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Tube-based coalitional MPC with plug-and-play features ⋆

E. Masero *, P. R. Baldivieso-Monasterios **, J. M. Maestre *,***, P. A. Trodden **, E. F. Camacho *

* *Dept. of Ingeniería de Sistemas y Automática, Universidad de Sevilla, Spain (e-mails: {evamasero, pepemaestre, efcamacho}@us.es)*
** *Dept. of Automatic Control and Systems Engineering, University of Sheffield, UK (e-mails: {p.baldivieso, p.trodden}@sheffield.ac.uk)*
*** *Dept. of Systems Science, Graduate School of Informatics, Kyoto University, Japan (e-mail: pepemaestre@sys.i.kyoto-u.ac.jp)*

Abstract: This paper presents a distributed setting of model predictive control (MPC) to manage linear multi-agent systems consisting of coupled subsystems. Specifically, local controllers can work in coalitions to improve performance and handle plug-and-play events. This study provides insight into a coalitional MPC strategy based on optimized tubes that handles plug-in and plug-out subsystems. Moreover, we explore an inherent robustness gap to absorb disturbances not covered by the tubes without having to group local controllers. A comparison of our approach with centralized and decentralized MPC is reported using an illustrative example.

Keywords: Model predictive control, distributed control, multi-agent systems, time-varying systems, robust control, plug-and-play events.

1. INTRODUCTION

Model predictive control (MPC) is a control strategy that optimizes control actions in a finite horizon according to an objective function (Maciejowski, 2002). The ability to handle constraints, uncertainties, and multi-variable systems has turned MPC into a successful control strategy, leading to its implementation in distributed multi-agent systems (Maestre and Negenborn, 2014). Distributed MPC techniques involve partitioning the system into subsystems that are managed by local MPC controllers, giving rise to the so-called agents. According to a communication control network, agents share information to improve its local cost while contributing to the global objective. A dynamic control topology that allows enabling and disabling communication links in real-time is the basis for coalitional MPC methods (Fele et al., 2017). Specifically, coalitional control allows local controllers form *coalitions* to find a balance between performance and communication effort. Depending on the way coalitions are selected, coalitional schemes can be classified into: i) *top-down* architectures, where a supervisory layer decides the cooperation topology, and ii) *bottom-up* structures, in which coalitions are formed at the agent level without global knowledge (Baldivieso-Monasterios and Trodden, 2021). The current work follows a top-down coalitional setting of model predictive controllers.

One of the main challenges in coalitional control and other distributed settings is dealing with disturbances caused by the dynamic coupling between subsystems. In this regard, robust control techniques are employed when designing controllers. An approach is to consider couplings as bounded additive uncertainties to ensure stability and suitable global performance (Richards and How, 2007). The most conservative fashion for modeling the presence of uncertainties is min-max MPC (Scokaert and Mayne, 1998), which minimizes the control input over the worst-case of the disturbances. The idea of tubes proposed in (Langson et al., 2004) has also become popular for ensuring robust stability for constrained linear systems (Mayne et al., 2005; Trodden and Richards, 2010). Nevertheless, a significant downside is incurred by tightening local constraint sets by margins that can conservatively go beyond the disturbances that subsystems will experience. To reduce conservatism, further tube-based methods have been explored. Raković et al. (2012) focus on optimizing the diameters of the state and control tubes online, Riverso and Ferrari-Trecate (2012) propose to apply tube-based control twice to exploit the region of attraction of the subsystem for the planned state trajectories of neighbors, Lucia et al. (2015) develop a contract-based scheme in which subsystems share to their neighbors a sequence of the coupling variables that will hold in the future, and Trodden and Maestre (2017) propose to reject dynamic couplings via optimized tubes to outer-bound their disturbance sets more accurately. A similar idea to the latter is here followed, but upgraded to a coalitional setting.

Many control approaches are limited in their ability to adapt to physical changes in a system. However, *plug-and-play* (PnP) control describes the control system's ability to

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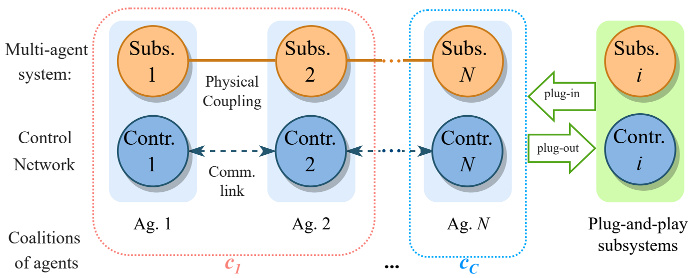

Fig. 1. Proposed scheme with plug-and-play features for coupled subsystems, where there is a set of coalitions $(c_1, c_2, \ldots, c_C)$ of local controllers.

automatically reconfigure when a new component is added to or removed from the system (Stoustrup, 2009). Multiple interpretations arise around the term plug-and-play. In fault-tolerant control, it is used for automatic recovery of the control objective after a failure (Bodenburg et al., 2014). In micro-grids applications, PnP operations are allowed to adapt control actions in real-time according to varying network conditions (Dörfler et al., 2014), and are more related to the concept of neutral interactions among the distributed generation units Riverso et al. (2014b), rather than on robustness against subsystem couplings. In this latter context, which is more relevant to the topic of this work, Riverso et al. (2014a) and Lucia et al. (2015) propose control strategies capable of handling PnP subsystems offline, provided that feasibility and stability is not compromised. Otherwise, PnP operations are rejected to ensure overall system's recursive feasibility and stability.

This work presents a tube-based coalitional model predictive control scheme with PnP features for linear multi-agent systems (see Fig. 1). We consider that subsystems can join or leave the system in real time, and switching dynamics may be introduced. The formation of coalitions allows for automatic reconfiguration of the control system and avoids the rejection of PnP operations, unlike the studies mentioned above. The set of coalitions is selected in real time to accommodate disturbances and find a trade-off between performance and cooperation burden. Similarly to (Trodden and Maestre, 2017), agents can scale down their constraint sets and share their scaling factors among neighbors to reconfigure the disturbance sets. Our approach goes a step further in exploring an inherent robustness gap to cover disturbances not considered by tubes (*e.g.*, PnP events) and avoid regrouping agents. The robustness gap is created by having each agent employ two different scaling factors: a public one that is implemented and broadcast to the neighborhood, and a private one that contains confidential information. Finally, a case study demonstrates the benefits of this coalitional approach.

