more useful to us when dealing with deductions. The main limitation of Pfähler's methodology is that the results are strongly influenced by the order or the application of tax deductions within the tax base, because the redistributive effect of each piece is measured against the previous calculation. It could even be the case that a deduction that is progressive when measured against gross income seems regressive when measured against gross income minus a previous deduction. But if the order of the application is changed, the opposite may apply. The reference to a fixed benchmark, as Lambert does, removes the distortions introduced by the sequential approach of Pfähler.

Nevertheless, the reasoning is not the same when we split the effect of tax schedule and tax credits, because the application of them is always sequential: first (and always) we apply the tax schedule, and then (and only in some cases) we apply tax deductions; therefore it makes sense to keep the sequential order, as Pfähler does. However, if we want to differentiate the effect of each tax credit, since their application is not sequential we should use Lambert's approach, even though he did not take into account the effect of tax credits.

Furthermore, the use of concentration indices in Lambert's methodology allows applying it to real taxes; however he does not include a re-ranking term, so his overall redistribution effect should be interpreted as the difference between the Gini index of gross income and the concentration index of net income, while the RS index is usually defined as the difference between both Gini indices. This limitation can be easily overcome by subtracting the re-ranking term ($R = G_{Y-T} - C_{Y-T}$) at the end of the equation.

Taking all these into account, equation [12] proposes a new version of Lambert's equation:

$$\Pi^{RS} = \frac{\overline{B-T}}{\overline{Y-T}}\left[(C_B - C_{B-S}) + (C_{B-S} - C_{B-S+C})\right] - \frac{\overline{T}}{\overline{Y-T}}\left[\frac{\overline{Y-A}}{\overline{B}}(G_Y - C_{Y-A}) + \frac{\overline{Y-D}}{\overline{B}}(G_Y - C_{Y-D})\right] - R \quad [12]$$

Expressing each unweighted redistributive effect as $\Pi^{RS}_{X,Y}$ we can write expression [12] as:

$$\Pi^{RS} = \frac{\overline{B-T}}{\overline{Y-T}}\Pi^{RS}_{B,B-S} + \frac{\overline{B-T}}{\overline{Y-T}}\Pi^{RS}_{B-S,B-S+C} - \frac{\overline{T}}{\overline{B}}\left(\frac{\overline{Y-A}}{\overline{Y-T}}\Pi^{RS}_{Y,Y-A} + \frac{\overline{Y-D}}{\overline{Y-T}}\Pi^{RS}_{Y,Y-D}\right) - R \qquad [13]$$

The equation can be easily generalized for m tax credits and n tax deductions (with this term we refer also to allowances and exemptions, since they operate the same way), as shown in equation [7]:

$$\Pi^{RS} = \frac{\overline{B-T}}{\overline{Y-T}}\Pi^{RS}_{B,B-S} + \frac{\overline{B-T}}{\overline{Y-T}}\sum_{i=1}^{m}\frac{\overline{B-S+C_i}}{\overline{B-T}}\Pi^{RS}_{B-S,B-S+C_i} - \frac{\overline{T}}{\overline{B}}\sum_{i=1}^{n}\frac{\overline{Y-D_i}}{\overline{Y-T}}\Pi^{RS}_{Y,Y-D_i} - R \qquad [14]$$

However this equation still has a limitation which is particularly relevant nowadays: it only considers the application of a single rate schedule to a comprehensive definition of income. Currently most income taxes split income (at least) in two parts (notably labour and capital income in dual income taxes), so we transform expression [14] to include the effect of l tax schedules:

$$\Pi^{RS} = \frac{\overline{B-T}}{\overline{Y-T}}\sum_{i=1}^{l}\frac{\overline{B-S_i}}{\overline{B-S}}\Pi^{RS}_{B,B-S_i} + \frac{\overline{B-T}}{\overline{Y-T}}\sum_{i=1}^{m}\frac{\overline{B-S+C_i}}{\overline{B-T}}\Pi^{RS}_{B-S,B-S+C_i} - \frac{\overline{T}}{\overline{B}}\sum_{i=1}^{n}\frac{\overline{Y-D_i}}{\overline{Y-T}}\Pi^{RS}_{Y,Y-D_i} - R \qquad [15]$$

The reasoning applied here is exactly the same used for splitting deductions and tax credits, i.e. we consider the effects of each tax schedule against the tax base. Still, since we are measuring the overall redistributive effect, it would not make sense to calculate the effect of each tax schedule against its own tax base, because we are interested in measuring the effect of each part of the tax on the comprehensive income of individuals, even if it is not taxed in a comprehensive way.

