Algebraic framework for outage information management in distribution networks

M. Rodríguez-Montañés, J. A. Rosendo-Macías, *Senior Member, IEEE*, A. Gómez-Expósito, *Fellow, IEEE*, and G. Tévar

***Abstract*—During faulty conditions, a sustained and heterogeneous stream of information, coming mainly from field signaling devices and customer calls, arrives at the Distribution Management System. This data should be promptly and systematically processed so that the faulted section can be isolated and service restored to customers as soon as possible. This paper presents a matrix-based framework allowing the status of the distribution grid and associated protection, signaling and switching devices to be compactly modeled and tracked. By means of algebraic expressions, the proposed information management system can infer the existence of a blackout (loss of load) and assist the operator in decision making during the identification and isolation of the affected areas. The proposed framework considers the potential presence of embedded distributed generation. With the help of both a tutorial system and a real-life case study, application examples are discussed to illustrate the potential of the proposed methodology.**

***Index Terms*—Service interruption, Distribution Management System, Fault location, Reliability management**

I. Notation

A. Static information

$\mathcal{N}$	Set of nodes in the functional graph (**G**), comprising five categories of components, $\{\mathcal{P}, \mathcal{D}, \mathcal{G}, \mathcal{L}, \mathcal{Z}\}$
$\mathcal{P}$	Subset of $\mathcal{N}$ with protection functionality
$\mathcal{P}_s$	Subset of $\mathcal{P}$ with signaling capability
$\mathcal{P}_r$	Subset of $\mathcal{P}$ with reclosing capability
$\mathcal{D}$	Subset of $\mathcal{N}$ with fault-detection functionality
$\mathcal{D}_o$	Subset of $\mathcal{D}$ with directional fault-detection capability
$\mathcal{G}$	Subset of $\mathcal{N}$ representing distributed generation
$\mathcal{L}$	Subset of $\mathcal{N}$ representing (passive) loads
$\mathcal{W}$	Subset of $\mathcal{N}$ with sectionalizing (switching) functionality
$\mathcal{Z}$	Subset of $\mathcal{N}$ representing impedance-type elements

Vector arrays N, P, P_s, P_r, D, D_o, G, L, W and Z binary encode the respective sets in callygraphic symbol.

B. Dynamic information

$\mathit{\Pi}$	Subset of $\mathcal{P}_s$ with trigger evidence
$\mathit{\Delta}$	Subset of $\mathcal{D}$ with meaningful signal, either downstream, $\mathit{\Delta}_d$, or upstream, $\mathit{\Delta}_u$
$\mathit{\Lambda}$	Subset of $\mathcal{L}$ with service interruption evidence, either off, $\mathit{\Lambda}_{off}$, or on, $\mathit{\Lambda}_{on}$
$\mathit{\Sigma}$	Set of all the available signals, $\{\mathit{\Pi}, \mathit{\Delta}, \mathit{\Lambda}\}$

M. Rodríguez-Montañés and G. Tévar are with Endesa, Spain; J. A. Rosendo-Macías and A. Gómez-Expósito are with the Department of Electrical Engineering, University of Seville, Spain.

Vector arrays Π, Δ and Λ encode the respective sets $\mathit{\Pi}$, $\mathit{\Delta}$, $\mathit{\Lambda}$.

C. Inferred knowledge

$\mathcal{F}_x$	Subset of $\mathcal{N}$ whose failure is compatible with the dynamic signal set $x \in \mathit{\Sigma}$
$\mathcal{T}_x$	Subset of $\mathcal{P}$ whose triggering decision is compatible with the dynamic signal set $x \in \mathit{\Sigma}$
$\mathcal{U}$	Subset of $\mathcal{P}$ whose triggering decision is not compatible with available information

Vector arrays F_x, T_x and U encode the respective sets $\mathcal{F}_x$, $\mathcal{T}_x$ and $\mathcal{U}$.

D. Functions, matrices and operators

diag	Matlab-like function that returns a square diagonal matrix with the elements of a vector array
nnz	Function that returns the number of non-zero elements in an array
I	Identity matrix
S	Successors or Path matrix of graph **G**
C	Boolean matrix linking each node in $\mathcal{N}$ (columns) with the protection device in $\mathcal{P}$ (rows) that clears a fault on the component represented by that node
r	Vector with the expected failure rates of the components represented by the nodes in $\mathcal{N}$
$\circ$	Hadamard (element-wise) product of matrices
$\setminus$	Symbol for set subtraction
$[\cdot]_{ix}$	i-th row of a matrix

II. Introduction

In order to keep reliability indices (such as SAIFI/SAIDI) within legally binding limits, any customer service interruption originated by a permanent fault in the distribution system must be detected by the Distribution Management System (DMS) as quickly as possible. This is straightforward if the fault is cleared by the circuit breaker (CB) located at the head of the MV feeder, whose status is continuously monitored at the DMS, but is far from trivial for instance in case of a hidden conductor breakdown or fuse tripping. In many cases, the service interruption can only be detected when customers complaint calls are received.

