# Heavy-ion induced quasi-elastic reactions in view of the NUMEN project

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**Abstract.** Double charge exchange (DCE) reactions induced by heavy ions and other direct reactions characterized by same projectile and target are crucial tools to access information relevant for neutrinoless double beta decay nuclear matrix elements. In this context the NUMEN project aims to investigate, for each system of interest, not only the DCE channel but also the whole set of reactions promoted by the same projectile/target interaction in the same experimental conditions and within the same theoretical framework. An example of the application of such a multi-channel approach is presented here.

## 1. Introduction

The neutrinoless double beta decay ($0\nu\beta\beta$) is nowadays the most promising resource to establish the Majorana nature of neutrinos and potentially shed light on the absolute neutrino mass and hierarchy. A critical aspect is that the associated Nuclear Matrix Elements (NME) must be known with good accuracy, even if the intrinsic many-body nature of the involved states of the parent and daughter nuclei makes this task particularly challenging. A comparison of the

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results of NME calculations, obtained within various nuclear structure frameworks, indicates that significant differences are found, which makes the present situation not satisfactory [1, 2, 3].

The NUMEN (NUclear Matrix Elements for Neutrinoless double beta decay) project [4, 5, 6, 7] proposes to use heavy-ion induced Double Charge Exchange reactions (HI-DCE) as a tool to access quantitative information, relevant for $0\nu\beta\beta$ decay NME. DCE reactions are characterized by the transfer of two units of charge, leaving the mass number unchanged, and can proceed by a sequential nucleon-transfer mechanism or by exchange of two isovector mesons, in an uncorrelated or correlated way. Despite $0\nu\beta\beta$ decays and HI-DCE are mediated by different interactions, they present a number of similarities. The key aspects are that initial and final nuclear states are the same and the transition operators in both cases present a superposition of short-range isospin, spin-isospin and rank-two tensor components with a relevant available momentum (100 MeV/c or so). In addition, NUMEN aims at the exploration of all the relevant reaction channels promoted by the projectile/target interaction. They include elastic and inelastic scattering, single and double nucleon transfer reactions and single charge exchange reactions [8, 9, 10, 11, 12, 13, 14].

NUMEN is conceived in a long-range time perspective, in the view of a comprehensive study of many candidate systems for $0\nu\beta\beta$ decay. Moreover, the project has promoted a renewal of the INFN-LNS research infrastructure and a specific R&D activity on detectors, materials and instrumentation [15, 16].

The NUMEN experimental campaigns, conducted so far at INFN-LNS, using the K800 Superconducting Cyclotron to accelerate beams and the MAGNEX large acceptance magnetic spectrometer [17, 18, 19, 11] for the detection of the ejectiles have brought to first results, giving encouraging indication on the capability of the proposed technique to access relevant quantitative information.

## 2. The DCE reaction mechanism

A microscopic theory of DCE based on semi-classical eikonal approximation was developed in Ref. [20] showing that the DCE differential cross sections can be factorized as the product of reaction and structure parts in closure approximation and in the low-momentum-transfer limit, corresponding to very forward detection angles, and that a further factorization in terms of target and projectile NMEs can be performed.

The description of the complete nuclear reaction mechanism of DCE needs a formulation based on quantum-mechanical microscopic reaction theory which accounts for the possible competition of different processes feeding the DCE channel. Indeed, HI-DCE can proceed either via multi-nucleon transfer (TDCE), mediated by mean-field, or through the exchange of correlated or uncorrelated mesons. In principle, the TDCE mechanism can be described within distorted wave Born approximation (DWBA) or coupled channels methods. However, the practical implementation of a sufficiently complete calculation has not been possible until very recent times [21], due to the fact that at least fourth order nucleon transfer scheme is required. For the meson exchange mechanisms, the theoretical framework formerly introduced for SCE by Lenske [22], has been recently extended to second order [23, 24], describing the successive exchange of two uncorrelated charged mesons. This formalism reveals a remarkable similarity of DSCE with $2\nu\beta\beta$ decay, although a much richer multipole spectrum is accessed in HI-DCE. A completely new reaction mechanism, named Majorana mechanism (MDCE), has been recently introduced in Refs. [25, 26]. The MDCE mechanism relies on neutral mesons induced nucleon-nucleon short range correlations. The included correlations bring to a single step reaction mechanism and suggest an intriguing connection of this formalism with $0\nu\beta\beta$.

