Revista Mexicana de Física **40**, *Suplemento 1 (1994) 318–329*

On the use of baryon mappings to derive nuclei from quarks

S. PITTEL
Bartol Research Institute, University of Delaware
Newark, Delaware 19716, USA

J. M. ARIAS
Departamento de Física Atómica, Molecular y Nuclear
Universidad de Sevilla, Apartado 1065, 41080 Sevilla, Spain

J. DUKELSKY
Grupo de Física Nuclear, Facultad de Ciencias
Universidad de Salamanca, 37008 Salamanca, Spain

AND

A. FRANK
Instituto de Ciencias Nucleares
and
Laboratorio de Cuernavaca, Instituto de Física, UNAM
Apartado postal 70-543, 04510 México, D.F., México
Received 6 January 1994; accepted 11 April 1994

ABSTRACT. A consistent baryon mapping of colorless three-quark clusters is proposed and tested in the context of a three-color extension of the Lipkin model. The results suggest that baryon mappings may provide a practical means of deriving nuclei from constituent quark models.

RESUMEN. Se propone en forma consistente un mapeo bariónico de agrupamientos de tres quarks acoplados a color cero. El mapeo se pone a prueba en el contexto de una extensión de tres colores del modelo de Lipkin. Los resultados sugieren que los mapeos bariónicos podrian dar lugar a métodos prácticos de derivación de la estructura de los núcleos a partir de modelos de quarks constituyentes.

PACS: 21.60.Gx; 21.60.Fw; 21.30.+y

1. INTRODUCTION

The traditional view of nuclear physics is based on the assumption that the QCD interactions that cluster quarks into nucleons completely decouple from the residual interactions between nucleons responsible for nuclear structure. Recent experiments by the EMC collaboration, however, indicate that the structure of the nucleon is in fact modified in a nuclear medium, suggesting that there is some coupling between the nucleon and nuclear scales. As a consequence, there is currently great interest in trying to develop methods to derive the properties of nuclei *directly* from QCD. Establishing such a link between QCD and nuclear physics should also help to elucidate where to look for explicit quark effects in nuclei.

At present, it is not feasible to establish this connection starting from QCD. A somewhat less-ambitious starting point is the use of QCD-inspired constituent quark models [1]. Such models have been applied with considerable success to nuclear systems with very few particles, but they have not yet been implemented for many-nucleon systems. The reason is that in a nuclear environment quark triplets cluster into spatially-localized nucleons, a scenario that cannot be described with existing many-body techniques.

In this talk, we explore the use of mapping methods as a practical means of accomplishing this. The idea is to map colorless three-quark clusters, which do not satisfy exact fermion anticommutation rules, onto triplet-fermions (baryons) that do. Such a mapping leads from the original multi-quark hamiltonian to an effective baryon hamiltonian, which rigorously incorporates the physics of the quark Pauli principle. In addition, since it is a hamiltonian for interacting fermions, it is amenable to the usual fermion many-body techniques [2].

Several groups have recently addressed the issue of baryon mappings [3–5]. Pittel, Engel, Dukelsky and Ring [3] proposed a two-step mapping that was specifically tailored to two-quark interactions. Their mapping cannot be applied, however, to systems with strong three-quark interactions. At roughly the same time, Nadjakov [4] suggested an alternative baryon mapping that should be applicable to systems dominated by three-quark interactions, but not to systems with strong two-quark interactions. What is needed is a mapping that *consistently* treats *both* two- and three-quark interactions. We have now succeeded in developing such a mapping, which we briefly describe in Section 2.

In Section 3, we describe a simple model that we have developed to test our consistent mapping. The model we have chosen is a three-color extension of the well-known Lipkin model [6], which in its traditional version has been used extensively to test various nuclear many-body techniques. In Section 4, we apply our mapping to this model for two baryons and present the results. The bottom line is that it seems to work perfectly.

