EVALUATION OF THE STIFFNESSES OF THE ACHILLES TENDON AND SOLEUS FROM THE APPARENT STIFFNESS OF THE TRICEPS SURAE.

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Abstract: The triceps surae plays an important role in the performance of many sports. Although the apparent average mechanical properties of the triceps surae may be a satisfactory parameter for estimating the training level of an athlete, a knowledge of the mechanical properties of the individual constituents of the triceps surae (in particular the Achilles tendon and soleus) permits a more detailed and in-depth control of the effects of training from more physically-based parameters. The objective of the present work is therefore the estimation of the individual viscoelastic properties (stiffness and viscosity) of soleus and Achilles tendon from the apparent properties of the triceps surae obtained by free vibration techniques. Different procedures have been developed and discussed, showing a high degree of robustness in the predictions. The results obtained for a non-oriented set of subjects present a high level of variability, depending on the training conditions and anthropometric features, although the corresponding average values compare well with data previously reported in the literature, particularly those associated to the tendon stiffness.

Keywords: Achilles tendon, soleus, triceps surae, stiffness, viscoelastic properties.

1. Introduction.

Free vibration techniques have been widely used for the assessment of musculo-articular properties. An excellent comprehensive review has recently been presented by Ditroilo et al [1], see also [2].

With reference to the viscoelastic properties (stiffness and viscosity) of the triceps surae (TS in what follows), París-García et al. [3] have developed and studied two devices and the corresponding methodologies to measure, in vivo, the apparent stiffness and viscosity of the triceps surae muscle-tendon complex.These two methods, based on the free vibration technique, followed earlier proposals from Fukashiro et al. [4] and Babic and Lenarcic [5], respectively.

The main objective of the study in [3] was to clarify similarities and differences between the two methods and the reproducibility, consistency and physical interpretation of the results obtained with both when applied to the same set of persons, since each approach involves a different position of the subject and consequently analyzes a different movement.

The study showed that the two methods led to similar trends in the results (e.g. the subject presenting higher values with one method also presented higher figures with the other), although the actual values obtained were clearly different. Thus, it was concluded that both methods are consistent in themselves and the values obtained are useful for comparison purposes, allowing, for instance, the effect of training on a control population to be assessed. However, a sufficiently accurate quantitative correlation between both methods was not found. The key point for these conclusions was having used the same set of subjects, and two similar equipments (in terms of devices for recording data) to apply the two methods.

Independently of the method used, only the apparent properties of the TS (the properties being represented by a single stiffness and a single viscosity) were obtained in [3]. These properties, stiffness and viscosity, are by themselves representative enough to estimate the evolution and performance of the TS, for instance, to track the training level after a period of inactivity (e.g. after surgery).

Nevertheless, knowing the actual mechanical properties of the individual constituents of the TS can provide more detailed information about the actual capacities of an athlete and his or her training level at a certain point in training or injury recovery protocol.

In the present work, of the three constituents of the TS, the gastrocnemius, the soleus and the Achilles tendon, attention will be focused on the mechanical properties of the latter two (soleus and Achilles tendon). The measurements in [3], which are the starting point for the present work, were performed using equipments in which the knee formed 90º. With such a position of the knee, the gastrocnemius is not working and consequently is not involved in the measurements obtained in the tests. Specific studies including the gastrocnemius can be found in previous studies (see for example [6]).

Different procedures have been used to separate the mechanical properties of the soleus and Achilles tendon from the global properties of the TS. Following Hill's model [7] of the TS, the soleus and the Achilles tendon can be modelled as two springs in series in the TS system as will be detailed in Section 3. Springs and dampers are frequently used in the literature to model human body kinematic [8]. Thus, a first set of approaches is based on the individual stiffness values of the two constituents under analysis. An alternative set of procedures, which has been explored for the first time, is based on the compliance values of the individual constituents. The second set of procedures, as will be seen later on, allows a linear regression to be used instead of the non-linear one necessarily associated to the procedures based on the stiffness values. In this work, it has been observed that there is a certain influence of the fitting procedure on the final results.

Although the authors have implemented procedures for the stiffness separation for both methods described in [3], for the sake of conciseness they will be applied here to only one of the methods, that used in [4]. The conclusions obtained are in any case applicable to separate viscoelastic properties of the components from the apparent values of these properties, independently of the method used to measure the apparent properties, if the same muscular model is used (see section 3).

As pointed out by Ditroilo et al [1], there is a certain number of important aspects to consider in the separation of properties, such as control of the perturbation of the amplitude, an accurate measurement of the lever arms of the feet, the number of cycles to be used in the fitting procedure, etc. All these important aspects have been analyzed herein.

In Section 2, a brief summary of the procedure developed in [3] to measure the global apparent values of the stiffness and viscosity of the TS is outlined. In Section 3 the details of Hill's model of the TS will be introduced together with some more complex alternative models. In Section 4 two sets of procedures, three based on the stiffness values and two based on the compliance values, are introduced. Finally, the results obtained in the different tests are presented in section 5, while a complete discussion with previous existing results reported in the literature is featured in section 6.

2. Methods

Different procedures have been proposed to obtain the apparent stiffness of the TS, see for example [4,5]. From that value, the results of separation procedure for obtaining the individual stiffnesses of the Achilles tendon and soleus depend only on the particular constitutive model assumed for the TS.

For the estimation of the apparent TS stiffness, the oscillating part is assumed to behave as a damped single degree of freedom (DOF) system. Under this assumption, the reaction force can be expressed by (1), being measured with a load cell.

$$F_m = e^{-\gamma t}(A_F sen\omega_D t + B_F \cos\omega_D t) + Mg \qquad (1)$$

where F_m is the measured force at the reaction point, A_F and B_F are constants related with the amplitude of the oscillation, γ is the damping coefficient, ω_D is the damped frequency, and M is the mass involved in the oscillation (g is the gravity acceleration). Details about the oscillating and static masses involved in the vibration can be found in [3]. These parameters were obtained in [3] by a least squares fitting procedure of the recorded data and allow the apparent stiffness k and viscosity c of the TS to be evaluated by means of the following relations [3,4]:

$$c = \frac{2MR^2}{r^2}\gamma \quad (2)$$

$$k = \frac{MR^2}{r^2}(\omega_D^2 + \gamma^2) \quad (3)$$

where R and r are the forefoot and rearfoot distances, respectively. These distances have to be carefully measured due to their influence (to the power of two) in equations (2,3). A particular procedure for the determination of *R* and *r*, proposed in [9], was used in [3].