## 3. The multi-channel approach

As a typical feature of direct reactions, DCE cross sections critically depend on the ion-ion initial and final state interaction and represent a minor component of the outgoing flux in heavy-ion collisions. In addition, the coupling between a specific reaction channel and the elastic channel and other direct processes makes it necessary to study all of them in a comprehensive approach. Such a requirement demands for sophisticated data analyses, where different reaction channels are studied in the same context.

The actual implementation of such a multi-channel approach has been rarely used in the analyses of data, mainly due to the complexity of the problem from both the experimental and the theoretical side. From the experimental side, in fact, different reaction cross sections should be measured under the same laboratory conditions. From the theory side, a high consistency is required in order to simultaneously describe the different degrees of freedom, selectively activated by the various direct reactions, in the same microscopic approach.

A clear advantage of the multi-channel approach is the possibility to use a broad and correlated ensemble of experimental data in the analyses, thus reducing the need of free parameters in the adopted theoretical models. For such reasons the NUMEN project proposed, and is actually adopting, a multi-channel approach, in which DCE cross sections are analyzed together with elastic and inelastic scattering, single and multi-nucleonic transfer reactions and SCE reactions. In the next Section an example of application of such an approach to the study of the $^{18}$O + $^{40}$Ca collision at 15 MeV/u incident energy is presented.

## 4. Application to the $^{18}$O + $^{40}$Ca case at 15 MeV/u

A case where such a multi-channel approach has been applied is the exploration of the $^{18}$O + $^{40}$Ca collision at 15 MeV/u incident energy. This system represents the pilot case in the NUMEN project and is the first system that has been studied in a comprehensive approach both from the experimental and theoretical side [27, 8, 28, 29].

### *4.1. Elastic and inelastic scattering*

The elastic and inelastic channels have been analysed in a recent work [8]. Fig. 1 summarizes the obtained results for the elastic and inelastic transition to the first $2^+$ state of $^{18}$O at 1.982 MeV. One-channel calculations (optical model OM and DWBA (DOM)) fails to reproduce the cross sections at large angles. Coupled-channel calculations (CC), using the Sao Paulo optical potential (SPP), provides a good description of both elastic and inelastic angular distributions in a unique reaction framework and without the need of adjusting free parameters. The adopted coupling model includes the ground and the $2_1^+$ state at 1.982 MeV in $^{18}$O and the ground, the $3_1^-$ and $2_1^+$ states at 3.737 MeV and 3.904 MeV, respectively, in $^{40}$Ca.

In addition, following the approach of Ref. [30], in this study we have implicitly incorporated the effect of channel couplings in the elastic optical potential by means of an effective polarization potential term, through an average local and $L$-independent polarization potential, named trivially equivalent local potential (TELP). Adding the TELP to the SPP bare optical potential used in CC calculations, a coupled channel equivalent polarization potential (CCEP) is obtained [31, 32]. Such CCEP, which treats the couplings in an effective way, has been used in the one-channel calculations shown in Fig. 1, also producing a good agreement with the data. This method, even if not complete, avoids the complications raising from higher order calculations.

### *4.2. One- and two-nucleon transfer reactions*

In the same experimental run, the one- and two-nucleon transfer channels populated in the $^{18}$O + $^{40}$Ca collision at 15 MeV/u have been measured. The details of analysis of the one-nucleon transfer reactions has been reported in Ref. [28] while the two-proton transfer reaction has been studied in Ref. [33]. The comparison between experimental angular distributions and

