Daylight factor calculation: Predictive method of the sky component in a courtyard under overcast sky conditions

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Abstract

The main aim of this article is to offer a predictive method to determine the most significant values of a sky component on the floor of circular or square courtyards of variable sizes under overcast sky conditions to be calculated quickly and precisely. Firstly, analytical geometry is used to calculate the sky component on the floor of the courtyards under study considering the variable height/width ratios. Empirical expressions are offered subsequently to fit the results obtained using analytical geometry, thus establishing the predictive method.

Keywords: daylighting, sky component, courtyard, overcast sky, predictive method.

1. Introduction and objective.

1.1. State of the art.

Courtyards are a key element in architecture, as they allow daylight into buildings and help to ventilate their rooms. The study of the proportions of inner courtyards has been analysed in the classic literature on daylighting [1] and in research using computer simulation [2,3].

In the field of architecture, courtyards and atria are two similar elements in terms of behaviour and efficiency when receiving daylight. Aldawoud et al. [4] carried out a comparative study highlighting the differences between them. In their conclusions the authors stated that low buildings with open courtyards display better energy performance than those with atria, as opposed to tall buildings, where atria would be more effective.

However, for the purposes of daylight requirements, courtyards and atria differ considerably [5]. In courtyards it is essential to provide lighting to the floor and vertical walls, while in atria it is necessary to provide lighting for adjoining spaces.

Early literature on courtyards studied the incidence of solar radiation. Mohsen [6] analysed different forms of houses with courtyards and developed a mathematical model that simulates the interaction occurring on courtyard floors as a result of the incidence of the sun. Subsequent studies, collected in Wright et al. [7], draw up methods for predicting daylight factors in courtyards and atria.

The performance of daylight inside a venue can be determined by observing the daylight factors occurring on its interior surfaces. Daylight factor (*DF*) is the ratio of daylight illumination at a given point on a given plane, from an obstructed sky of assumed or known illuminance distribution, to the light received on a horizontal plane from an unobstructed hemisphere of this sky, expressed as a percentage [8]. Direct sunlight is excluded for both values of illumination.

As seen in Hopkinson et al. [1], daylight factors can be defined as the sum of three components: the sky component (*SC*), which represents the fraction of daylight from the sky, the externally reflected component (*ERC*), generated by exterior reflective surfaces illuminated by the sky, and the internally reflected component (*IRC*), produced by the reflection of light on the interior surfaces of the venue (1).

$$DF = SC + ERC + IRC \quad (1)$$

Among the most notable predictive methods it is worth highlighting Lynes's research [9], which establishes that the average daylight factor on all the surfaces of an atrium (DF_{avs}) is given by (2):

$$DF_{avs} = \frac{WT_gT_f\theta}{2A(1-R)} \quad (2)$$

As a complementary method to predict illuminance on the ground, Littlefair [10] established that the average daylight factor for the base of an atrium (DF_{av}) is given by (3):

$$DF_{av} = \frac{WT_gT_f\theta}{A(1-R^2)} \quad (3)$$

where W is the area of the atrium roof aperture in m^2, T_g is the diffuse visible transmittance of the glazing, corrected for dirt on the glazing, T_f is a factor to allow for light blocked by the atrium roof structure, θ is the angle of visible sky in degrees, A is the total area of the atrium surfaces and R is the average reflectance of these surfaces.

The study of illuminance in atria was expanded with research by Littlefair and Aizlewood [11], who established the contribution of the average daylight factor (DF_{sav}) in the adjoining space (4):

$$DF_{sav} = \frac{2A_WT_sDF_v}{A_s(1-R_s^2)} \quad (4)$$

where A_w is the net area of the glazing between the space and the atrium (m^2), T_s is the diffuse visible transmittance of this glazing, A_s is the total area (m^2) of the room surfaces, R is the average reflectance of the room, and DF_v is the daylight factor on the vertical centre line of a rectangular atrium.

In most research on predictive methods for daylight factors, the sky component (*SC*), which represents the fraction of daylight from the sky, is calculated with analytical formulations. In a study on a vertical plane, Littlefair and other researchers use the Seshadri formula [12], which represents the sky component at a certain point as can be observed in (5):

$$SC = \frac{2.3}{14\pi}(\beta - \beta' cos\gamma) + \frac{2.2}{7\pi}(sin\beta - sin\beta' cos^2\gamma) \quad (5)$$

where:

$$\beta' = tan^{-1}\frac{w}{2\sqrt{d^2+y^2}}$$

$$\beta = tan^{-1}\frac{w}{2y}$$

where *w* is the length of the wall on which the point is found, *d* the length of the opposite wall, and *y* the distance between this point and the upper edge of the courtyard.

In most studies on sky components the Tregenza algorithms are used [13]. This equation is used by the International Commission on Illumination to assess the accuracy of lighting computer programs [14]. One of the Tregenza algorithms, used when evaluating simulation programs, makes it possible to calculate the sky component produced by a vertical opening, as observed in (6):

$$SC = \frac{1.5\big(b - c\cos(a)\big) + 2\arcsin(\sin(b)\sin(a)) - \sin(2a)\sin(c)}{7\pi} \quad (6)$$

In contrast, the sky component produced by a horizontal opening is calculated using a different Tregenza algorithm, as observed in (7):

$$SC = 1.5z\,\big(d\sin(a) + c\sin(b)\big) + z\pi + z(\sin(2b)\sin(c) + \sin(2a)\sin(d)) - 2z\Big(\arcsin\big(\cos(\alpha)\cos(a)\big) + \arcsin\big(\sin(\alpha)\cos(b)\big)\Big) \quad (7)$$

where:

$$z = \frac{1}{7\pi}$$

In both equations, variables *a*, *b*, *c*, *d* and *α* are determined by the shape of the opening and the position of the point.

As can be observed in this brief state of the art, most predictive methods are based on the analytical formulation of the sky component, since there are no empirical expressions established for it.

Nevertheless, the development of predictive methods for daylight factors can lead to errors, as the calculation of the reflected component is performed by an empirical hypothesis of daylight factor reflection [15]. Given the complexity of calculating the reflection of light, the formulation of this hypothesis cannot be based on analytical calculations.

Following on with the evolution of predictive methods, and based on Littlefair's proposal, Calcagni et al. [16] developed a study which determined a method for predicting daylight factors in atria. One of the most relevant recent studies on atria was produced by Kim et al. [17]. The authors evaluated the illuminance at the centre of the atrium and on exterior balconies. The results obtained are significant as they show how the daylight factors decrease from the centre of the atrium to its perimeter.

