

Quadrics Theorems as an Introduction to Geometry, Parametric Design and Digital Fabrication

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The Caterpillar gallery was a teaching innovation project intended to integrate geometry, parametric design and digital fabrication at the earliest stage of the undergraduate training period. This paper shows the contents, based on this project, of the workshop carried out within the 33rd eCAADe conference, Vienna 2015. The geometrical principles -stemming from certain quadrics theorems-, the parametric definition and the digital fabrication of the reduced-scale model executed in the workshop are outlined and illustrated.

Keywords: *Architectural geometry, Quadrics, Parametric Design, Digital Fabrication, Education*

INTRODUCTION

The use of computation in architectural design has definitely opened a new paradigm in architecture (Terzidis, 2003). The focus of the new design strategy has moved from the object to the process itself. Algorithms acquire the role of the new means of representation as the language which translates human thinking for the combination power of computer-based processes. Indeed scripting languages integrated in CAD systems go beyond visual and mouse-based operations establishing a new way of interacting with the geometry involved in the project. On the other hand, the parallel development of digital fabrication perfectly matches with digital design tools, allowing a totally digital architectural process, from conception to materialisation.

This new paradigm places geometry in a new position with new roles. The explicit use of geometry is almost the only link between programming/scripting languages and architectural spatial relationships and

forms. Compared with the conventional paradigm, this means major changes in the way in which geometry is applied, represented and even managed along the different stages of a project. In fact, the new relationships between applied geometries -descriptive, projective, algebraic...- and related disciplines in architecture -CAAD, programming, digital fabrication...- foster the emergence of a new discipline which integrates all of them together, Architectural Geometry (Pottmann, 2007).

The ways of thinking, conceiving, developing and materialising an architectural work are affected by the new paradigm. In most architecture schools, training matching this new concept is provided within postgraduate programmes. Nevertheless, just because the fundamentals are affected, the authors of this paper strongly believe -after years of tested teaching experience- that the implementation of the digital realm must take place at the beginning of the undergraduate training period, which is exactly the

time when students shape their design methodology.

The Caterpillar gallery (Narvaez-Rodriguez, Martin-Pastor, & Aguilar-Alejandre, 2015) was a project intended to produce this implementation under a carefully-planned pedagogical structure to avoid gaps of knowledge and competences. It was a ten-meter-long built full-scale prototype which facilitated the integration of digital tools in a course about the Fundamentals of Architectural Geometry for undergraduate first-year students. This paper shows the contents, based on this project, of the workshop carried out within the 33rd eCAADe conference, held in the Vienna University of Technology (Austria) in September 2015. The geometrical principles -based on quadrics theorems-, the parametric definition and the digital fabrication of the 1:6 scale model executed in the workshop are outlined and illustrated.

GEOMETRICAL PRINCIPLES AND PARAMETRIC DEFINITION

As an exercise to be addressed by beginner students, the design of the pavilion was intended to respond to a series of requirements; simplicity, constructive efficiency to be built and assembled by the students, and spatial interest both from the installation itself and the environment relationship. At the same time, the piece must contain the intended geometrical

concepts for the course, including an introduction to algorithmic thinking applied to architectural design, human scale control, vector geometry, basic curves, developable surfaces, intersections, true size and shape for fabrication...

The result was not a closed-ended design but a generative law based on the use of developable surfaces and simple geometrical operations, which allowed students to propose a variety of solutions. The parameterisation of a quadratic surface theorem by Gaspard Monge (Taibo, 1983), regularly used in teaching to give explanation to classic vaults, is the starting point to address exploration. The statement basically says that if two quadratic surfaces are circumscribed about a third, the intersection curve decomposes into two planar curves.

