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# SENSITIVITY EXAMINATION OF THE SIMULATION RESULT OF DISCRETE EVENT DYNAMIC SYSTEMS WITH PERTURBATION ANALYSIS*

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*Simulation completed with perturbation analysis provides a new approach for the optimal control of queuing network type systems. The objective of this paper is to calculate the sensitivity range of finite zero-order perturbation, that is, to determine the maximum and minimum size of perturbation within which zero-order propagation rules can be applied. By the introduction of the concept of virtual queue and first and second level no-input and full-output matrices, an algorithm is provided which can solve this task efficiently in transfer lines and in relatively small general networks when short simulation run is required and the sensitivity of the individual sample path is in question. The implementation of the algorithm with the help of conventional simulation languages is also discussed and presented in an example.*

**Key words:** Simulation, perturbation analysis, queuing networks.

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## 1. INTRODUCTION

The complex nature of design, planning and control of modern, highly automated production systems raises various challenging problems in the field of operations management. The difficulty of their examination comes from the fact that these are generally discrete event dynamic systems (DEDS) in the majority of the cases with a stochastic nature. Queuing network representation seems to provide appropriate modelling framework but their exact analytical examination can be performed just in very limited cases. Usually the combined application of simulation and exact optimization algorithm is required.

One of the most challenging solution of this problem seems to be the recently developed perturbation analysis (PA) which can provide gradient information from a single simulation experiment [4]. The idea is to perform a simulation experiment, and via an algorithm an estimate can be derived about the derivative of a performance measure of the system with respect to one of its parameters [6]. This gradient information can be used for iterative improvement of system performance [8],[11].

Various intriguing problems have been solved since the first presentation of the method. Propagation rules for infinitesimal and finite perturbations [5], examination of multi-class networks [1], various suggestions for avoiding or at least smoothing the effect of discontinuities are extending the applicability of the method [7]. Researchers of this field, however, have mostly concentrated on generating and/or propagating perturbations, but have avoided the examination of validity range within which the gradient information is correct. The infinitesimal approach deals with this problem by simply saying that the size of the perturbation is small enough not to hurt the deterministic similarity. The finite approach calculates accurately the effect on the performance measures or on other activities with higher order propagation rules, but it also fails to provide information about the validity [5]. The effect of a specific perturbation is calculated correctly but if the perturbation changes the calculation has to be performed again.

The objective of this paper is to calculate the sensitivity range of zero-order perturbation, that is, to determine the maximum and minimum size of perturbation within which zero-order propagation rules can be applied. The practical significance of this problem is, that within the determined sensitivity limits a linear extrapolation of the change of the examined performance measure based on the gradient information is accurate [3].

An intuitive illustration of this problem can be given by linear programming (LP). The basic idea of the optimization process in LP is that an initial basis

solution is generated and an iterative improvement is performed with the help of the shadow prices. The shadow price is actually the gradient of the objective function with respect to the right hand side parameters. The validity range of the shadow price imposes limits on the improvement and also provides important information about the limiting constraints [2]. With PA we may try to construct a similar process. The simulation results can be considered as initial solutions. The gradient information can be gained by the propagation rules. The objective of this paper is to derive the validity range of the gradient.

## 2. PROBLEM DEFINITION

Suppose we have a transfer line type queuing network consisting of $M$ resources and the same number of queues, illustrated in Fig. 1. Every resource ($R_j$) is preceded by a queue ($Q_j$) with $c(j)$ capacity. ($c(j)$ includes the place in $R_j$ as well.) At the queues the first-in-first-out (FIFO) rule is applied.

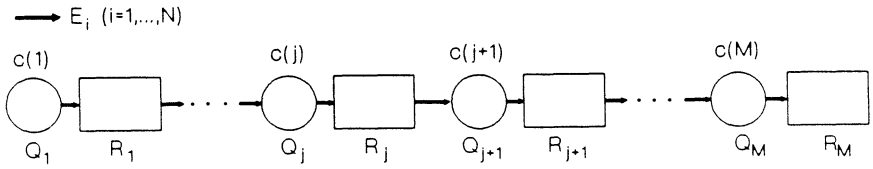


**Figure 1.**
Transfer line type queuing network.

$N$ entities ($E_i$) enter the network and follow a flow-shop type process. The order of entities at every resource is the same. The operation time of $E_i$ at $R_j$ in a specific sample path is $t^{\omega}_{i,j}$ (shortly $t_{i,j}$)[1].

A simulation experiment is performed. As a result a time schedule of the activities is gained and summarized in a matrix $B$ where $b_{i,2j-1}$ denotes the beginning time and $b_{i,2j}$ the ending time of the operation of $E_i$ at $R_j$.

A single finite perturbation is introduced at $E_x$ on $R_y(\delta_{x,y})$ and its effect is examined on the throughput time ($T$), that is, on the total operation time

[1] Since all the calculations introduced here, refer to a specific sample path we abandon $\omega$ when it is not misleading.

necessary to finish the manufacturing of $N$ entities. We would like to determine the gradient of the throughput time with respect to the change of the operation time $t_{x,y}$ and the validity range of this gradient.

Our analysis is based on the **event sequence table** introduced by Ho and Cassandras [6]. The principle of this table is that every resource participate in one of three mutually exclusive events, that is,

- performing an operation on an entity (OP),
- being idle and waiting for the arrival of an entity (NI),
- being blocked by the proceeding part of the process (FO).