Daylight factors are the simplest and most common medium to evaluate the performance of daylight, as they express the potential illuminance in the worst possible scenario, under overcast sky conditions when there is less daylight. Nevertheless, it is important to mention other methods for daylight evaluation, such as daylight autonomy [18], developed by Reinhart et al.

1.2. Aim.

The aim of this study is to offer an empirical formulation for the rapid and accurate prediction of sky component values on a courtyard floor, depending on the height/width ratio and whether the floor is circular or square. The evaluation of the sky component is carried out at different points of the courtyard floor. Thus, in the absence of reflected components, it is possible to determine the daylight factors occurring at the bottom of a courtyard, obtaining the performance of daylight according to the geometrical characteristics of the courtyard.

The originality and usefulness of the predictive method developed in this research, as well as its innovation and differences from earlier formulations, are all summarised in the following points:

1. The method makes it possible to predict sky component values for different points of a courtyard, with no need to resort to the analytical formulations of Seshadri or Tregenza [12,13], which can at times be complex to apply.
2. The method calculates the sky component for a given point, with several different values possible, unlike other formulations which determine the average value on a surface.
3. The method ignores reflected components to prevent a considerable margin of error resulting from the empirical interpretation of these components and to determine the daylight factors in the absence of reflection, depending exclusively on the shape of the courtyard.

2. Calculation methodology.

2.1. Choosing the calculation conditions.

By definition, the calculation of daylight factor components is carried out considering an obstructed sky of assumed or known illuminance distribution, excluding direct sunlight. Therefore, to calculate the sky component the definition of traditional overcast sky is used.

The overcast sky model, used in methodology, is that defined by Moon-Spencer [19], where the luminance values are distributed in accordance with the following law (8):

$$L_\theta = \frac{L_Z \cdot (1 + 2sin\theta)}{3} \quad (8)$$

Where L_Z is the luminance at the zenith of the sky vault and θ the projection angle. This implies that the lowest luminance value in an overcast sky vault occurs on the horizon, and is equivalent to a third of the maximum luminance at the zenith (9):

$$L_0 = \frac{L_Z}{3} \quad (9)$$

The formulation established by Moon-Spencer corresponds to the definition of overcast sky accepted by the CIE [20], which is known as traditional overcast sky: Sky type 16.

2.2. Choosing the calculation model.

Two calculation models were used for this study. The first was for a circular courtyard (model 1), with a 3 metre diameter and with a height varying between 3 and 15 metres. The variation of the height of the model was measured at 1 metre intervals. Thus, the proportions for the courtyard are defined in terms of height/diameter.

The second calculation is for a square courtyard (model 2), 3 metres wide and with a height varying between 3 and 15 metres. As in the previous model, height variations were measured at 1 metre intervals, meaning the ratios for the courtyard are defined in terms of height/width.

A height/width ratio of 1/1 to 5/1 was used for all trials as it is considered to be the most common ratio in the design of courtyards.

Both calculation models have an opening at the zenith matching the measurements of the courtyard floor. As there is no glazing in the courtyard there is no risk of altering the properties of light: spectral composition, propagation, direction, by being reflected and transmitted while passing through glazing.

As the aim of this study is the analysis of the sky components in the calculation models, the properties of materials and surfaces,- reflection, transmission and absorption-, are not significant.

2.3. Choosing the calculation methods.

Calculation with analytical geometry provides an easy resolution of the maximum sky component observed on the floor of a circular courtyard. The sky component is defined as the fraction of the whole of the sky vault visible from the observation point. The fraction of visible sky occurring in a circular courtyard is defined as a spherical domain in the spherical coordinate system, meaning that the sky component is easy to determine.

In the analysis of the maximum and minimum sky components for the floor of a square courtyard, the daylighting algorithms established by Tregenza [13] are used.

The Tregenza algorithms are used to assess the accuracy of lighting computer programs [14], and therefore it is not necessary to compare the results of these equations using simulation programs. However, the study of a physical model under an artificial sky might lead to error, as established by Thanachareonkit et al. [21].

Subsequently an empirical expression is established to calculate the formula developed using analytical geometry with a low margin of error.

3. Analytical calculation of the sky component.

3.1. Calculation of the maximum sky component on the floor of a circular courtyard using analytical geometry.

The maximum sky component on the floor of a circular courtyard is at its centre (point A_{CC}), with the greatest luminance since the largest section of sky is visible from there (fig. 1).

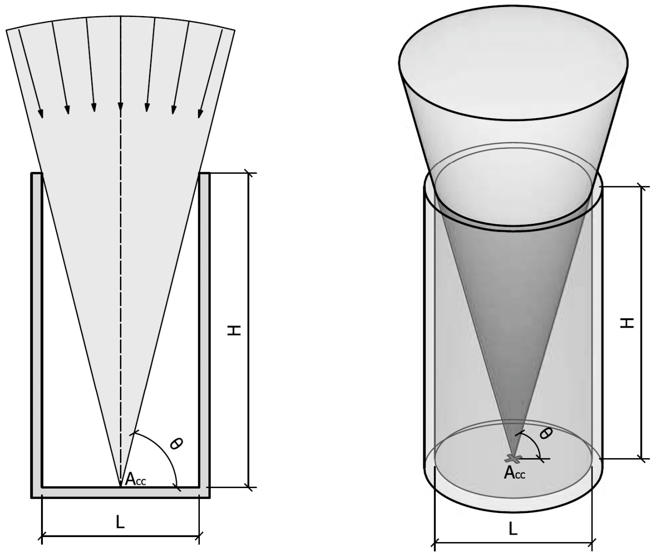

Figure 1: Maximum sky component on the floor of a circular courtyard.