2. Baryon mapping of quark systems

2.1. General remarks

Our starting point is a nonrelativistic model of constituent quarks. We denote the quark creation and annihilation operators by $q^\dagger_{1a}$ and q_{1a}, respectively. The first subscript denotes the color quantum number and the second all the rest. These operators satisfy the usual fermion anticommutation relation

$$\left\{q_{1a},\, q^\dagger_{2b}\right\} = \delta_{1a,2b} \equiv \delta_{12}\,\delta_{ab}. \tag{1}$$

QCD considerations suggest that a realistic quark hamiltonian may include up to three-body interactions, all of which are color scalars. Such a hamiltonian can always be expressed in terms of the following colorless one-, two- and three-body operators, respectively:

$$A_{ab} = \sum_{1} q^{\dagger}_{1a} q_{1b}; \tag{2}$$

$$B_{abcd} = \sum_{12345} \epsilon_{123}\, \epsilon_{145}\, q^{\dagger}_{2a}\, q^{\dagger}_{3b}\, q_{5d}\, q_{4c}; \tag{3}$$

and

$$C_{abcdef} = \sum_{123456} \epsilon_{123}\, \epsilon_{456}\, q^{\dagger}_{1a}\, q^{\dagger}_{2b}\, q^{\dagger}_{3c}\, q_{6f}\, q_{5e}\, q_{4d}. \tag{4}$$

The ϵ_{123} quantities are antisymmetric tensors that guarantee the colorless nature of these operators.

The idea of a baryon mapping is to replace this problem by an *equivalent* one involving triplet-fermions or baryons. We denote the creation and annihilation operators of the baryon space by $\Lambda^{\dagger}_{1a2b3c}$ and Λ_{1a2b3c}, respectively. They, by definition, satisfy the multi-index anticommutation relation

$$\{\Lambda_{1a2b3c},\, \Lambda^{\dagger}_{4d5e6f}\} = \delta(1a2b3c, 4d5e6f), \tag{5}$$

where

$$\begin{aligned}\delta(1a2b3c, 4d5e6f) \equiv\ & \delta_{1a,4d}\delta_{2b,5e}\delta_{3c,6f} + \delta_{1a,5e}\delta_{2b,6f}\delta_{3c,4d} + \delta_{1a,6f}\delta_{2b,4d}\delta_{3c,5e} \\ & -\delta_{1a,4d}\delta_{2b,6f}\delta_{3c,5e} - \delta_{1a,5e}\delta_{2b,4d}\delta_{3c,6f} - \delta_{1a,6f}\delta_{2b,5e}\delta_{3c,4d}.\end{aligned} \tag{6}$$

Since the operator $\Lambda^{\dagger}_{1a2b3c}$ (Λ_{1a2b3c}) creates (annihilates) a baryon that *corresponds* to three quarks in the states $1a, 2b$ and $3c$, it is antisymmetric under the interchange of its quark indices, *e.g.*, $\Lambda^{\dagger}_{1a2b3c} = -\Lambda^{\dagger}_{2b1a3c}$, *etc.*

The space generated by these baryon operators is in fact larger than the original quark space. It includes a subset of states that are fully antisymmetric under the interchange of quark indices and in one-to-one correspondence with the original states of the quark space; this is referred to as the *physical subspace.* But it also contains states that are not fully antisymmetric under quark interchange. These states, called *unphysical*, have no counterparts in the original quark space. They are a pure artifact of the mapping and provide the principal difficulty in developing a *practical* baryon mapping.

As we will discuss shortly, there are several possible mappings that can be developed, all of which exactly preserve the physics of the original problem *in the physical-baryon subspace.* Where they differ is in their predictions for unphysical states. Clearly, for a mapping to be of practical use, the unphysical states must lie high in energy relative to the physical states. Otherwise, it will be difficult to disentangle the physical states of interest from those that are unphysical, particularly in the presence of variational approximations.

So far, we have not discussed how to guarantee that the physics of the original problem is preserved by the mapping. Here we follow the Belyaev-Zelevinski prescription, whereby the mapping is defined so as to exactly preserve the commutation relations for physical operators in the original quark space.

There is a simple way to accomplish this. Consider the colorless one-body operator A_{ab} defined in (2.2). The commutator $[A_{ab},\ A_{cd}]$ is exactly preserved if this operator is mapped according to

$$A_{ab} = \sum_{1} q^{\dagger}_{1a} q_{1b} \rightarrow \frac{1}{2} \sum_{123cd} \Lambda^{\dagger}_{1a2c3d} \Lambda_{1b2c3d}. \tag{7}$$

Since the operators in (2.3–2.4) can be rewritten in terms of colorless one-body operators, it would seem that we could also apply (2.7) to them and achieve our goal. Unfortunately, this is not the case.