Once the existence of a permanent fault is confirmed, the DMS operator launches a systematic procedure, in coordination with the mobile maintenance teams, in order to isolate the faulted zone and restore the service to as many customers as possible. This is achieved through adjacent healthy feeders with spare capacity [1] or, if the opportunity arises, by resorting to embedded distributed generation [2], [3]. The restoration process requires that the underlying circuit connectivity (static information) be known in advance and that the stream of alarms, tripping signals, customer calls and other evidence received in real time (dynamic information) be duly processed [4].

In the upcoming smart grid paradigm [5], the amount of information received from distribution automation devices, distributed generators, customers meters and calls, etc. will be overwhelming. Therefore, more sophisticated procedures should be developed in order to efficiently and reliably distill valuable knowledge from the stream of data arriving from the field. As the fault isolation and restoration processes cannot be fully automated, accurate and updated knowledge about the true status of the whole set of network components will be indispensable to assist the DMS operator in informed decision making.

In [6], [7] Artificial Neural Networks are applied to the determination of reliability indices for distribution systems. State enumeration methods are applied in [8] to compute the expected frequency and duration of faults in radial networks, while the same approach is adopted in [9] to determine the optimal location of switching elements so as to keep reliability indices under control. In [10], graph theoretic tools in combination with load flow solutions are used to assess the impact of remotely operated switching devices on restoration sequences and resulting quality indices. In [11], graph-based search and state enumeration techniques are combined to obtain reliability indices and to choose suitable ways of enhancing those indices. In [12], Monte Carlo simulation is adopted to evaluate the impact of distribution automation on reliability indices. Closed-form expressions for the reliability indices in radial networks are developed in [13]. In [14], a real-time methodology for operative configuration inference is presented based on power flow analysis and fuzzy logic techniques. Most works base their analysis on the availability of expected failure rates [15], [16] and characteristic duration of individual tasks [17], [18].

The methodology presented in this work relies on the so-called functional graph, an extension of the customary electrical graph explicitly considering all the protection, detection and switching elements involved in MV distribution systems. Characteristic matrices and arrays are built allowing algebraic expressions to be applied from which the existence of a service interruption, along with its most likely geographical extent, can be systematically inferred from existing evidence. The main goal is to help and guide the DMS operator in decision making during the fault detection and isolation stages. The proposed matrix-based framework can be applied also to other related problems, such as computation of reliability indices, optimal placement of devices to improve reliability [19], service restoration or maintenance teams scheduling, which are out of the scope of a single paper.

III. Outage management background: information sources and major stages

A. Static information: network structure

Even though MV distribution systems are structurally meshed, they are radially operated with the help of strategically located normally-open switches. This way, every system is characterized by a preferred configuration of radial feeders, determined during the planning stage, which is seldom changed. Only for maintenance or repairing tasks is the default configuration modified, which is then returned to the initial state whenever possible.

In this work, the preferred radial configuration is assumed to be the initial (healthy) state for every feeder of interest, before any alarm or evidence of fault is received at the DMS. In the sequel, only a single MV feeder will be considered, as the analysis could be applied in a decoupled fashion to other feeders, if necessary. The following static information is available for a given feeder:

- Electrical connectivity (one-line diagram), showing how every secondary substation (SS) is fed from the busbars of the primary substation through feeder sections [20]. In turn, each SS aggregates the set of LV customers downstream [21] and the embedded distributed generators (DG) in case of prosumers [22], [23], whose operating status is known. Service interruption affecting a SS necessarily implies a blackout for the associated LV customers [13].
- The location, role and operating status of every protection device (PD), fault passage detector (FPD) and switching device (SD).
- Failure rate (expected value) for every network component, obtained from historic records [24], [25].

B. Dynamic information

1) Protection and fault detection signalling: Upon the occurrence of a fault, the involved PD may, or may not depending on the type, inform the DMS that it has issued a triggering signal [26]. In this work, it will be assumed that all PDs along a radial feeder are duly coordinated so that the fault is cleared only by the closest PD located upstream [27]. It is also assumed that DGs get disconnected in islanded mode, i.e. when the feeder is disconnected from the primary system. The PD can additionally be equipped with the automatic reclosing capability, by which a delayed circuit breaker closing is issued aimed at avoiding permanent interruption of service in case of temporary faults [28].