Under overcast sky conditions, as defined by Moon-Spencer (8), the luminance on a horizontal plane is expressed as (10):

$$L_{\theta XY} = \frac{L_z(1 + 2sin\theta)}{3} sin\theta \ (10)$$

The fraction of sky visible from the centre of the floor of the circular courtyard is defined by the spherical domain determined by projecting a circumference delimiting the opening from the observation point. This domain is defined as a spherical cap:

$$\int_{\theta}^{\frac{\pi}{2}} \int_{\varphi}^{2\pi} R^2 \, sin\theta \, d\theta d\varphi \; (11)$$

It is considered in (11) that θ depends on the ratio of the height of the courtyard to the diameter:

$$\theta = arctan\frac{2H}{L} \; (12)$$

As the domain corresponds to a spherical cap covering the entire azimuth, the result is:

$$\int_{0}^{2\pi} d\varphi = 2\pi \; (13)$$

From (10) and (13) in (11) it can be concluded that the luminance projected on a point of a domain equal to a spherical cap, under overcast sky conditions is:

$$L_P = 2\pi R^2 \int_{\theta}^{\frac{\pi}{2}} \frac{L_z(1 + 2sin\theta)}{3} cos\theta sin\theta \, d\theta \; (14)$$

This gives rise to (14):

$$L_P = \frac{2\pi L_z R^2}{3} \int_{\theta}^{\frac{\pi}{2}} (1 + 2sin\theta) cos\theta sin\theta \, d\theta = \frac{2\pi L_z R^2}{3} \left(\int_{\theta}^{\frac{\pi}{2}} \frac{sin2\theta}{2} d\theta + \int_{\theta}^{\frac{\pi}{2}} 2sin^2\theta cos\theta \, d\theta \right)$$

$$= \frac{2\pi L_z R^2}{3} \cdot \left(\left[\frac{-cos2\theta}{4} \right]_{\theta}^{\frac{\pi}{2}} + 2 \cdot \left[\frac{sin^3\theta}{3} \right]_{\theta}^{\frac{\pi}{2}} \right) \; (15)$$

The luminance generated by the entire sky vault is equal to the domain of expression (15) from 0 to $\pi/2$:

$$L_{XY} = \frac{2\pi L_z R^2}{3} \cdot \left(\left[\frac{-cos2\theta}{4} \right]_{0}^{\frac{\pi}{2}} + 2 \cdot \left[\frac{sin^3\theta}{3} \right]_{0}^{\frac{\pi}{2}} \right) = \frac{7\pi L_z R^2}{9} \; (16)$$

From (16) it can be deduced that the illuminance on a horizontal plane under overcast sky conditions (17) is that established by Hopkinson et al. [1]:

$$E_{XY} = \frac{L_{XY}}{R^2} = \frac{7\pi L_z}{9} \; (17)$$

The luminance projected on the centre of the courtyard floor is calculated from the height and diameter variables. From (12) in (15):

$$L_P = \frac{2\pi L_z R^2}{3} \cdot \left(\left[\frac{1}{4} + \frac{cos2\left(arctan\frac{2H}{L}\right)}{4} \right] + 2 \cdot \left[\frac{1}{3} - \frac{sin^3\left(arctan\frac{2H}{L}\right)}{3} \right] \right) \; (18)$$

As stated earlier, the sky component is defined as the fraction of the entire sky vault visible from the observation point. Combining (18) and (16) it can be deduced that:

$$SC_{ACC} = \frac{\frac{2\pi L_z R^2}{3} \cdot \left(\left[\frac{1}{4} + \frac{cos2\left(arctan\frac{2H}{L}\right)}{4} \right] + 2 \cdot \left[\frac{1}{3} - \frac{sin^3\left(arctan\frac{2H}{L}\right)}{3} \right] \right)}{\frac{7\pi L_z R^2}{9}} = \frac{11 + 3cos2\left(arctan\frac{2H}{L}\right) - 8sin^3\left(arctan\frac{2H}{L}\right)}{14} \; (19)$$

Expression (19) represents the sky component at the centre of the floor of a circular courtyard. For the purposes of this study this is referred to as analytical method for model 1 of the sky component at point A_{CC} (AM.1A_{CC}).

3.2. Calculation of the maximum sky component on the floor of a square courtyard using the Tregenza equation.

The maximum sky component on the floor of a square courtyard is found at its centre (point A_{SC}). The greatest luminance is obtained here since it is possible to observe the greatest fraction of visible sky (fig. 2).

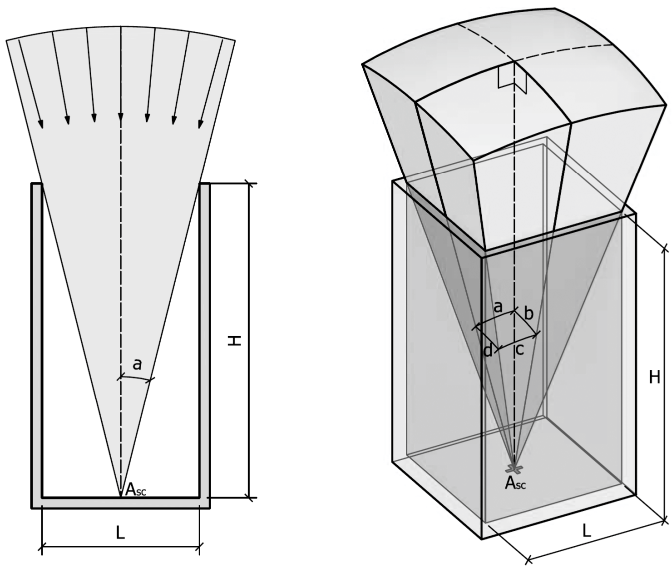

Figure 2: Maximum sky component on the floor of a square courtyard.

As can be seen in figure 2, the sky component inciding on the central point of the floor of a square courtyard can be defined as the projection of four openings that are symmetrical about the axis of the zenith. The sky component on a horizontal point directly below the corner of a rectangular unglazed opening under a CIE overcast sky is given by the Tregenza equation [13]:

$$SC = 1.5z\left(d\sin(a) + c\sin(b)\right) + z\pi + z(\sin(2b)\sin(c) + \sin(2a)\sin(d)) - 2z\left(\arcsin\left(\cos(\alpha)\cos(a)\right) + \arcsin\left(\sin(\alpha)\cos(b)\right)\right) \quad (20)$$

Where:

$$z = \frac{1}{7\pi}$$

$$\alpha = \arctan\left(\frac{\tan(a)}{\tan(b)}\right)$$

And angles a, b, c and d are in radians as shown in figure 2.

Considering a square opening, it can be stated that:

$$a = b = \arctan\left(\frac{L}{2H}\right)$$

$$c = d = \arctan\frac{L}{2\sqrt{H^2 + \left(\frac{L}{2}\right)^2}}$$

$$\alpha = \arctan\left(\frac{\tan(a)}{\tan(b)}\right) = \frac{\pi}{4}$$

Therefore the Tregenza equation is (21):

$$SC = 3zcsin(a) + z\pi + 2zsin(2a)sin(c) - 4zarcsin\left(sin\left(\frac{\pi}{4}\right)cos(a)\right) \quad (21)$$

Given that the sky component occurring on the centre of the floor of the courtyard equals the sum of the four openings coinciding on the axis of the zenith, it can be deduced that the maximum sky component on the floor of a square courtyard is:

$$SC_{ASC} = \frac{4}{7\pi}\left(3csin(a) + \pi + 2sin(2a)sin(c) - 4arcsin\left(sin\left(\frac{\pi}{4}\right)cos(a)\right)\right) \quad (22)$$

For the purposes of this study, expression (22) is referred to as analytical method for model 2 of the sky component at point A_{SC} (AM.2A_{SC}).