Since a one-body operator contains just one creation and one annihilation operator, it cannot incorporate information on the quark Pauli principle. Thus, when we apply this simple mapping, we find that (*i*) it reproduces all quark dynamics in the physical subspace, *but* (*ii*) it invariably leads to unphyical states lower in energy than the physical states of interest.

To incorporate quark Pauli effects in a practical way, we *must* map the multi-quark creation and annihilation operators that appear in the two- and three-body interactions directly. What this means is that we must find a mapping that preserves the commutation relations between the colorless one-, two- and three-body operators *simultaneously.*

2.2. Colorless baryons

The baryon operators introduced in the previous subsection all have color. We know, however, that it is possible to describe all of the relevant physics solely in terms of colorless baryons. The relevant operators for colorless baryons can be defined according to

$$\Lambda^{\dagger}_{abc} = \frac{1}{6} \sum_{123} \epsilon_{123} \Lambda^{\dagger}_{1a2b3c}, \tag{8}$$

and

$$\Lambda_{abc} = \frac{1}{6} \sum_{123} \epsilon_{123} \Lambda_{1a2b3c}. \tag{9}$$

They are fully symmetric under the interchange of their indices and satisfy the anticommutation relation

$$\{\Lambda_{abc},\ \Lambda^{\dagger}_{def}\} = \tfrac{1}{6}\, S(abc, def), \tag{10}$$

where

$$S(abc, def) \equiv \delta_{ad}\delta_{be}\delta_{cf} + \delta_{ae}\delta_{bf}\delta_{cd} + \delta_{af}\delta_{bd}\delta_{ce} + \delta_{ad}\delta_{bf}\delta_{ce} + \delta_{ae}\delta_{bd}\delta_{cf} + \delta_{af}\delta_{be}\delta_{cd}. \tag{11}$$

In what follows, we carry out our mapping directly from the space of colorless quark operators to the space built out of these colorless baryon operators.

2.3. The hermitean baryon mapping

First we present the results obtained using the simple mapping described earlier, in which the colorless quark operators are first expressed as products of colorless one-body operators A_{ab} and then mapped according to (2.7). A subsequent truncation to colorless baryons is then achieved by making the replacements

$$\begin{aligned} \Lambda^{\dagger}_{1a2b3c} &\rightarrow \epsilon_{123}\Lambda^{\dagger}_{abc}, \\ \Lambda_{1a2b3c} &\rightarrow \epsilon_{123}\Lambda_{abc}, \end{aligned} \tag{12}$$

as discussed in Ref. [3].

Implementing the above prescription leads to the following results:

$$A_{ab} \rightarrow A^{\mathrm{h}}_{ab} = 3\sum_{cd} \Lambda^{\dagger}_{acd}\Lambda_{bcd}; \tag{13}$$

$$\begin{aligned} B_{abcd} \rightarrow B^{\mathrm{h}}_{abcd} = {} & 12\sum_{f} \Lambda^{\dagger}_{abf}\Lambda_{cdf} \\ & -9\sum_{efgh} \Lambda^{\dagger}_{aef}\Lambda^{\dagger}_{bgh}(\Lambda_{cef}\Lambda_{dgh} + \Lambda_{def}\Lambda_{cgh}); \end{aligned} \tag{14}$$

$$\begin{aligned} C_{abcdef} \rightarrow C^{\mathrm{h}}_{abcdef} = {} & 36\ \Lambda^{\dagger}_{abc}\Lambda_{def} \\ & -36\sum_{ghi}(\Lambda^{\dagger}_{gha}\Lambda^{\dagger}_{bci} + \Lambda^{\dagger}_{ghb}\Lambda^{\dagger}_{cai} + \Lambda^{\dagger}_{ghc}\Lambda^{\dagger}_{abi}) \\ & \qquad (\Lambda_{dgh}\Lambda_{efi} + \Lambda_{egh}\Lambda_{dfi} + \Lambda_{fgh}\Lambda_{dei}). \end{aligned} \tag{15}$$

We refer to this as the *hermitean baryon mapping* and include a superscript h to distinguish it from the improved *nonhermitean baryon mapping* to follow.