In addition to PDs, with capability to detect and clear fault currents, FPD are being also deployed to speed up the identification of faulted components [29]. In absence of DG, FPD will detect and signal only downstream faults. However, an operational DG located downstream the FPD can contribute to the current of a fault located upstream the FPD. In this work, a DG is considered as non operational and, thus, excluded from $\mathcal{G}$, when it is out of service, disconnected, or not able to contribute enough fault current to be detected by any FPD (e.g., in case of low solar irradiance for PV generation). More sophisticated FPD can be directional, which means that they inform whether the fault is upstream or downstream.

Any other monitoring device which can remotely provide information on the occurrence and value of a fault current, such as micro-PMUs [30], could be also handled as a directional FPD.

2) Customer feedback: Note that some PDs, such as fuses, do not inform the DMS. Therefore, a fault with loss of load may still exist even in absence of signals from PDs or FPDs. In those cases, the only way for the operator to be aware of a fault is through customer calls. Moreover, during the identification phase, the operator may call back some selected customers to check if the interruption of service persists. With the advent of smart meters, this can be done more quickly and reliably by directly polling those devices through the available communication links [31].

C. Extracting knowledge from upcoming information

In absence of any evidence (signals, alarms or customer calls), it is assumed that there is no service interruption. Otherwise, a process begins comprising the following major stages:

- Make sure that the fault is permanent. In case of a customer call, it is important to discard in-house faults. Reciprocally, customers can be contacted to fully confirm the loss of load.
- Once the permanent fault is confirmed, the operator must narrow as much as possible the suspected area, based on the upcoming information [17], [32].
- If the available information is not sufficient to uniquely identify the faulted section or component, then the operator must lead and coordinate a lengthy and costly trial-and-error identification process, mostly based on one or several mobile teams locally operating the sectionalizing switches. In this process, the operator should duly balance the benefits of a given switching action, in terms of the new knowledge potentially gained, against its cost (e.g. switch wear and tear) and duration, keeping always in mind the whole sequence of actions.
- Eventually, the faulted zone is identified and isolated, and the service is restored through alternative adjacent paths.

The first three stages will be illustrated in section V.

IV. Algebraic methodology for information processing

A. Functional graph of a MV radial feeder

In this work, a functional oriented graph $\mathbf{G} = \{\mathcal{N}, \mathcal{E}\}$ is adopted, comprising all relevant components (branch sections, network devices or SS) of the MV radial feeder. Each node $n_i \in \mathcal{N}$ refers to a unique component, out of n, whereas $(n_i, n_j) \in \mathcal{E}$ if the i-th component is the immediate predecessor of j in the radial structure.

Figure 1 shows the one-line diagram of a simple feeder where each number represents a node ($n = 25$). Nodes $\{2, 3, 5, 7, 10, 13, 15, 18, 21, 24\}$ correspond to line sections, $\{1, 4\}$ refer to CBs (PDs with reclosing capability), $\{6\}$ is a FPD, $\{12, 23\}$ are fuses (PDs without signaling or reclosing capability), $\{9, 11, 17, 20\}$ are SD, $\{14, 16, 19, 22, 25\}$ are SS (loads) with passive customers connected to, and node $\{8\}$ represents a DG directly connected to the MV grid.

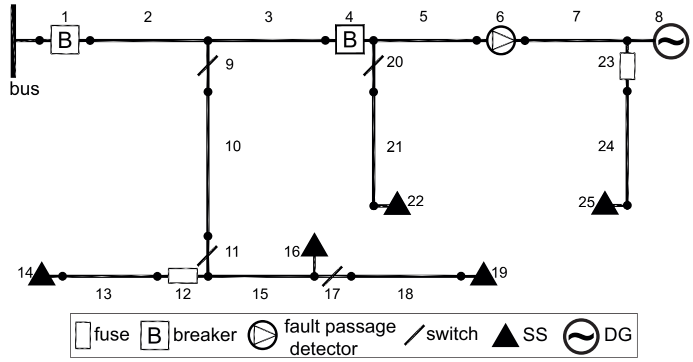

Fig. 1. One-line diagram of the tutorial example

The reader is referred to the nomenclature section where relevant subsets of $\mathcal{N}$ are introduced. Primary distribution substations are usually equipped with an automatic CB at the head of each feeder, with signaling and reclosing capabilities, which means that this component (n_1 in the example) belongs to both $\mathcal{P}_s$ and $\mathcal{P}_r$. Table I presents the elements of the subsets of $\mathcal{N}$ for the tutorial example.