3.3. Calculation of the minimum sky component on the floor of a square courtyard using the Tregenza equation.

The minimum sky component on the floor of a square courtyard is located in the corner (point B_{SC}) with least luminance given that the smallest fraction of sky is visible (fig. 3).

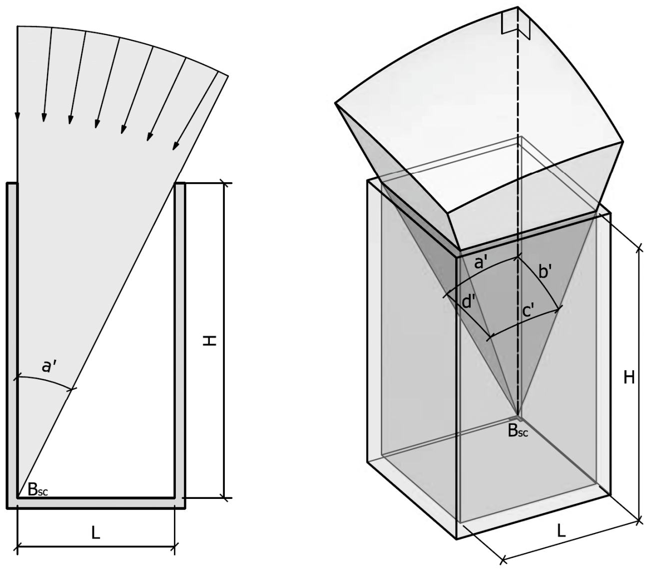

Figure 3: Minimum sky component on the floor of a square courtyard.

From (21) it can be deduced that the sky component in the corner of the square floor is:

$$SC_{BSC} = \frac{1}{7\pi}\left(3c'sin(a') + \pi + 2sin(2a')sin(c') - 4arcsin\left(sin\left(\frac{\pi}{4}\right)cos(a')\right)\right) \quad (23)$$

Where:

$$a' = b' = arctan\left(\frac{L}{H}\right)$$

$$c' = d' = arctan\frac{L}{\sqrt{H^2 + L^2}}$$

For the purposes of this study, expression (23) is referred to as analytical method for model 2 of the sky component at point B_{SC} (AM.2B_{SC}).

3.4. Calculation of an intermediate sky component on the floor of a square courtyard using the Tregenza equation.

An intermediate sky component on the floor of a square courtyard is located at the centre of the edge of the floorplan (point C_{SC}) where luminance provides an intermediate value between the maximum and minimum (fig. 4).

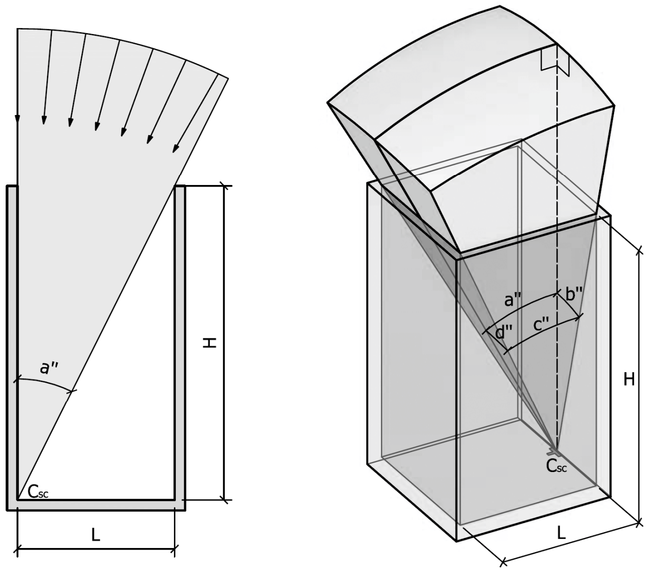

Figure 4: Intermediate sky component on the floor of a square courtyard.

From (20) it can be deduced that the sky component in the centre of the edge of the square floorplan is:

$$SC_{CSC} = 2\Big[1.5z\left(d''sin(a'') + c''sin(b'')\right) + z\pi + z(sin(2b'')\,sin(c'') + sin(2a'')\,sin(d'')) \\ - 2z\left(arcsin\big(cos(\alpha)cos(a'')\big) + arcsin\big(sin(\alpha)cos(b'')\big)\right)\Big] \quad (24)$$

Where:

$$z = \frac{1}{7\pi}$$

$$a'' = arctan\left(\frac{L}{H}\right)$$

$$b'' = arctan\left(\frac{L}{2H}\right)$$

$$c'' = arctan\frac{L}{\sqrt{H^2 + L^2}}$$

$$d'' = arctan\frac{L}{2\sqrt{H^2 + \left(\frac{L}{2}\right)^2}}$$

$$\alpha = arctan\left(\frac{tan(a'')}{tan(b'')}\right) = \arctan(2)$$

For the purposes of this study, expression (24) is referred to as the analytical method for model 2 of the sky component at point C_{SC} (AM.2C_{SC}).

4. Predictive methods of the sky component.

4.1. Predictive method of the maximum sky component on the floor of circular courtyards.

4.1.1. Deduction of the predictive method.

Using a curve fitting study of expression (19), an empirical formula can be deduced for the maximum sky component on the floor of a circular courtyard, depending on the H/L variable and the limit values of the expression according to analytical geometry. The following limits are established depending on the assessment of the sky component at point A_{CC} according to the analytical geometry equation:

$$\lim_{H\to 0} SC_{ACC} = 1 \quad (25)$$

$$\lim_{H\to \infty} SC_{ACC} = 0 \quad (26)$$

$$\lim_{L\to 0} SC_{ACC} = 0 \quad (27)$$

$$\lim_{L\to \infty} SC_{ACC} = 1 \quad (28)$$

where L is the diameter of the courtyard and H its height.