Although, as mentioned, the separation process is independent of the procedure for evaluating the apparent stiffness of the TS, it was shown in [3] that the TS stiffnesses obtained for the same person using both equipments proposed in [4,5] are not comparable with each other.

Thus, although the values obtained with each of these two methodologies are representative by themselves, and similar tendencies were observed in the measurements obtained with them, comparison between the values obtained with the two methodologies is not plausible. To clarify this question was one of the main objectives in [3], since previous published results based on the two methodologies might be misleading on this question.

The separation of the global properties of the TS in the individual constituents needs a previous understanding of the TS behaviour from a modelling point of view, which will be addressed in the following section devoted to the TS model by Hill.

Let us finally stress again that in the present work the procedure suggested in [4] has been used to get the TS apparent stiffness, although the procedure for separation of individual properties of the components of the TS is applicable to values of the apparent stiffness of the TS obtained by any other procedure.

3. Relevant aspects of Hill's model and alternatives.

Despite the multiple proposals to represent the dynamics of the muscles (Winters and Woo [10]), the phenomenological model based on the original ideas by Hill [7] has historically dominated the tendon-muscle complex analysis. In addition to the reasonably satisfactory results derived from its use, its simplicity and low computational cost are positive aspects of Hill's model.

There are, basically, two alternatives emerging from Hill's ideas, which are schematically represented in Figure 1. Both models have a parallel elastic component (PEC), a serial elastic component (SEC) and a contractile component (CC), the difference between the two models being the relative position of the PEC inside the scheme. In the first model (Figure 1a) the PEC is parallel to both the CC and the SEC, whereas in the second one (Figure 1b) it is parallel only to the CC. The contribution of the parallel elastic component (PEC) can be neglected in the oscillation movement of the present study, as done in previous works (van Ingen Schenau *et al*. [11] or Fukashiro [4]), both alternatives in Figure 1 then leading to the same configuration.

Despite its popularity, in the resolution of the muscle-tendon systems incorporating Hill's model, ordinary differential equations appear in a natural way. This fact, in most cases, makes the incorporation of additional elements (springs and/or dampers) to the model a difficult task. To simplify the analysis, tools like fuzzy logic have been applied, O'Brien [12].

Other proposals, such as those based on Huxley's models (Huxley [13]), are more complex and need a higher number of model parameters. However, Bogert et al. [14] showed that, in most cases, the ability of Hill's model to predict the forces at the muscles surpasses other more physiologically based models.

Multiple proposals have tried to modify Hill's model, either altering the behaviour of some of the elements of the model or incorporating new elements to it. For example, Siebert et al. [15] analyzed the incorporation of a non-linear behaviour in the elastic components of Hill's model, which makes the two alternatives shown in Figure 1 significantly different. Gunther et al. [16] investigated the incorporation of a dissipation element in the Achilles tendon, which appears in parallel with the elastic component, finding that its presence is crucial for suppressing the natural frequencies which typically appear in the resolution of Hill's model. A very interesting work by Winters and Stark [17] shows the benefits and drawbacks of varying the complexity of muscle-tendon models, including both Hill's and Huxley's models. Also of great interest, from a mathematical point of view, are the conclusions of the study by Scovil and Ronsky [18] in which they demonstrated that Hill's models are quite sensitive to parameter perturbations, obtaining for some constitutive parameters variations in the results much higher than those in the parameter. This is of crucial importance when performing statistical robustness and stability analyses. In particular, the influence of at least 14 parameters in the sensitivity of different muscle systems based on Hill's models is analyzed in [18].

Of great importance therefore is the use of robust numerical procedures and techniques for fitting the experimental results to the model predicted response. The work by Ortiz et al. [19], published in a different context, has been used in the present study as a guide for defining robust regression techniques when fitting experimental data with model predictions.

Having a good idea of the range of values of the different parameters defining the individual constituents of the muscle tendon complex helps to better fit the experimental data to the model results. Previous works thus give experimental values for tendons. For example, Abrahams [20] analyzed the influence of the strain rate in the tendon stiffness, Fukashiro et al. [21] used ultrasonography to obtain the stiffness of the human Achilles tendon *in vivo* and Sharkey et al. [22] made an interesting proposal for a testing machine allowing the measurement of the mechanical properties of the muscle-tendon complex. Finally, Wren et al. [23] presented an extensive experimental program of tensile testing in human Achilles tendons. The range of values reported was used as a guide to start the fitting procedure, obtaining an efficient search for the optimum values of the parameters.

4. Separation of soleus and Achilles tendon mechanical properties from those of the TS.

The separation of the soleus and Achilles tendon components from the global apparent value of the stiffness can be done, following Hill's model, by means of two different approaches: i) using the stiffness values or ii) their inverse values, namely, the compliances. Both approaches will be explored in Sections 4.1 and 4.2, respectively.

4.1 Procedures for separation using stiffness values.

The relationship between the global behaviour of the MTC and its individual constituents is schematically represented in Figure 2. Following Hill's model, with the previously mentioned assumption and neglecting the PEC contribution, the model has a spring in series with a damper. The spring includes the spring associated to the elastic behaviour of the Achilles tendon and a set of springs in parallel (in series with the Achilles tendon) representing the elastic behaviour of the soleus.

While the Achilles tendon is assumed to have a constant value of the stiffness (k_t), the soleus is assumed to have a stiffness (k_m) which is proportional to the load that is being transferred by the system. Thus, the total stiffness of the soleus (k_m) can be obtained from a unitary stiffness value (k_{ss}) multiplied by the total load (f) passing through the MTC.

$$k_m = k_{ss} \cdot f \qquad (4)$$

The relationship between the equivalent global stiffness (k) and the individual stiffnesses (k_{ss} and k_t) can be easily obtained from Figure 2, with two springs in series. On one hand, for the apparent TS system the elongation and the associated force are related by the equivalent stiffness k by means of

$$u(k) = \frac{f}{k} \qquad (5)$$

On the other hand, for the components in series, both with the same force (f), the following relation applies:

$$u(k_t, k_m) = u(k_t) + u(k_m) = \frac{f}{k_t} + \frac{f}{k_m} \qquad (6)$$

Identifying displacements in (5) and (6), in order to have an equivalent behaviour, an expression of k in terms of its individual constituents (k_m and k_t) can be easily obtained, and using (4) in terms of (k_{ss} and k_t):

$$k = \frac{k_t k_m}{k_t + k_m} = \frac{k_t k_{ss} f}{k_t + k_{ss} f} \qquad (7)$$

In equation (7) k and f are considered known (see [3] for example). The unknowns in (7) are the stiffness of the Achilles tendon and the unit stiffness of the soleus, respectively k_t and k_{ss}. They will be evaluated by means of least squares fitting between experimental data and equation (7).