The event sequence table contains the order of these events at the resources. The event sequence table changes if any of its event disappears or new one appears as a consequence of any change of the system control variables. **Deterministic similarity** means that the event sequence table of the original and perturbed sample path are equal.

Based on these definitions our problem can be expressed on the following way:

We suppose that $\delta_{x,y}$ is not infinitesimal, but small enough not to hurt the deterministic similarity. We want to determine the set of $\delta_{x,y}$ which satisfies this requirement.

## 3. EVENT SEQUENCE AND VIRTUAL QUEUE

We will assign numeric values to the activities of the event sequence table and group NI (no-input) and FO (full-output) activities in different matrices. First, however, we have to introduce the concept of the 'virtual queue'.

Entities may wait for a resource either in a queue or, in case of blocking, in an other resource. Let's suppose that $E_i$ is blocked in $R_j$ (Fig. 2.). This practically means that $E_i$ joins to the $c(v)$ entities waiting for the release of $R_v$, by this way transforming $R_j$ into a part of the queue of $R_v$. If in the meantime entities are entering in $Q_j$ then they will join a line which now consists of the entities waiting in $Q_v$, $R_j$ and $Q_j$ for the release of $R_v$. If $Q_j$ is full and blocks $R_p$ then the entities in $R_p$ and in $Q_p$ join the same line, transforming $R_p$ and $Q_p$ as well into part of the virtual queue of $R_v$.

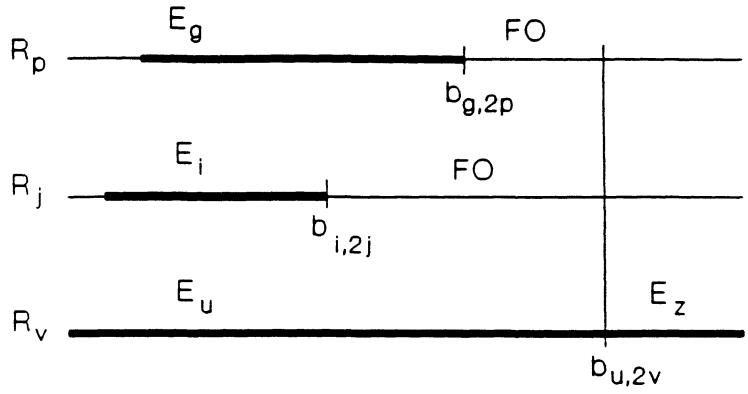


**Figure 2.**
Illustration of the virtual queue.

*Definition 1:*

The virtual queue consists of all those resources and queues in which the entities are waiting at the same time for the release of the same resource.

In further discussions, let $\boldsymbol{V(t)} \subset \{1, \ldots, M\}$ denote the index set of the resources belonging to a virtual queue of $\boldsymbol{R_v}$ $\boldsymbol{v} \in \boldsymbol{V(t)}$, as $\boldsymbol{E_u}$ causes blocking in it when $b_{u,2v-1} \leq t \leq b_{u,2v}$. If we move upstream in this virtual queue, then the utmost resource which is blocked (by $\boldsymbol{E_g}$) will be $\boldsymbol{R_p}$ $\boldsymbol{p} \in \boldsymbol{V(t)}$.

A state of a queue changes if the number of entities in the queue changes [6]. This will be true for the virtual queue as well. The only difference is that even if the capacity of all the resources is one, the state of the virtual queue may change by more than one. It may occur when a queue joins the virtual queue and contains more than one entity. For example in Fig. 2. all the entities in $Q_p$ join the virtual queue of $R_v$ at $b_{g,2p}$. It may also occur that more than one resource and their queues join or leave the virtual queue at the same time.

The capacity of the virtual queue is the sum of the capacity of all the queues participating in it (In Fig. 2. $c'(v) = c(v) + c(j) + c(p)$). An extreme case of the virtual queue is a transfer line where the last workstation is the bottleneck of the system and blocks all the other machines.

The pseudo code of the algorithm to find the resource and entity $(R_v, E_u)$ which are responsible for the appearance of the virtual queue containing $R_j, E_i$ is represented in Fig. 3.

```
fin:= 0
WHILE j + 1 ≤ M AND i − c(j + 1) > 0 AND fin:= 0 DO
        BEGIN
        IF b_{i−c(j+1),2(j+1)} > b_{i,2j} THEN
            BEGIN
            j := j + 1
            i := i − c(j)
            END
        ELSE
            BEGIN
            u := i + c(j)
            v := j − 1
            fin:= 1
            END
        END
store (v, u)
```

**Figure 3.** Pseudo code of the algorithm for searching $E_u, R_v$.

The last resource element of the virtual queue and the entity residing in it $(R_p, E_g)$ can be found by the algorithm given in Fig. 4. It is assumed that $R_v$ and $E_u$ is already known and the search is started at $E_i$ in $R_j$ $j \in V(t)$.

```
fin:= 0
WHILE j − 1 > 0 AND i + c(j) ≤ N AND fin:= 0 DO
        BEGIN
        IF b_{i,2j} < b_{u,2v} THEN
            BEGIN
            i := i + c(j)
            j := j − 1
            END
        ELSE
            BEGIN
            g := i − c(j + 1)
            p := j + 1
            fin:= 1
            END
        END
store (p, g)
```

**Figure 4.** Pseudo code of the algorithm for searching $E_g, R_p$.

## 4. QUANTITATIVE ANALYSIS OF THE FO AND NI EVENTS

We will define two FO and two NI matrices based on whether the event (FO or NI) in question is **experienced at** or **caused by** $E_i$ in $R_j$.