It can be observed that the variable in the expression (19) is H/L, which means that the predictive method must be established following the same parameters. Based on the previous statement, an equation is established in compliance with the limits set in (25), (26), (27) and (28):

$$SC_{ACC} \cong \frac{1}{1 + k \cdot \left(\frac{H}{L}\right)^2} \quad (29)$$

where *k* is an empirical coefficient that determines the curvature of the equation.

Through an empirical trial it is established that coefficient *k* is roughly equal to 3. Therefore, it is deduced that (30):

$$SC_{ACC} \cong \frac{1}{1 + k \cdot \left(\frac{H}{L}\right)^2} = \frac{1}{1 + 3 \cdot \left(\frac{H}{L}\right)^2} = \frac{L^2}{L^2 + 3H^2} \quad (30)$$

For the purposes of this study, expression (30) is referred to as the empirical method for model 1 of the sky component at point A_{CC} (EM.1A_{CC}).

4.1.2. Evaluation of the predictive method.

The precision of expression (30) is evaluated in figure 5, which shows the sky component at point A_{CC} on the floor of a circular courtyard of variable height, using analytical (19) and empirical (30) expressions:

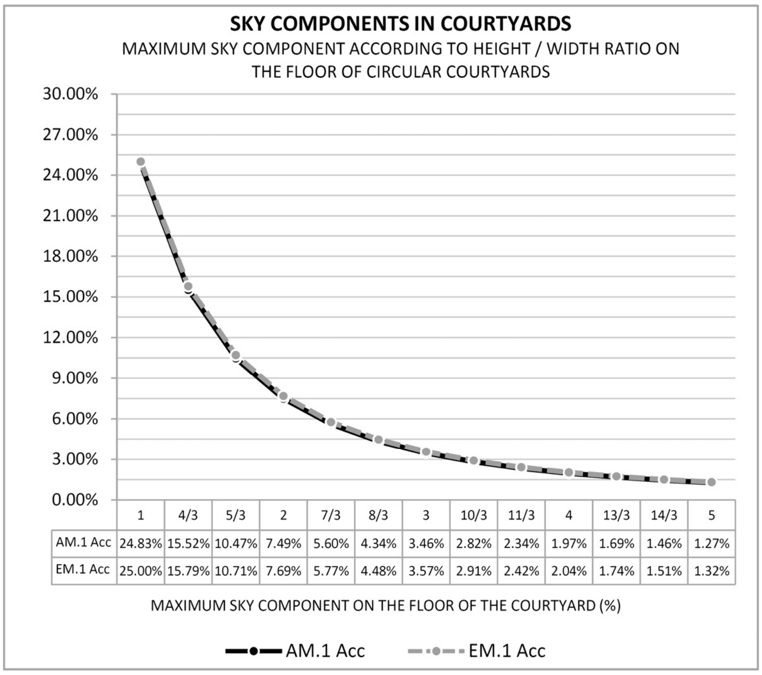

Figure 5: Sky component at point A_{CC} on the floor of circular courtyards, according to analytical and empirical methods.

As can be observed in figure 5, the results of expression (30) are considerably closer to those of the expression using analytical geometry (19) for any height/width ratio of a circular courtyard. Figure 6 shows the relative difference between the empirical and analytical methods.

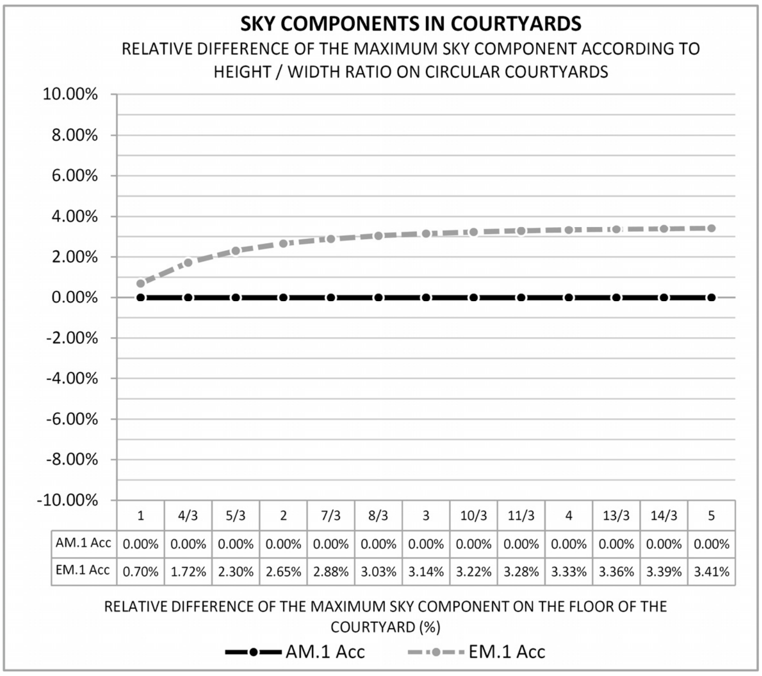

Figure 6: Relative difference of the sky component at point A_{CC} on the floor of circular courtyards, according to empirical method compared with analytical method.

As can be observed in figure 6, the average difference between the analytical and empirical methods in the study intervals is 2.80%, while the maximum difference, 3.41%, is observed for the courtyard with the greatest height. Maximum difference increases to 3.57% for a courtyard of undefined height exceeding that of the study sample. It can be concluded that the margin of error for the empirical method is low and therefore, acceptable.

4.2. Predictive method of the maximum sky component on the floor of square courtyards.

4.2.1. Deduction of the predictive method.

Through the curve fitting study of expression (22), an empirical formula is deduced for the maximum sky component on the floor of a square courtyard, depending on the H/L variable and the limit values of the Tregenza expression. It is established that the limits according to the assessment of the sky component at point A_{SC}, following the Tregenza equation, are the same as in the previous model, (25), (26), (27) and (28), and therefore the predictive method is used in expression (29).

As with the previous deduction, the variable of the predictive method is H/L. Coefficient *k'* is different as the fraction of visible sky is greater in the square courtyard than in the circular one, considering the same height and a side measurement equal to that of the diameter. Therefore it is deduced that:

$$\lim_{H\to\infty}\frac{SC_{ASC}}{SC_{ACC}}=\frac{4}{\pi}\rightarrow\lim_{H\to\infty}\frac{1+3\cdot\left(\frac{H}{L}\right)^2}{1+k'\cdot\left(\frac{H}{L}\right)^2}=\frac{4}{\pi}\rightarrow k'=\frac{3\pi}{4}\qquad(31)$$

where L is the width of the courtyard and H its height.