The paper is structured as follow: §2 introduces the problem setting, §3 describes the tube-based coalitional MPC approach, §4 presents the control algorithm, §5 details PnP operations, §6 reports results on a case study, and §7 provides the conclusions.

Notation: Sets $\mathbb{R}_{0+}$ and $\mathbb{R}_+$ ($\mathbb{N}_{0+}$ and $\mathbb{N}_+$) refers to non-negative and positive real numbers (integers). For two sets $\mathcal{X}, \mathcal{Y} \subseteq \mathbb{R}^n$, the Cartesian product is $\mathcal{X} \times \mathcal{Y} \triangleq \{(x, y) : x \in \mathcal{X},\ y \in \mathcal{Y}\}$. If $\{\mathcal{X}_i\}_{i \in \mathcal{N}}$ is a finite family of sets indexed by $\mathcal{N} = \{1, \ldots, N\}$, then the Cartesian product $\times_{i \in \mathcal{N}} \mathcal{X}_i$ is defined as $\mathcal{X}_1 \times \cdots \times \mathcal{X}_N = \{(x_1, \ldots, x_N) : x_1 \in \mathcal{X}_1, \ldots, x_N \in \mathcal{X}_N\}$. The image of a set $\mathcal{X} \subseteq \mathbb{R}^n$ under a linear mapping $A : \mathbb{R}^n \mapsto \mathbb{R}^m$ is given by $A\mathcal{X} \triangleq \{Ax : x \in \mathcal{X}\}$. For sets $\mathcal{X}, \mathcal{Y} \subseteq \mathbb{R}^n$, the Minkowski sum is $\mathcal{X} \oplus \mathcal{Y} \triangleq \{x + y : x \in \mathcal{X},\ y \in \mathcal{Y}\}$, and the Pontryagin difference is $\mathcal{X} \ominus \mathcal{Y} \triangleq \{z \in \mathbb{R}^n : \mathcal{Y} \oplus \{z\} \subseteq \mathcal{X}\}$ for $\mathcal{Y} \subseteq \mathcal{X}$. $||x||_a$ represents l_a-norm of the vector $x \in \mathbb{R}^n$ with $a \in \mathbb{N}_+$, and $||x||_Q^2 = x^\top Q x$, with Q being a weight matrix.

2. PROBLEM SETTINGS

This section provides an overview of the dynamics and constraints of the subsystems, defines the coalitional structure given by the cooperation topology of the control network, and also details the coalitions' dynamics and constraints.

2.1 Subsystem dynamics and constraints

The system is composed of $\mathcal{N} = \{1, 2, \ldots, N\}$ dynamically coupled subsystems i, whose discrete-time dynamics are:

$$\begin{aligned} x_i(k+1) &= A_{ii} x_i(k) + B_{ii} u_i(k) + w_i(k), \\ w_i(k) &= \sum_{j \in \mathcal{M}_i} \big(A_{ij} x_j(k) + B_{ij} u_j(k)\big) + w_i^{\mathrm{e}}(k), \end{aligned} \tag{1}$$

where $k \in \mathbb{N}_+$ is the discrete-time index, $x_i \in \mathbb{R}^{q_i}$ and $u_i \in \mathbb{R}^{r_i}$ are the state and input vectors, respectively, subject to constraints $\mathcal{X}_i$ and $\mathcal{U}_i$, $w_i \in \mathbb{R}^{q_i}$ is the coupling through states and inputs with neighbors $j \in \mathcal{M}_i \triangleq \{j \in \mathcal{N} \backslash \{i\} : A_{ij} \neq 0 \vee B_{ij} \neq 0\}$ with $A_{ij} \in \mathbb{R}^{q_i \times q_j}$, $B_{ij} \in \mathbb{R}^{q_i \times r_j}$, and the external noise w_i^{e} is assumed to be bounded by $\mathcal{W}_i^{\mathrm{e}}$.

Assumption 1. $\mathcal{X}_i, \mathcal{W}_i^{\mathrm{e}} \subset \mathbb{R}^{q_i}$ and $\mathcal{U}_i \subset \mathbb{R}^{r_i}$ are compact convex sets that contain the origin in their interiors.

2.2 Control network and cooperation topology

Each subsystem i is managed by an agent that has access to local information and can coalesce with its neighbors to improve performance and handle unplanned disturbances such as plug-and-play events. Let us describe the control network by an undirected graph $(\mathcal{N}, \mathcal{L})$ with $\mathcal{N}$ agents and $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$ cooperation links. Each link $l_{ij} = \{i, j\} = \{j, i\} = l_{ji} \in \mathcal{L}$, which connects agents i and j, is assumed to provide a bidirectional information flow that can be enabled or disabled by the control scheme. Each enabled link involves a fixed cooperation cost $\mathrm{c}_{\text{link}} \in \mathbb{R}_+$. The set of active links at time instant k defines the cooperation topology $\Lambda \subseteq \mathcal{L}$, which is determined by a balance between performance and cooperation costs. Being $|\mathcal{L}|$ total number of links, there are $2^{|\mathcal{L}|}$ different topologies that are grouped into a set $\mathcal{T} = \{\Lambda_1, \Lambda_2, \ldots, \Lambda_{2^{|\mathcal{L}|}}\}$. For convenience, Λ_1 is the decentralized topology with all links disabled: $\{0, 0, 0\}$, and the centralized topology $\Lambda_{2^{|\mathcal{L}|}}$ denotes full network cooperation: $\{1, 1, 1\}$. The cardinality of Λ is denoted by $|\Lambda|$ and provides the number of active links in the topology.