### 4.1. Full-output matrices

*Definición 2*

The first level full-output matrix (FO(1)) expresses the duration of the FO $R_j$ has to **endure**, while it contains $E_i$.

A FO event exists when an entity can not leave the resource because the queue (either real or virtual) it has to enter is full. On Fig. 5(a), and 5(b), it can be seen that $E_i$ wants to leave $R_j$ and go to $R_{j+1}$ but $Q_{j+1}$ is full. Since the capacity of $Q_{j+1}$ is $c(j+1)$, $R_j$ will be blocked until $E_{i-c(j+1)}$ can not leave $R_{j+1}$ and free a place in $Q_{j+1}$. This event may occur at $b_{i-c(j+1),2(j+1)}$ if $j+1 \notin V(t)$ or at $b_{u,2v}$ if $j+1 \in V(t)$. The first level full-output matrix ($fo_{i,j,1}$) denotes the length of the FO event and is calculated as follows.

$$(1) \qquad fo_{i,j,1} = \begin{cases} b_{i-c(j+1),2(j+1)} - b_{i,2j} & \text{if } j+1 \notin V(t) \\ b_{u,2v} - b_{i,2j} & \text{otherwise} \end{cases}$$

If $fo_{i,j,1} > 0$ then FO really exists when $E_i$ wants to leave $R_j$ and the blocking of $R_j$ lasts $fo_{i,j,1}$. If $fo_{i,j,1} \leq 0$ then there is no FO and $fo_{i,j,1}$ expresses the available time to avoid it.

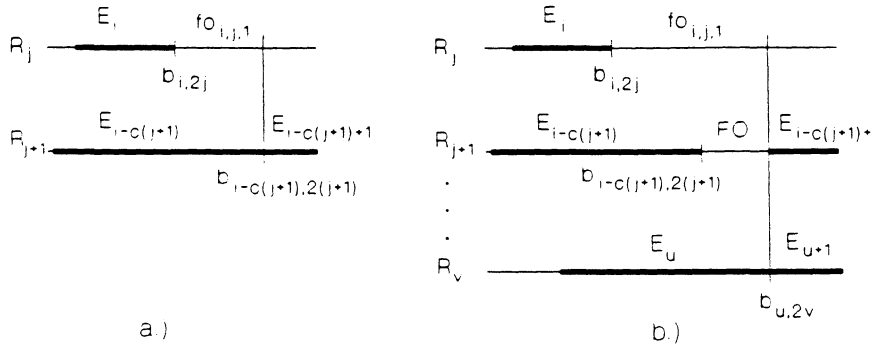


**Figure 5.**
Calculation of the FO(1) matrix.

*Definition 3*

The second level full-output matrix (FO(2)) expresses the duration of the FO $R_j$ **causes**, while it contains $E_i$. The calculation of $fo_{i,j,2}$ is as follows (fig 6(a), 6(b)):

If. $(j-1) \notin V(t)$ then.

$$fo_{i,j,2} = b_{i+c(j),2(j-1)} - b_{1,2j} \tag{2}$$

If. $(j-1) \in V(t)$ then.

$$fo_{i,j,2} = \begin{cases} b_{g,2p} - b_{i,2j} & \text{if. } b_{g+1,2(p-1)} \leq b_{u,2i} \\ b_{g+1,2(p-1)} - b_{i,2j} & \text{otherwise} \end{cases} \tag{3}$$

where $(R_p, E_g)$ is the last in this virtual queue.

If $fo_{i,j,2} < 0$, then FO really exists when $E_g$ wants to leave $R_p$. If $fo_{i,j,2} \geq 0$, then there is no FO and $fo_{i,j,2}$ expresses the available time to avoid it.

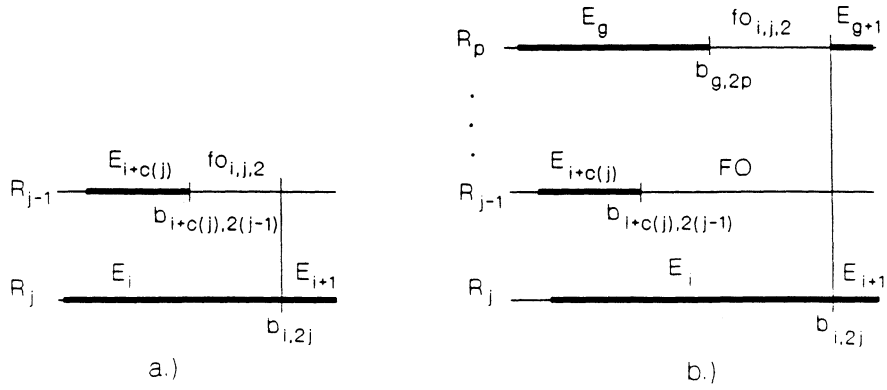


**Figure 6.** Calculation of the FO(2) matrix.

## 4.2. No-input matrices

*Definition 4*

The first level no-input matrix (NO(1)) expresses the duration of the NI $R_j$ has to **endure** after completing the operation on $E_i$.