2.4. The nonhermitean baryon mapping

The nonhermitean baryon mapping arises when we require the simultaneous preservation of *all* (anti)commutation relations. As noted earlier, we expect it to provide a more satisfactory description of unphysical states, through implicit incorporation of quark Pauli effects. A detailed discussion on how this mapping was obtained can be found in Ref. [7]. Here, we simply present the final results:

$$A_{ab} \rightarrow A^{\mathrm{nh}}_{ab} = 3\sum_{cd} \Lambda^{\dagger}_{acd}\Lambda_{bcd}; \tag{16}$$

$$\begin{aligned} B_{abcd} \rightarrow B^{\mathrm{nh}}_{abcd} = {} & 12\sum_{e} \Lambda^{\dagger}_{abe}\Lambda_{cde} \\ & +9\sum_{efgh} \Lambda^{\dagger}_{aef}\Lambda^{\dagger}_{bgh}(\Lambda_{cde}\Lambda_{fgh} + \Lambda_{efg}\Lambda_{cdh}); \end{aligned} \tag{17}$$

$$C_{abcdef} \to C^{\mathrm{nh}}_{abcdef} = 36\Lambda^{\dagger}_{abc}\Lambda_{def} + 36\sum_{ghi}(\Lambda^{\dagger}_{gha}\Lambda^{\dagger}_{bci} + \Lambda^{\dagger}_{ghb}\Lambda^{\dagger}_{cai} + \Lambda^{\dagger}_{ghc}\Lambda^{\dagger}_{abi})\Lambda_{ghi}\Lambda_{def}. \tag{18}$$

The superscript nh denotes that these are the nonhermitean baryon images.

We should emphasize here that the sets of mapping equations given in this and the preceding subsections can be applied to *any* colorless constituent quark hamiltonian written in uncoupled form.

3. THE THREE-COLOR LIPKIN MODEL

The three-color Lipkin model is based on the well-known Lipkin model [6], which has been used extensively in nuclear physics to test many-body approximation methods. Since many of the characteristics of the three-color Lipkin model are already in the original one, we first devote a few lines to reviewing it.

The Lipkin model has two levels, each Ω-fold degenerate, separated by an energy Δ. It is assumed that in the unperturbed ground state, $N=\Omega$ particles occupy *all* the single-particle states of the lower level. The fermion creation and annihilation operators of the model are written as $q^{\dagger}_{\sigma m}$ and $q_{\sigma m}$ respectively, where σ characterizes whether the particle is in the lower level, $\sigma = -$, or in the upper one, $\sigma = +$, and m denotes which of the Ω degenerate states of that level the particle occupies.

The hamiltonian of the model can be expressed as

$$H = H_1 + H_2, \tag{19}$$

$$H_1 = \frac{\Delta}{2}\sum_{m}(q^{\dagger}_{+m}q_{+m} - q^{\dagger}_{-m}q_{-m}), \tag{20}$$

$$H_2 = -\frac{\chi}{\Omega}\sum_{m_1 m_2}\{q^{\dagger}_{+m_1}q^{\dagger}_{+m_2}q_{-m_2}q_{-m_1} + q^{\dagger}_{-m_1}q^{\dagger}_{-m_2}q_{+m_2}q_{+m_1}\}. \tag{21}$$

In addition to the one-body term, it contains a two-body interaction that scatters pairs of particles among the two levels, without changing their m values.

This model can be solved exactly using group theoretical techniques for any value of Ω and any values of the parameters Δ and χ. The set of all possible one-body operators built from its creation and annihilation operators generates the Lie algebra $\mathrm{U}(2\Omega)$. The structure of the problem suggests a decomposition

$$\mathrm{U}(2\Omega) \supset \mathrm{U}(\Omega) \otimes \mathrm{U}(2). \tag{22}$$

The Lipkin hamiltonian can be rewritten solely in terms of the $\mathrm{U}(2)$ generators and all the states belong to a definite irreducible representation of $\mathrm{U}(2)$ (or $\mathrm{SU}(2)$). This in turn implies that the hamiltonian matrix can be analytically evaluated using the well known $\mathrm{SU}(2)$ angular momentum algebra.