Definition 2. Given a current topology Λ_{cur}, the set of the potential successor topologies $\mathcal{T}_{\text{new}} \subseteq \mathcal{T}$ is defined as:

$$\mathcal{T}_{\text{new}} \triangleq \{\Lambda \in \mathcal{T} : \operatorname{dist}(\Lambda_{\text{cur}}, \Lambda) \leqslant 1\} \subseteq \mathcal{T}, \tag{2}$$

where $\operatorname{dist}(\Lambda_{\text{cur}}, \Lambda)$ denotes the Hamming distance between two topologies.

2.3 Coalition dynamics and constraints

The cooperation topology defines the coalitional structure of the control network as follows.

Definition 3. (Coalitional structure). A cooperation topology Λ partitions the system $\mathcal{N} = \{1, \ldots, N\}$ into a set of non-overlapping coalitions $\mathcal{C} = \{c_1, \ldots, c_C\}$ with $C \leqslant N$, which cover all subsystems: $\bigcup_{c\in\mathcal{C}} c = \mathcal{N}$. Each coalition $c \in \mathcal{C}$ is a non-empty cluster of subsystems that range from the grand coalition $c = \mathcal{N}$ to a single subsystem $c = i$.

The discrete-time dynamics of each coalition $c \in \mathcal{C}$ is

$$\begin{aligned} x_c(k+1) &= A_{cc}x_c(k) + B_{cc}u_c(k) + w_c(k), \\ w_c(k) &= \sum_{d\in\mathcal{M}_c} \big(A_{cd}x_d(k) + B_{cd}u_d(k)\big) + w_c^{\mathrm{e}}(k), \end{aligned} \tag{3}$$

where $x_c = (x_i)_{i\in c}$ and $u_c = (u_i)_{i\in c}$ are, respectively, aggregate state and input vectors, $A_{cc} = [A_{ij}]_{i,j\in c}$ and $B_{cc} = [B_{ij}]_{i,j\in c}$ are state and input matrices, and disturbance term w_c represents coupling with neighbors plus external noise w_c^{e}. The neighbors of coalition c are gathered in set $\mathcal{M}_c \triangleq \{d \in \mathcal{C}\backslash c : A_{cd} \neq 0 \vee B_{cd} \neq 0\}$.

Assumption 4. State and input constraint sets of a coalition are $\mathcal{X}_c = \times_{i\in c} \mathcal{X}_i$ and $\mathcal{U}_c = \times_{i\in c} \mathcal{U}_i$, respectively.

3. TUBE-BASED COALITIONAL MPC

Given a coalitional structure $\mathcal{C}$, we consider that coalitions can scale down their constraint sets and reconfigure disturbance sets to reduce conservatism of tube-based approaches. The constraint sets of coalition $c \in \mathcal{C}$ are scaled by factors $\alpha_c \in [0, 1]$ and $\beta_c \in [0, 1]$ as:

$$\mathcal{X}_c(\alpha_c) = \alpha_c\mathcal{X}_c, \quad \mathcal{U}_c(\beta_c) = \beta_c\mathcal{U}_c, \tag{4}$$

where the original set of state constraint is $\mathcal{X}_c = \mathcal{X}_c(1)$ and the set of input constraint is $\mathcal{U}_c = \mathcal{U}_c(1)$. Note that states x_c and inputs u_c satisfy constraints provided that $\alpha_c \in [0, 1]$ and $\beta_c \in [0, 1]$.

Given the dynamics (3) and constraints (4) of coalition c, the disturbance w_c is constrained by the set:

$$\mathcal{W}_c(\alpha, \beta) \triangleq \bigoplus_{d\in\mathcal{M}_c} \big(A_{cd}\mathcal{X}_d(\alpha_d) \oplus B_{cd}\mathcal{U}_d(\beta_d)\big) \oplus \mathcal{W}_c^{\mathrm{e}}, \tag{5}$$

where (α, β) represents the dependence of the scaling factors of all neighbors $d \in \mathcal{M}_c$, and external noise is also assumed to be constrained by set $\mathcal{W}_c^{\mathrm{e}}$. The original disturbance set $\mathcal{W}_c(1, 1)$ depends on the original constraint sets $\mathcal{X}_d(1)$ and $\mathcal{U}_d(1)$ for all $d \in \mathcal{M}_c$.

Definition 5. The set Ω_c is robust control invariant (RCI) for the dynamics $x_c(k + 1) = A_{cc}x_c(k) + B_{cc}u_c(k) + w_c(k)$ with constraint sets $(\mathcal{X}_c, \mathcal{U}_c, \mathcal{W}_c)$ if $\Omega_c \subseteq \mathcal{X}_c$ and there exists a control law $u_c = \mu(x_c(k)) \in \mathcal{U}_c$ such that $x_c(k) \in \Omega_c$ implies $x_c(k + 1) \in \Omega_c, \forall w_c \in \mathcal{W}_c$. The control law is said to be invariance-inducing over the set Ω_c.

Since the size of Ω_c depends on the size of $\mathcal{W}_c(\alpha, \beta)$, its existence defines a set $\Omega_c(\mathcal{W}_c)$ such that:

$$\Omega_c(\mathcal{W}_c) \subseteq \mathcal{X}_c(\alpha_c), \quad \mu\big(\Omega_c(\mathcal{W}_c)\big) \subseteq \mathcal{U}_c(\beta_c). \tag{6}$$

In tube approaches, the existence of invariant sets equips controllers with robustness to handle disturbances from coupling subsystems. For that reason, the following assumption is considered as starting point.

Assumption 6. There exists an RCI set $\Omega_c(\mathcal{W}_c(1, 1)) \subseteq \mathcal{X}_c(1)$ for all $c \in \mathcal{C}$ in the decentralized cooperation topology, *i.e.,* for all current and future subsystems.

3.1 Inherent robustness gap

For accommodating locally disturbances not covered by tubes without any serious reconfiguration of the control system, we create an extra robustness gap by using two different scaling factors:

- *Public factors* $\alpha_i^{\mathrm{p}}, \beta_i^{\mathrm{p}} \in [0, 1]$, which are shared with the network and parameterize constraint sets.
- *Private factors* $\alpha_i^{\mathrm{r}}, \beta_i^{\mathrm{r}} \in [0, 1]$, which are confidential and individually optimized by each controller.