A no-input event exists when a resource is empty and waits for an incoming entity. It may occur just after the termination of an operation (Fig. 7(a).) or after the release of the resource from blocking (Fig. 7(b).).

If. $(j-1) \notin V(t)$ and $j \notin V(t)$ then.

$$ni_{i,j,1} = b_{i+1,2(j-1)} - b_{i,2j} \tag{4}$$

If. $(j-1) \notin V(t)$ and $j \in V(t)$ then.

$$ni_{i,j,1} = b_{i+1,2(j-1)} - b_{i,2j} - fo_{i,j,1} \tag{5}$$

If $ni_{i,j,1} > 0$. then NI really exists when $E_i$ leaves $R_j$ and $ni_{i,j,1}$ expresses the time $R_j$ has to wait for $E_{i+1}$. If $ni_{i,j,1} \leq 0$, then there is no NI and $ni_{i,j,1}$ expresses the available time to avoid it. When both $R_j$ and $R_{j+1}$ belong to a virtual queue, then $ni_{i,j,1}$ has no meaning because $E_{i+1}$ can never cause NI at $R_j$.

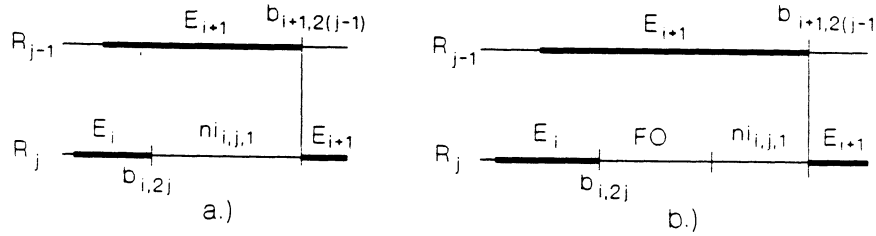


**Figure 7.**
Calculation of the NI(1) matrix.

*Definition 5*

The second level no-input matrix (NO(2)) expresses the duration of the NI **caused** by $R_j$ while performing operation on $E_i$.

The calculation of $ni_{i,j,2}$ is as follows (fig 8(a). 8(b)):

$$ni_{i,j,2} = \begin{cases} b_{i-1,2(j+1)} - b_{i,2j} & \text{if } j+1 \notin V(t) \\ b_{u,2i} - b_{i,2j} & \text{otherwise} \end{cases} \tag{6}$$

If $ni_{i,j,2} < 0$. then NI really exists when $E_{i-1}$ ($E_u$) leaves $R_{j+1}$ ($R_i$) and $ni_{i,j,2}$ expresses the time $R_{j+1}$ ($R_i$) has to wait for $E_i$ ($E_{u+1}$). If $ni_{i,j,2} \geq 0$. then there is no NI and $ni_{i,j,2}$ expresses the time we have to avoid it.

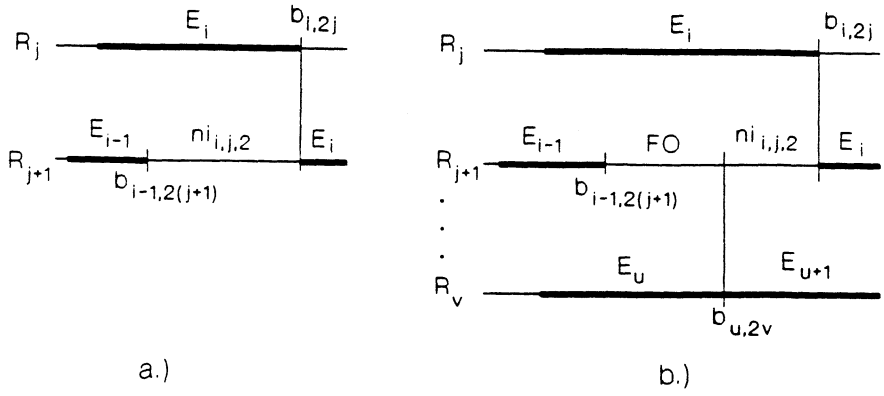


**Figure 8.**
Calculation of the NI(2) matrix.

## 5. CALCULATION OF THE VALIDITY RANGE

In case of zero-order perturbation the size of $fo_{i,j,k}$ and $ni_{i,j,k}$ may change if the perturbation appears only at one of the two entities which participate in the calculation of $fo_{i,j,k}$ and $ni_{i,j,k}$. If it appears at both entities then the size of $fo_{i,j,k}$ and $ni_{i,j,k}$ will not change.

Let $\mathbf{Z}$ denote a set of the $(i,j)$ index-pairs belonging to those $E_i$ at $R_j$ which $fo_{i,j,k}$ and $ni_{i,j,k}$ $(k = 1,2)$ change due to the effect of the introduced perturbation.

