Advances on clusters and correlations in nuclear structure and reactions

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Abstract. Recent advances obtained in the last few years by the Theoretical Nuclear Physics group in Padova with various collaborators on alpha-cluster models and on nuclear correlations in stable and unstable light nuclei are reviewed in this contribution. The algebraic cluster model assumes triangular and tetrahedral arrangement of α particles for ^{12}C and ^{16}O respectively. The description of the low-lying states achieved in this model, that is a consequence of the requirement of discrete symmetries, is extremely good. We have made several calculations of α-transfer form factors and reaction cross-sections obtaining a good agreement with available data, thus corroborating the main hypotheses of the model [1]. We have speculated about smoking-gun nuclear fluorescence experiment that might shed light on the exact spatial arrangement of alpha particles in ^{12}C [2]. We will also talk about the successful predictions on the positioning of ^{29}F on the southern shore of the island of inversion [3] and on recent calculations on its halo character and dipole response [4, 5]. Extension of these calculations to ^{31}F [6], pointing out the presence of a halo, are also discussed.

1. Introduction

It is the purpose of this contribution to summarize some peculiar aspects of cluster models and few-body systems, such as Borromean systems, that have been studied in Padova, Italy, in the last few years, mainly in collaboration with Spanish and Japanese institutions.

Cluster models arise from the necessity to model certain aspects of the structure of light nuclei, that are too light to allow for a full implementation of the collective model and cannot yet be fully tamed by the shell model, neither in the classic core+valence approximation, nor by modern-day ab initio shell model calculations. Only recently shell model approaches based on
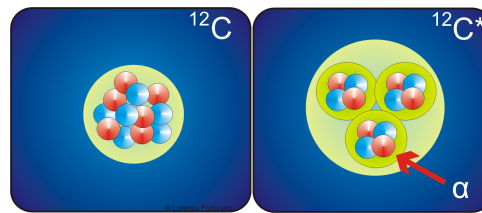
Figure 1. α-cluster configurations in ^{12}C (right) apparently are important not only in excited states, close to threshold, as it was thought a few years ago, in contrast to spherical shell model configurations (left), but also in low-lying energy states.

Monte Carlo and configuration interaction methods, using chiral EFT interactions, [7] start to show emergence of $\alpha-$cluster configurations from first principles. Other approaches, including for example symmetry adapted no-core shell model [8] and density functional theories [9], show it as well.

Since the very beginning of nuclear structure and reaction studies, it was soon realized that the α particle plays a crucial role in the constitution of nuclei, because it is a very tight spin-0 boson composed of four fermions in the first s-shell. Not only it gives rise to one of the main decay modes, due to Q-value considerations, but it might be thought of as a building block of certain light nuclei. The pattern of binding energies in light nuclei ($A \leq 20$) shows peaks in correspondence of α-conjugate nuclei. Initially, it was thought that cluster configurations are seen more often in excited states, close to the corresponding separation threshold. Ikeda (see Ref. [10] for a review) proposed several diagrams to highlight the merging of low-energy shell model configurations, with high-lying cluster configuration (See Fig. 1 for a pictorial representation), but more recently it is becoming evident that cluster structures might explain some features of the whole spectrum of nuclei like ^{12}C, for example. The well-developed methods of molecular physics, heavily based on symmetries and group theory, might turn very useful for the description of these systems. Iachello and Bijker [11, 12, 13, 14] started a very successful program of application of algebraic methods to several systems, like ^{12}C, ^{16}O and ^{20}Ne and extended it to some odd nuclei, like ^{13}C, where additional complications arise.

The paper is divided as follows: we will start with summarizing a proposal for a crucial polarization experiment aimed at establishing a connection between symmetry and geometry in ^{12}C, then we will summarize a few works on the application of an improved version of the algebraic cluster model to treat reactions involving α-clusterized nuclei. Then, we will discuss a series of work on three-body models applied to the heaviest known halo nuclei among the fluorine isotopes, ^{29}F and ^{31}F.