TABLE I
SUBSETS OF THE WHOLE SET OF NODES $\mathcal{N}$

Subset	Node
$\mathcal{L}$	14,16,19,22,25
$\mathcal{G}$	8
$\mathcal{P}$	1,4,12,23
$\mathcal{P}_s \subseteq \mathcal{P}$	1
$\mathcal{P}_r \subseteq \mathcal{P}$	1,4
$\mathcal{D}$	6
$\mathcal{D}_o \subseteq \mathcal{D}$	6
$\mathcal{W}$	9,11,17,20

The main goal in this context is to assure the continuity of power supply from the feeder head to all SS feeding LV customers (subset $\mathcal{L}$), which is compromised when any component unexpectedly fails and the associated PD upstream clears the fault (note that each component is characterized by a failure rate, r_i) . Therefore, it is of paramount importance to properly identify the relative location of each node in $\mathcal{L}$ with respect to the remaining elements of $\mathcal{N}$. This will be achieved with the help of the matrices and arrays introduced in the sequel.

B. Matrix-related data structures

For any subset of nodes $\mathcal{X} \subseteq \mathcal{N}$, an $n \times 1$ binary array X is defined, such that:

$$X_i = \begin{cases} 1 & \text{if } n_i \in \mathcal{X} \\ 0 & \text{if } n_i \notin \mathcal{X} \end{cases}$$

This way, N is an array composed of n ones. Note that sets are represented with calligraphic fonts while matrix or vector arrays are denoted with the respective latin character.

The cornerstone of the proposed method is the so-called path matrix, defined for a tree with a given root and arbitrary edge orientations [33]. Such matrix can be arranged in upper triangular form provided the nodes are ordered in the right sequence. In this work, all edges of the tree are oriented downwards for convenience, and the resulting path matrix will be more properly denoted as the successors matrix, S. Accordingly, the element $S_{ij} = 1$ if and only if the node n_j belongs to the subtree of **G** rooted at n_i (i.e., n_j is a successor or descendant of n_i). By definition, $S_{ii}= 1$ for any n_i. Algebraically, matrix S is obtained from [33],

$$S = -A^{-1} \tag{1}$$

where A is the well-known incidence matrix. In practice, though, the non null elements of matrix S can be more efficiently obtained by applying graph-based search algorithms.

An *extraction* of rows and columns of S, determined by any two subsets $\mathcal{X}, \mathcal{Y} \subseteq \mathcal{N}$, is defined as follows:

$$S\left(\mathcal{X}, \mathcal{Y}\right)_{ij} = \begin{cases} S_{ij} & \text{if } n_i \in \mathcal{X} \text{ and } n_j \in \mathcal{Y} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

which, algebraically, can be obtained from,

$$S\left(\mathcal{X}, \mathcal{Y}\right) = \text{diag}\left(X\right) \cdot S \cdot \text{diag}\left(Y\right) \tag{3}$$

In addition, the $n \times n$ binary matrix C is introduced, such that $C_{ij} = 1$ when a fault at node $n_j \in \mathcal{N}$ triggers the PD node $n_i \in \mathcal{P}$. Assuming perfect coordination of all PDs, n_i is the closest node in $\mathcal{P}$ located upstream of n_j. By applying the expression (16) in the Appendix to the set $\mathcal{P}$ the following expression for C is obtained:

$$C_{ij} = \begin{cases} 1 & \text{when } \left[S\left(\mathcal{P}, \mathcal{P}\right) \cdot \left(S - I\right)\right]_{ij} = 1 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Note that, as a PD does not clear its own fault, $S_{ii} = 0$.

When a fault at node n_j is cleared by the PD node n_i the service is interrupted to the subset of nodes $\mathcal{L}^{(j)} \subseteq \mathcal{L}$, which can be algebraically obtained as the j-th column of the matrix $[S^t\left(\mathcal{P}, \mathcal{L}\right) \cdot C]$. As $\left[C \cdot r\right]_i$ is the tripping probability of PD node $n_i \in \mathcal{P}$, $\left[S^t\left(\mathcal{P}, \mathcal{L}\right) \cdot C \cdot r\right]_i$ is the interruption probability of SS node $n_i \in \mathcal{L}$. Figure 2 shows, superimposed, the binary matrices S (dots) and C (circles) for the tutorial example. Note that there is no circle in the first column, as a fault of the CB at the head (n_1) is not cleared by any feeder component. Table II collects, for any faulted component in the tutorial example, the node that clears the fault and the nodes whose service is interrupted as a consequence.