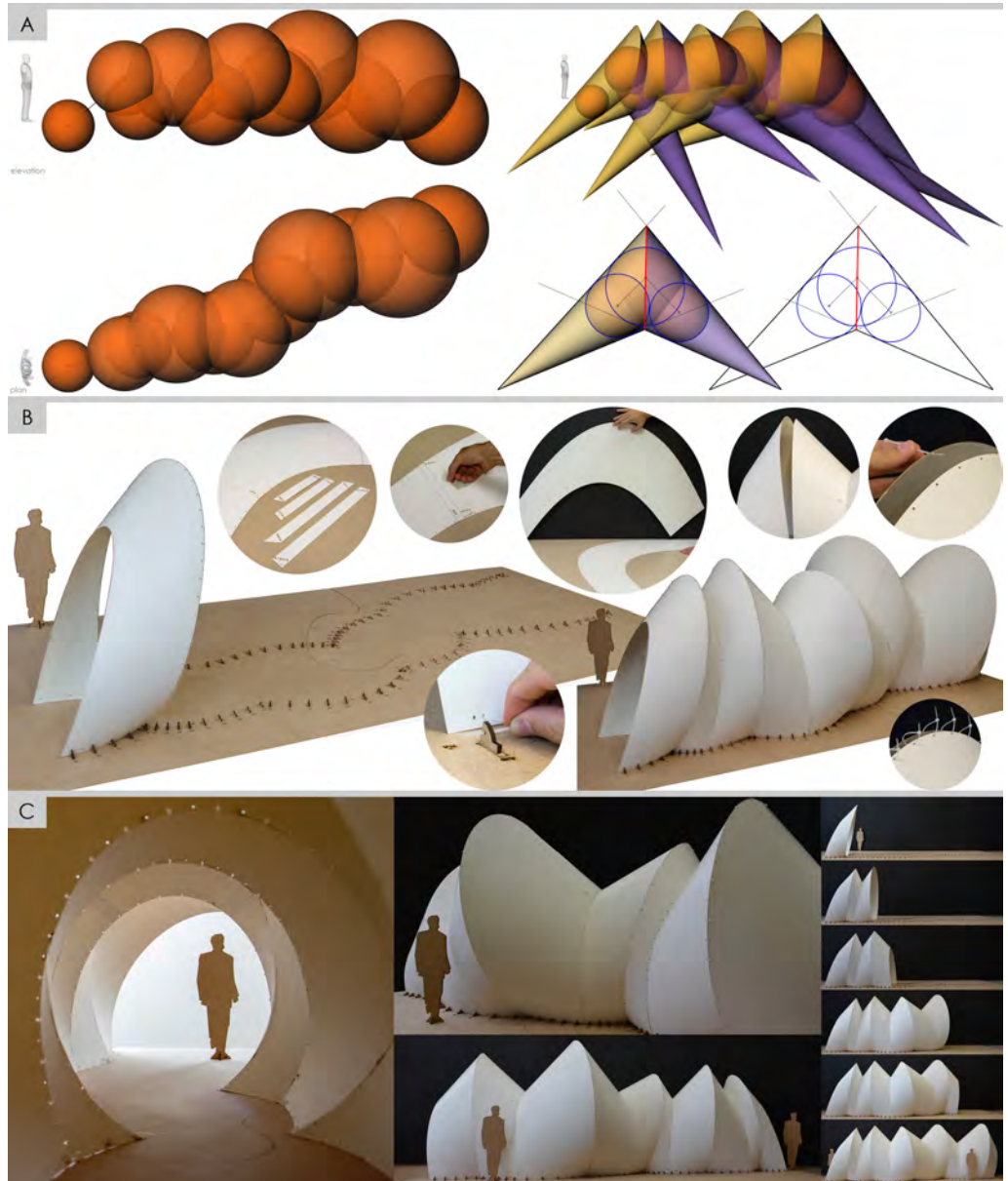
In order to work with simple and developable surfaces, the quadrics chosen are rotational cones or cylinders circumscribed about spheres to force the intersections to be planar curves, particularly ellipses. The spheres are placed at the vertices of a polygonal path with an elevation by way of zig-zag lines and a plan defining the route to follow, as the idea was to create a longitudinal space. Moreover, the diameters of the spheres control and define the spaciousness of the project at every point of the path at two different levels; at the ground level and at the average human size. Finally, the shell is defined by keeping



Figure 1
Pictures of the built full-scale prototype, The Caterpillar gallery, University of Seville (Spain), 2014.

Figure 2

A-left. Human scale and space control with the set of spheres at the two levels. A-top-right. Set of cones circumscribed about the spheres to define the shell. A-bottom-right. Second quadrics theorem showing the two-dimensional construction to simplify the parametric definition. B. Different stages of the assembly process; the conical fractions are linked to the ground through the use of wooden wedges and to the other fractions with the use of nylon cable ties. Excepting the rigid position of the wedges along the intersection of the cones with the ground, the rest of the linkages can be directly executed, as the drillings for the ties are laid out to geometrically constrain the surfaces to acquire the desired shape. C. Pictures of the resulting model at 1:6 scale.



every conical fraction located between the two planar intersection curves with the adjacent cones and the ground floor plane.

The parametric definition also takes advantage of another quadric theorem which states that the two planar curves resulting from the previous theorem are projected onto the plane of symmetry of the two quadrics as two straight lines. This simplifies, even more, the three-dimensional geometrical relationships to an easy two-dimensional construction. The spheres turn into circles and the rest of the elements, cones, cylinders and their intersections can be represented as straight lines. In addition, this is a good exercise for students to get trained in the use of projective properties of three-dimensional geometries.

DIGITAL FABRICATION AND ASSEMBLY

The impact of digital fabrication laboratories in architectural education can also be channelled to produce benefits from the earliest stage of the training period, introducing students to the relationships among geometry, software, the production process and the control over building components and materials (Celani, 2012).

In this case, due to the possibility of unrolling the surfaces defining the project, all building components can be laid out on a plane. Consequently they can be obtained from sheets or panels of different materials, according to their functions and thicknesses required, and fabricated with a laser cutter or a three-axis CNC milling machine.

The structural behaviour differences between the built full-scale prototype and the 1:6 scale model must be pointed out. At 1:1 scale, the shell was materialised with five-millimetre-thick plywood panels. The fractions of conical surface are not self-supporting. They are used as components that, once assembled, bring about the emergence of the desired structural behaviour without any kind of auxiliary structure. This behaviour is difficult to simulate at a reduced scale, therefore conclusions on this matter are not reliable at 1:6 scale. Nevertheless, essential

conclusions related with the assembly strategy can be achieved. The figures illustrate different stages of this process.

CONCLUSION

The quadrics theorems stated in this paper, in the particular case of cones and cylinders circumscribed about spheres, provide effective and appropriate geometrical principles to integrate geometry, parametric design and digital fabrication at the earliest stage of the undergraduate training period.

Geometry plays new roles within the digital realm in contemporary architecture which must be translated to the educational context. Although The Caterpillar gallery was a successful innovation project in this sense, this is a field which still needs research, development and testing in many other architecture schools. Workshops, such as the one showed in this paper, where lecturers, researchers and students from different countries are involved, foster interesting and motivating discussions to keep advancing on this topic.

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