### Theorem 1

$Q_j$, $R_j$ $j = 1,\ldots,M$ and $E_i$ $i = 1,\ldots,N$ determine a transfer line type queuing network with $c(j)$ queue capacities and FIFO queuing disciplines. A perturbation $(\delta_{x,y})$ is introduced on $E_x$ at $R_y$, and has an effect of $\delta_{i,j}$ on $E_i$ at $R_j$. Deterministic similarity holds as long as

$$\begin{array}{cc} \text{MAX}(ni_{i,j,k}, fo_{i,j,k}) \leq \delta_{x,y} \leq & \text{MIN}(ni_{i,j,k}, fo_{i,j,k}) \\ ni_{i,j,k} \leq 0 & ni_{i,j,k} > 0 \\ fo_{i,j,k} \leq 0 & fo_{i,j,k} > 0 \end{array} \tag{7}$$

$$i,x \in \{1,\ldots,N\}; \quad j,y \in \{1,\ldots,M\};$$
$$i,j \in \mathbf{Z}; \quad k = 1,2;$$

*Proof*

Due to the propagation of $\delta_{x,y}$ the resulting $\delta_{i,j}$ may have the following effects on the FO and NI events,

If $\delta_{i,j} > 0$, then while $R_j$ is working on $E_i$

- FO and NI may disappear after $R_j$,
- FO may appear on the preceding resources,
- NI may appear on the subsequent resources.

If $\delta_{i,j} < 0$, then while $R_j$ is working on $E_i$

- FO and NI may appear after $R_j$,
- FO may disappear on the preceding resources,
- NI may disappear on the subsequent resources.

Identifying these cases in the FO(1), FO(2), NI(1) and NI(2) matrices directly comes (7).

■

## 6. EXTENSION OF THE PROBLEM

The calculation introduced in the previous sections is valid only in transfer line type networks. It is easy, however, to generalize the method to be valid for general networks (GN). Fig. 9. shows the type of network we want to deal with. We kept the notations of Fig. 1., but completed it with the routing parameter $(r_{i,j})$. There are two basic problem we have to cope with,

- entities may follow different routes in the network,
- entities may overtake other entities.

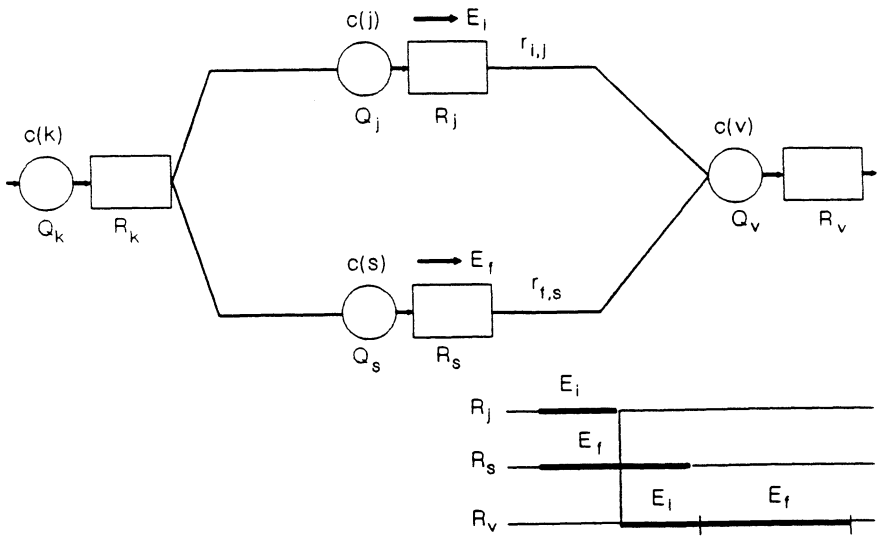


**Figure 9.**
Illustration of a general queuing network.

### 6.1. The effect of different routes

The possibility of the different routes makes necessary to revise that simple index identification of entities and resources we introduced at the transfer line type queuing networks.

Since in GN entities may follow different routes, the order of entities at the various resources may be distinct. Let us suppose that $E_i$ is the $i-$th entity on $R_j$ and leaving $R_j$ it goes to $R_v$. Since $R_v$ can be visited by entities from other resources as well we have no guarantee that $E_i$ will also be the $i-$th at $R_v$. This phenomena causes the difficulty. In transfer lines we can identify entities by their place in the order at the various resources. At the calculation of the validity range we implicitly used this fact. To cope with GN we have to introduce a general entity identification.

Let $n_{i,j}$ denote the number in the order of $E_i$ at $R_j$ $i = 1, \ldots, N$, $j = 1, \ldots, M$ and $n^*_{k,j}$ $k = 1, \ldots, N$ the identification of the $k-$th entity at $R_j$.

The $n_{i,j}$ and $n^*_{i,j}$ matrices can be obtained from the simulation output. From the $B$ matrix we can easily derive $n_{i,j}$ if we put in increasing order $b_{i,2j-1}$ or $b_{i,2j}$ $i = 1, \ldots, N$. In a transfer line case $n_{i,j} = i$ and $n^*_{(i,j)}$ identifies $E_i$.

A similar approach should be applied for the identification of resources. At the transfer line $E_i$ always moves from $R_j$ to $R_{j+1}$, in GN, however, it may move to any $R_s$ $s = 1, \ldots, M$. The information of the routing in a sample path will be expressed in the routing matrix.

Let $r_{i,j}$ denote the index of the resource $E_i$ enters after completing the service at $R_j$ and $r^*_{i,j}$ the index of the resource $E_i$ left before arriving to $R_j$.

The $r_{i,j}$ and $r^*_{i,j}$ matrices can be obtained from the simulation output. We can derive $r_{i,j}$ from the $B$ matrix if we put in increasing order $b_{i,2j-1}$ or $b_{i,2j}$ $j = 1, \ldots, M$. The serial of $j$ determines the route of $E_i$. In the transfer line case $r_{i,j} = j + 1$ and $r^*_{i,j} = j - 1$.