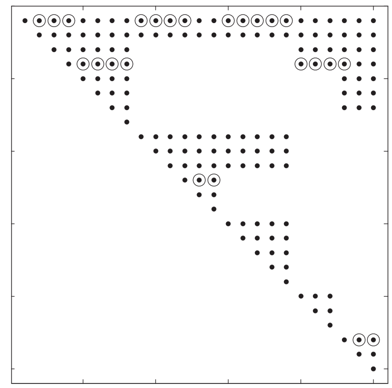

Fig. 2. Visualization of matrices S($\cdot$) and C($\circ$)

TABLE II
NODES INVOLVED IN EVERY POSSIBLE FAULT

Faulted node	Clearing PD	Interrupted SS
2,3,4,9,10,11,12, 15,16,17,18,19	1	14,16,19,22,25
5,6,7,8,20,21,22,23	4	22,25
13,14	12	14
24,25	23	25

C. Knowledge extraction

Given the static data (tree graph and associated matrices S and C, along with the sets $\mathcal{L}$, $\mathcal{G}$, $\mathcal{P}$, $\mathcal{P}_s$, $\mathcal{P}_r$, $\mathcal{D}$, $\mathcal{D}_o$, and $\mathcal{W}$, any other information dynamically collected at the DMS must be processed in real time to produce as much knowledge as possible regarding the faulted zone and interrupted customers. Each source of information is discussed below separately.

1) Protection tripping signals: Let Π be the set of PDs from which a tripping signal is received (if the PDs are duly coordinated, this set is either empty or contains at most one element). The set $\mathcal{T}_\Pi \subseteq \mathcal{P}$, denoting the PDs whose tripping decision is compatible with the information provided by Π is obtained as follows: if $\text{nnz}\left(\Pi\right) = 1$ (i.e., a field signal is received) then $\mathcal{T}_\Pi = \Pi$; otherwise $\mathcal{T}_\Pi = \mathcal{P} \setminus \mathcal{P}_s$, which is simply the set of PDs without signaling capability.

2) Fault passage detection signals: As stated above, both directional (i.e., oriented) and nondirectional FPDs are considered.

In the presence of downstream DG, nondirectional FPDs signaling provides only the evidence of a fault, but gives no clues about its location. Therefore, useful information about the occurrence of a downstream fault is obtained only from oriented FPDs, or nondirectional FPDs in absence of DG downstream, which are stored in $\mathcal{D}_d \in \mathcal{D}$. Algebraically, the i-th element of D_d will be 1 when $n_i \in \mathcal{D}$ and whenever, $n_i \in \mathcal{D}_o$ or $\text{nnz}\big(\left[S\left(\mathcal{D}, \mathcal{G}\right)\right]_{ix}\big) = 0$. FPDs in $\mathcal{D}_d$ for which downstream fault signaling is received, are stored in Δ_d.

Similarly, univocal evidence of upstream faults can only be signaled by nodes $n_i \in \mathcal{D}_o$ with DG downstream, which are stored in $\mathcal{D}_u \subseteq \mathcal{D}_o$. Algebraically, the i-th element of D_u will be 1 when $\text{nnz}\big(\left[S\left(\mathcal{D}_o, \mathcal{G}\right)\right]_{ix}\big) > 0$. FPDs in $\mathcal{D}_u$ for which upstream fault signaling is received, are stored in Δ_u.

Let $\mathcal{F}_\Delta \subset \mathcal{N}$ denote the minimal set of nodes whose fault is consistent with the information provided by Δ_d and Δ_u. A node n_i will belong to $\mathcal{F}_\Delta$ when the following four conditions are simultaneously satisfied:

1) It is downstream of all nodes in Δ_d:

$$\left[\left(S^t - I\right) \cdot \Delta_d\right]_i = \text{nnz}\left(\Delta_d\right)$$

2) It is not downstream of any node in $\mathcal{D}_d$ which is not in

$\varDelta_d$:

$$\left[\left(S^t - I\right) \cdot \left(D_d \circ \overline{\Delta_d}\right)\right]_i = 0$$

3) It is not downstream of any node in $\varDelta_u$:

$$\left[S^t \cdot \Delta_u\right]_i = 0$$

4) It is simultaneously downstream of every node in $\mathcal{D}_u$ which is not in $\varDelta_u$:

$$\left[S^t \cdot \left(D_u \circ \overline{\Delta_u}\right)\right]_i = \mathrm{nnz}\left(D_u \circ \overline{\Delta_u}\right)$$

The set $\mathcal{T}_\Delta \subset \mathcal{P}$ contains the set of PD nodes whose tripping is compatible with the information provided by $\varDelta_d$ and $\varDelta_u$ (i.e., with a fault originated by a node in the set $\mathcal{F}_\Delta$). In other words, the i-th element of T_Δ will be 1 when $\left[C \cdot F_\Delta\right]_i > 0$.

Table III shows, for any faulted node in the tutorial example, the possible signals provided by PDs and FPDs, along with the knowledge which can be algebraically extracted from this information.