2. Polarization experiments on ^{12}C

By drawing on quantum chemistry and molecular physics methods, we have proposed a way to determine the shape of a nuclear molecule with application to ^{12}C. Many models assume or predict α-cluster structures for this nucleus, either in some of the states or for the whole spectrum [11, 13]. Some adopt a linear configuration of three α-clusters, others an equilateral triangle, others again a bent (isosceles) geometry. The number of models and the complications in each have become a jungle and we need a way out that can be found in the methods usually employed in quantum chemistry. Any geometrical arrangement implies a certain degree of symmetry and can be classified according to one of the known discrete point-groups. In this case the nuclear molecule is made up of three α-particle "atoms" and the number of possible geometries is limited to 5. Either the particles are placed on a line, at equal or different distances from each other, or they are on a plane, forming an equilateral, iscosceles (either acute or obtuse) or scalene

name	shape	group	Γ_{vib}	Patterns
linear $=$		$\mathcal{D}_{\infty h}$	$A_{1g} + A_{1u} + E_{1u}$	
linear $\neq$		$\mathcal{C}_{\infty \nu}$	$2A_1 + E_1$	
equilateral		$\mathcal{D}_{3h}$	$A_1' + E'$	
isosceles		$\mathcal{C}_{2\nu}$	$2A_1 + B_1$	
scalene		$\mathcal{C}_s$	$3A'$	

Figure 2. All possible arrangements of three α clusters, their point-group and list of normal modes of vibration. The patterns indicate the expected behaviour of scattered parallel and perpendicular intensities for all rotational states inside each vibrational band. From Ref. [2].

triangle. These shapes conform to the centrosymmetric group, $D_{\infty h}$, the non-centrosymmetric, $C_{\infty \nu}$, the dihedral group D_{3h}, the cyclic group $C_{2\nu}$ and the simple involution group C_s for the least symmetric case. Now, for each linear or non-linear configuration one has $3N - 5$ or $3N - 6$ vibrational degrees of freedom, Γ_{vib}, that can be determined by straightforward application of group theoretical methods. The results are collected in Fig. 2 for the five different configurations. Each geometry produces a number of vibrational modes, indicated with the Mulliken notation [16]. Forgetting about details, those in red are one-dimensional totally-symmetric modes, others are non-totally symmetric modes either one dimensional (A, B) or doubly-degenerate modes $(E$'s). To each vibrational mode, there corresponds a rotational band (with special and different allowed states, depending on the symmetry!), including that ground state band that is totally symmetric.

When one shines electromagnetic radiation to such a sample of molecules with a certain geometry and random orientation, some might be absorbed and re-emitted to the same or lower frequency. If we collect the outgoing radiation at 90° through a polarizer, we will find some intensity in the parallel plane, $I_\parallel$, and some in the perpendicular plane, $I_\perp$. If the initial radiation is linearly polarized, and if we define the ratio of the quantities above, i.e. the depolarization ratio $\rho = \frac{I_\parallel}{I_\perp}$, it is a remarkable theorem of electromagnetism and quantum mechanics that, whatever the underlying point-group symmetry, the outgoing radiation can only have:

$$\rho = 3/4 \quad \text{for non-totally symmetric modes,} \tag{1}$$

$$0 \leq \rho \leq 3/4 \quad \text{for totally symmetric modes} \tag{2}$$

where the latter is usually closer to the lower side. The patterns in the last column of Fig. 2 indicate in a schematic way what is the expected outcome of scattered parallel to perpendicular intensities for all the states in each band. Except lines 2 and 4, that give the same scenario, the others are mutually exclusive. Measuring the depolarization ratio for all of the states of the system would allow to classify the spectrum according to any of the symmetries listed in the table and therefore, at least in principles, would allow to determine the shape of the nuclear molecule.