Therefore the predictive method of the sky component at point A_{SC} is expressed:

$$SC_{ASC}\cong\frac{1}{1+k'\cdot\left(\frac{H}{L}\right)^2}=\frac{1}{1+\frac{3\pi}{4}\cdot\left(\frac{H}{L}\right)^2}=\frac{L^2}{L^2+\frac{3\pi}{4}H^2}\cong\frac{L^2}{L^2+2.4H^2}\qquad(32)$$

For the purposes of this study expression (32) is referred to as the empirical method for model 2 of the sky component at point A_{SC} (EM.2A_{SC}).

4.2.2. Evaluation of the predictive method.

The precision of expression (32) is evaluated in figure 7, which represents the sky component at point A_{SC} on the floor of a square courtyard of variable heights, using the Tregenza (22) and empirical (32) expressions:

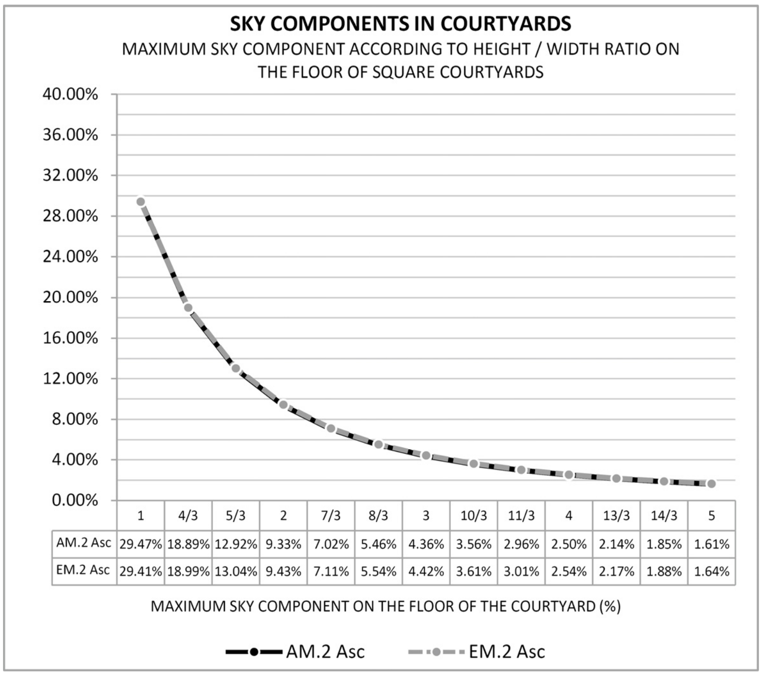

Figure 7: Sky component at point A_{SC} on the floor of square courtyards, according to analytical and empirical methods.

As can be observed in figure 7, the results of expression (32) are very close to those of the Tregenza (22) expression for any height/width ratio of the square courtyard. Figure 8 shows the relative difference between the empirical method and the Tregenza equation.

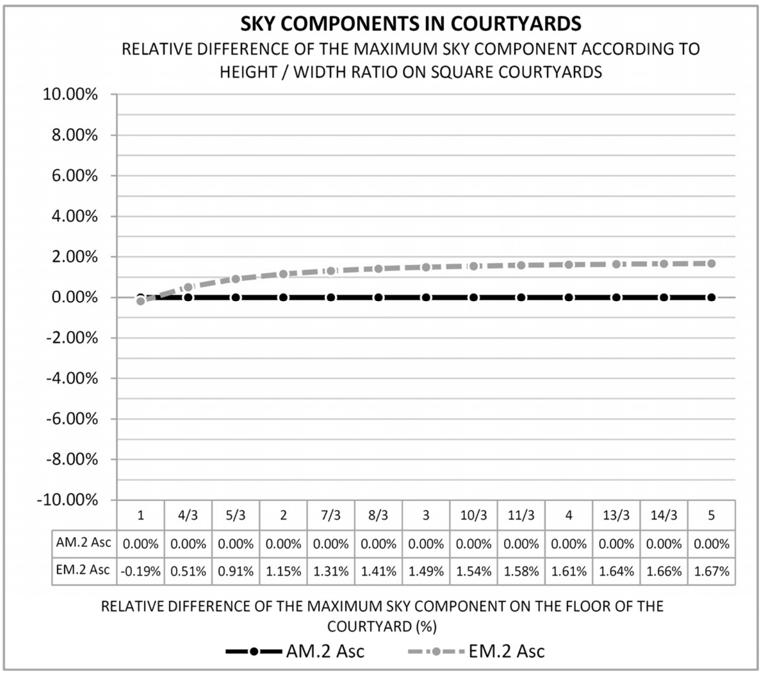

Figure 8: Relative difference of the sky component at point A_{SC} on the floor of square courtyards, according to empirical method compared with analytical method.

The average difference between the Tregenza equation and the empirical method in the study interval is 1.28%, while the maximum difference, 1.67%, is observed in the case of the courtyard with the greatest height. The maximum difference increases to 1.78% for a courtyard with undefined height exceeding that of the study sample. It is concluded that the margin of error of the empirical method is low, and therefore acceptable.

4.3. Predictive method of the minimum sky component on the floor of square courtyards.

4.3.1. Deduction of the predictive method.

Using the curve fitting study of expression (23) an empirical formula can be deduced for the minimum sky component on the floor of a square courtyard, depending on the H/L variable and the limit values of the Tregenza expression. The following limits are established following the evaluation of the sky component at point B_{SC} according to the Tregenza algorithms:

$$\lim_{H\to 0} SC_{BSC} = \frac{1}{4} \quad (33)$$

$$\lim_{H\to \infty} SC_{BSC} = 0 \quad (34)$$

$$\lim_{L\to 0} SC_{BSC} = 0 \quad (35)$$

$$\lim_{L\to \infty} SC_{BSC} = \frac{1}{4} \quad (36)$$

where L is the width of the courtyard and H its height.

The variables considered are those established in expression (32). Based on the previous statement, an equation is established that complies with the limits set in (33), (34), (35) and (36):

$$SC_{BSC} \cong \frac{1}{4 + k' \cdot \left(\frac{H}{L}\right)^2} \quad (37)$$

where k' is an empirical coefficient that determines the curvature of the function, defined in (31).

$$SC_{BSC} \cong \frac{1}{4 + k' \cdot \left(\frac{H}{L}\right)^2} = \frac{1}{4 + \frac{3\pi}{4} \cdot \left(\frac{H}{L}\right)^2} = \frac{L^2}{4L^2 + \frac{3\pi}{4} H^2} \cong \frac{L^2}{4L^2 + 2.4H^2} \quad (38)$$

For the purposes of this study expression (38) is referred to as the empirical method for model 2 of the sky component at point B_{SC} (EM.2B_{SC}).