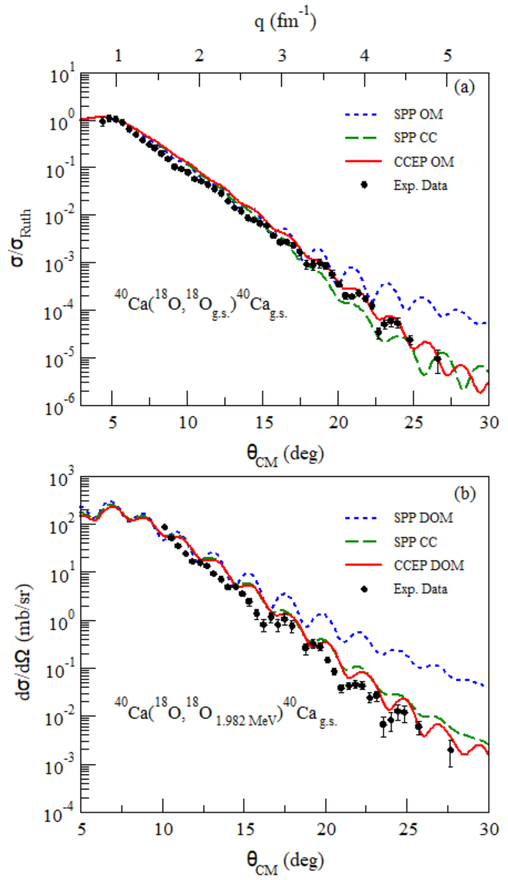


**Figure 1.** Panel (a): Cross section angular distribution of the $^{18}O$ + $^{40}Ca$ elastic scattering at 15 MeV/u in terms of its ratio to the Rutherford cross section $\sigma_{Ruth}$. Panel (b): Angular distribution of the inelastic channel $^{40}Ca(^{18}O,^{18}O_{1.982})^{40}Ca_{g.s.}$. In both plots, the blue dotted line shows the optical model (OM) and distorted wave Born approximation (DWBA, here indicated as DOM) calculations with SPP, the green dashed line shows the CC calculations with SPP and the red solid line shows the OM and DOM calculations with coupled channel equivalent polarization potential (CCEP). Figure from Ref. [8].

theoretical calculations is shown in Fig. 2 for the one-nucleon transfer transition to the ground state $^{40}Ca(^{18}O,^{17}O_{g.s.})^{41}Ca_{g.s.}$. Other cases are reported in Ref. [28]. The cross section angular distributions show bell-shaped curves peaked at very forward angles and with contribution of many $\ell$-transfer components. Reaction calculations using shell model derived spectroscopic amplitudes and initial state interaction derived from the elastic/inelastic scattering analysis previously performed provide an excellent description of the data, without the need of any normalization factor. This is a strong indication that both the reaction inputs and the nuclear structure information are in place for a reliable description of the one-nucleon transfer. In particular, it is observed that the inclusion of the couplings (through CCBA calculations or through the simplified DWBA approach with CCEP) is not relevant in the explored angular range.

Also the $^{40}Ca(^{18}O, ^{20}Ne)^{38}Ar$ two-proton transfer data have been measured and analysed in the same experimental conditions. The calculations have been performed considering shell model spectroscopic amplitudes extracted within the approach used for the single-nucleon transfer case and reaction calculations which include both the simultaneous and the sequential transfer of the two particles and their sum (see Ref. [33]). Again the inputs used for the reaction calculations are the same as those used for the scattering and single-nucleon transfer case. As shown in Ref. [33], the calculations give a satisfactory description of the data, strengthening the confidence on the adopted theoretical approach.

### *4.3. Single charge exchange reaction*
Within the multi-channel study of the $^{18}O$ + $^{40}Ca$ collision, the $^{40}Ca(^{18}O, ^{18}F)^{40}K$ single charge exchange reaction has also been explored in a consistent way. Details on the experiment

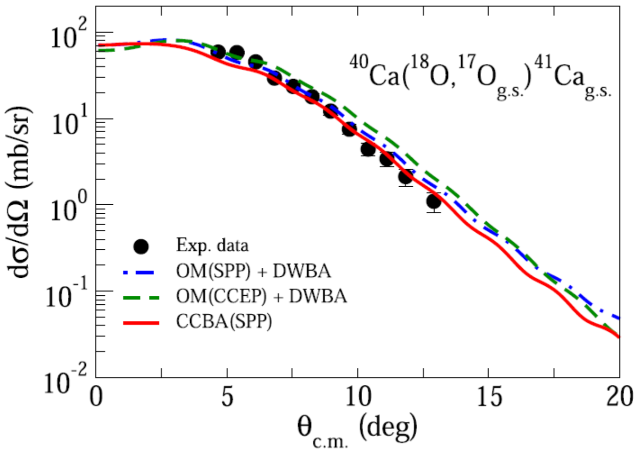


**Figure 2.** Comparison between theoretical and experimental one-neutron transfer angular distribution for the transition $^{40}Ca(^{18}O,^{17}O_{g.s.})^{41}Ca_{g.s.}$. The OM(SPP)+DWBA, OM(CCEP)+DWBA and CCBA(SPP) calculations are indicated by the dash-dotted blue, dashed green, and solid red lines, respectively. Figure from Ref. [28].