TABLE III
KNOWLEDGE EXTRACTION FROM PD AND FPD SIGNALS

Faulted node	$\varPi$	$\varDelta_d$	$\varDelta_u$	$\mathcal{T}_\Pi$	$\mathcal{T}_\Delta$
2,3,4,9,10,11,12,15, 16,17,18,19	1	-	6	1	1,4,12
1,5,13,14,20,21,22,	-	-	6	4,12,23	4,12
7,8,23,24,25	-	6	-	4,12,23	4,23
6	-	-	-	4,12,23	4

3) Service status of secondary substations: The nodes in $\mathcal{L}$ for which its service status is certain are classified in two subsets: $\varLambda_{off}$ (service interrupted) and $\varLambda_{on}$ (in service). The knowledge set $\mathcal{T}_\Lambda \subseteq \mathcal{P}$ contains the PD nodes whose tripping is compatible with this information.

If the service interruption is originated by the tripping of a PD, it must be located upstream of all nodes in $\varLambda_{off}$. Reciprocally, any PD located upstream of the nodes in $\varLambda_{on}$ has not originated the interruption. Therefore, the i-th element of T_Λ will be 1 when $\mathrm{nnz}\left(\left[S\left(\mathcal{P}, \varLambda_{off}\right)\right]_{ix}\right) = \mathrm{nnz}\left(\varLambda_{off}\right)$ and $\mathrm{nnz}\left(\left[S\left(\mathcal{P}, \varLambda_{on}\right)\right]_{ix}\right) = 0$.

Based on the three information sources (signals from PDs, FPDs and customer calls), the set $\mathcal{T}_\Sigma \subseteq \mathcal{P}$, containing the PDs whose tripping is fully compatible with all available information, can be algebraically obtained as follows:

$$T_\Sigma = \left(T_\Pi \circ T_\Delta \circ T_\Lambda\right) \tag{5}$$

V. APPLICATION TO IDENTIFICATION AND ISOLATION OF FAULTED ZONE

A. Identification of faulted status

Under normal conditions, signaling-related sets ($\varPi$, $\varDelta_d$ and $\varDelta_u$), as well as the set of interrupted customers ($\varLambda_{off}$) are empty. Figure 3 presents the proposed procedure to follow by the DMS to determine the supply status, which can be either the normal state, as a default initial state or after having been recovered from a temporary fault (successful reclosing), or a faulted state, after permanent interruption is confirmed and bounded. The procedure initiates when service interruption

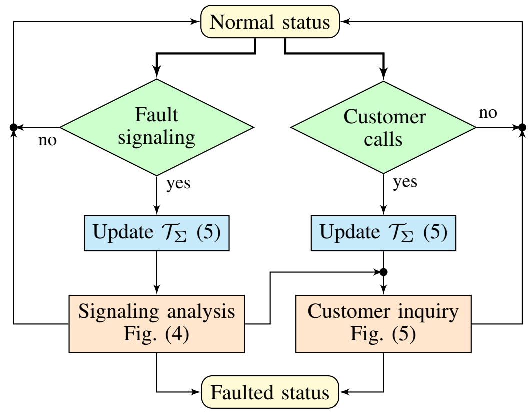

Fig. 3. Faulted status identification procedure

evidence comes from remote signaling devices or customer calls, as illustrated below:

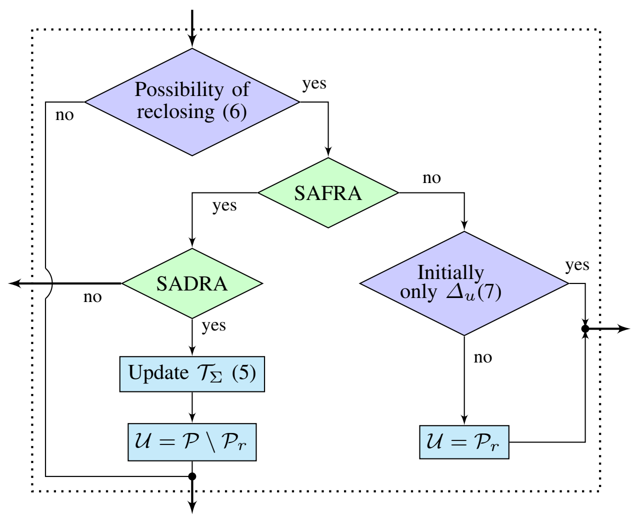

Fig. 4. Signaling analysis module

1) Fault signaling: When a fault signal is received from a PD or FPD, before entering the module for signaling occurrence deduction, $\mathcal{T}_\Sigma$ must be determined according to expression (5) with updated terms. This module, illustrated in Figure 4, shows three possible outcomes depending on the possibility of reclosing and the sequence of signals in the event. Those possible signals may arrive either after a first fast attempt (SAFRA stands for this Signaling After a Fast Reclosing Attempt), or later, after a delayed attempt (SADRA stands for Signaling After Delayed Reclosing Attempt).