3. Reactions involving clusterized nuclei

In a series of works [1, 15], we have applied the algebraic cluster model with the idea of calculating reaction observables and compare with cross-section data. We have calculated densities and
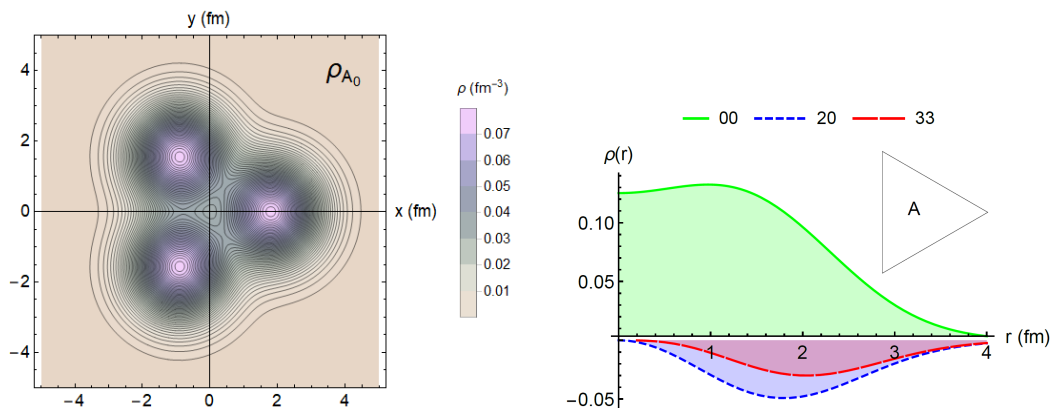
Figure 3. Ground state density of ^{12}C in an equilateral triangle model with three gaussian centers (right) and radial functions of the first few terms of the expansion in spherical harmonics (left). Notice that only those terms allowed by the $\mathcal{D}_{3h}$ symmetry are present. From Ref. [1].
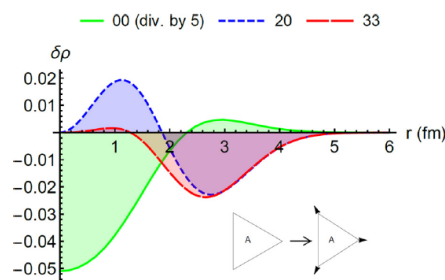

Figure 4. Expansion of the transition density from the ground state to the states of the Hoyle band (that is interpreted as the vibration excitation with one A-type phonon). From Ref. [1].

transition densities for an equilateral triangular configuration of three α particles in ^{12}C. Each particle is given a gaussian density distribution, properly adjusted to experimental input. This amounts to adjusting a parameter, β, that gives the distance of each α particle from the center of the triangle. We take $\beta = 1.82$ fm. These densities are expanded in a series of spherical harmonic functions, revealing the intrinsic structure. Fig. 3 shows the ground state density and the corresponding expansion. Notice that not all combinations of angular momentum quantum numbers $(\lambda\mu)$ are allowed, but only those that respect the triangular symmetry. The same expansion can be made for densities of the vibrational bands and for transition densities connecting two different bands (for example the ground state band and the Hoyle band, as in Fig. 4).

The knowledge of transition densities in the laboratory frame allows to construct form factors and, from those, to calculate inelastic ^{12}C-α cross sections in the distorted wave Born approximation. This geometrical model with rotational and vibrational d.o.f. is able to give a reasonable description of these cross-sections, without any adjustment. In Fig. 5 one can see the elastic scattering cross section and the cross-sections for the transitions from the ground state to the first excited 2^+ and 3^+ states. The good agreement is another piece of evidence in favour of the equilateral triangular model. It appears that the role of the underlying fermions does not seem to be relevant in order to describe the spectrum and the reaction observables involving clusters.
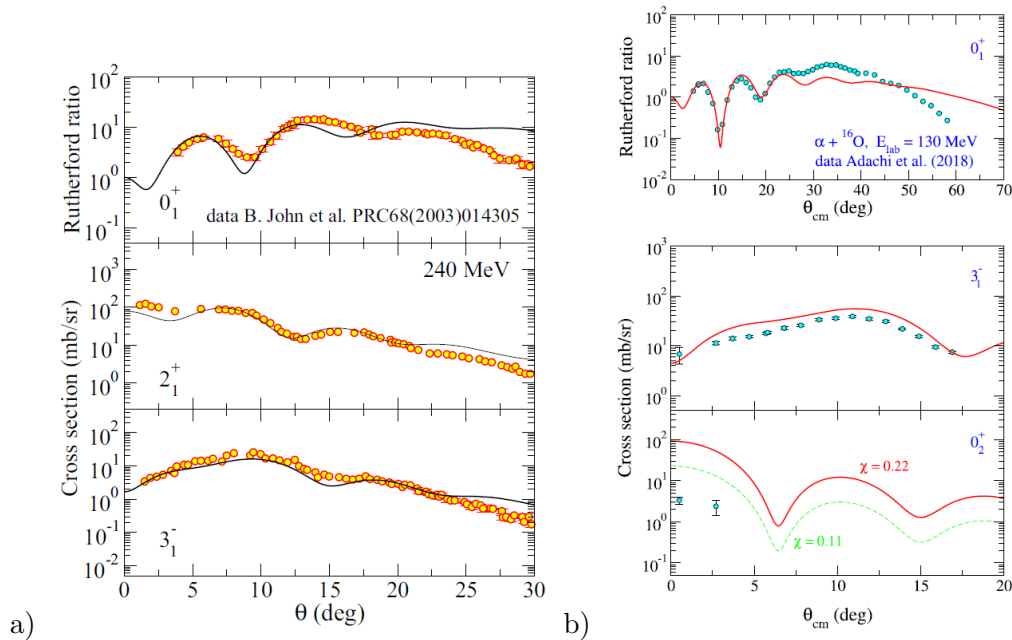
Figure 5. a) Differential cross section for the elastic scattering (upper panel) and for the transitions between the ground state and the first excited 2^+ and 3^+ states at 240 MeV bombarding energy (lower panels). Data are retrieved from EXFOR. Figure from Ref. [1]. b) Differential $\alpha + {}^{16}O$ cross section at 130 MeV for the elastic scattering (upper panel) and the population of the 3^- and 0^+ states. From Ref. [1, 15].
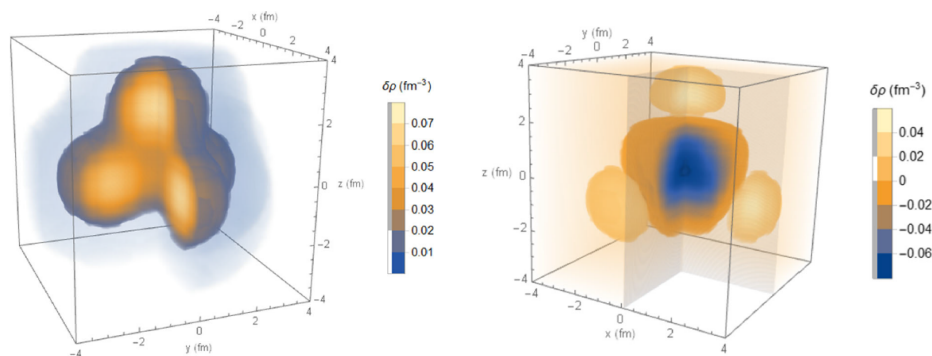