Equation [15] can be also expressed as a weighted sum of partial progressivity effects. Using equation [7] and operating we have:

$$\Pi^{RS} = \frac{\overline{B-T}}{\overline{Y-T}}\frac{\overline{B}}{\overline{B-S}}\sum_{i=1}^{l}\frac{\overline{S_i}}{\overline{B}}\Pi^{K}_{B,B-S_i} - \frac{\overline{B}}{\overline{Y-T}}\sum_{i=1}^{m}\frac{\overline{C_i}}{\overline{B}}\Pi^{K}_{B-S,B-S+C_i} - \frac{\overline{Y}\overline{T}}{\overline{B}(\overline{Y-T})}\sum_{i=1}^{n}\frac{\overline{D_i}}{\overline{Y}}\Pi^{K}_{Y,Y-D_i} - R \qquad [16]$$

This equation gives us all the information we need for each "piece": its progressivity (measured by the Kakwani index), its weight in monetary terms (measured by the "gross average rate" inside the summation operator) and the weight of the group in the structure of the tax (measured by the quotient outside the summation). It is important to emphasize that only tax schedules have positive weights, while the weights for tax deductions and tax credits have to be negative in order to compensate the contribution of negative Kakwani indices: when a tax deduction (or a tax credit) is more concentrated than gross income (or the tax base), their Kakwani index is negative, but this means that the contribution to overall progressivity is positive.

Finally, using again formula [7] to decompose total Π^{RS} in [16], and then isolating total Π^{K}, we obtain an expression for total progressivity as a weighted sum of partial progressivities:

$$\Pi^{K} = \frac{\overline{B-T}}{\overline{T}}\frac{\overline{B}}{\overline{B-S}}\sum_{i=1}^{l}\frac{\overline{S_i}}{\overline{B}}\Pi^{K}_{B,B-S_i} - \frac{\overline{B}}{\overline{T}}\sum_{i=1}^{m}\frac{\overline{C_i}}{\overline{B}}\Pi^{K}_{B-S,B-S+C_i} - \frac{\overline{Y}}{\overline{B}}\sum_{i=1}^{n}\frac{\overline{D_i}}{\overline{Y}}\Pi^{K}_{Y,Y-D_i} \qquad [17]$$

4. An illustration for Spain

To illustrate the potential of the methodology we apply it to the Spanish Personal Income Tax. We choose two years, 2006 and 2007, in which we find two important changes in the structure of the tax. On one hand, family allowances were transformed into tax credits (although legally defined as allowances, their work actually as tax credits); and on the other hand, a dual tax schedule was approved (even though the tax already had a flat rate for capital gains generated in more than one year). Table 1 offers a summary of the design for each year.

Table 1. Structure of the Spanish Personal Income Tax (2006-2007)

Concept		**2006**	**2007**
Allowances, exemptions and deductions	**Rental income deduction** (D_R)	Deduction of 50% of gross income from household rental	
	Personal and family allowances (D_F)	Personal allowances, allowances for dependent children and parents, allowances for disabilities, allowance for joint taxation	Allowance for joint taxation
	Labour income deduction (D_L)	Deduction for work income	
	Pension allowances (D_P)	Allowances for pension schemes (with absolute limits) and compensatory pensions to ex-spouses	Allowances for pension schemes (with absolute and relative limits) and compensatory pensions to ex-spouses
Tax schedules	**Progressive schedule** (S_P)	General income: all income except capital gains and losses generated in more than one year	General income: labour income, self-employment income and rental income
	Flat rate (S_F)	Special income: capital gains and losses generated in more than one year (15% rate)	Savings income: capital income and capital gains and losses (18% rate)
Tax credits	**Personal and family tax credits** (C_F)	-	Personal tax credit, tax credit for dependent children and parents, tax credits for disabilities
	Tax credit on housing investment (C_H)	15%-25% of the amounts invested in acquisition of the main dwelling, with the amount limited to EUR 9,040	15% of the amounts invested in acquisition of the main dwelling, with the amount limited to EUR 9,015
	Other tax credits (C_O)	Tax credits on entrepreneurial investment, donations and other	

Since equation [16] is the one that gives more information, we choose it ant adapt it to our variables:

$$\Pi^{RS} = \frac{\overline{B-T}}{\overline{Y-T}}\frac{\overline{B}}{\overline{B-S}}\left(\frac{\overline{S_P}}{\overline{B}}\Pi^K_{B,B-S_P} + \frac{\overline{S_F}}{\overline{B}}\Pi^K_{B,B-S_F}\right) - \frac{\overline{B}}{\overline{Y-T}}\left(\frac{\overline{C_F}}{\overline{B}}\Pi^K_{B-S_P-S_F,B-S_P-S_F-C_F} + \frac{\overline{C_H}}{\overline{B}}\Pi^K_{B-S_P-S_F,B-S_P-S_F-C_H} + \frac{\overline{C_O}}{\overline{B}}\Pi^K_{B-S_P-S_F,B-S_P-S_F-C_F}\right) - \frac{\overline{Y}\overline{T}}{\overline{B}(\overline{Y-T})}\left(\frac{\overline{D_F}}{\overline{Y}}\Pi^K_{Y,Y-D_F} + \frac{\overline{D_R}}{\overline{Y}}\Pi^K_{Y,Y-D_R} + \frac{\overline{D_P}}{\overline{Y}}\Pi^K_{Y,Y-D_P}\right) - R \quad [18]$$