The bottom output, leading to faulted status, is achieved either when no PD in $\mathcal{T}_\Sigma$ has reclosing capability (i.e., eq. (6) not fulfilled),

$$\left(T_\Sigma\right)^t \cdot P_r > 0 \tag{6}$$

or when persistent signaling SAFRA and SADRA is received after reclosing. In the latter case, additional knowledge can be obtained from:

- The mere reception of SAFRA means that the tripped PD has reclosing capability. For future use, it is convenient to

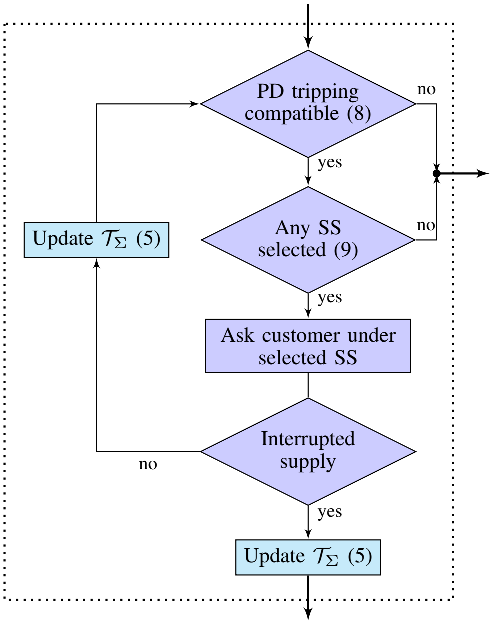

Fig. 5. Customer inquiry module

store this information in an explicit fashion, by defining the set $\mathcal{U}$ containing the PDs whose tripping is not compatible with a permanent interruption situation, taking into account all the additional knowledge not stored in $\mathcal{T}_\Sigma$. In this case, $\mathcal{U} = \mathcal{P} \setminus \mathcal{P}_r$.

- The possible change of SAFRA contents with respect to the initial signaling, originated by the DG disconnection after the initial fault event to avoid islanding, by updating $\mathcal{T}_\Sigma$ using Δ_d from SAFRA, and $\mathcal{D}_d$ for the disconnected DG.

The leftmost output, leading to normal state, is reached when SAFRA is received but delayed reconnection succeeds, giving no SADRA after the expected delayed reconnection time.

Finally, if $\mathcal{T}_\Sigma$ contains any recloser fulfilling (6), but no SAFRA is received, then neither normal nor faulted status can be concluded in this module, being it necessary to enter the customer inquiry module, as shown in Figure 5. However, some additional knowledge could still be extracted, except for certain cases with embedded DG.

When the reclosing fails under persistent faulty conditions and initial signaling was not only driven by DG, SAFRA would not be feasible. Algebraically,

$$\mathrm{nnz}(\Pi + \Delta_d) = 0 \tag{7}$$

So, if (7) is not fulfilled, which implies that signals other than Δ_u were received, SAFRA would only be missing in two cases, a) when the tripping PD has not reclosing capability; b) when it has, and it succeeded in the reclosing attempt. In other words, permanent service interruption status can only be explained by the actuation of a PD without reclosing capability, which is an additional knowledge algebraically expressed as $\mathcal{U} = \mathcal{P}_r$.

If $\mathcal{T}_\Sigma$ contains a PD not shared with $\mathcal{U}$, it is possible that the acting PD remains permanently open. This condition implies that

$$(T_\Sigma)^t \cdot U < \mathrm{nnz}(T_\Sigma) \tag{8}$$

Otherwise, the recovery of supply by reconnection of the PD is the only situation compatible with the knowledge created so far.

As long as (8) is fulfilled, to ascertain the supply status, it will be necessary to assess the service status of the SS (set $\mathcal{L}$), in order to confirm a service interruption with the help of customer feedback. For this purpose, the sequence of customers to contact can be based on recorded failure rates for the set of components (nodes) protected by each PD. Nodes corresponding to largest (positive) values in the vector

$$S^t \left(\mathcal{T}_\Sigma \setminus \mathcal{U}, \mathcal{L}\right) \cdot C \cdot (r \circ F_\Delta) \tag{9}$$

should be ranked first.