Figure 6. Left: 3D density profile of ${}^{16}O$ built as a collection of four α particles with gaussian density distribution at the vertexes of a tetrahedron. Right: 3D transition density from ground to first excited A-type band (breathing mode). From Ref. [15].

One can also model ${}^{16}O$ with four α particles at the vertexes of a tetrahedron (See left panel in Fig. 6) and play the same game [15], i.e. calculating transition densities (See right panel of Fig. 6), form factors and cross-sections. The right part of Fig. 5 shows some results on elastic and inelastic cross-sections involving ${}^{16}O$. Data are less abundant than in the case of carbon, but the agreement is sufficiently encouraging to believe that the model allows a reasonable description of the involved reaction channels. For future investigations, one might study the excitation of higher-lying levels and include multistep processes in the coupled channel reaction
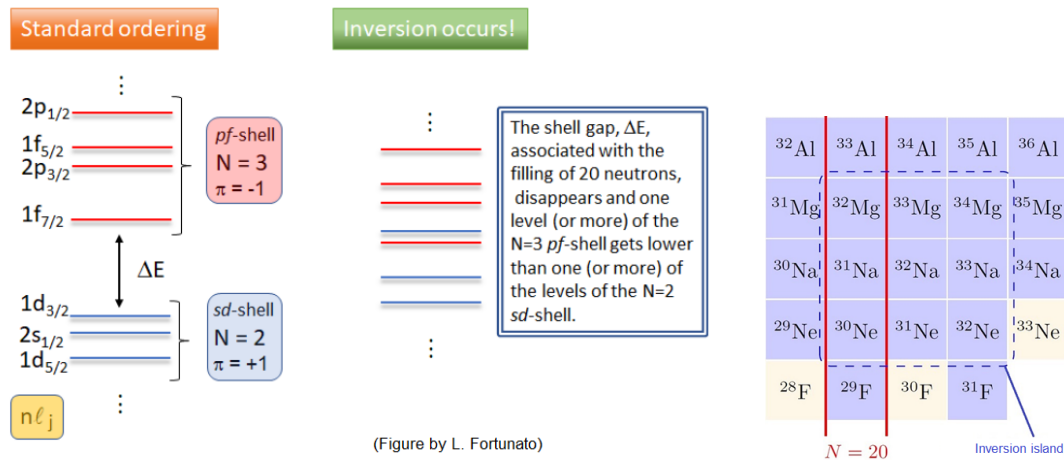
Figure 7. Mechanism of level inversion and portion of the nuclear chart with the island of inversion. Adapted from Ref. [3].

description. Another direction for future investigations would be to include α-transfer reactions like $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$.