We apply equation [18] to the cross-section data for 2006 and 2007 of the Spanish Personal Income Tax Return Panel, an expanded panel that represents the Spanish population of taxpayers of the 1999-2007 period for the personal income tax[3]. Table 2 shows the results for 2006.

Table 2. Decomposition of the Reynolds-Smolensky index of redistribution (2006)

	Kakwani index	Own weight	Group weight	Redistribution effect	% of total redistribution
Rental income deduction (D_R)	0.172859	0.000029	-0.266543	-0.000001	0.00
Personal and family allowances (D_F)	-0.422173	0.211090		0.023753	52.43
Labour income deduction (D_L)	-0.418736	0.081251		0.009068	20.02
Pension allowances (D_P)	0.156165	0.022553		-0.000939	-2.07
Sum of exemptions, allowances and deductions				***0.031882***	***70.37***
Progressive schedule (S_P)	0.012900	0.213693	0.830822	0.002290	5.06
Flat rate (S_F)	0.288883	0.030943		0.007427	16.39
Sum of tax schedule				***0.009717***	***21.45***
Personal and family tax credits (C_F)	-	-	-0.810175	-	-
Tax credit on housing investment (C_H)	-0.307269	0.017536		0.004365	9.64
Other tax credits (C_O)	-0.047714	0.001714		0.000066	0.15
Sum of tax credits				***0.004432***	***9.78***
Re-ranking				***-0.000727***	***-1.61***
Total				**0.045303**	**100.00**

On aggregate, we see that more than 70% of the redistributive effect is given in 2006 by the deductions applied to the tax base, more than 20% by the tax schedules and slightly less than 10% by tax credits, being the re-ranking effect -1,61%.

If we analyse the detail for the tax deductions, we find that personal and family allowances contribute more than half of the total redistributive effect, while the labour income deduction contributes 20%. If we see the detail of these two pieces, we see that their progressive effect ("Kakwani index") is similar (because they apply similar amounts to all levels of income), but the effect of personal and family allowances is higher because they are higher in absolute terms (measured by "Own weight"). The other two pieces of the tax base are not significant, but regressive (positive Kakwani indices on the tax base imply that higher deductions are applied on higher income levels).

Regarding tax schedules, we see that the progressivity effect of the flat rate applied on certain capital gains is much higher than the effect of the progressive schedule applied on most of the income. The reason is twofold. On one hand, those capital gains are almost entirely concentrated on the top decile; on the other hand, the rate applied to these capital gains increased from 15% in 2006 to 18% in 2007, so many taxpayers decided to realize capital gains before the reform took place. Consequently, although a flat rate is applied on capital gains, it falls almost entirely on the richest taxpayers, therefore acting as a two-rate schedule that applies a zero-rate to most taxpayers and a positive rate only to the rich. Even though the monetary weight of the progressive schedule is much higher, it does not compensate the much lower progressivity, being the flat rate more redistributive than the progressive schedule.

[3] For a detailed description of the Spanish PIT Returns Panel see Onrubia and Picos (2011).

Finally, both types of tax credits are progressive in relation to the tax base (negative values of the Kakwani index). However, the tax credit on housing investment is much more progressive and much more relevant, contributing almost 10% to final redistribution.

Table 3 shows the decomposition for 2007.

Table 3. Decomposition of the Reynolds-Smolensky index of redistribution (2007)

	Kakwani	Own weight	Group weight	Redistribution effect	% of total redistribution
Rental income deduction (D_R)	0.041092	0.003654	-0.211276	-0.000032	-0.07
Personal and family allowances (D_F)	-0.365956	0.032942		0.002547	5.45
Labour income deduction (D_L)	-0.480326	0.102106		0.010362	22.17
Pension allowances (D_P)	0.077649	0.021919		-0.000360	-0.77
Sum of exemptions, allowances and deductions				***0.012518***	***26.78***
Progressive schedule (S_P)	-0.001749	0.226920	1.092943	-0.000434	-0.93
Flat rate (S_F)	0.271928	0.031143		0.009256	19.80
Sum of tax schedule				***0.008822***	***18.87***
Personal and family tax credits (C_F)	-0.380972	0.063678	-0.988235	0.023974	51.29
Tax credit on housing investment (C_H)	-0.168716	0.013450		0.002243	4.80
Other tax credits (C_O)	0.027454	0.001483		-0.000040	-0.09
Sum of tax credits				***0.026176***	***56.00***
Re-ranking				***-0.000774***	***-1.66***
Total				**0.046742**	**100.00**