It is important to stress that k_t and k_{ss}, are unknowns of a different nature. While in the Achilles tendon, k_t is a real stiffness value (measured in force/length, e.g.: *kN/m*), k_{ss} represents the stiffness per unit load in the soleus (e.g.: (*kN/m*)/*kN* or simply m^{-1}). For a better physical understanding of the parameters k_t and k_{ss}, Figure 3 represents equation (7), showing the non-linear dependence of the total stiffness of the MTC (k) on the total load (f) and a saturation level for high values of f.

The stiffness per unit load in the soleus k_{ss} represents the slope of the curve at the origin,

$$k_{ss} = \lim_{f \to 0} \frac{dk(f)}{df} \qquad (8)$$

while the Achilles tendon stiffness k_t represents the horizontal asymptote of the curve for high values of the total force f.

$$k_t = \lim_{f \to \infty} k(f) \quad (9)$$

With the two elastic elements (Achilles tendon and soleus) in series, the lower stiffness is the one that controls the apparent stiffness of the TS. With low values of the total transmitted force f, the stiffness of the soleus ($k_{ss} \cdot f$) is lower than the stiffness of the Achilles tendon k_t. Thus, the total stiffness k is controlled by the stiffness of the soleus. By contrast, at higher values of f, the stiffness of the soleus ($k_{ss} \cdot f$) is much higher than the constant value of the Achilles tendon k_t. Thus, the total stiffness k is now controlled by the stiffness of the Achilles tendon k_t. Finding experimental results close to the horizontal asymptote depends on the total stiffness that can be developed by the soleus. This fact is important as an accurate determination of both values (k_t and k_{ss}) should be carried out, having experimental results for low and high values of f.

A difficulty inherent to the method now appears clearly defined, due to the need to perform tests close to the ideal conditions for determining k_{ss} (when f tends to zero) and k_t (when f tends to ∞). Basically, low weight values (5 kg) do not produce a sufficiently good quality in the oscillation while high weight values (above 35 or 40 kg) are physically difficult to maintain in the knee with the proposed configuration.

The least squares error function, *error*(k) to be minimized is therefore written as:

$$error(k) = \sum_{i=1}^{n} \left[k_{exp}(f_{exp}) - k(k_t, k_{ss}) \right]^2 \qquad (10)$$

A variation on the standard least squares procedure is the so-called trimmed least squares, in which an iterative procedure is defined by means of a least squares fitting excluding, at each iteration, the data with the worst residual (unlike in the case of the predicted analytical equation). A number of iterations around one half of the total number of data gives this method a high robustness (see Ortiz *et al* [19]).

After each iteration, a new regression line is obtained, and consequently a new list of residuals for each piece of data. The datum (only one) with the highest residual is then discarded for the next iteration.

The results of the least squares (LS) and trimmed least squares (TLS) will be shown in Section 5.1.

4.2 Procedures for separation using compliance values.

The inverse of equation (7) gives the relationship between the compliances of the TS (k^{-1}) and the compliances of the individual constituents, the soleus (k_{ss}^{-1}) and the Achilles tendon (k_t^{-1}).

$$\frac{1}{k} = \frac{k_t + k_{ss} f}{k_t k_{ss} f} = \frac{1}{k_{ss}} \frac{1}{f} + \frac{1}{k_t} \quad \Rightarrow \quad k^{-1} = k_{ss}^{-1} f^{-1} + k_t^{-1} \qquad (11)$$

The representation of equation (11) is a straight line (Figure 4) whose slope is the compliance of the soleus, per unit inverse load (f^{-1}) k_{ss}^{-1}, and the value at the origin (f^{-1}=0) is the compliance of the Achilles tendon k_t^{-1}.

Although strictly speaking, both procedures, using stiffnesses or compliances, are mathematically equivalent, the presence of a linear relation in the case of compliances allows more robust techniques to be used, specially developed for linear regression, which will be addressed in what follows.

Three different fitting procedures have been analyzed here: i) one equivalent to that used (eq.10) for stiffness values (Section 4.2.1); ii) one based on the minimization of distances (Section 4.2.2), and iii) the median-median line approach (Section 4.2.3).

4.2.1 Least squares.

The quadratic error function, *error*(k^{-1}), to minimize is then:

$$error(k^{-1}) = \sum_{i=1}^{n}\left[k_{exp}^{-1}(f_{exp}^{-1}) - k^{-1}(k_t^{-1}, k_{ss}^{-1})\right]^2 \qquad (12)$$

Results obtained using (12) will not necessarily coincide with those obtained using (10). Experimental results with the highest residuals have the highest weight in the least squares fitting procedure and, if stiffness is replaced by compliance, all variables will have the inverse numerical value and thus the residuals will change.

4.2.2 Minimum Distance.

It is well known that in the presence of outliers (an observation numerically distant from the rest of the data), using an error function based on the minimum absolute residual gives a better estimation than using the quadratic residual. In that case, an alternative to equation (12) is the error function "$error2(k^{-1})$" expressed as:

$$error2(k^{-1}) = \sum_{i=1}^{n}\left|k_{exp}^{-1}(f_{exp}^{-1}) - k^{-1}(k_t^{-1}, k_{ss}^{-1})\right| \qquad (13)$$

4.2.3 Median-median line

The robustness of the median as an estimator in the presence of outliers, Beaton and Tukey [24], has also led to an easier alternative to linear regression. This proposal divides the set of data in the (k^{-1}, f^{-1}) space into nine regions, which result from the intersection of the three subsets, defined along each axis, containing the same number of data.

The medians of the subsets with the lowest and highest values of both axes define two points respectively: (f_1^{-1}, k_1^{-1}) and (f_2^{-1}, k_2^{-1}), and the following straight line in the (k^{-1}, f^{-1}) space:

$$k^{-1}(f^{-1}) = \left(\frac{k_2^{-1} - k_1^{-1}}{f_2^{-1} - f_1^{-1}}\right) f^{-1} + \left(\frac{k_1^{-1} f_2^{-1} - k_2^{-1} f_1^{-1}}{f_2^{-1} - f_1^{-1}}\right) \qquad (14)$$

The drawback of this procedure is that one third of the values are not taken into account in the process. In the particular case under analysis, the excluded data may be those with the best quality, as mentioned in [3], due to the fact that they have been obtained with intermediate values of the weight. Different problems were reported in [25] (and also in Section 4.1) for tests with the highest and lowest weights.

