PENROSE AND HIS VISION ON MATHEMATICAL PLATONISM

DANIEL HEREDIA
Universidad de Sevilla

1. INTRODUCTION

Much has been said about mathematical platonism since Paul Bernays presented his article "On Platonism in mathematics" in 1935 and this topic has remained strongly in debates in both the philosophy of mathematics and science in general. Roger Penrose adopts a particular position with respect to this doctrine and in this work we will see some of the characteristics to try to understand his position more clearly.

For this, the work has been divided into several sections. Section 2 contains what Penrose says directly about platonism, and can see some illustrative mathematical examples, such as the Mandelbrot set or the tessellation problem. Section 3 contains Penrose's three-world theory, which serves to understand another aspect of his platonism. In section 4 we will see how Penrose's theory of the three worlds has evolved over the years. Section 5 is dedicated to the analysis of some of the criticisms that Penrose platonism has received, in this case those carried out by Steiner (2000) and Feferman (1995). Said analysis will be focused on my own response to both criticisms.

2. DISCUSSION: PENROSE AND PLATONISM

It is striking that platonism is fundamental in the conglomerate of Penrose's thought and that, however, it does not constitute a very large part of his work. And it is not that he hesitates when it comes to declaring himself a follower of this doctrine (at least on a mathematical level). In fact, he defines himself as a platonist on numerous occasions. Let's look at an example in which he speaks a little more extensively about the platonism to which he subscribes:

> Yet the matter is perhaps not quite so straightforward as this. As I have said, there are things in mathematics for which the term "discovery" is indeed, much more appropriate than "invention", such as the examples just cited. These are the cases where much more comes out of the structure than is put into it in the first place. One may take the view that in such cases the mathematicians have stumbled upon "works of God". However, there are other cases where the mathematical structure does not have such a compelling uniqueness, such as when, in the midst of a proof of some result, the mathematician finds the need to introduce some contrived and far from unique construction in order to achieve some very specific end. In such cases no more is likely to come out of the construction than was put into it in the first place, and the word "invention" seems more appropriate than "discovery". These are indeed just "works of man". On this view, the true mathematical discoveries would, in a general way, be regarded as greater achievements or aspirations than would the "mere" inventions (Penrose, 1991: 134).

We can really find few texts similar to those cited in Penrose's work in which he gives an account of his particular platonism. The platonism that we see in the quote is not far from the most basic, that is, that promulgated by Plato. Is it correct, therefore, to define [or include] Penrose as a platonist in the broadest sense of the term, understood as a faithful follower of Platonic ideas? Although at first it may seem so, the truth is that Penrose's thought contains traces that make him different from any common platonist.

As we can see in Penrose et al. (2008: 14-15), only refers to the platonism of mathematical concepts. Mathematical concepts are eternal and immovable. This has always translated (that is, from any type of platonism) into the independence of ideas with respect to the physical world. Although this idea fits into his approach, it is no less true that he is not so categorical when it comes to disregarding human work[69]. Precisely this aspect poses a problem in his exposition, since he seems to be clearly positioned, but the nuance he adds places him in a position that is difficult to define.

[69] For actual works about the role of human with respect to mathematics see Ferreirós (2016, 2023a, 2023b).

Mathematical concepts have their own existence and truth, always and forever. But he also recognizes the important role that construction plays. This construction cannot be understood in the same way as the Brouwerian intuitionists understood it (since they are in favor of the total dependence of mathematics on human thought). For many, Penrose's position –a hybrid between platonism and a certain type of constructivism– is not fully explained (Herce, 2014: 63).

For my part, I do not consider it that way, at least to the highest degree. Although I can perceive the philosophical complication in which he unintentionally gets involved (in my understanding), it is also recognizable that Penrose offers enough examples with which his position is sufficiently clear. One of the most illuminating is the one concerning the Mandelbrot set[70].