4.3.2. Evaluation of the predictive method.

The accuracy of expression (38) is assessed in figure 9, which represents the sky component at point B_{SC} on the floor of a square courtyard of variable heights, using the Tregenza (23) and empirical (38) expressions:

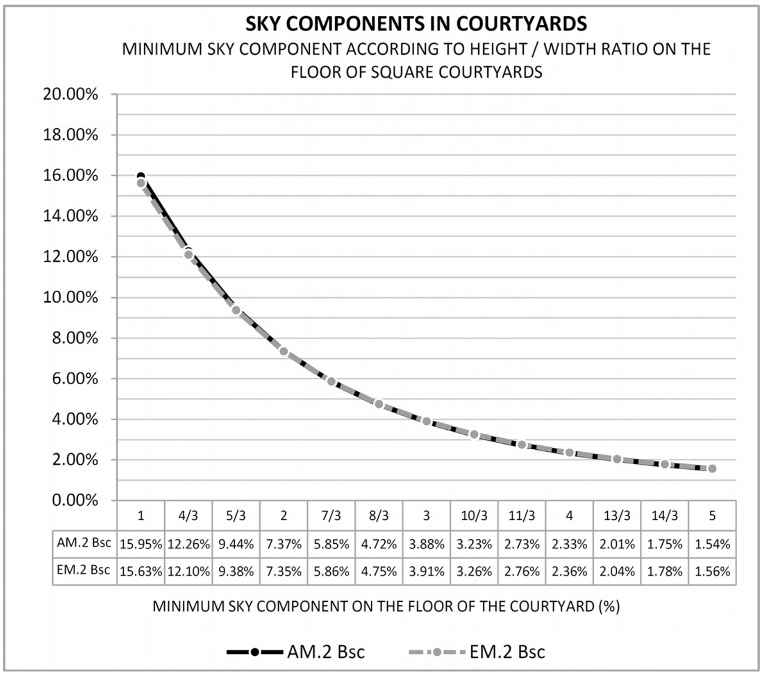

Figure 9: Sky component at point B_{SC} on the floor of square courtyards, according to analytical and empirical methods.

As can be observed in figure 9, the results of expression (38) are very close to those of the Tregenza equation (23) for any height/width ratio for a square courtyard. Figure 10 shows the relative difference between the empirical method and the Tregenza equation.

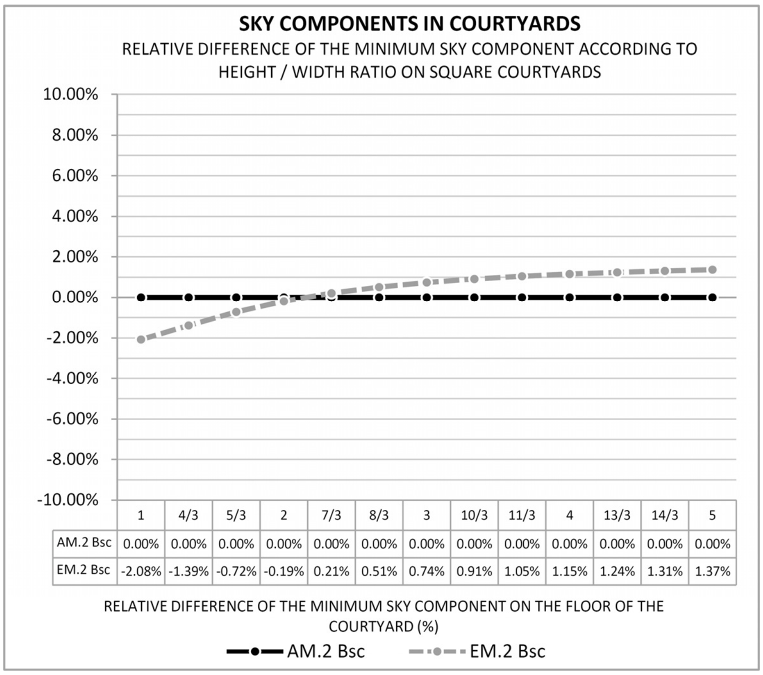

Figure 10: Relative difference of the sky component at point B_{SC} on the floor of square courtyards, according to empirical method compared with analytical method.

The average difference between the Tregenza equation and the empirical method in the study interval is 0.99% while the maximum difference is observed for the courtyard with the lowest height, at 2.08%. The maximum difference increases to 2.35% for a courtyard with a height equal to approximately 1/3 of its width, outside the study sample. It can be concluded that the margin of error of the empirical method is low and therefore acceptable.

4.4. Predictive method of an intermediate sky component on the floor of square courtyards.

4.4.1. Deduction of the predictive method.

Using the curve fitting study of expression (24) an empirical formula can be deduced for the minimum sky component on the floor of a square courtyard, depending on the H/L variable and the limit values of the Tregenza expression. The following limits are established according to the evaluation of the sky component at point B_{SC} using the Tregenza algorithms:

$$\lim_{H\to 0} SC_{BSC} = \frac{1}{2} \quad (39)$$

$$\lim_{H\to \infty} SC_{BSC} = 0 \quad (40)$$

$$\lim_{L\to 0} SC_{BSC} = 0 \quad (41)$$

$$\lim_{L\to \infty} SC_{BSC} = \frac{1}{2} \quad (42)$$

where L is the width of the courtyard and H its height.

The variables considered are those established in expression (32). Based on the previous statement, a function is established to comply with the limits set in (39), (40), (41) and (42):

$$SC_{BSC} \cong \frac{1}{2 + k' \cdot \left(\frac{H}{L}\right)^2} \quad (43)$$

where *k'* is an empirical coefficient that determines the curvature of the function, defined in (31).

$$SC_{BSC} \cong \frac{1}{2 + k' \cdot \left(\frac{H}{L}\right)^2} = \frac{1}{2 + \frac{3\pi}{4} \cdot \left(\frac{H}{L}\right)^2} = \frac{L^2}{2L^2 + \frac{3\pi}{4} H^2} \cong \frac{L^2}{2L^2 + 2.4H^2} \quad (44)$$

For the purposes of this study expression (44) is referred to as the empirical method for model 2 of the sky component at point C_{SC} (EM.2C_{SC}).