5. Results.

Following the previous paragraph, Section 5.1 summarizes the results obtained using the stiffness values while subsection 5.2 presents the results obtained using the alternative methods based on compliances. The least squares fitting have been implemented in a program using *Mathematica* [26]

For all approaches, using a set of data $\{(x_1,y_1), (x_2,y_2),... (x_i,y_i),... (x_N,y_N)\}$, to be fitted to a generic function $y^*(x)$, the regression coefficient R^2 ($0<R^2<1$) is defined as:

$$R^2 = 1 - \frac{\sum_{i=1}^{N}\left(y_i - y^*(x_i)\right)^2}{\sum_{i=1}^{N}\left(y_i - \bar{y}_i\right)^2} \qquad (15)$$

5.1 Results using stiffness values.

The starting point for the separation of Achilles tendon and soleus properties from those of the TS is a set of experimental list of data (f, k), where f is the force passing through the TS and k is the total apparent stiffness of the TS. These experimental results have been obtained using the methodology developed by the same authors in [3] in which the degree of freedom corresponds to the vertical displacement of the lower leg. An example (for one subject) of the experimental data (f, k) which will be used in this paper to apply the separation procedures is shown in Table 1, with a set of data having the force f (N) and the apparent stiffness k (kN/m) values, respectively.

The Achilles tendon is assumed to behave as a single linear elastic spring. Then, considering that the gastrocnemius does not play any relevant role in the particular test under analysis (due to the 90º angle of the knee during the test), the only significant contribution to the total apparent viscosity of the TS comes from the soleus.

Figure 5 shows the experimental data from Table 1 together with the curve determined by equation (7) in which k_t and k_{ss} have been obtained by a least squares fitting procedure, using (10).

The experimental methodologies developed in [3], from which data in Table 1 have been obtained, need to use different weights (which are added to the weight of the part of the body under oscillation) to increase the force passing through the TS. Remember that following Hill's model, the stiffness of the soleus is proportional to the load passing through it, unlike the stiffness of the Achilles tendon, which can be considered to be almost constant. Thus, it is necessary to apply different weights to discriminate both stiffnesses.

The level of repeatability of the results obtained by a certain value of the weight applied was really satisfactory. Thus, the values of k for each weight appear almost in a vertical line (the results corresponding to a 15 kg weight have been circled as an indication) and they are not too different from each other.

Two curves, one using the information from all tests and the other using only mean values for each weight, have been represented in figure 5. As can be observed, both curves are almost coincident but with a difference in the regression coefficient R^2, which is 0.86 for the curve fitted with all individual values and 0.95 for the curve using the mean values. This fact shows that although there might be a certain dispersion of the results for each constant weight value, as in the case selected, the set of mean values per weight accurately fits the assumed model in equation (7) with a regression coefficient R^2 close to 1.

From the least squares fitting procedure using all individual test data in Table 1, the following values of k_t and k_{ss} were obtained: k_t=376.4 kN/m and k_{ss}=489.2 (kN/m)/kN. When the mean values were used in the fitting process instead of individual values, the results changed slightly to: k_t=391 kN/m and k_{ss}=467 (kN/m)/kN, with a 3.9% and 4.5% difference, respectively.

It is important to notice that each experimental datum (f, k) is the result of a previous fitting procedure between the experimentally recorded force *vs* time curve and the assumed oscillation model, see [3] for further information. This fact makes the actual fitting procedure accumulate the uncertainties of the previous fitting step. Thus, the obtained regression coefficients R^2 may be considered to be very satisfactory.

The trimmed least squares procedure previously defined was also applied to these stiffness values. 16 points were iteratively eliminated from the original data set, evaluating at each iteration a new fitting curve and a new list of residuals to eliminate the following point.

Figure 6 shows the values of k_t and k_{ss} for each iteration, together with the value of R^2. After each iteration, the regression factor R^2 increases (with the exception of iteration 3), moving from an original value of R^2=0.865 with the complete set of data to R^2=0.993 for the last (16th) iteration. Figure 7 shows the first (Figure 7a) and 16th (Figure 7b) iteration results graphically. It is noteworthy that only in the first 5 or 6 iterations are there significant changes in the values of k_t and k_{ss}, coinciding with the elimination of the major outliers, after which the values remain reasonably constant.

5.2 Results using compliance values.

The starting point for the separation of Achilles tendon and soleus properties from those of the TS is now the inverse values of the set of data shown in Table 1, f^{-1} (N^{-1}) and the compliance k^{-1} (m/kN).

As stated in Section 4.2, the graphical representation of the data in Table 2 in the space (f^{-1}, k^{-1}) can now be fitted using linear regression.

Figure 8 shows the data associated to compliance values (the inverse data of Table 1) together with the linear fitting. The regression coefficient obtained using all individual values was R^2=0.84, which is almost equal to that obtained in the case of fitting the individual stiffness data (R^2=0.86, see Figure 5). When mean values were used for the linear regression, instead of individual values, the resulting regression coefficient was R^2=0.95.

From Figure 8, when using individual values for the linear regression, the compliance of the Achilles tendon can be evaluated as the value at the origin, which leads to k_t^{-1}=0.002412 m/kN. The value of the compliance of the soleus per unit inverse load can be estimated from the slope of the fitted line, which leads to k_{ss}^{-1}=2.2689 (m·N)/kN. The values obtained in the case where mean values are used are, respectively, k_t^{-1}=0.002295 m/kN and k_{ss}^{-1}=2.3742 (m·N)/kN, which are 4.8% and 5.9% different, respectively, from those obtained considering all points and not just mean values.

The inverse of these values (k_t^{-1} and k_{ss}^{-1}) gives rise, respectively, to the stiffness of the Achilles tendon and the stiffness of the soleus per unit load, these values being, when the individual values are used, $k_t = 414.6$ kN/m and $k_{ss} = 0.4407$ kN/(m·N)=440.7 kN/(m·kN). When using mean values the results are $k_t = 435.7$ kN/m and $k_{ss} = 0.4212$ kN/(m·N)=421.2 kN/(m·kN)

Finally, Achilles tendon and soleus compliances were evaluated using the median-median line. Figure 9 shows the nine quadrants (red dashed lines) in which compliance data are equally divided. The medians of the two extreme quadrants define the line which gives the compliance values of the Achilles tendon k_t^{-1}=0.003197 m/kN and soleus k_{ss}^{-1}=1.6524 (m·N)/kN. The corresponding stiffnesses are $k_t = 312.8$ kN/m and $k_{ss} = 605.2$ kN/(m·kN).