Alternatively, the model can be solved using standard shell-model techniques. Here, solutions are limited to Ω values for which the size of the hamiltonian matrix is tractable. In those cases for which both algebraic and shell-model solutions can be generated, the results are in complete agreement.

The three-color model involves three sets, one for each color, of standard two-level Lipkin models. Again the lower levels are assumed to be completely filled in the unperturbed ground state, which in this case contains $N = 3\Omega$ particles. The creation and annihilation operators now include a label 1 that represents the color quantum number and are thus written as $q^{\dagger}_{1\sigma m}$ and $q_{1\sigma m}$, respectively. The model hamiltonian includes one-body, two-body *and* three-body interactions, all colorless, which scatter particles coherently among the levels.

$$H = H_1 + H_2 + H_3, \tag{23}$$

$$H_1 = \frac{\Delta}{2}\sum_{1m}(q^{\dagger}_{1+m}q_{1+m} - q^{\dagger}_{1-m}q_{1-m}), \tag{24}$$

$$H_2 = -\frac{\chi_2}{\Omega}\sum_{12345,m_1m_2}\epsilon_{123}\epsilon_{145}\{q^{\dagger}_{2+m_1}q^{\dagger}_{3+m_2}q_{5-m_2}q_{4-m_1} + q^{\dagger}_{4-m_1}q^{\dagger}_{5-m_2}q_{3+m_2}q_{2+m_1}\}, \tag{25}$$

and

$$H_3 = -\frac{\chi_3}{\Omega^2}\sum_{123456,m_1m_2m_3}\epsilon_{123}\epsilon_{456}\{q^{\dagger}_{1+m_1}q^{\dagger}_{2+m_2}q^{\dagger}_{3+m_3}q_{6-m_3}q_{5-m_2}q_{4-m_1} + q^{\dagger}_{4-m_1}q^{\dagger}_{5-m_2}q^{\dagger}_{6-m_3}q_{3+m_3}q_{2+m_2}q_{1+m_1}\}. \tag{26}$$

The model contains three parameters, one for each of the terms in the hamiltonian. Whenever $\chi_2 \gg \chi_3$ *and* Δ, the system will be dominated by two-quark correlations. Whenever $\chi_3 \gg \chi_2$ *and* Δ, it will be dominated by three-quark correlations. Thus, it has a rich enough structure to make it useful as a test of our proposed mapping.

The group structure of this model is significantly more complex than for the usual Lipkin model. Now the set of one-body operators generates a Lie algebra U(6Ω), and the structure of the model suggests a classification of states in terms of the chain

$$\mathrm{U}(6\Omega) \supset \mathrm{U}(\Omega)\otimes\mathrm{U}(6) \supset \mathrm{U}(\Omega)\otimes\mathrm{U}(3)\otimes\mathrm{U}(2). \tag{27}$$

The group U(3) is essential in this classification, since all physically admissible states should be colorless, *i.e.*, they should belong to the (Ω,Ω,Ω) representation of U(3) (or equivalently the $(\lambda,\mu) = (0,0)$ scalar representation in Elliott's SU(3) notation). The complication is that different U(6) representations can contain these states and, moreover, for each of them several U(2) representations are connected by the hamiltonian. Nevertheless, we have succeeded in generating algebraic solutions for this model for both $\Omega = 2$ and 3, by appropriate commutator manipulations.

As for the usual Lipkin model, shell-model techniques can also be used here to obtain exact solutions. Such solutions have now been generated for all Ω values up to $\Omega = 6$.

4. TEST OF BARYON MAPPING ON THE THREE-COLOR LIPKIN MODEL

Here, we apply the colorless baryon mappings developed in Section 2 to the three-color Lipkin model. We carry out the analysis for $\Omega = 2$ only, for which the number of baryons is likewise 2. Diagonalization of the effective baryon hamiltonian can be carried out exactly for this case, leading to a direct test of the mapping.