Applying these notations (1), (2), (3), (4), (5) and (6) can be transformed to be valid for general queuing networks. The transformation can be seen in the appendix.

### 6.2. The effect of overtake

Overtake of entities may occur as an effect of perturbation at assembly type queues. Let's suppose that both $E_i$ from $R_j$ and $E_f$ from $R_s$ goes to $R_v$ and $b_{i,2j} < b_{f,2s}$ (Fig. 9.). As a consequence of the FIFO rule $n_{i,v} < n_{f,v}$, that is, $E_i$ will get to $R_v$ before $E_f$. If the perturbation changes the relationship of $b_{i,2j}$ and $b_{f,2s}$ the relationship of $n_{i,v}$ and $n_{f,v}$ will also change, violating this way the deterministic similarity. To avoid this, we have to ensure that the perturbation be small enough not to cause overtake of entities.

Let $\mathbf{A}$ denote the index set of the assembly type queues and $v \in \mathbf{A}$. If $Q_v$ visited by $E_i$ from $R_j$ and $E_f$ from $R_s$, then $ot_{i,f,v}$ will denote the time of $E_f$ for not to be passed by $E_i$ on $R_v$ as a consequence of perturbation.

If $ot_{i,f,v} < 0$, then $n_{f,v} < n_{i,v}$, that is, $E_f$ can overtake $E_i$. If $ot_{i,f,v} \geq 0$, then $n_{f,v} > n_{i,v}$, that is, $E_i$ can overtake $E_f$. The overtake matrix is calculated as follows,

(8) $$ot_{i,f,v} = b_{f,2s} - b_{i,2j}$$

Let $\mathbf{W}$ denote a set of the $(i, f)$ index-pairs belonging to those $E_i$ and $E_f$ which $ot_{i,f,v}$ $v \in \mathbf{A}$ changes due to the effect of the introduced perturbation.

From the point of view of the change of order of entities at the resources, deterministic similarity holds as far as the perturbation is small enough not to cause overtake, that is, if

$$
\begin{array}{cc}
\mathrm{MAX}(ot_{i,f,v}) \leq \delta_{x,y} & \leq \mathrm{MIN}(ot_{i,f,v}) \\
ot_{i,f,v} \leq 0 & ot_{i,f,v} > 0
\end{array} \tag{9}
$$

### 6.3. Validity range for general queuing networks

The extension of theorem 1 for general queuing networks is as follows.

**Theorem 2**

$Q_j$, $R_j$ $j = 1, \ldots, M$ and $E_i$ $i = 1, \ldots, N$ determines a general queuing network with $c(j)$ queue capacities and FIFO queuing disciplines. A perturbation $(\delta_{x,y})$ is introduced on $E_x$ at $R_y$, and has an effect of $\delta_{i,j}$ on $E_i$ at $R_j$. Deterministic similarity holds as long as

$$
\begin{array}{ccc}
\mathrm{MAX}(ni_{i,j,k}, fo_{i,j,k}, ot_{i,f,v}) \leq \delta_{x,y} & \leq \mathrm{MIN}(ni_{i,j,k}, fo_{i,j,k}, ot_{i,f,v}) \\
ni_{i,j,k} \leq 0 & ni_{i,j,k} > 0 \\
fo_{i,j,k} \leq 0 & fo_{i,j,k} > 0 \\
ot_{i,f,v} \leq 0 & ot_{i,f,v} > 0
\end{array} \tag{10}
$$

$$
\begin{array}{c}
i, f, x \in \{1, \ldots, N\}; \quad j, v, y \in \{1, \ldots, M\}; \\
i, j \in \mathbf{Z}; \quad i, f \in \mathbf{W}; \quad k = 1, 2
\end{array}
$$

*Proof*

Completing the proof of theorem 1 with the limits imposed by the overtake restrictions, we can get (10).

■

## 7. IMPLEMENTATION OF THE ALGORITHM

Based on theorem 1 the validity range can be calculated by the following algorithm:

1. Various preliminary calculations are performed to obtainthe $B^*$, $R$, $R^*$, $N$, $N^*$ furthermore the NI, FO, OT matrices and the **A** set.
2. The perturbation is propagated and the **Z** and **W** set is determined.

3. With theorem 1 or 2 the validity range is calculated.

The algorithm is polynomial type but, due to the matrices necessary to the calculation, a careful data organization is required. If $N$ is the number of entities, $M$ is the number of resources and $F$ is the number of assembly type nodes then, the required number of input data is approximately $N(8M + FN) + F$. This formula shows that we have a quadratically increasing data requirement. In case of transfer line the quadratic tag is canceled out ($F = 0$).

As a consequence, in systems where $N$ is high (i.g. in communication systems where the number of arriving entities may easily increase $10^4$) data management may slow down the calculation. In manufacturing systems, especially when technology intensive parts are produced in small lots (e.g. FMS), $N$ may not exceed even $10^2$ and the algorithm can be organized easily. The computational time is insignificantly small.

The results provided by the algorithm are valid only for a specific sample path. Considering the stochastic similarity, the convergence properties of a single perturbation should be regarded here as well [3].