6 Discussion.

Table 2 summarizes the mean values of k_t and k_{ss} obtained by means of all proposed fitting procedures for the right and left legs and for all tested subjects. It becomes clear, from data in Table 2, that the median-median line fitting procedure yields different results than the other proposed procedures. The reasons for this different behaviour were outlined at the end of Section 4. In Table 2, the global mean values and standard deviations of k_t and k_{ss} have been calculated both including this procedure (the row with the superscript 1-5) and not including it (the row with the superscript 1-4). The results show a significant decrement in the standard deviation, without a significant variation in the mean value, when excluding the median-median line procedure. Therefore, once the results obtained with the median-median line procedure are discarded, the rest of the results are quite similar, showing a satisfactory robustness and consistency.

For more detailed information, figures 10 and 11 show, respectively, the values of k_t and k_{ss} for both legs of all tested subjects. The bars denote the mean values and the lines the ranges (min-max) obtained for each subject using all previously introduced fitting procedures (excluding the mean-mean line, for the reasons mentioned above).

The first observation for all plots in figures 10 and 11 is that the definition of a mean value of k_t or k_{ss} for an unrelated set of people has a limited representativity, due to the high values of the standard deviation obtained (values of the standard deviation included in the figures). It is clear, from figures 10 and 11, that each subject has different values of k_t and k_{ss} due to their different sex, age, weight, height, training level, etc. It is also observable in figures 10 and 11 that the ranges of variation, using any of the proposed fitting procedures, both for k_t and k_{ss}, are very low in comparison with their mean values. Subjects with different mean values do not share values in their range (min-max) of measured values.

Figures 10 and 11 also show two important facts. The first one is that values for both legs are not equal, this fact being clearly associated to the declared laterality of each subject, which makes both legs not physiologically equal. The second one is that, although not equal, at least the trend is similar for both legs. Those subjects having the highest values of k_t or k_{ss} in one leg (in comparison with the rest of the subjects) have also the highest values of k_t or k_{ss} in the other leg (also in comparison with the other subjects).

In Table 2 the results reported in the literature for k_t and k_{ss} (in Refs.[4,5]) have also been included for the sake of completeness. Although, as mentioned previously in this Section, the determination of a mean value of an unrelated set of people might be a meaningless parameter, in the sense already explained, the lack of information on the individual values of the results reported in the literature leaves the comparison of mean values as the only possibility. Even more, while [5] distinguishes between the values of k_t and k_{ss} for both legs, [4] reports one single value of k_t and k_{ss}. It is also remarkable, as pointed out in [3], that the methodologies proposed in [4] and [5] should yield different results for the same parameter (k_t or k_{ss}) if they are applied to the same subject, as done in [3]. As the data used in the present work are based on measurements obtained using a methodology equal to that proposed in [4], more confident comparisons should be made with results obtained in [4].

With all these previous comments in mind, limiting the representativity of mean values, Table 2 shows for the set of 10 subjects used in the present work a global mean value of k_t=382 kN/m (right leg) and k_t=366 kN/m (left leg), while the result reported in [4] for both legs is k_t=364 kN/m, which is quite similar. In contrast, values for k_{ss} obtained in the present work are k_{ss}=452 (kN/m)/kN (right leg) and k_{ss}=487 (kN/m)/kN (left leg) which significantly differ from the k_{ss}=611 (kN/m)/kN reported in [4].

Results in [5] using a different methodology from that used in the present work (and in [4]), and obviously applied to a different set of people, are very similar for k_t or k_{ss} to those reported in [4] (see Table 2). Thus, to summarize, the results in terms of mean values obtained in the present work are of the same order in the values of k_t and clearly differ in the values of k_{ss}.

An explanation for this difference may be associated to the training level of the subjects. Of the ten tested subjects in the present study, subjects 7 and 8 had the best training level of all. Subjects 7 and 8 gave the highest values of k_{ss} for both legs, around 600 (kN/m)/kN, the other subjects being much more sedentary and giving lower values. Ref [5] reports a mean k_{ss} value of 665 (kN/m)/kN and 669 (kN/m)/kN for the right and left legs respectively, and these results are associated to a set of ten trained male subjects. This observation is connected to the known fact that the stiffness of the soleus is more sensitive to the training status of the person than the stiffness of the Achilles tendon.

7. Conclusions

Several procedures have been developed for the evaluation of the stiffness of the Achilles tendon and the stiffness per unit load of the soleus. The evaluation of these stiffness values is based on the previous knowledge of the equivalent global stiffness properties of the triceps surae muscle-tendon-complex [3].

Knowledge of the stiffness properties of these two individual constituents of the triceps surae (the gastrocnemius not being involved in the oscillation due to the 90º position of the knee in the test) allows many questions, such as tracking the training level after a period of inactivity (after surgery or injury), the efficiency of a particular strategy of training in the improvement of the triceps-surae properties, etc., to be clarified.

The values of stiffness of the Achilles tendon and the soleus have been evaluated by means of different fitting procedures for the apparent data (stiffnesses or compliances) of the triceps surae. The fitting procedures have proved to be very robust in the determination of the initial slope k_{ss} (the stiffness per unit load of the soleus) and the horizontal asymptote k_t (the stiffness of the Achilles tendon). Only one procedure, the median-median line was shown to be inaccurate, due to the nature of the procedure which discards, in this particular case, the best quality data.

Comprehensive data have been reported for individual subjects, each leg and different fitting procedures, which might help researchers to use them as benchmark data.

The results obtained in the present work have been compared with others in the literature (mean values only) and details about the representativity and comparability of these results have been discussed. The large intervals found for the stiffnesses of the Achilles tendon and the soleus give only a limited representativity to these mean values. In any case, the mean value found for the Achilles tendon stiffness turned out to be very similar to others presented in the literature. On the other hand, the soleus stiffness is very much affected by the training level, so that only people with similar training status should be compared.

With the present and previous works of the authors, it has been shown that results for the individual constituents of the triceps surae are influenced by several different aspects which have to be taken into account, such as the methodology for obtaining the apparent mechanical properties of the triceps surae [3] or the measurement of the lever arms of the foot and the fitting procedure itself [9].