At start, all public and private scaling factors are set equal to 1 for all $i \in \mathcal{N}$. Then private factors are online optimized by agents to tighten their constraints and the idea is that agents share larger public scaling factors to their neighbors. In this way, the robustness gap is created by setting public factors as outer bounds of private scaling factors:

$$\alpha_i^{\mathrm{p}+} = \begin{cases} \alpha_i^{\mathrm{p}}(1 - \rho_\alpha) + \alpha_i^{\mathrm{r}}\,\rho_\alpha, & \text{if } \alpha_i^{\mathrm{r}} \leqslant \alpha_i^{\mathrm{p}}, \\ \alpha_i^{\mathrm{r}}, & \text{if } \alpha_i^{\mathrm{r}} > \alpha_i^{\mathrm{p}}, \end{cases} \tag{7}$$

$$\beta_i^{\mathrm{p}+} = \begin{cases} \beta_i^{\mathrm{p}}(1 - \rho_\beta) + \beta_i^{\mathrm{r}}\,\rho_\beta, & \text{if } \beta_i^{\mathrm{r}} \leqslant \beta_i^{\mathrm{p}}, \\ \beta_i^{\mathrm{r}}, & \text{if } \beta_i^{\mathrm{r}} > \beta_i^{\mathrm{p}}, \end{cases} \tag{8}$$

where $\alpha_i^{\mathrm{p}+}$ and $\beta_i^{\mathrm{p}+}$ denote, respectively, the state and input public scaling factors of agent i at instant $k + 1$, and tuning parameters $\rho_\alpha, \rho_\beta \in (0, 1)$.

The formation of coalitions is based on the public information of the agents involved, which *frozen* their public factors $(\alpha_i^{\mathrm{p}}, \beta_i^{\mathrm{p}})$. Hence, the coalition scaling factors $(\alpha_c^{\mathrm{p}}, \beta_c^{\mathrm{p}})$ hold with the corresponding values to be fulfilled:

$$\alpha_c^{\mathrm{p}}\mathcal{X}_c = \bigtimes_{i\in c} \alpha_i^{\mathrm{p}}\mathcal{X}_i, \qquad \beta_c^{\mathrm{p}}\mathcal{U}_c = \bigtimes_{i\in c} \beta_i^{\mathrm{p}}\mathcal{U}_i. \tag{9}$$

Remark 7. Public scaling factors of all $i \in c$ are *frozen* while in coalition, unless private factors of any $i \in c$ are greater than public factors. In the latter case, factors $(\alpha_c^{\mathrm{p}}, \beta_c^{\mathrm{p}})$ are recomputed as (9) with the updated factors of those agents $i \in c$ whose $\alpha_i^{\mathrm{r}} > \alpha_i^{\mathrm{p}}$ and $\beta_i^{\mathrm{r}} > \beta_i^{\mathrm{p}}$ to avoid misleading their neighbors about the real disturbances.

3.2 Preliminaries of the tube-based approach

Approaches based on tubes are characterized by dealing with a nominal system with no uncertainties:

$$z_c(k + 1) = A_{cc}z_c(k) + B_{cc}v_c(k),$$

where v_c is the control input obtained by an MPC controller to regulate the nominal state z_c, and handling error dynamics:

$$e_c(k + 1) = A_{cc}e_c + B_{cc}\,\mu_c\big(e_c(k)\big) + w_c,$$

with $e_c(k) = x_c(k) - z_c(k)$ contained in an invariant set, in this work, the RCI set is $\Omega_c(\mathcal{W}_c(\alpha^{\mathrm{p}}, \beta^{\mathrm{p}}))$.

The nominal constraint sets depend on RCI sets as follows:

$$\begin{aligned} \mathcal{Z}_c(\alpha_c^{\mathrm{p}}) &\triangleq \mathcal{X}_c(\alpha_c^{\mathrm{p}}) \ominus \Omega_c(\mathcal{W}_c(\alpha^{\mathrm{p}}, \beta^{\mathrm{p}})), \\ \mathcal{V}_c(\beta_c^{\mathrm{p}}) &\triangleq \mathcal{U}_c(\beta_c^{\mathrm{p}}) \ominus \mu\big(\Omega_c(\mathcal{W}_c(\alpha^{\mathrm{p}}, \beta^{\mathrm{p}}))\big), \end{aligned} \tag{10}$$

but we can employ an outer bounding of the RCI set to reduce the computational effort of calculating Ω_c explicitly every time the size of $\mathcal{W}_c(\alpha^{\mathrm{p}}, \beta^{\mathrm{p}})$ changes. To this end, a linear programming problem is solved to obtain the RCI set Ω_H, as proposed by Raković et al. (2007):

$$\min\{\epsilon : \phi \in \Phi\}, \tag{11}$$

where $\phi = (\mathbf{M}_H, a, b, \epsilon)$ and $\Phi = \{\phi : \mathbf{M}_H \in \mathcal{M}_H,\ \Omega_H \subseteq a\,\mathcal{X},\ \mu(\Omega_H) \subseteq b\,\mathcal{U}, a \in [0,1], b \in [0,1],\ q_x a + q_u b \leqslant \epsilon\}$ with $\mathbf{M}_H = (M_0, \ldots, M_{H-1})$, and weights q_x and q_u. Here, scaling constants a and b tighten, respectively, the state and input constraint sets to guarantee: $\Omega_H \subseteq a\,\mathcal{X}$ and $\mu(\Omega_H) \subseteq b\,\mathcal{U}$ (see (Raković et al., 2007) for more details).