and data analysis are given in Refs. [8, 34]. Charge exchange cross section calculations have been performed in DWBA using the CCEP tested against the scattering data and form factors extracted from double folding of a nucleon–nucleon isovector interaction with QRPA transition densities. The direct SCE mechanism with microscopic nuclear structure inputs describes the order of magnitude and shape of the observed cross sections without introducing scaling factors, as it was found necessary in the former heavy-ion-induced SCE reactions on the same target at 35 MeV [35] or the pioneering ($^{18}$O,$^{18}$F) studies of Refs. [36, 37].

The slight underestimation of the calculations with respect to the experimental data shown in Ref. [8] has probably a two-fold origin. On a nuclear structure side, there is some room left for refinements of the nuclear response functions by going deeper into the core excitation effects, e.g., in the multi-phonon approach of [38] or by second RPA methods of [39]. On the reaction side, the competing sequential nucleon transfer mechanism should be included [29]. However, the quite successful description of the higher excitation energy region by the direct SCE mechanism suggests a rather insignificant role of nucleon transfer, at least in that spectral region, as indeed it is expected from the matching conditions for transfer processes [40].

### *4.4. Double charge exchange reaction*

The $^{40}$Ca($^{18}$O,$^{18}$Ne)$^{40}$Ar double charge exchange reaction at 15 MeV/u incident energy was the first DCE reaction to be measured and analysed as pilot system within the NUMEN project. The energy spectrum and cross section angular distribution for the ground to ground state transition and a preliminary data analysis based on the experimental evidences were shown for the first time in Ref. [27]. In that paper also the motivation of the study of DCE reactions in view of their connection to $0\nu\beta\beta$ decay NME was discussed for the first time. In Fig. 3 the $^{40}$Ca($^{18}$O,$^{18}$Ne)$^{40}$Ar DCE angular distribution for the $^{40}Ca_{g.s.} \rightarrow {}^{40}Ar_{g.s.}$ transition is reported and compared with DSCE and MDCE calculations.

The formalism of the DSCE calculations and the details of the application to the $^{40}$Ca($^{18}$O, $^{18}$Ne)$^{40}$Ar case have been given in Ref. [23]. The optical potential was derived from the experimental data for the elastic and inelastic channels of the same incident system [8]. The magnitude of the measured cross section is almost perfectly well reproduced without the need for adjustments by scaling factors. However, the calculations show a more pronounced diffraction pattern than the one observed in the experimental data, and an underestimation of the cross section at small angles. This behaviour might indicate contributions by other reaction mechanisms like the MDCE.

First numerical MDCE results have also been performed, employing the decay–MDCE scheme

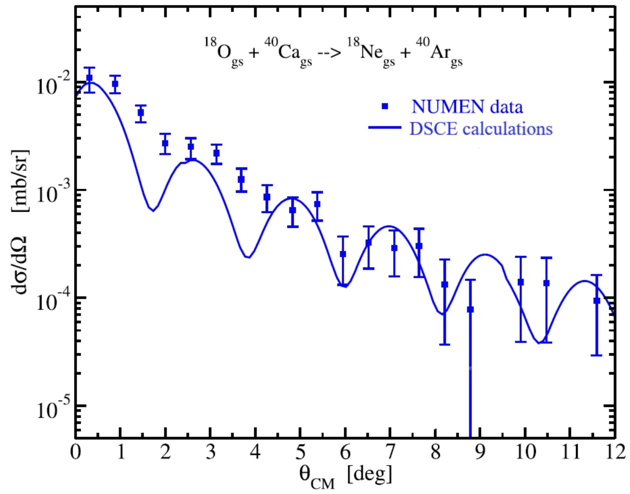


**Figure 3.** DCE angular distribution for the reaction $^{40}$Ca ($^{18}$O, $^{18}$Ne$_{g.s.}$)$^{40}$Ar$_{g.s.}$ at 15 MeV/u. The experimental data are from Ref. [27] and the DSCE calculation details are described in Ref. [23]
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in closure approximation and using only the diagonal pion potentials. Refs. [26]) and [29] report on the adopted formalism the preliminary results. The contribution of the MDCE mechanism especially at small angles appear to be relevant.

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