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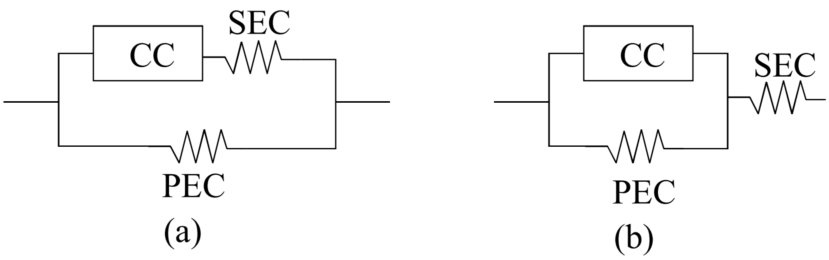

Figure 1. Schemes of two alternatives of Hill's model.

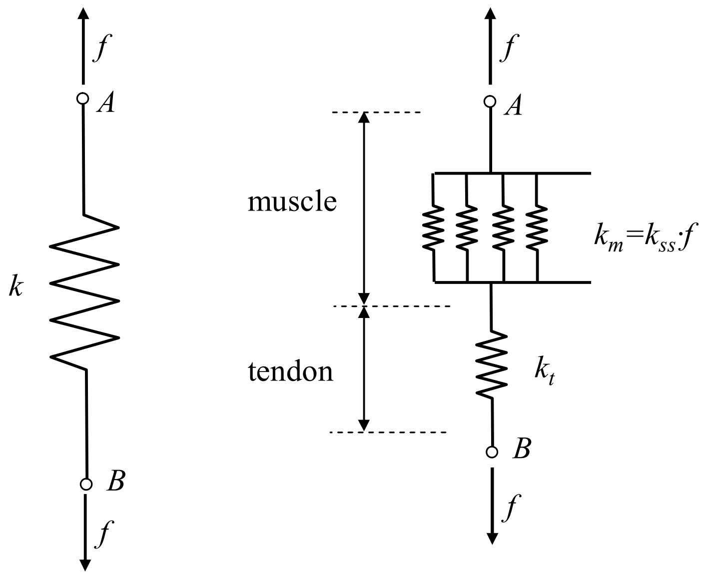

Figure 2. The Triceps Surae scheme a) apparent behaviour and b) individual constituents.

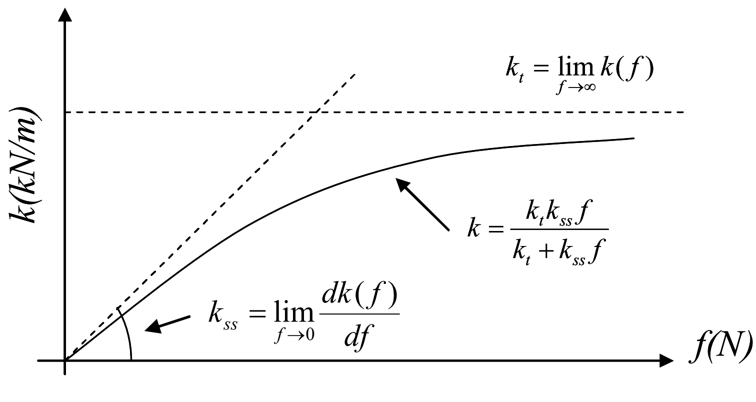

Figure 3. Graphical representation of equation (4) and meaning of k_{ss} and k_t.

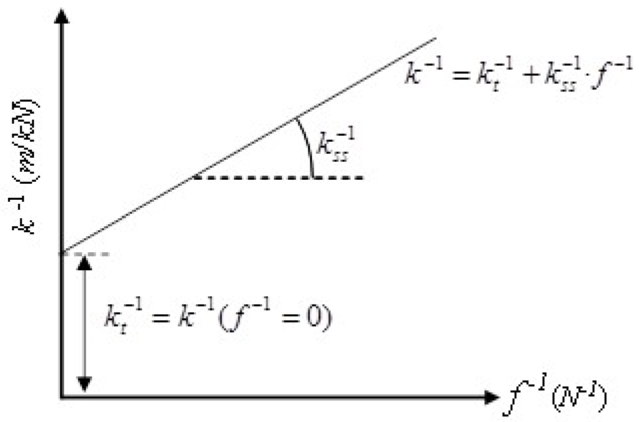

Figure 4. Graphical representation of equation (8) and meaning of k_{ss}^{-1} and k_t^{-1}.

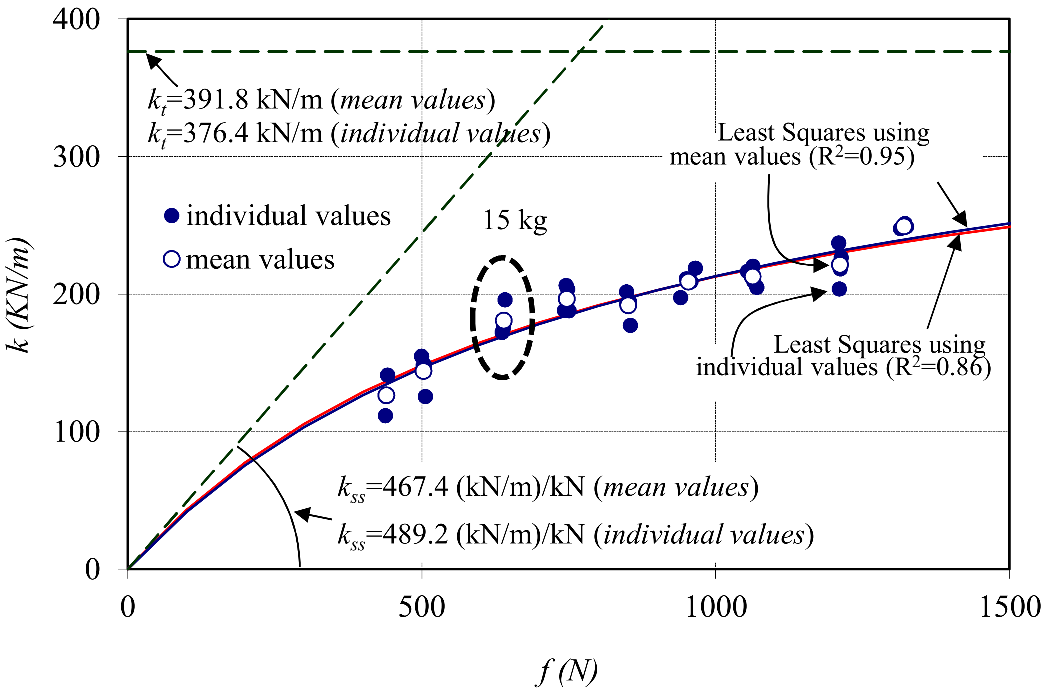

Figure 5. Stiffness (k) vs force (f) in the TS and results of Least Squares fitting of the Achilles tendon stiffness (k_t) and soleus stiffness per unit load (k_{ss}).