Therefore, constraint sets (10) can be substituted by:

$$\begin{aligned} \mathcal{Z}_c(\alpha_c^{\mathrm{p}}, a_c) &= \mathcal{X}_c(\alpha_c^{\mathrm{p}}) \ominus a_c \mathcal{X}_c(\alpha_c^{\mathrm{p}}) = (1-a_c)\mathcal{X}_c(\alpha_c^{\mathrm{p}}),\\ \mathcal{V}_c(\beta_c^{\mathrm{p}}, b_c) &= \mathcal{U}_c(\beta_c^{\mathrm{p}}) \ominus b_c \mathcal{U}_c(\beta_c^{\mathrm{p}}) = (1-b_c)\mathcal{U}_c(\beta_c^{\mathrm{p}}). \end{aligned} \tag{12}$$

with $a_c \in [0,1]$ and $b_c \in [0,1]$.

3.3 Tube-based coalitional MPC problem

The objective of coalition $c \in \mathcal{C}$ is to lead the state to the origin while fulfilling the constraints. We propose a coalitional MPC scheme based on optimized tubes to manage coalition dynamics (3) with the control policy:

$$u_c(k) = \mu_c\big(e_c(k)\big) + v_c^{\mathrm{o}}\big(z_c(k)\big), \tag{13}$$

where $\mu_c(e_c(k))$ is the RCI control law, and v_c^{o} is the first element of optimal control sequence. The control sequence $V_c^{\mathrm{o}}(k) \triangleq \{v_c^{\mathrm{o}}(k), \ldots, v_c^{\mathrm{o}}(k+N_{\mathrm{p}}-1)\}$, the state sequence $Z_c^{\mathrm{o}}(k) = \{z_c^{\mathrm{o}}(k), \ldots, z_c^{\mathrm{o}}(k+N_{\mathrm{p}})\}$, and private factors $(\alpha_c^{\mathrm{r}}, \beta_c^{\mathrm{r}})$ are obtained by solving the following nominal MPC problem in the prediction horizon N_{p}:

$$\min_{V_c(k), \alpha_c^{\mathrm{r}}, \beta_c^{\mathrm{r}}} J_c\big(Z_c(k), V_c(k)\big) + \rho_\alpha \alpha_c^{\mathrm{r}} + \rho_\beta \beta_c^{\mathrm{r}}, \tag{14}$$

with weights $\rho_\alpha, \rho_\beta \in \mathbb{R}_+$, and subject to constraints:

$$\begin{aligned} & z_c(k+t+1) = A_{cc} z_c(k+t) + B_{cc} v_c(k+t)\\ & z_c(k) = \tilde{z}_c(k), \quad t = 0, \ldots, N_{\mathrm{p}}-1,\\ & z_c(k+t) \in \alpha_c^{\mathrm{r}} \mathcal{Z}_c(\alpha_c^{\mathrm{p}}, a_c), \quad t = 1, \ldots, N_{\mathrm{p}}-1,\\ & z_c(k+N_{\mathrm{p}}) \in \Omega_c^{\mathrm{f}}(\alpha_c^{\mathrm{p}}, \beta_c^{\mathrm{p}}, a_c, b_c),\\ & v_c(k+t) \in \mathcal{V}_c(\beta_c^{\mathrm{p}}, b_c), \quad t = 0, \ldots, N_{\mathrm{p}}-1\\ & \alpha_c^{\mathrm{r}} \in [0,1], \quad \beta_c^{\mathrm{r}} \in [0,1], \end{aligned} \tag{15}$$

for all $c \in \mathcal{C}$, where $\tilde{z}_c(k)$ is the current state value, and the set Ω_c^{f} is the terminal state constraint.

Assumption 8. Terminal set $\Omega_c^{\mathrm{f}}(\alpha_c^{\mathrm{p}}, \beta_c^{\mathrm{p}}, a_c, b_c)$ is positively invariant for dynamics $z_c(k+1) = A_{cc} z_c(k) + B_{cc} v_c(k)$ under the control law $v_c = K_c^{\mathrm{f}} z_c$, that is, $(A_{cc} + B_{cc} K_c^{\mathrm{f}})\Omega_c^{\mathrm{f}} \subseteq \Omega_c^{\mathrm{f}}$ with $\Omega_c^{\mathrm{f}} \subseteq \mathcal{Z}_c(\alpha_c^{\mathrm{p}}, a_c)$ and $K_c^{\mathrm{f}} \Omega_c^{\mathrm{f}} \subseteq \mathcal{V}_c(\beta_c^{\mathrm{p}}, b_c)$.

The function $J_c(Z_c(k), V_c(k))$ in problem (14) is:

$$\sum_{t=0}^{N_{\mathrm{p}}-1} \underbrace{\left(\|z_c(k+t)\|_{Q_c}^2 + \|v_c(k+t)\|_{R_c}^2\right)}_{l_c\left(z_c(k+t), v_c(k+t)\right)} + \underbrace{\|z_c(k+N_{\mathrm{p}})\|_{P_c}^2}_{f_c\left(z_c(k+N_{\mathrm{p}})\right)},$$

where stage cost $l_c(\cdot)$ weight the nominal state and input vectors by matrices $Q_c \succ 0$ and $R_c \succ 0$, respectively, and terminal cost $f_c(\cdot)$, with $P_c \succ 0$, is designed such that

$$z_c(k)^\top P_c z_c(k) - z_c(k+1)^\top P_c z_c(k+1) \geqslant l_c\big(z_c(k+1), K_c^{\mathrm{f}} z_c(k+1)\big),$$

therefore, $z_c(k)^\top P_c z_c(k)$ is a control Lyapunov function.

4. CONTROL ALGORITHM

We present a top-down coalitional MPC algorithm based on optimized tubes and an extra robustness margin.