As can be seen in the last row, the total redistributive effect of the tax is higher in 2007 than in 2006. However the main differences arise in the decomposition: most of the redistributive effect (56%) is now provided by the tax credits, as a result of the move from personal and family allowances to tax credits. Still, the effect of the treatment of personal and family circumstances gives almost the same redistributive effect before and after the reform, around 51-52%.

The reform of the tax schedules gives surprising results: paradoxically, the progressive schedule has a regressive effect. This is due to the fact that in 2007 it is applied to a smaller portion of income than 2006, not including capital income or capital gains. Since this kind of income is more concentrated on the top deciles, and it is not taxed by the progressive schedule, the average rate derived from the application of the progressive schedule can be higher for lower deciles, when measured against whole income (as we do).

The effect of the other "pieces" of the tax is less important. Tax credits on housing investment have a much smaller progressive effect in 2007 than in 2006 (this is not due to the change in percentages, since it only applies to new investments, but to the fact that the tax base is now less unequal because no personal and family allowances are applied); pension allowances have a lower effect in 2007 (probably because of the application of lower limits); and rental deductions and other tax credits have also negligible effects. Finally, re-ranking is negative and similar in both years.

Finally, in order to see the advantage of the benchmark approach against the sequential approach, we decompose the indirect redistributive effect of 2006 by using both methodologies. For the sequential methodology, we apply it in four different orders, to show

how they affect to the results. Table 4 shows the comparison in percentage of the indirect redistributive effect (which is in Table 2: 0.031882).

Table 4. Decomposition of the indirect redistributive effect in 2006 (benchmark approach vs. sequential approach)

Concept	Benchmark approach	Sequential approach, being the sequence:			
		rental-personal-labour-pension	personal-labour-pension-rental	labour-pension-rental-personal	pension-rental-personal-labour
Rental income deduction ($\mathrm{D_R}$)	0.00	0.00	0.00	0.00	0.00
Personal and family allowances ($\mathrm{D_F}$)	74.50	64.70	64.70	80.51	66.04
Labour income deduction ($\mathrm{D_L}$)	28.44	34.97	34.96	21.21	36.03
Pension allowances ($\mathrm{D_P}$)	-2.94	0.34	0.34	-1.72	-2.06
Total indirect effect	100.00	100.00	100.00	100.00	100.00

The table clearly shows how the sequential approach depends strongly on the adopted sequence. The main difference arise when $\mathrm{D_F}$ and $\mathrm{D_L}$, the most important deductions, swap positions: when $\mathrm{D_L}$ is applied before $\mathrm{D_F}$ (3^{rd} sequence in the table), its redistributive effect is much less strong (21,21%) than when it is applied after (around 35%). The reason is that, since both deductions are quite progressive, the first one makes the benchmark for the second one more unequal, so the progressivity of the second seems higher in relative terms (the effect of both using the benchmark approach is somewhere in the middle). In turn, pension allowances are progressive in sequences 1 and 2, and regressive in sequences 3 and 4 (as in the benchmark approach).

5. Concluding remarks

As we said in the introduction, since its publication in 1990, the decomposition methodology proposed by Pfähler has been commonly applied in most analysis of income tax redistribution and progressivity. Its simplicity of calculation (based on the computation of Gini and concentration indices) and its easy interpretation explain its success. However this methodology in its original form (and in Lambert’s adaptation) has significant limitations that restrict its application to complex real-world tax structures.

In this paper we provide an alternative to overcome the methodological limitations pointed out. Our methodology avoids the problem of sequentiality when the tax has several deductions/allowances or tax credits, making indifferent the order in which they are computed within each group. In addition we present an alternative decomposition adapted to the currently mainstream dual income class structures, with two differentiated kinds of taxable income, and consequently two tax schedules. Finally, our objective to adapt the methodology to real-world taxes has led us to include the re-ranking effects caused by the existence of differentiated treatments based on non-income attributes.

The empirical analysis included in the previous section, focusing on the last Spanish PIT reform, allows observing the improvements provided by the use of the alternative methodology. The dual structure of the new Spanish PIT, along with the replacement of the personal and family allowances by tax credits, clearly illustrate its potential.

In the process of drafting of the proposal different decomposition alternatives have arisen. Some of them seem very suggestive to us, since they can improve the explanatory potential of the decomposition. Further developments of these alternatives will be a line of work for our future research on this topic.

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