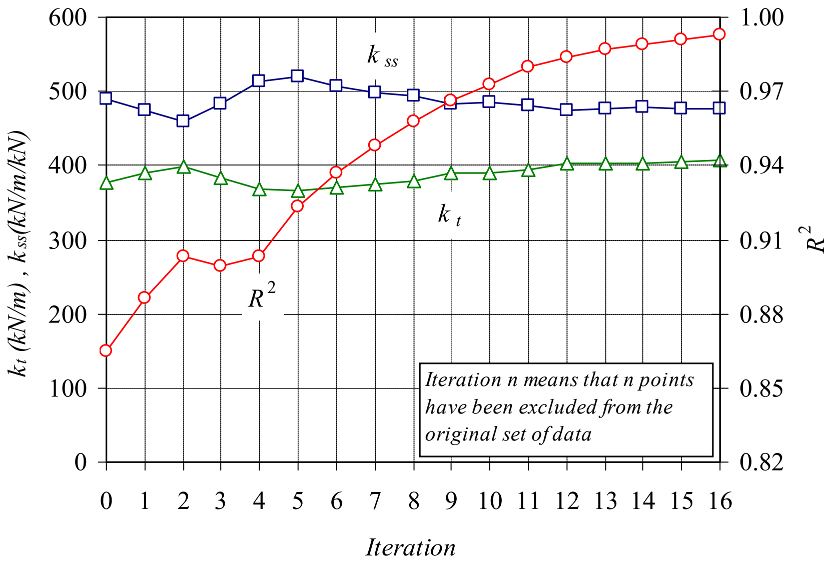

Figure 6. Results of k_t, k_{ss} and R^2 for the Trimmed Least Squares procedure.

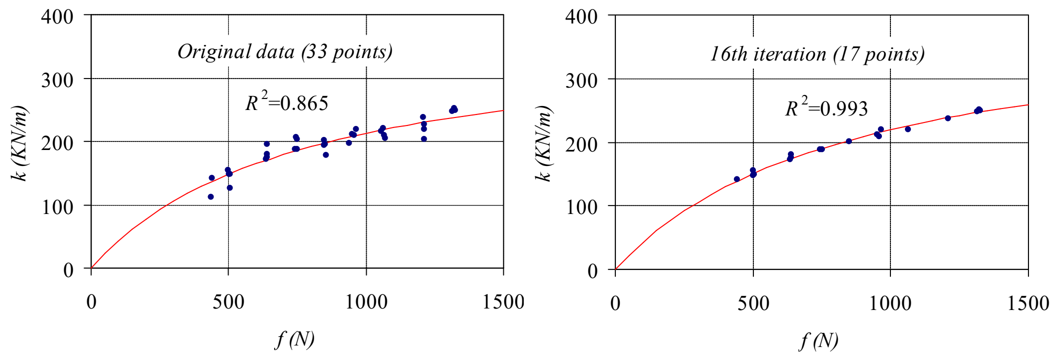

Figure 7. Fitting of experimental results in the a) first and b) 16th iteration of the TLS procedure.

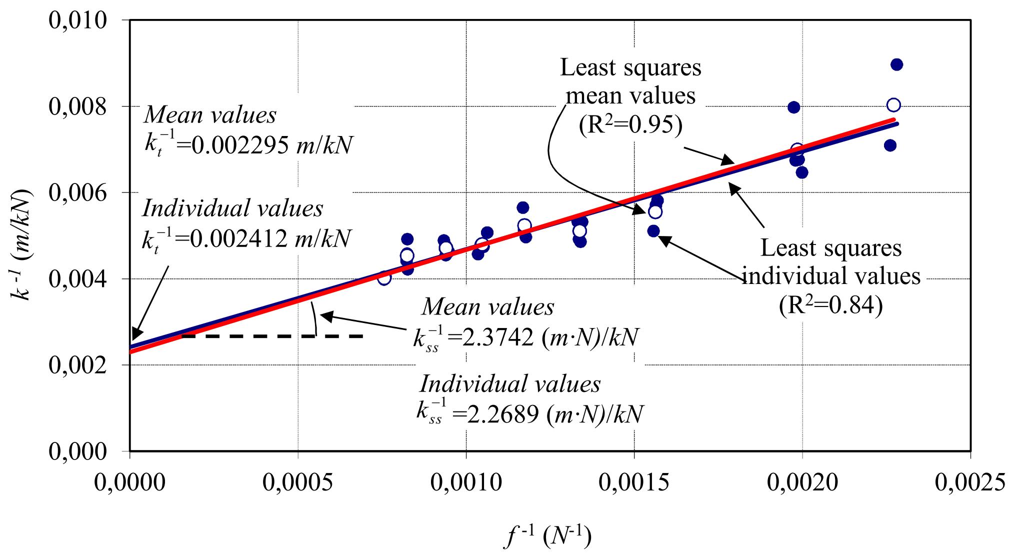

Figure 8. Least Squares fitting of the Achilles tendon compliance (k_t^{-1}) and soleus compliance per unit load (k_{ss}^{-1}).

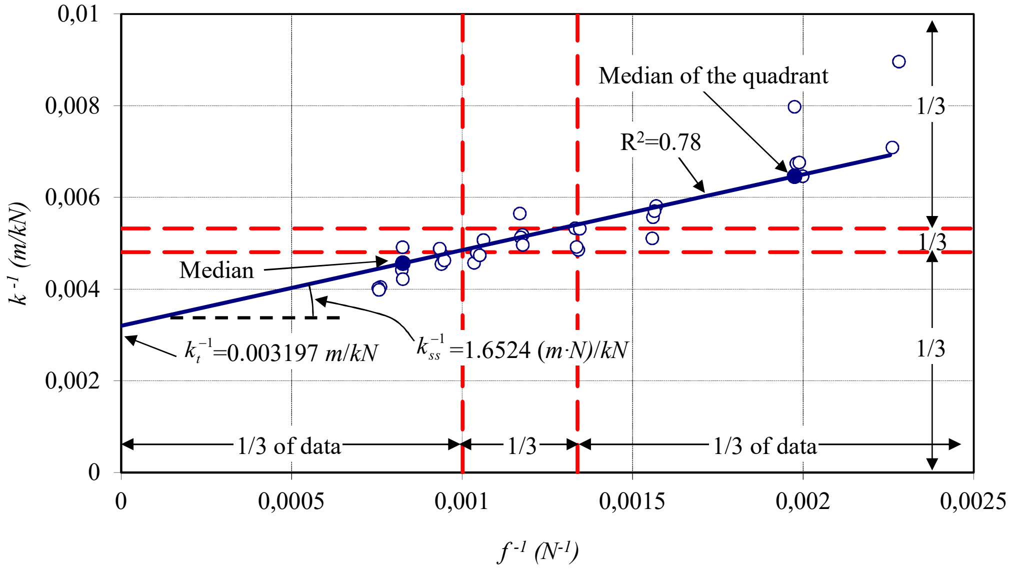

Figure 9. Achilles tendon compliance (k_t^{-1}) and soleus compliance per unit load (k_{ss}^{-1}) using the median-median line.