The *supervisory layer* is responsible for selecting the cooperation topology every $T_{\mathrm{up}} \in \mathbb{N}_+$ time instants (and when needed) to improve performance and guarantee recursive feasibility. The algorithm is described as follows:

Alg. 1: Supervisory layer

Initial data: $\mathcal{X}_i, \mathcal{U}_i, K_i^{\mathrm{f}}, H_i,\ \forall i, N_{\mathrm{p}}, \rho_\alpha, \rho_\beta, \tau_\alpha, \tau_\beta, \mathrm{c}_{\mathrm{link}}$
Start: $\Lambda_{\mathrm{cur}} = \Lambda_{\mathrm{cen}}, z_i(0) = x_i(0), \alpha_i^{\mathrm{p}}, \beta_i^{\mathrm{p}}, \alpha_i^{\mathrm{r}}, \beta_i^{\mathrm{r}} = 1, \forall i \in \mathcal{N}$
Inputs: $\Lambda_{\mathrm{cur}}, \alpha_i^{\mathrm{p}}, \beta_i^{\mathrm{p}},\ \forall i \in \mathcal{N}$. **Output:** Λ_{new}

1: Given Λ_{cur}, measure $\tilde{x}_c$ and $\tilde{z}_c$ for all $c \in \mathcal{C}$.
2: Calculate $\mathcal{T}_{\mathrm{new}}$ as (2) with Λ_{cur}.
3: **for** each $\Lambda_{\mathrm{new}} \in \mathcal{T}_{\mathrm{new}}$ **do**
4: Compute $\mathcal{W}_c$ as (5), Ω_c by (11), and $\Omega_c^{\mathrm{f}}, \forall c \in \mathcal{C}$.
5: **if** $\nexists \Omega_c$ for any c **then**
6: Mark Λ_{new} as infeasible, and go to Step 3.
7: **end if**
8: **for** each $c \in \mathcal{C}$ **do**
9: Solve (14) with $\tau_\alpha, \tau_\beta = 0$ and $\alpha_c^{\mathrm{r}}, \beta_c^{\mathrm{r}} = 1$ to obtain control sequence U_c via (13) and $\Gamma_c = \sum_{t=1}^{N_{\mathrm{p}}} (l_c(x_c(k+t), u_c(k+t)) + c_{\mathrm{link}}|\Lambda_c|$.
10: **end for**
11: Calculate the cost $\Gamma_\Lambda = \sum_{c \in \mathcal{C}} \Gamma_c$ for Λ_{new}.
12: **end for**
13: **if** all $\Lambda_{\mathrm{new}} \in \mathcal{T}_{\mathrm{new}}$ are marked as infeasible **then**
14: Any c with $\nexists \Omega_c$ clusters with neighbor $d \in \mathcal{M}_c$ with the largest $\mathcal{W}_d$, and update Λ_{cur}.
15: Go to Step 2.
16: **else**
17: Select topology $\Lambda_{\mathrm{new}} \in \mathcal{T}_{\mathrm{new}}$ with lowest cost Γ_Λ.
18: Sent Λ_{cur} to the lower layer (Alg. 2).
19: **end if**

In the *lower layer*, each $c \in \mathcal{C}$ implements Alg. 2 according to the current topology Λ_{cur} and public scaling factors:

Alg. 2: Lower control layer

Initial data: $\mathcal{X}_c, \mathcal{U}_c, H_c, K_c^{\mathrm{f}}, N_{\mathrm{p}}, \tau_\alpha, \tau_\beta, \sigma_c = 0,\ \forall c \in \mathcal{C}$.
Inputs: $\Lambda_{\mathrm{cur}}, x_c, \alpha_c^{\mathrm{p}}, \beta_c^{\mathrm{p}},\ \forall c \in \mathcal{C}$
Outputs: $\alpha_c^{\mathrm{p}+}, \beta_c^{\mathrm{p}+}, x_c^+, z_c^+$

1: Compute $\mathcal{W}_c$ by (5), Ω_c by (11), and Ω_c^{f}.
2: Solve (14) to obtain v_c^{o} and α_i^{r} and $\beta_i^{\mathrm{r}}, \forall i \in c$.
3: Apply u_c to get x_c^+ and v_c^{o} to obtain z_c^+.
4: **if** $\alpha_i^{\mathrm{r}} > \alpha_i^{\mathrm{p}}$ and $\beta_i^{\mathrm{r}} > \beta_i^{\mathrm{p}}$ for any $i \in c$ **then**
5: Set $\alpha_i^{\mathrm{p}+} = \alpha_i^{\mathrm{r}}$ and $\beta_i^{\mathrm{p}+} = \beta_i^{\mathrm{r}}$.
6: Update α_c^{p} and β_c^{p} by (9).
7: Execute Alg. 1.
8: **else if** $c = \{i\}$, and $\alpha_i^{\mathrm{r}} \leqslant \alpha_i^{\mathrm{p}}$ and $\beta_i^{\mathrm{r}} \leqslant \beta_i^{\mathrm{p}}$ **then**
9: Get $\alpha_i^{\mathrm{p}+}$ (7), $\beta_i^{\mathrm{p}+}$ (8), and active flag $\sigma_c = 1$.
10: Share $\alpha_c^{\mathrm{p}+}, \beta_c^{\mathrm{p}+}$ with all $c \in \mathcal{M}_c$.
11: Update $\mathcal{W}_c^+$ and Ω_c^+.
12: **end if**
13: **if** $\sigma_c = 1$, $\exists \Omega_c^+$, and $x_c^+ - z_c^+ \in \Omega_c^+, \forall c \in \mathcal{C}$ **then**
14: Set $\sigma_c = 0$ and share $\alpha_c^{\mathrm{p}+}, \beta_c^{\mathrm{p}+}$ with all $d \in \mathcal{M}_c$.
15: **else**
16: Hold current scaling factors as $\alpha_c^{\mathrm{p}+}$ and $\beta_c^{\mathrm{p}+}$.
17: **end if**

5. PLUG-AND-PLAY SUBSYSTEMS

We consider that subsystems can join and leave the system in real time, so switching dynamics may be introduced. This fact may force the redesign of the current cooperation topology for feasibility and performance reasons. When a new subsystem $i = N+1$ is plugged at time instant k_{plug}, the system yields: $\mathcal{N}(k) = \{1, 2, \ldots, N+1\}$. Due to dynamic couplings, the disturbances of neighbors $j \in \mathcal{N}_i(k)$

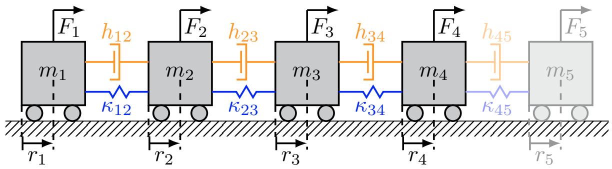

Fig. 2. System compound of an array of coupled trucks.

grow, and recursive feasibility may be lost. To prevent that, plug-ins are only executed when the supervisory layer is triggered, and cooperation topology is updated according to Alg. 1 at the infinitesimal instant after the plug-in, k_{plug}^{+}. Conversely, when a subsystem i is disconnected from the system, it results in $\mathcal{N}(k) = \{1, 2, \ldots, i-1, i+1, \ldots, N\}$. Since disturbances of neighbors $d \in \mathcal{M}_i$ decrease, Alg. 1 could be executed to select another topology that improves performance. Otherwise, the current topology could be held, which is computationally less expensive.