4. Fluorine isotopes in the island of inversion

Some interesting aspects of nuclear structure of several fluorine isotopes close to the neutron drip-line have also been studied by our research team. We have concentrated our efforts on ^{29}F and ^{31}F, because these are Borromean systems and they are also lying on the southern border of the island of inversion, a portion of the nuclear chart where a merging of the N= 2 and N= 3 nuclear major shells determines an inversion of usual level sequence, see Fig. 7. Another reason why this region is interesting is due to the fact that it crosses the $N = 20$ neutron magic number. Somewhat ahead of the times, in Ref. [4], we had made a speculative investigation on the structure of ^{29}F using the hyperspherical three-body model developed by the Seville group, i.e. postulating a $^{27}\text{F}+n+n$ structure with two- and three-body interactions and solving the eigenvalue problem in hyperspherical coordinates. The application of the Transformed Harmonic Oscillator (THO) formalism allows to compute continuum states in a treatable manner. We had identified four possible scenarios (standard, intruder, degenerate and inverted) for the structure of ^{29}F, i.e. for the list of energy levels and resonances, when an almost coeval set of experiments added further information, allowing to refine the $n-^{27}\text{F}$ interaction. It turned out that the inverted scenario is the preferred one at the moment [3]. Therefore, according to our calculations, it fully belongs to the island of inversion. The ground state is dominated by the $(2p_{3/2})^2$ component with 57.5%, followed by $(1d_{3/2})^2$ with 28.1% and $(1f_{7/2})^2$ with $\sim$6%. We then investigated several properties of the refined model for this nucleus and, in particular, the dipole response [5], obtaining useful information for planning new experiments. The total dipole strength integrated up to 6 MeV, about 1.59 e^{2}fm^2, amounts to more than the 80% of the total cluster sum rule. We have also moved one step further studying the properties of the next bound isotope, ^{31}F in Ref. [6]. In the latter paper, we make a plethora of hypotheses on the structure using different interactions and level orders, but the main point is that, whatever interactions we choose, we get a large radius whenever the level inversion occurs as depicted in the red oval in Fig. 4, that indicates that this system is also a halo nucleus. In this figure, we can also see that the $R = r_0 A^{1/3}$ systematics on lower mass fluorine isotopes, indicated with a red shaded area (the solid line is the fit and the dashed lines are 99.99% C.L. lines) is clearly
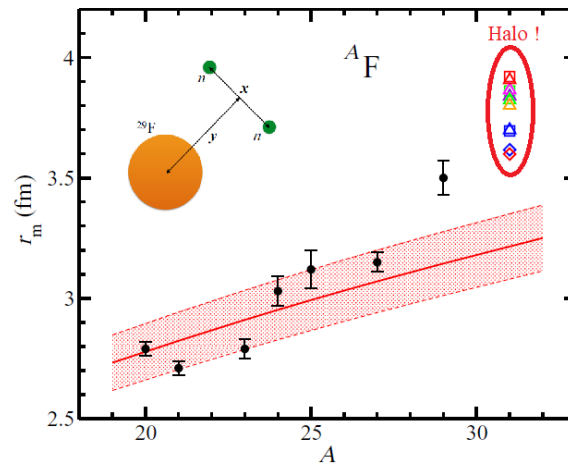
Figure 8. Three-body models for ^{31}F allow to study the formation of a halo, all models indicates that this is the case. Black points are experimental data or averages for the matter radius (indicating for example that ^{29}F is a halo nucleus). Colored points correspond to the different models that we used, one may take the variation as an indication of the theoretical error. The solid red line indicates the $A^{1/3}$ fit as a reference. From Ref. [6].

surpassed by the values of ^{29,31}F. Therefore, our models indicate that this is a halo nucleus that is also belonging to the island of inversion.

5. Conclusions

Clustering and pairing correlations in stable as well as in unstable nuclei are a hot topic where several new challenges can be found. Despite having a solid history, these two aspects of nuclear structure and reactions are still not completely understood: they are very fruitful, showing many facets, and they do interplay with other topics such as formation of a halo, shell evolution, level inversions and new magic numbers at the driplines, nucleon-nucleon interactions and few- and many-body methods in general.

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