4.3.2. Evaluation of the predictive method.

The accuracy of expression (44) is assessed in figure 11, which represents the sky component at point C_{SC} on the floor of a square courtyard of variable heights, using the Tregenza (24) and empirical (44) expressions:

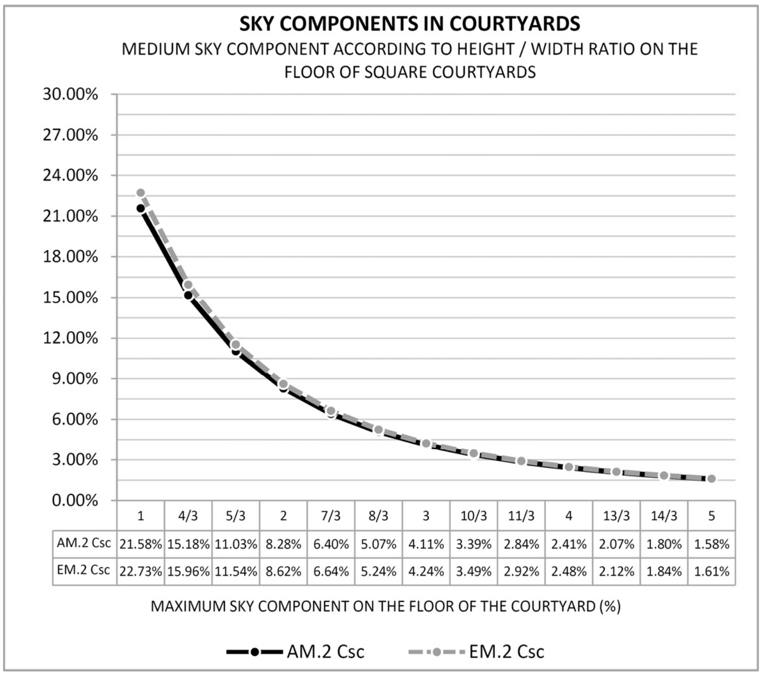

Figure 11: Sky component at point C_{SC} on the floor of square courtyards, according to analytical and empirical methods.

As can be observed in figure 11, the results of expression (44) are close to those of the Tregenza equation (24) for any height/width ratio for a square courtyard. Figure 12 shows the relative difference between the empirical method and the Tregenza equation.

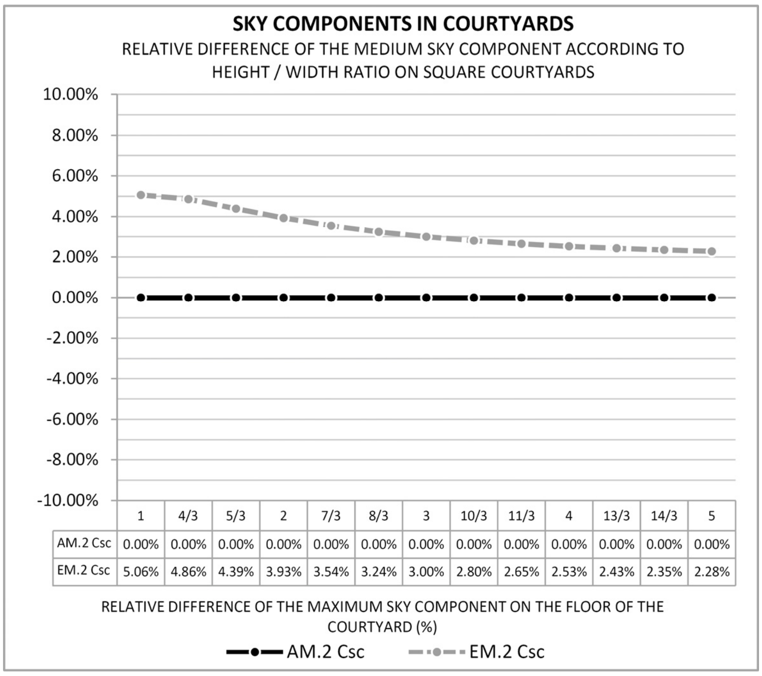

Figure 12: Relative difference of the sky component at point C_{SC} on the floor of square courtyards, according to empirical method compared with analytical method.

The average difference between the Tregenza equation and the empirical method in the study interval is 3.31% while the maximum difference, 5.06%, is observed for the courtyard with the lowest height. It can be concluded that the margin of error of the empirical method is low and therefore acceptable.

5. Conclusion

5.1. Results.

Under CIE overcast sky conditions, considering a circular courtyard with a known height/diameter ratio, it can be concluded that the maximum sky component on the floor of the courtyard can be defined using empirical expression (30), calculating an average margin of error for the sample of 2.80% and a maximum of 3.57% for the analytical formulation.

Under CIE overcast sky conditions, considering a square courtyard with a known height/width ratio it can be concluded that the maximum sky component on the floor of the courtyard can be defined using empirical expression (32), calculating an average margin of error for the sample of 1.28%, and a maximum of 1.78% for the Tregenza equation.

Under CIE overcast sky conditions considering a square courtyard with a known height/width ratio, it can be concluded that the minimum sky component on the floor of the courtyard can be established using empirical expression (38), with an average margin of error for the sample of 0.99% and a maximum value of 2.35% for the Tregenza equation.

Under CIE overcast sky conditions considering a square courtyard with a known height/width ratio, it can be concluded that the intermediate value of the sky component on the floor of the courtyard can be established using empirical expression (44), with an average margin of error for the sample of 3.31% and a maximum value of 5.06% for the Tregenza equation.

5.2. Discussion.

The predictive methods concluded in this research provide a precise approximation for the sky component assessed for different points of a courtyard. Unlike earlier studies which determined average values for daylight factors, these predictions are carried out for the different points of the surface under study, and provide more detailed information on the illuminance distribution caused by an overcast sky.

Specifically the predictive methods of the points assessed in the perimeter of the courtyard (B_{SC} and C_{SC}) also make it possible to find the sky component on the vertical planes, determining the illuminance caused by the sky visible on different points of the floorplan and courtyard walls.

Therefore, these tools for predicting the sky component make it possible to roughly calculate the illuminance distribution of all the planes of a courtyard, in the absence of reflected components.

As was expressed in the objectives of study, the methods did not include the reflected components in order to prevent a considerable margin of error resulting from the empirical interpretation of these components and to determine the daylight factors in the absence of reflection and depending exclusively on the shape of the courtyard.

The practical application of these predictive methods can be very broad. They can be easily implemented in computer programs, although they were primarily created to estimate courtyard daylighting performance for architects, using simple calculations.

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