6. ILLUSTRATIVE EXAMPLE

We use the system presented in (Trodden and Maestre, 2017), where four trucks are coupled by springs and dampers with their neighbors, as depicted in Fig. 2. At time instant k_{plug}, a fifth truck is connected to the system. Each truck $i \in \mathcal{N}(k)$ has a state x_i, which is composed of position displacement r_i and velocity v_i, and follows continuous-time dynamics:

$$\underbrace{\begin{bmatrix} \dot{r}_i \\ \dot{v}_i \end{bmatrix}}_{\dot{x}_i} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{m_i}\sum_{j\in\mathcal{M}_i}\kappa_{ij} & -\frac{1}{m_i}\sum_{j\in\mathcal{M}_i}h_{ij} \end{bmatrix}\begin{bmatrix} r_i \\ v_i \end{bmatrix} + \begin{bmatrix} 0 \\ 100 \end{bmatrix} u_i + w_i,$$

where each truck's controller applies a horizontal force $F_i = u_i\,[0, 100]^\top$ with u_i being the control input, and the disturbance w_i is defined as:

$$w_i = \sum_{j\in\mathcal{N}_i}\begin{bmatrix} 0 & 0 \\ \frac{1}{m_i}\sum_{j\in\mathcal{M}_i}\kappa_{ij} & \frac{1}{m_i}\sum_{j\in\mathcal{M}_i}h_{ij} \end{bmatrix}\begin{bmatrix} r_j \\ v_j \end{bmatrix} + w_i^{\text{e}},$$

where the external noise $|w_i^{\text{e}}| \leqslant [2.5\text{e}-3,\, 2.5\text{e}-3]^\top$. A discrete-time model with sample time $T_{\text{s}} = 0.2\,s$ that approximates the continuous-time model is employed to simulate and control each truck i. As for model parameters in simulations, masses [kg]: $m_1, m_3 = 3$, $m_2, m_4 = 2$, and $m_5 = 5$; spring constants [N/m]: $\kappa_{12} = 0.5$, $\kappa_{23} = 0.7$, $\kappa_{34} = 1$, and $\kappa_{45} = 0.8$; and damping factors [Ns/m]: $h_{12} = 0.3$, $h_{23} = 0.4$, $h_{34} = 0.5$, and $h_{45} = 0.2$.

The objective is to lead trucks from initial states $x_1(0) = [1.5, 0]^\top$, $x_2(0) = [-0.5, 0]^\top$, $x_3(0) = [1, 0]^\top$, $x_4(0) = [-1, 0]^\top$, and $x_5(k_{\text{plug}}) = [1, 0]^\top$ to their equilibrium positions, considering constraints $|r_i| \leqslant 4\,\text{m}$, $|v_i| \leqslant 1\,\text{m/s}$, and $|u_i| \leqslant 0.5\,\text{N/kg}$, and handling a plug-in truck at $k_{\text{plug}} = 16\,\text{s}$. The weighting matrices for each truck are $Q_i = I$ and $R_i = 100$, and are aggregated as $Q_c = \text{diag}(Q_i)_{i\in c}$ and $R_c = \text{diag}(R_i)_{i\in c}$. The LQR terminal controller $K_c^{\text{f}} = \text{diag}(K_i^{\text{f}})_{i\in c}$, where $K_1^{\text{f}} = [-0.0365, -0.0460]$, $K_2^{\text{f}} = [-0.0334, -0.0443]$, $K_3^{\text{f}} = [-0.0335, -0.0446]$, $K_4^{\text{f}} = [-0.0306, -0.0441]$, $K_5^{\text{f}} = [-0.0363, -0.0463]$, and the terminal weight matrix $P_c = \text{diag}(P_i)_{i\in c}$, where

$$P_1 = \begin{bmatrix} 4.3327 & -2.7765 \\ -2.7765 & 3.9817 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 4.2137 & -2.7240 \\ -2.7240 & 3.9148 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 4.2203 & -2.7763 \\ -2.7163 & 3.9196 \end{bmatrix},$$
$$P_4 = \begin{bmatrix} 4.1235 & -2.6833 \\ -2.6833 & 3.8755 \end{bmatrix}, \quad P_5 = \begin{bmatrix} 4.3305 & -2.7743 \\ -2.7743 & 3.9825 \end{bmatrix}.$$

Fig. 3. Formation of coalitions.

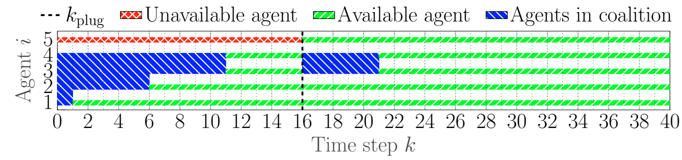

Table 1. Comparison of costs.