The main idea that Penrose intends to extract from the exposition of the Mandelbrot set consists of making it clear that a simple mathematical description can give rise to a very complex mathematical problem. Underlying this approach is the idea that, in fact, mathematics holds much more within itself than it may at first seem:

> […] It would seem that this structure is not just part of our minds, but it has a reality of its own. Whichever mathematician or computer buff chooses to examine the set, approximations to the *same* fundamental mathematical structure will be found. It makes no real difference which computer is used for performing calculations (provided that the computer is in accurate working order), apart from the fact that differences in computer speed and storage, and graphic display capabilities, may lead to differences in the amount of fine detail that will be revealed and in the speed with which that detail is produced. The computer is being used in essentially the same way that the experimental physicist uses a piece of experimental apparatus to explore the structure of the physical world. The Mandelbrot set is not an invention of the human mind: it

[70] This set is presented in the following terms:

Let c be any complex number. Starting from c, a sequence is constructed by induction:

z0 = 0 is the initial term

zn+1 = zn2 + c is the induction ratio

If this sequence is bounded, then it is said that *c* belongs to the Mandelbrot set, and if not, it is excluded from it (Herce, 2014: 57). The sequence is bounded when all its terms are greater than or equal to *c*.

> was a discovery. Like Mount Everest, the Mandelbrot set is just *there*! (Penrose, 1991: 132; italics in the original).

In Penrose's description there are two points that should be taken into account: a) the elaborate shape that the set has despite coming from a mathematical rule of concrete simplicity and b) it is not the product of human design. With the first of the characteristics we have something not very different from other mathematical problems. With respect to the second, we are accepting the principles of platonism, although without explaining the union of both points. Let's see what is the complete description of the set that Penrose offers us.

We have that the Mandelbrot set is developed from a simple rule and from there it extends, reaching infinite variety and unlimited complication. This would happen, even if the human being had never found it. However, the crux of the matter for Penrose is not only in this independence, but also in the fact that human beings managed to find it. Therefore, he is appealing to a special relationship that human beings have with mathematics. If this relationship were not special, how would it be possible for human beings to have the ability to find such independent entities and their relationships, even though these may become so complex?.

In any case, it is worth noting that Penrose does not place all the emphasis of the relationship on the complexity of mathematics. The important thing is that we have the ability to think about the properties that mathematics has, without having to resort to its most complex approaches. Penrose argues that this relationship is the one that differentiates us from machines: the entirety of mathematics cannot be translated in terms of computability, because it is obvious that there is something beyond that computational processes do not have access to!

Another example of the special way that humans have of relating to mathematics is the so-called tessellation problem. As with the previous example, the Penrosean exposition of this problem is to argue against strong AI. However, the attack in this example is more direct than the previous one, in the sense that with the Mandelbrot set we can dispense with the perspective of the machines. With the problem of tessellation, on the contrary, the same does not happen. Let's see why.

The tessellation problem is a geometric problem which states that, in a Euclidean plane composed of polygonal shapes, they can cover said plane in its entirety in an overlapping manner (this way of covering the plane is what is known as tessellation). The question has an answer, but the result is obtained through the handling of “real” numbers. That this type of numbers is definitive for solving the problem is dramatic for the machines, since they cannot operate through them. This is precisely why Penrose brings up this specific problem. We are facing a limitation of machines that does not affect human beings!:

> As a curious fact, the computational insolubility of the tiling problem depends upon the existence of certain sets of polyominoes called *aperiodic* sets-which will tile the plane *only non-periodically* (i.e. in a way so that the completed pattern never repeats itself no matter how far it is extended) (Penrose, 2012: 45; italics in the original).