The algorithm was tested on various examples. We provide the results of a small system similar to the one in Fig. 9. The queue capacities used in the calculation are the following, $c(k) = 1$, $c(j) = 1$, $c(s) = 2$, $c(v) = 1$. 10 entity with identifications from 1 to 10 enter the system at $Q_k$ and may randomly choose between $R_j$ and $R_s$. They leave the system at $R_v$. The simulation was performed with the SIMAN simulation language. The identification of entities and the resulting $B$ matrix of a sample path can be seen in Table 1. Note that the change of $E_8$ and $E_7$ is not a coincidence. It reflects that the order of entities at the entrance of the system are not the same as at the exit.

**Table 1.**

The simulation output ($B$ matrix)

| ID | $BEG_k$ | $END_k$ | $BEG_j$ | $END_j$ | $BEG_s$ | $END_s$ | $BEG_v$ | $END_v$ |
|---|---|---|---|---|---|---|---|---|
| $E_1$ | 0.0 | 3.0 | 3.0 | 8.0 | ——— | ——— | 8.0 | 20.0 |
| $E_2$ | 3.0 | 6.0 | ——— | ——— | 6.0 | 12.0 | 20.0 | 24.0 |
| $E_3$ | 6.0 | 10.0 | 10.0 | 17.0 | ——— | ——— | 24.0 | 30.0 |
| $E_4$ | 10.0 | 14.0 | ——— | ——— | 20.0 | 25.0 | 30.0 | 33.0 |
| $E_5$ | 14.0 | 21.0 | 24.0 | 27.0 | ——— | ——— | 33.0 | 35.0 |
| $E_6$ | 24.0 | 29.0 | ——— | ——— | 30.0 | 38.0 | 38.0 | 43.0 |
| $E_8$ | 34.0 | 39.0 | 39.0 | 44.0 | ——— | ——— | 44.0 | 57.0 |
| $E_7$ | 29.0 | 34.0 | ——— | ——— | 38.0 | 46.0 | 57.0 | 61.0 |
| $E_9$ | 39.0 | 42.0 | 44.0 | 47.0 | ——— | ——— | 61.0 | 66.0 |
| $E_{10}$ | 44.0 | 52.0 | ——— | ——— | 57.0 | 64.0 | 66.0 | 71.0 |

**Table 2.**

Results of the calculations of perturbation analysis

| Introduction | | SLOPE | Validity range results | | | | |
|---|---|---|---|---|---|---|---|
| RES. | IDEN. | | LIMIT | IDEN. | ID/R | TYPE | |
| $R_k$ | $E_1$ | 1.0 | $-\infty$ | ——— | ——— | ——— | LL |
| | | | $\infty$ | ——— | ——— | ——— | UL |
| $R_k$ | $E_2$ | 0.0 | $-2.0$ | $E_3$ | $R_k$ | FO(1) | LL |
| | | | 3.0 | $E_5$ | $R_k$ | FO(1) | UL |
| $R_k$ | $E_3$ | 0.0 | $-2.0$ | $E_3$ | $R_k$ | FO(1) | LL |
| | | | 3.0 | $E_5$ | $R_k$ | FO(1) | UL |
| $R_k$ | $E_4$ | 0.0 | $-\infty$ | ——— | ——— | ——— | LL |
| | | | 3.0 | $E_5$ | $R_k$ | FO(1) | UL |
| $R_k$ | $E_5$ | 0.0 | $-\infty$ | ——— | ——— | ——— | LL |
| | | | 3.0 | $E_5$ | $R_k$ | FO(1) | UL |
| $R_k$ | $E_6$ | 1.0 | $-1.0$ | $E_9$ | $E_7$ | OT($v$) | LL |
| | | | 1.0 | $E_6$ | $R_k$ | NI(2) | UL |
| $R_k$ | $E_7$ | 1.0 | $-1.0$ | $E_9$ | $E_7$ | OT($v$) | LL |
| | | | 2.0 | $E_8$ | $E_7$ | OT($v$) | UL |
| $R_k$ | $E_8$ | 1.0 | $-1.0$ | $E_9$ | $E_7$ | OT($v$) | LL |
| | | | 2.0 | $E_8$ | $E_7$ | OT($v$) | UL |
| $R_k$ | $E_9$ | 0.0 | $-\infty$ | ——— | ——— | ——— | LL |
| | | | 2.0 | $E_9$ | $R_k$ | FO(1) | UL |
| $R_k$ | $E_{10}$ | 0.0 | $-14.0$ | $E_{10}$ | $R_k$ | FO(1) | LL |
| | | | 5.0 | $E_{10}$ | $R_k$ | NI(2) | UL |

Since SIMAN can directly communicate with FORTRAN [10] the programs for calculating the validity range were written in this programming language. The results can be seen in Table 2. The effect of 10 perturbation was studied. The perturbations were introduced in $R_k$ at every entity independently. The SLOPE column shows the gradient of the through put time with respect to the operation time of the respective entities at $R_k$. These slopes are certainly valid only within the limits provided by the LIMIT columns. If the perturbation exceeds these limits the deterministic similarity is hurt, that is, higher order propagation rule should be applied. The IDEN. and ID/R columns of the result section identify the place, while the TYPE column identifies the reason of the limits. For example if we introduce a perturbation at $E_6$ in $R_k$ then the lower limit will be caused

by an overtake between $E_9$ and $E_7$ while the upper limit will be the result of a NI caused by $E_6$ when it is in $R_k$.

The algorithm was implemented in a practical case as well. The gradient of the throughput time and of the waiting time in queues were calculated for production control purposes in a continuous steel manufacturing process. The approximately 40 entities (steel slabs) and four resources (machines) did not cause any memory management problem and the insignificant computation time allowed on line gradient calculation. For further details about this project see Koltai *et al.* [9].