The colorless states of the model, following the mapping, can be expressed as

$$\Lambda^{\dagger}_{\sigma_1 m_1 \sigma_2 m_2 \sigma_3 m_3} \Lambda^{\dagger}_{\sigma_4 m_4 \sigma_5 m_5 \sigma_6 m_6} |0\rangle.$$

The total number of two-baryon states that can be formed is 52. In contrast, the total number of colorless six-quark states in the Lipkin model is 20. That the two-baryon space is larger than the six-quark space was anticipated in our earlier discussion. The two-baryon space includes not only physical states (in one-to-one correspondence with the states of the quark model) but unphysical states as well.

The general three-color Lipkin hamiltonian (3.5–3.8) can be mapped in either nonhermitean or hermitean form, using the results of the previous section. We will be particularly interested in the nonhermitean mapping, since it is expected to provide a more practical incorporation of quark Pauli effects. However, in the results that follow, we consider both, to see whether our expectations are realized.

In order to assess the feasibility of "pushing up" unphysical states with respect to physical states with the nonhermitean mapping, it is important to have a criterion for distinguishing one from the other. This can be done by introducing a Majorana operator [8], analogous to the one used in boson mappings.

Consider the square of the quark number operator,

$$N^2 = \sum_{\sigma_1 \sigma_2 m_1 m_2} q^{\dagger}_{\sigma_1 m_1} q_{\sigma_1 m_1} q^{\dagger}_{\sigma_2 m_2} q_{\sigma_2 m_2} \,. \tag{28}$$

This operator can be mapped both in hermitian and nonhermitean form. The Majorana operator is defined as the difference between the two resulting images,

$$M = N^2_{\mathrm{h}} - N^2_{\mathrm{nh}}. \tag{29}$$

Clearly, it has a zero expectation value for all physical states. Equally important, its expectation value for all unphysical states is positive-definite, making it useful as a means of distinguishing physical from unphysical states.

In Figures 1–2, we present some representative results of our test calculations for two different choices of the model parameters. In both, we show the algebraic results obtained prior to the mapping (denoted *exact*) and the results obtained after the nonhermitean (*nh*) and hermitean (*h*) mappings. In the spectra that refer to diagonalization after the mapping, we distinguish physical from unphysical states by using the Majorana operator of (4.2). Physical states are indicated by solid lines and unphysical states by dashed lines. We use a heavy solid line to denote degenerate (or nearly degenerate) solutions, and indicate to the right the number of physical (P) and unphysical (U) states at that energy. We only show the low-energy portions of the spectra corresponding to $E < 0$.