As a historical note, it should be said that the insolubility in computational terms of the tessellation problem was already exposed prior to Penrose's work. Robert Berger's research in 1966, as an extension of Hao Wang's arguments in 1961, gave an account of the insoluble solution. But this should only count as a historical note, since Penrose does not intend to make any claim about this problem. If he highlights it, it is because he considers that it has had a special importance in his intellectual life. Proof of how it has influenced his thinking is reflected in two of his contributions to mathematics: the endless triangle and the endless ladder (the latter made together with that of his father Lionel). Apart from the contributions on this matter by Berger and Wang, Penrose also highlights the influence of the work of the Dutch painter M. C. Escher, with whom our author came to have a personal relationship.

Although Penrose insists on using scientific examples to justify his platonism, we can perceive that this topic leads to a discourse that ultimately responds to internal convictions. And this is where, I think, Penrose is clear. While his defense becomes a bit scattered, it is not because his (to continue with the same expression) inner conviction is unclear, but rather because he reaches the point where he can no longer account for it, scientifically speaking.

In my opinion, I think that Penrose does not overdefine the type of platonism that he defends precisely to avoid entering into a debate that concerns the work of those who dedicate themselves to philosophy. He prefers to continue talking about physics, a field in which he feels safe, rather than risk entering the field of metaphysics. However, ending up entering this area, as we have seen, seems inevitable, as can be seen in his theory of the three worlds.

3. MORE ASPECTS OF THE DISCUSSION

The originality of his approach resides and can be located in the context in which he exposes his ideas, but also holds a significant weight the conception of reality that is handled with respect to this matter. Penrose comes to expose a scheme in which he speaks of three worlds, which has a certain similarity with those of Popper[71]. Briefly, we must know that these worlds are the physical, the mental and the Platonic-mathematical, being easily identifiable what kind of entities is in each of them. The three worlds keep three mysteries, which are: 1) the mystery that links the physical world with a small portion of the Platonic-mathematical world, which functions as its foundation, 2) the mystery that links the mental world with a small part of the physical structures that function as their physical substrate and 3) the mystery that links the mental world with a small fraction of the Platonic-mathematical world[72].

These worlds are connected with each other making the reality of each one of them wider. Our author understands that this disposition of the worlds may not be adequate for a debate in which it is intended to make things clear, instead of getting bogged down in another discussion derived from it. However, this first intention does not prevent debates from continuing to arise, since the outline of its is broad, inviting discussion.

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71 As he himself recognizes, also indicating that his model is not identical to that of Popper (Penrose, 2012: 433).

72 These three mysteries are cited by Herce (2016: 10).

Rubén Herce (2014) finds an aspect in the posture of Penrose, in relation to what we are seeing, which is interesting to comment. Penrose in his exposition of the ideas of the intuitionists points out that he does not share with these the dependence of mathematics in relation to the human mind. Mathematical entities have their own existence (as he explains in his theory of the three worlds). But it is also true that it recognizes a degree of construction. He quickly points out that this construction does not directly relate to intuitionism, but that this constructivism would have to do with classical mathematics. However, as Herce points out, the relationship between constructivism (which Penrose defends) and mathematical platonism (which he also defends) is not entirely clear (Herce, 2014: 40). From there, whatever we want to say about this issue would belong to the plane of conjecture. I share the opinion of Rubén Herce, who says about this the following words:

> [...] According to my opinion the mathematical construction would be inside the mental world: after the mind has reached the mathematical objects through intuition, construction would fit (Herce, 2014: 41).

Although everything seems excessively metaphysical, and we are entering into another fundamental philosophical problem (that is, realism), what Penrose intends is to base his arguments on mathematics and physics, since these can provide judgments that bring us closer to reality. The starting point for this is twofold: the theories accepted by the scientific community and the results of the experiments. This may seem very schematic and hermetic, but in reality it is stated in looser terms:

> […] this double starting point is not an immovable basis, but rather has the solidity of plate tectonics: received theories are revised through the development of new experiments and experimental data are subject to reinterpretation. There is a continuous flow of theories, experiments and interpretation, where the human being plays the fundamental role. In this access to reality, the human being formulates theories, prepares experiments, interprets data and judges the appropriateness of what needs to be changed: the theory, the experiments or the interpretation. So theories and experiments are not only the starting point, but also a point of continuous return through interpretation and judgment. The revision of a theory will depend on the scientific judgment about how fundamental the data provided by the experiments are (Herce, 2014: 42).