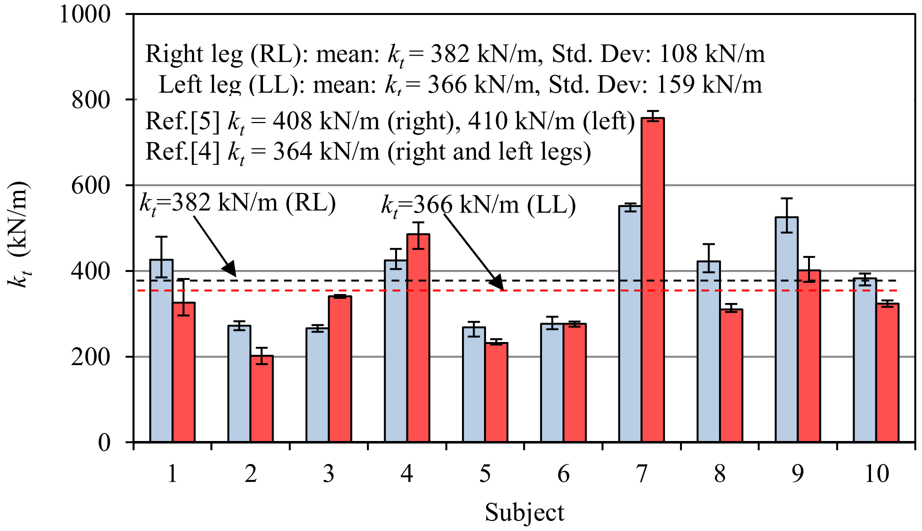

Figure 10. Stiffness of the tendon, k_t (kN/m) for all subjects (the range of values shown for each subject excludes the median-median line result) and both legs.

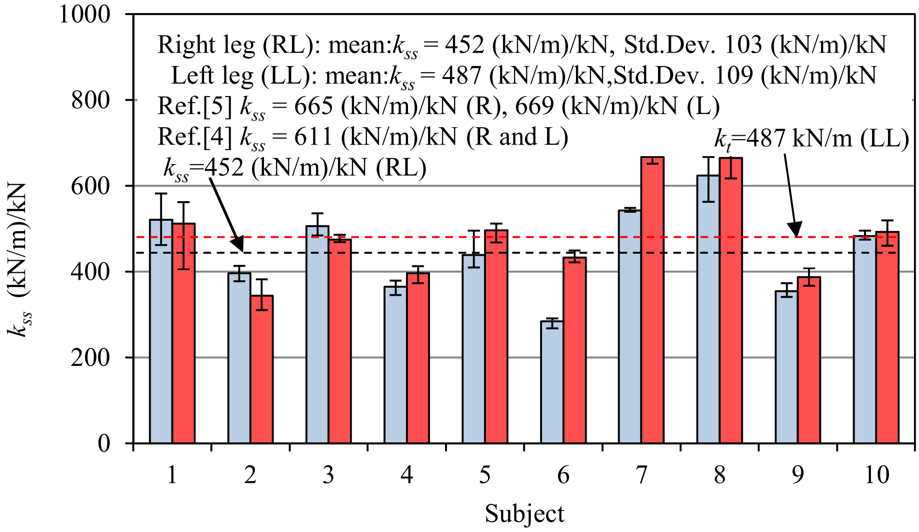

Figure 11. Stiffness per unit load of the soleus, k_{ss} (kN/m)/kN for all subjects (the range of values shown for each subject excludes the median-median line result) and both legs.

Table 1. Set of values (*f*, *k*) corresponding to a subject, used to apply the separation procedures.

f (N)	*k* (kN/m)	*f* (N)	*k* (kN/m)	*f* (N)	*k* (kN/m)
438.283	111.601	743.257	188.133	1063.99	220.050
442.067	141.105	745.767	206.140	1064.97	209.539
504.868	148.384	748.536	203.454	1054.45	216.375
502.694	147.936	849.429	193.141	1070.34	204.849
506.307	125.429	855.161	177.188	1210.85	203.584
500.163	154.790	852.990	195.085	1212.17	218.368
640.874	179.787	848.855	201.541	1214.16	226.406
641.870	195.869	965.603	218.838	1209.82	237.161
637.199	172.084	958.124	208.929	1314.83	247.453
639.068	175.483	950.517	211.187	1325.13	248.985
750.803	187.828	940.687	197.373	1322.24	251.336

Table 2. Comparison of k_t and k_{ss} values with previous results.

	Left leg		Right leg	
Fitting procedure	k_t (kN/m)	k_{ss} (kN/m)/kN	k_t (kN/m)	k_{ss} (kN/m)/kN
Present work(1)	376	475	386	443
Present work(2)	360	495	382	453
Present work(3)	369	483	376	461
Present work(4)	358	495	385	449
Present work(5)	424	443	408	427
Mean / St.Dev. (1-5)	377/27.1	478/21.5	387/12.1	447/12.9
Mean / St.Dev. (1-4)	366/8.1	487/9.6	382/4.4	452/7.4

Ref. [4] (6)	k_t=364 kN/m, k_{ss}=611(kN/m)/kN (mean of both legs)			
Ref. [5] (7)	410	669	408	665

(1) Mean value of 10 subjects using the least squares procedure with stiffness.
(2) Mean value of 10 subjects using the least squares procedure with compliances.
(3) Mean value of 10 subjects using the trimmed least squares procedure with compliances.
(4) Mean value of 10 subjects using the minimum distance procedure with compliances.
(5) Mean value of 10 subjects using the median-median line procedure with compliances.
(6) Mean value of 6 subjects and left/right legs.
(7) Mean value of the left leg for 10 trained male subjects.