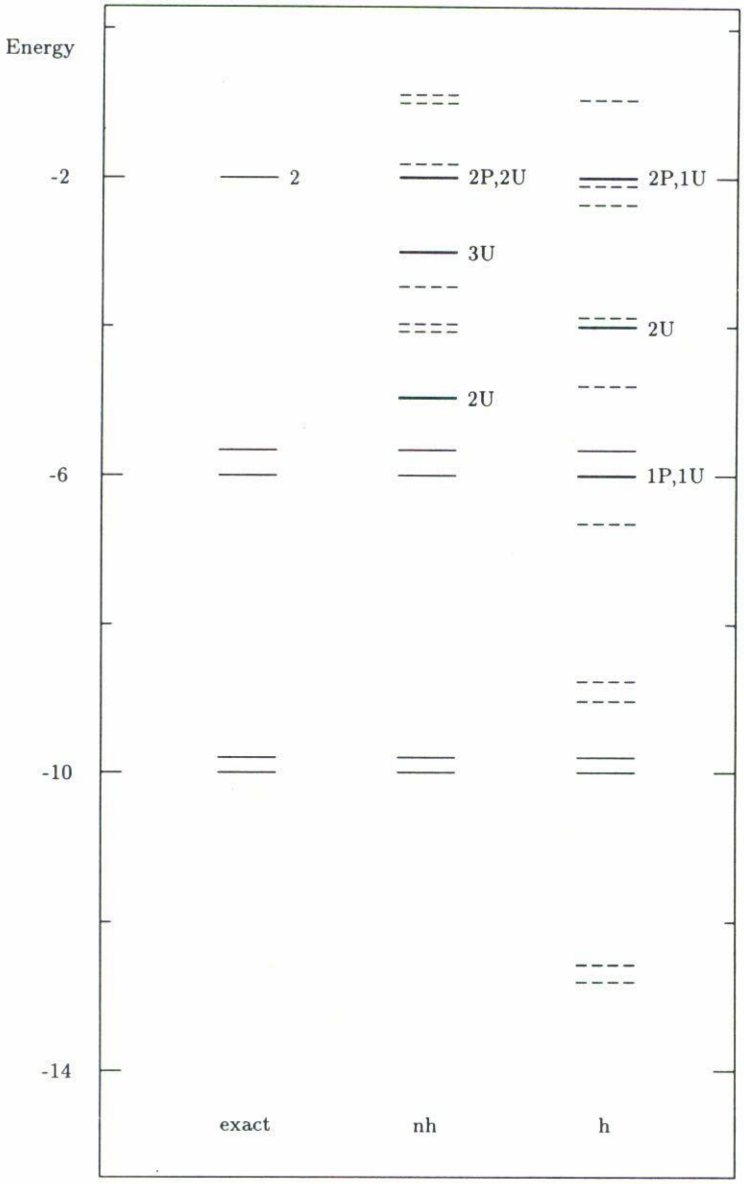

FIGURE 1. Calculated spectra of the three-color Lipkin model for $\Omega = 2$, $\Delta = 0$, $\chi_2 = 1$ and $\chi_3 = 0$. Only the levels with $E < 0$ are shown. The spectrum denoted *exact* refers to a diagonalization of the hamiltonian in the original quark space. Degenerate levels in this spectrum include to the right the degeneracy. The spectra denoted nh and h refer to results obtained following nonhermitean and hermitean triplet-fermion mappings, respectively. Physical states in the mapped spectra are denoted by solid lines and unphysical states by dashed lines. Heavy solid lines indicate degenerate (or nearly degenerate) solutions; to the right are given the number of physical and unphysical states at that energy.