Reality, Penrose defends, although it is there, that does not mean we should conceive it in a static way. Therefore, the way to get there cannot be either. The dynamic feature of Penrosean reality is explained in the metaphysical conception that he elaborates, that is, that of the three worlds.

Penrose takes the idea of the three worlds from Popperian theory, although he insists that the approaches are different (Penrose, 2012: 433). The theory consists, as its name indicates, that there are three independent worlds with different contents, but that are interrelated with each other. The scheme of worlds is understood as follows (Penrose, 2012: 434):

- Mental world: this world is the one we know most directly, since it is made up of our conscious perceptions. However, the mental world is the least accessible to science (at least, for now). The content of this world is made up of ideas (be they feelings such as pain, memories, such as ideas of objects in the physical world).
- Physical world: as its name indicates, this world contains everything that is physical (chairs, tables, brains, atoms...). For Penrose, the mental world is more direct than the physical world, although he admits that the latter is increasingly less alien to us, thanks to what science tells us about it.
- Platonic mathematical world: in this world we would find mathematical ideas, in the broadest sense. That is, it contains both the mathematics that we know (the natural numbers and the operations that we can perform with them); such as mathematics to which we do not have direct access (the number pi in its entirety, solutions to unsolved mathematical problems and even those that have not yet been posed). Penrose recognizes that this world is difficult for many to accept (Penrose, 2012: 434), but this does not prevent him from giving it great importance. It is precisely the belief in this world that Penrose considers himself a platonist. But before delving into this matter, let's see how the interrelation between the three worlds occurs.

The way in which the three worlds are interrelated is by emerging from each other, although there is no original world, since this interrelation would be cyclical (so seeing a possible beginning or end would be a useless task). In detail, the physical world would emerge from a part of the Platonic mathematical world, while the latter would emerge from the mental world, which, in turn, would emerge from the physical world. Far from thinking that the scheme he offers is clear, Penrose recognizes the difficulty of conceiving it and that is why he considers the interrelation to be *mysterious* [73](Penrose, 2012: 435).

Penrose does not go into too much detail to explain all the mysteries. In fact, he only does so with those that involve the Platonic mathematical world, which is, as we saw above, the most difficult to accept. Taking this last aspect into account and that Penrose is a mathematician, it makes it more understandable for him to add this clarification.

There is no doubt that understanding reality as composed of more than one world is difficult to explain, and even more so if said idea is presented in the form of a sketch. On the other hand, I think that it is inevitable to fall into insufficient explanations when we try to address reality, so pointing out Penrose for this seems unfair to me. It is true that a somewhat more extensive exposition may be required of him, but it must also be understood that, after all, he is speaking in metaphysical terms, a terrain with which he is not familiar. Let's move on to see the explanation of the mysteries that involve the Platonic mathematical world.

Penrose thinks that if the conception of the Platonic mathematical world is so complicated for many, it is precisely because they do not adequately know the scope of mathematics and the influence it has on the physical world[74]. To make this reach and influence manifest, Penrose resorts to physical theories, specifically Einstein's relativity and the Newtonian scheme. According to Penrose, physical theories

[73] This aspect has been hardly criticized, because Penrose does not offer many more explanations than those mentioned, which inevitably makes the scheme poor, in a way. For a criticism of this aspect see (Badía, 2008).

