

Addendum to the paper: Global Attractors for Multivalued Random Dynamical Systems, *Nonlinear Analysis* 48 (2002), 805-829

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The aim of this note is to clarify a misleading statement which appeared in our first paper [3] dealing with the concept of global random attractor for multivalued random dynamical systems, as well as in the subsequent work [4].

Using the notation and assumptions from [3], we have the following new definition.

Definition 1 *A closed random set $\omega \mapsto \mathcal{A}(\omega)$ is said to be a global random attractor of the MRDS G if:*

i) $G(t, \omega)\mathcal{A}(\omega) \supseteq \mathcal{A}(\theta_t\omega)$, for all $t \geq 0$, $\mathbb{P} - a.s$ (that is, it is negatively invariant);

ii) for all $D \subset X$ bounded,

$$\lim_{t \rightarrow +\infty} \text{dist}(G(t, \theta_{-t}\omega)D, \mathcal{A}(\omega)) = 0;$$

iii) $\mathcal{A}(\omega)$ is compact $\mathbb{P} - a.s$.

Notice that in [3, 4] a random attractor was required to be strictly invariant, i.e.

$$G(t, \omega)\mathcal{A}(\omega) = \mathcal{A}(\theta_t\omega), \quad \text{for all } t \geq 0, \mathbb{P} - a.s.$$

Theorem 2 *Let assumptions (H1) – (H2) hold (see [3]), the map $(t, \omega) \mapsto \overline{G(t, \omega)D}$ be measurable for all deterministic bounded sets $D \subset X$, and the map $x \in X \mapsto G(t, \omega)x$ have compact values. Then,*

$$\mathcal{A}(\omega) := \overline{\bigcup_{\substack{D \subset X \\ \text{bounded}}} \Lambda_D(\omega)} \tag{1}$$

is a global random attractor for G (measurable with respect to \mathcal{F}). It is unique and the minimal closed attracting set.

Moreover, if the map $x \mapsto G(t, \omega)x$ is lower semicontinuous for each fixed (t, ω) , then the global random attractor $\mathcal{A}(\omega)$ is strictly invariant, i.e., $G(t, \omega)\mathcal{A}(\omega) = \mathcal{A}(\theta_t\omega)$, for all $t \geq 0$.

Without assuming the lower semicontinuity of the map $x \mapsto G(t, \omega)x$, one can only prove (as it appears in [3]) the negative invariance of the sets defined in (1), which according to Definition 1 ensures the existence of the global attractor according to this definition.

Now, taking into account the additional lower semicontinuity assumed in Theorem 2, we will prove the strict invariance of the global random attractor given by (1).

Indeed, we first note that the set $\mathcal{A}'(\omega) = G(t, \theta_{-t}\omega)\mathcal{A}(\theta_{-t}\omega)$ is negatively invariant, since

$$\begin{aligned} \mathcal{A}'(\theta_t\omega) &= G(t, \omega)\mathcal{A}(\omega) \subset G(t, \omega)G(t, \theta_{-t}\omega)\mathcal{A}(\theta_{-t}\omega) \\ &= G(t, \omega)\mathcal{A}'(\omega). \end{aligned}$$

Moreover, the set $\mathcal{A}'(\omega)$ is \mathbb{P} -a.s. compact. This follows from the fact that $\mathcal{A}(\theta_{-t}\omega)$ is \mathbb{P} -a.s. compact, the map $x \mapsto G(t, \omega)x$ is upper semicontinuous and G has compact values (see Aubin and Cellina, [1, p.42], Proposition 3).

We need to prove (and this is the point missed in [3]) that $\mathcal{A}'(\omega)$ is measurable. Since $x \mapsto G(t, \omega)x$ is continuous, the map $(\omega, x) \mapsto G(t, \omega, x)$ is Caratheodory. Hence, $\omega \mapsto G(t, \omega)\mathcal{A}(\omega)$ is measurable with respect to the \mathbb{P} -completion of \mathcal{F} (see [2]). It follows from the proof of Theorem 3 in [3] that $\mathcal{A}(\omega)$ is the maximal random (w.r.t. the \mathbb{P} -completion of \mathcal{F}) negatively invariant and compact set. Hence, $\mathcal{A}'(\omega) \subset \mathcal{A}(\omega)$, and $\mathcal{A}(\omega)$ becomes strictly invariant.

We note that in the applications given in [3, 4] this condition is satisfied, so the global random attractor is in addition strictly invariant, as was stated in those papers.

Acknowledgements. We would like to thank Pedro Marín-Rubio for having pointed out that the paper contained this misleading statement.

References

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