Methods	J_{perf}	J_{coop}	J_{total}	J_{total}[%]
Cen. MPC	28.47	14.50	42.97	—
Coal. MPC	28.84	2.00	30.84	28.24 %

6.1 Simulated results

We consider simulations of length $N_{\text{sim}} = 40$, a prediction horizon $N_{\text{p}} = 10$, and a cost per link $c_{\text{link}} = 0.1$. We also define the supervisory layer period $T_{\text{up}} = 5$, parameter $\rho_\alpha = 0.1$, and weight $\tau_\alpha = 5\text{e}-5$. In Fig. 3, we present the formation of coalitions starting from the centralized topology. Since the supervisory layer decides the new topology every $T_{\text{up}} = 5$ instants, the coalitional structure is $\mathcal{C} = \{\{1\}, \{2, 3, 4\}\}$ for $k < 5$, and $\mathcal{C} = \{\{1\}, \{2\}, \{3, 4\}\}$ for $6 \leqslant k < 11$. Afterwards, agents work independently until $i = 5$ is introduced, causing the disturbance of its neighbor to increase. As a result, agent 4 forms a coalition with agent 3. This coalition is disbanded when disturbances decrease enough as a consequence of shrinking constraints sets of neighbors. In Fig. 4, we show the evolution of the public and private scaling factors of all agents. When agents 3 and 4 break their coalition at $k = 21$ (see Fig. 3), factor $\alpha_4^{\text{r}} > \alpha_4^{\text{p}}$. At next instant $k = 22$, it is updated $\alpha_4^{\text{p}} = \alpha_4^{\text{r}}$, and Alg. 1 is triggered to hold recursive feasibility. By chance, instant $k = 22$ coincides with the normal moment of execution of the supervisory layer. Afterwards, all agents work individually until the end. In Fig. 5, we illustrate the nominal and real state trajectories of agents and their tubes, which can shrink and grow due to changes of topology and factors α_i^{p} for all $i \in \mathcal{N}(k)$. For example, tube of agent 3 generally shrinks but, at instants $k = 11$ and $k = 21$, its disturbances increase due to its coalition breakups (see Fig. 3), so its tube grows to handle more uncertainty.

Finally, Table 1 shows a comparison of the performance, cooperation, and total cost for all MPC schemes:

$$J_{\text{perf}} = \sum_{k=1}^{N_{\text{sim}}} \|x(k)\|_Q^2 + \|u(k)\|_R^2, \quad J_{\text{coop}} = \sum_{k=1}^{N_{\text{sim}}} c_{\text{link}}|\Lambda(k)|.$$

As shown, centralized MPC provides the lowest performance cost at the expense of a high cooperation cost, while coalitional MPC yields a total cost ($J_{\text{total}} = J_{\text{perf}} + J_{\text{coop}}$) reduction of 28.24 %. We report that decentralized MPC cannot be implemented since it becomes infeasible. Therefore, coalitional MPC provides a suitable control framework to deal with plug-and-play operations online, while reducing the total cost.

7. CONCLUSIONS

We propose a tube-based coalitional MPC method with plug-and-play (PnP) features for linear multi-agent systems. Agents exchange information about the size of their

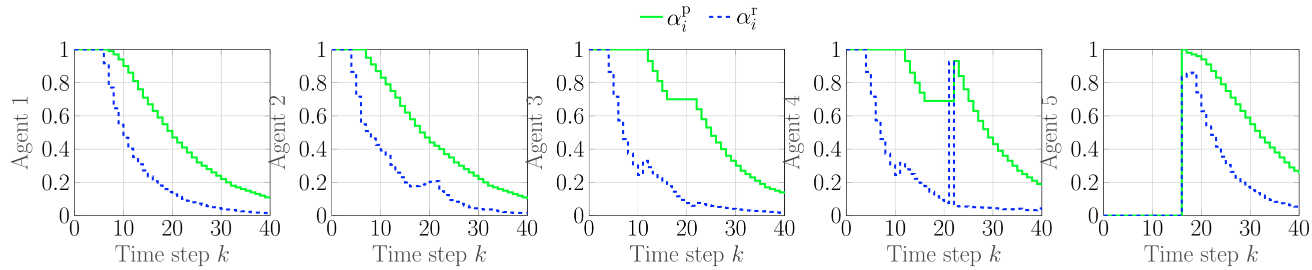

Fig. 4. Evolution of scaling factors in coalitional MPC.

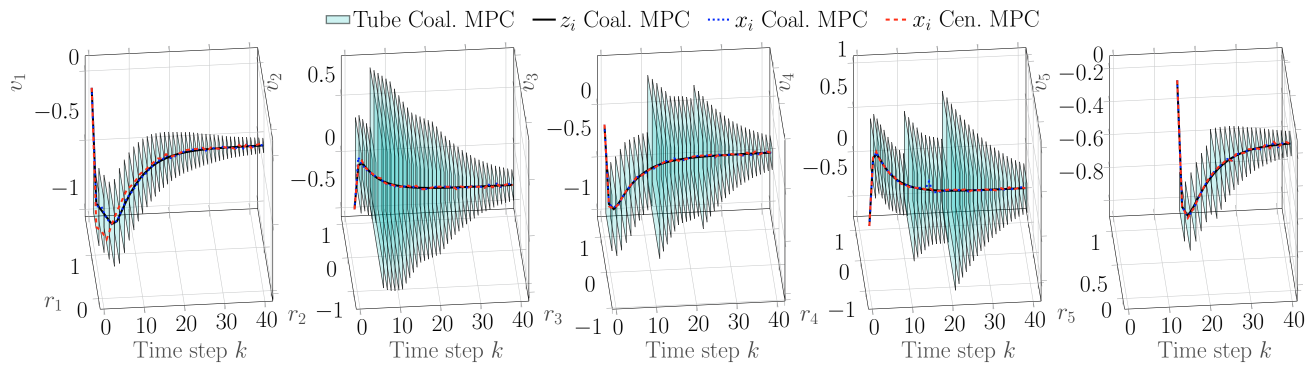

Fig. 5. Evolution of tubes cross sections for each agent.

state and input constraints to reconfigure their disturbance sets and reduce conservatism. Moreover, agents can form coalitions to improve their performance and handle PnP operations. As shown in simulations, our coalitional MPC outperforms centralized MPC in total cost and has a degree of flexibility for handling PnP using a distributed setting. Since tubes depend on disturbances, they shrink when states come close to the origin. Future work will include a bottom-up implementation of our proposal and a rigorous analysis of the recursive feasibility and stability.

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