[74] This statement is directly related to one of the most mainstream topics in the philosophy of mathematics today, that is, the applicability of mathematics in physics. For recent studies on this topic see Molinini (2020, 2021, 2022), Molinini and Panza (2022), Bueno and French (2018).

are a satisfactory test both to see the relationship of mathematics with the physical world (since physical theories have a marked mathematical foundation) and to perceive their scope (that is, their explanatory degree):

> Newton's gravitational theory had stood for some 250 years, and had achieved an extraordinary accuracy, of something like one part in ten million […]. An anomaly had been observed in Mercury's motion, but this certainly did not provide cause to abandon Newton's scheme. Yet Einstein perceived, from deep physical grounds, that one could do better, if one changed the very framework of gravitational theory […]. However, now, nearly 80 years after the theory was first produced, its overall precision has grown to something like ten million times greater. Einstein was not just "noticing patterns" in the behaviour of physical objects. He was uncovering a profound mathematical substructure that was already hidden in the very workings of the world (Penrose, 2012: 437).

Penrose himself recognizes that this is not a definitive argument to defend the existence of the Platonic mathematical world, but he is convinced that it is necessary to take it into account so that said world is not rejected *a priori* (Penrose, 2012: 438).

According to the platonism we have seen so far, we can see that it contains the other mystery that involves the Platonic mathematical world (that is, that of its relationship with the mental world). As we have seen, the theory of the three worlds tells us that the Platonic mathematical world emerges from the mental world. And under this statement the question arises: how can it be that a world whose contents are perfect emerges from one with imperfect contents? It is Penrose who realizes this conceptual conflict and the truth is that he cannot resolve it in a satisfactory way. He recognizes that the arrow in the scheme of the three worlds that represents the emergence of the Platonic mathematical world from the mental world necessarily leads to the idea that the latter is more originary than the former. However, Penrose, as a platonist, is totally contrary to this conception. How does he solve this problem? Well, downplaying the importance of the scheme to, in this way, try not to harm platonism:

> The essential point about the arrows […] is not so much their direction but the fact that in each case they represent a correspondence in which a *small* region of one world encompasses the *entire* next world (Penrose, 2012: 439; italics in the original).

With these types of statements it is easy to see that Penrose developed this scheme keeping in mind that it would not be definitive and that there would be nuances that would need to be outlined, especially those related to making his platonist position clear. For Penrose, although the scheme and theory of the three worlds belong to a conjectural and metaphysical plane, platonic existence, on the other hand, has an objective nature:

> […] To my way of thinking, Platonic existence is simply a matter of objectivity and, accordingly, should certainly not be viewed as something "mystical" or "unscientific", despite the fact that some people regard it that way (Penrose, 2006: 58).

These types of questions necessarily lead to a reworking, at least, of the three worlds scheme. Penrose certainly does so, not changing the principles of the theory excessively, although he does change the structure of the scheme.

Penrose in his reworking of the three-world scheme does not intend to make drastic changes with respect to the basic ideas of the theory. In fact, there are essentially no changes at all. The scheme does experience a subtle but significant difference. In the previous diagram, we could see that the arrows projected from one world to another, encompassing it in its entirety. The implication of this characteristic is that the relationship between the worlds occurs in a complete way, and this is something that Penrose does not say at any time. For this reason, he understands that the most convenient thing is to change the shape of the arrow, in such a way that they access a part of the world and not its entirety, which is what he defends at all times.

The aforementioned aspect is resolved satisfactorily, but, as we can see, the issue regarding the direction of the arrows does not change at all. This leads us to wonder why Penrose takes the trouble to restructure the scheme if with it he is going to continue without being able to explain a tremendously conflictive part with respect to the platonism that he defends.

Actually, this question is answered by the attitude that he shows regarding this topic. Although Penrose develops his ideas seriously and tries to give them visibility through this type of schemes, the truth is that at the end of the day it is a purely metaphysical question. That is to say, although he certainly believes in what he is exposing, there are clear limits to his explanations and these cannot be overcome through science or a scientific method. So Penrose, rather than making a mistake, I consider that he avoids it (not mixing purely metaphysical issues with science).