## 8. CONCLUSIONS

An algorithm was provided to calculate the validity range of deterministic similarity at the simulation of general queuing networks. The calculation provides information about the validity of the zero order gradient of the throughput time with respect to the operation time of entities at resources. The algorithm requires only the time schedule of the resources ($B$ matrix) which can be provided easily by any simulation language in use today. The information gained by the calculation can be attached to the simulation output, providing useful information for parametric analysis of the throughput time.

The results can also provide basis for the calculation of the validity range of higher order perturbations, for the computation of multi variable gradient information, and for the examination of the gradient information concerning other parameters and performance measures, which are all subject of our further research. We expect that any result in these areas might support the improvement of the planning and dynamic control of discrete event dynamic systems.

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## 10. APPENDIX

### a) The examination of virtual queue

The transformed version of the algorithm to look for $R_v$, $E_u$ is presented in Fig. 10. and the algorithm for finding $R_p$, $E_g$ belonging to $R_j$, $E_i$ when $R_v$, $E_u$ is already known, is given in figure 11.

To facilitate further discussion let $b^*_{i,2j-1}$ denote the beginning and $b^*_{i,2j}$ the ending time of the operation of the $i$–th entity that is, entity $n_{i,j}$ on $R_j$.

```
nr := r_{i,j}                          (next resource)
ec := n*_{n_{i,nr}-c(nr),nr}           (entity for comparison)
fin:= 0
WHILE nr ∈ {1,...,M} AND ec ∈ {1,...,N} AND fin:= 0 DO
        BEGIN
        IF b_{ec,2nr} < b_{i,2j} THEN
            BEGIN
            j := nr
            i := ec
            nr := r_{i,j}
            ec := n*_{n_{i,nr}-c(nr),nr}
            END
        ELSE
            BEGIN
            v := j
            u := i
            fin:= 1
            END
        END
store (v, u)
```

**Figure 10.**
Pseudo code of the algorithm for searching $E_u$, $R_v$ in general networks.

```
ec := n*_{n_{i,j}+c(j),j}              (entity for comparison)
pr := r*_{ec,j}                        (previous resorce)
fin:= 0
WHILE pr ∈ {1,...,M} AND ec ∈ {1,...,N} AND fin:= 0 DO
        BEGIN
        IF b_{ec,2pr} < b_{u,2v} THEN
            BEGIN
            i := ec
            j := pr
            ec := n*_{n_{i,j}+c(j),j}
            pr := r*_{ec,j}
            END
        ELSE
            BEGIN
            g := i
            p := j
            fin:= 1
            END
        END
store (p, g)
```

**Figure 11.**
Pseudo code of the algorithm for searching $E_g$, $R_p$ in general networks.

### b) Calculation of the FO(1) matrix

Recall the elements in figure 5a, 5b. In addition, let's suppose that $r_{i,j} = s$, that is $E_i$ goes to $R_s$ after completing the operation at $R_j$.

(1) $$fo_{i,j,1} = \begin{cases} b^*_{n_{i,s}-c(s),2s} - b^*_{n_{i,j},2j} & s \notin V(t) \\ b^*_{n_{u,v},2v} - b^*_{n_{i,j},2j} & \text{otherwise} \end{cases}$$

### c) Calculation of the FO(2) matrix

Recall the elements in figure 6a, 6b. In addition, let's suppose that $r^*_{i,j} = s$, that is $E_i$ visited $R_s$ before arriving to $R_j$.

If, $s \notin V(t)$ then,

(2) $$fo_{i,j,2} = b^*_{n_{i,s}+c(j),2s} - b^*_{n_{i,j},2j}$$

If, $s \in V(t)$ then

(3) $$fo_{i,j,2} = \begin{cases} b^*_{n_{g,p},2p} - b^*_{n_{i,j},2j} & \text{if } b^*_{n_{g,p}+1,2(p-1)} \geq b^*_{u,2v} \\ b^*_{n_{g,p}+1,2p} - b^*_{n_{i,j},2j} & \text{otherwise} \end{cases}$$

### d) Calculation of the NI(1) matrix

Recall the elements in figure 7a, 7b. In addition, let's suppose that $r^*_{i,j} = s$, that is $E_i$ visited $R_s$ before arriving to $R_j$.

If, $s \notin V(t)$ and $j \notin V(t)$ then,

(4) $$ni_{i,j,1} = b^*_{n_{i,s}+1,2s} - b^*_{n_{i,j},2j}$$

If, $s \notin V(t)$ and $j \in V(t)$ then,

(5) $$ni_{i,j,1} = b^*_{n_{i,s}+1,2s} - b^*_{n_{i,j},2j} - fo_{i,j,1}$$

If, $s \in V(t)$ then $ni_{i,j,1}$ has no meaning.

### e) Calculation of the NI(2) matrix

Recall the elements in figure 8a, 8b. In addition, let's suppose that $r_{i,j} = s$, that is $E_i$ goes to $R_s$ after completing the operation at $R_j$.

(6) $$ni_{i,j,2} = \begin{cases} b^*_{n_{i,s}-1,2s} - b^*_{n_{i,j},2j} & s \notin V(t) \\ b^*_{n_{u,v},2v} - b^*_{n_{i,j},2j} & \text{otherwise} \end{cases}$$