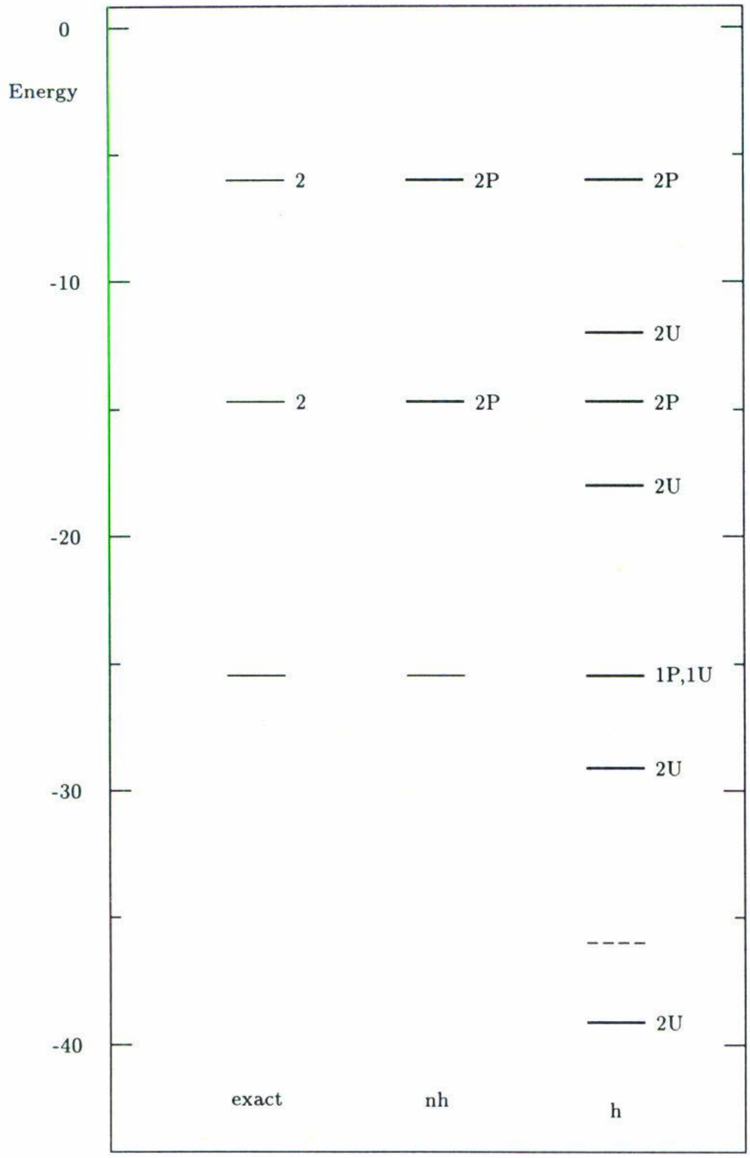

FIGURE 2. The same as Figure 1 except that the hamiltonian parameters used are $\Delta = 0$, $\chi_2 = 0$ and $\chi_3 = 1$.

Figure 1 shows our results for the choice $\Delta = 0$, $\chi_2 = 1$ and $\chi_3 = 0$, for which the system is dominated by two-quark correlations. Both the hermitean and nonhermitean mappings exactly reproduce the spectrum of states obtained by exact diagonalization of the quark model. Following the hermitean mapping, however, the lowest eigenvalues are unphysical. In contrast, when the nonhermitean mapping is used, the unphysical states are pushed up in energy, and the lowest four eigenvalues are physical. This is precisely what we had hoped would occur.

It is important, however, to see whether this also occurs in the presence of three-quark correlations. Thus, in Figure 2 we show results obtained for $\Delta = 0$, $\chi_2 = 0$ and $\chi_3 = 1$, namely for a system dominated by three-quark correlations. Exactly the same conclusions apply. Both mappings exactly reproduce the spectrum of physical states. The hermitean mapping, however, leads to unphysical states very low in energy, whereas the nonhermitean mapping yields them significantly raised.

We have also carried out calculations for mixed scenarios in which all three terms in the quark hamiltonian are active. All such calculations lead to the same general conclusion; our nonhermitean baryon mapping seems to provide a practical means of incorporating dynamical many-body correlations in multi-quark systems.

5. THE FUTURE

Despite the very promising results of our test calculations, there remains more to do before we can proceed to our ultimate goal, the derivation of nuclei from real constituent quark models.

We must still demonstrate the usefulness of our mapping in the presence of variational approximations, as will be required in real problems. In this respect, it is important that unphysical states are pushed up in energy by the nonhermitean mapping. In an approximate diagonalization of the mapped hamiltonian, the separation between physical and unphysical states is lost. Thus, only if the unphysical states lie relatively high in energy can we be confident that they will not mix appreciably into approximations to the low-lying states of interest. It may be possible to study this issue in the context of the three-color Lipkin model, by considering larger values of Ω than treated here.

We also need to show that our mapping can describe spatial three-quark correlations, as are certainly present in real quark models of nuclei. While the three-color Lipkin model contains dynamical corelations in the σ quantum number, it has no spatial degrees of freedom and thus no spatial correlations. Further tests along these lines are likewise underway.

ACKNOWLEDGMENTS

This work was supported by the National Science Foundation under grant #s PHY-9108011 and PHY-9303041, by NATO under grant # CRG.900466, by the Spanish DG-ICYT under project # PB92-0663 and by CONACYT, Mexico, under project # E-120.3475. We wish to express our appreciation to Etienne Caurier who carried out the shell-model calculations for the three-color Lipkin model that we briefly discussed.

REFERENCES

1. M. Oka, K. Shimizu and Y. Yazaki, *Nucl. Phys.* **A464** (1987) 700; K. Brauer, A. Faessler, F. Fernandez and K. Shimizu, *Nucl. Phys.* **A507** (1990) 599.

2. P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer, Berlin, 1980).
3. S. Pittel, J. Engel, J. Dukelsky and P. Ring, *Phys. Lett.* **B247** (1990) 185.
4. E.G. Nadjakov, *J. Phys. G: Nucl. Part. Phys.* **16** (1990) 1473.
5. J. Meyer, *J. Math. Phys.* **32** (1991) 2142.
6. H.J. Lipkin, N. Meshkov and A. Glick, *Nucl. Phys.* **62** (1965) 188.
7. S. Pittel, J.M. Arias, J. Dukelsky and A. Frank, BRI preprint # BA-93-65 (1993).
8. P. Park, *Phys. Rev.* **C35** (1987) 807.