4. CONCLUSIONS

An important philosophical question is to determine to what degree Penrose's platonism is legitimate. It is at least striking that Penrose's ideas have been the subject of numerous criticisms in all their aspects, but, on the other hand, his platonism has gone almost unnoticed.

In my opinion, the most elaborate and transcendent direct criticism is that carried out by Mark Steiner. Steiner's criticism focuses, above all, on the idea that Penrose does not adequately understand platonism. In what respect does Penrose misunderstand platonism? Steiner argues that Penrose, by focusing on mathematical thinking, makes a distinction between the concepts that Plato does not make in his approach to philosophy:

> Actually, I doubt that Plato is a good historical source for Penrose's view. Plato made no distinction between mathematical and other concepts which would imply that mathematical concepts have more reality than others. If anything, the opposite was true - mathematical concepts, having a foot both in the world of sense and in the world of the intellect, were inferior metaphysically to those concepts applying only to the intelligible world (particularly "The Idea of the Good"). For the same reason, though, Plato held that mathematical concepts were a good entree into the intelligible world, an entering wedge into metaphysics (Steiner, 2000: 134).

The “idea of Good” is the guarantor of Truth and not mathematics, so conceiving otherwise would be a mistake. In fact, Steiner goes further and thinks that the Penrosean exposition is more in line with Cartesian rather than Platonic philosophy (Steiner, 2000: 134), since Descartes does give priority to mathematics (specifically its concepts[75]).

Although on the one hand I think Steiner is right, on the other I think that part of the analysis is wrong. It is easy to concede that Penrose places the emphasis of his defense on mathematical thinking. But despite the special status he grants it, it is difficult to observe that he makes a distinction between mathematical concepts and others. Penrose himself knows that by doing so he would only be entering into difficult domain. One of the compelling reasons why he clings to the mathematical field is because this is his field of study. It is true that he defends a certain superiority of mathematics, but, however, it does not seem that he is speaking in absolute terms. Mathematics is those in which Penrose feels comfortable and those that allow him to more adequately explain the connection with platonism. We can see this idea expressed almost at the end of SOTM:

> Plato himself would have insisted that the ideal concept of "the good" or "the beautiful" must also be attributed a reality […], just as mathematical concepts must. Personally, I am not averse to such a possibility, but it has yet played no important part in my deliberations here. Issues of ethics, morality, and aesthetics have had no significant role in my present discussions, but this is no reason to dismiss them as being not, at root, as "real" as the ones I have been addressing. Clearly there are important separate issues to be considered here, but they have not been my particular concern in this book (Penrose, 2012: 439).

If Penrose considers that issues such as ethics, morality or aesthetics are still just as "real" as mathematics, how can he defend that there is a superiority in absolute terms as Descartes defends, in a certain way? Actually, I argue, he does not do it, but he understands what is the terrain that he dominates (and in which, of course, he believes) and he limits himself to giving explanations from this:

> […] It is only with this mental quality that I have been able to make the necessary strong claim: that it is essentially *impossible* that such a quality

[75] This is not an unfounded consideration, since Descartes literally went so far as to say: Mathematics accustoms us to recognizing the truth, because in it we find correct reasoning that you will not find anywhere else. Therefore, whoever has accustomed his ingenuity to mathematical reasoning will also be apt to investigate other truths, because reason is everywhere one and the same (Descartes, 2011: 457).

> can have arisen as a feature of mere computational activity nor can computation even properly simulate it-and I should emphasize that there is no suggestion here that there is anything special about *mathematical* as opposed to any other kind of understanding. The conclusion is that whatever brain activity is responsible for consciousness (at least in this particular manifestation) it must depend upon a physics that lies beyond computational simulation (Penrose, 2012: 433; italics in the original).

However, Steiner is convinced that Penrose is more in favor of Cartesianism than of platonism. To do this, he brings up a text by Hertz that says:

> One cannot escape the feeling that these mathematical formulae have an independent existence and intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than was originally put into them (Hertz, cif. by Steiner, 2000: 135).

Next, Steiner comments that both Descartes and Hertz maintain that "mathematical concepts contain "latent information" in such a way that they go beyond mere logical deduction" (Steiner, 2000: 135). Although the exposition is correct, the underlying criticism is once again incorrect. Although Penrose agrees with what Descartes and Hertz defend (which he does), to what extent does this place him outside platonism? The underlying idea in the cited phrase is the independence of the existence of numbers with respect to the physical world and this, as we have seen previously, is something that is within the characteristics of platonism. If there is one aspect in which Penrose does not hesitate to admit his adherence to platonism, it is this. Therefore, using this feature as a definition of Penrose's error is, to say the least, inappropriate.

Leaving aside Steiner's criticism, on the other we have Feferman's criticism of Penrosean platonism.

Following the same tone as one of his previous criticism, Feferman recognizes as a fact that the vast majority of mathematicians can support their way of conceiving mathematics in platonism. However, he also understands that this cannot be the only guarantee of his knowledge:

> […] While mathematicians may *conceive* of what they are talking about in Platonistic set-theoretical terms, these results show that such a conception is *not necessary* to secure confidence in the body of mathematical practice (Feferman, 1995: 10; italics in the original).

Although he initially agrees with Penrose, he later shows his disagreement, or rather a certain skepticism, with respect to understanding set theory through platonism. Gödel himself, a great defender of this position, would also at one point back down from his speculations:

> […] Indeed, Gödel himself, at least for a period in the 1930s, found this deeply troubling. In a previously unpublished lecture […], he said that: "The result of the preceding discussion is that our axioms [for set theory], if interpreted as meaningful statements, necessarily presuppose a kind of Platonism, which cannot satisfy any critical mind and which does not even produce the conviction that they are consistent." […]. And Gödel continued to take proof-theoretical approaches to consistency seriously throughout his life (Feferman, 1995: 10).

In Feferman's opinion, this position is so unsustainable that it can only lead Penrose towards a solo route:

> […] Incidentally, on p. 116 of SOTM, Penrose says that Paul Cohen, in the last section of his 1966 book on the independence of AC and CH from ZF set theory "reveals himself to be, like Gödel [and Penrose] a true Platonist for whom matters of mathematical truth are absolute and not arbitrary." While that is a reasonable inference from what Cohen said there, shortly after that, at a 1967 conference, he stated: "By now it may have become obvious that I have chosen the Formalist [as opposed to the Platonic Realist] position for set theory" (Cohen 1971, p. 13). As far as I know, that is still his view (Feferman, 1995: 10).

Although the main task of someone who presents their ideas is to convince with them and, therefore, have more followers than detractors, the possibility of the opposite occurring does not cease to exist. And it is not that Penrose has embarked on that solitary path that Feferman sees him on, but it is evident that his approaches are prone to discussion. In any case, I do not think Penrose has a problem seeing the situation of continuous discussion in which his position finds itself. I would even dare to say that the opposite is true, since his attitude is not characterized by wanting to give definitive answers (Penrose, 1991: 24).

The fact that Steiner's or Feferman's criticisms seem to me that, in principle, they do not put Penrose's platonism in too much trouble does not mean that his position fully convinces me (Heredia, 2020: 120). On the other hand, it does not seem entirely inappropriate to me either. The criticisms that go more in the direction of questioning Penrose's position as too metaphysical seem to me to have a tone that could be more in line with my own. But not in the same sense. I think that if he can be blamed for the fact that his ideas regarding platonism become metaphysical (normally used in a pejorative way) it is not because he gets into unnecessary debates, but, rather, for not fully getting into them. And it is not that I consider Penrose to be unaudacious in his approach (far from it), but when it comes to this particular matter I do miss the audacity that he displays in most of his work.

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