If any LV customer fed from the first SS in the priority list confirms the service interruption, then there is no need to continue, exiting the module through its bottom output. Otherwise, once the sets $\mathcal{T}_\Lambda$ and $\mathcal{T}_\Sigma$ are duly updated with the new information, another customer of the next node in $\mathcal{L}$, ranked according to (9), should be contacted. Eventually, if (8) is fulfilled or (9) returns a null vector during this process, it can be concluded that the fault was temporary (successful reclosing), exiting through the rightmost output.

2) Customer calls: When a permanent fault is not perceived by any FPD and it is cleared by a PD without signaling capability, the DMS has no immediate knowledge about the supply interruption status. Thus, when the DMS has no previous signaling and receives any customer call claiming supply interruption, the procedure in Figure 3 leads to the customer inquiry module (Figure 5). This module is initiated with $\mathcal{U} = \emptyset$ and Λ_{off} including the SS feeding the claiming customers. In this case, if normal status is concluded, the fault is limited to the LV subsystem fed by the SS, but is not caused by a PD tripping.

B. Identification of the suspicious node set

The information used to conclude the faulted status can also be used by the DMS to determine which elements are suspicious of having caused the fault, [34]. The suspicious node set (SNS), denoted as $\mathcal{F}_\Sigma \subset \mathcal{N}$, contains all nodes which are prone to have failed (note that one of them actually failed). Algebraically:

$$F_\Sigma = \left(C^t \cdot \left(T_\Sigma \circ \overline{U}\right)\right) \circ F_\Delta \tag{10}$$

which takes into account the nodes that could have caused the available information on tripping, either through positive (T_Σ) or negative knowledge (U), and the information about the failure acquired from FPDs, F_Δ.

The tutorial example can be used to illustrate the SNSs associated with different initial signals and the subsequent procedures. Table IV shows the different combinations of driving field signals (columns Π, Δ_d, Δ_u), and the intermediate information obtained during the detection procedure (fulfillment of (7), existence of SAFRA, and the inquired customers together with their answers). In the cases of faulted status, $\mathcal{F}_\Sigma$ is determined using (10) and shown in the rightmost column. A dash is shown in the case of normal status. Note that (6) is fulfilled in all the considered cases.

TABLE IV
SUSPICIOUS NODE SETS FOR DIFFERENT SCENARIOS INITIATED BY FIELD SIGNALS

Π	Δ_d	Δ_u	(7)	SAFRA	Selected SS (feed)	$\mathcal{F}_\Sigma$
1		6	No	Yes	-	2,3,4,9,10,11,12, 15,16,17,18,19
-	6	-	No	Yes	-	7,8,23
				No	25(on)	-
					25(off)	24,25
-	-	6	Yes	No	25(off)	5,20,21,22
					25(on),14(on)	-
					25(on),14(off)	13,14

The same tutorial case is used to illustrate the procedure after customer calls. Table V shows the procedure results for different customer claims and scenarios with and without DG (On or Off). When the claim is compatible with the lack of field signals, further customers are inquired if necessary to obtain $\mathcal{F}_\Sigma$. Note that claims of customer at n_{16} and n_{19} are not listed, as they are not compatible with the lack of previous signaling.

TABLE V
SUSPICIOUS NODE SETS FOR DIFFERENT SCENARIOS INITIATED BY CUSTOMER CALLS

SS claim	GD	Compatible with no signaling	Selected SS (feed)	$\mathcal{F}_\Sigma$
22	On	Yes	25(off)	6
	Off	Yes	25(off)	5,6,20,21,22
25	On	Yes	22(off)	6
	Off	Yes	22(off)	5,6,20,21,22
14	On	No	-	-
	Off	Yes	-	13,14

C. Reduction of the suspicious node set

To further reduce the SNS before starting the fault location task, the DMS can make use of two kinds of additional actions: asking customers for their supply statuses, and operating switching devices. With the information obtained from these actions, a previous SNS, $\mathcal{F}_\Sigma^{prev}$, can be reduced to $\mathcal{F}_\Sigma^{post}$. The consideration of the cost of the different actions is out of the scope of this paper.

By applying a binary search (divide and conquer) methodology, [35], the desiderable action should let the failure rate of $\mathcal{F}_\Sigma^{post}$ as close as possible to that of its complementary set, $\mathcal{F}_\Sigma^{prev} \setminus \mathcal{F}_\Sigma^{post}$, wich is fulfilled by finding the action that maximizes the product of both failure rates

$$\left[\left(F_\Sigma^{post}\right)^t \cdot r\right] \cdot \left[\left(F_\Sigma^{prev} \circ \overline{F_\Sigma^{post}}\right)^t \cdot r\right] \tag{11}$$

1) Selection of customers to inquire: When the SNS is partially downstream of a PD, some customer inquiries could be helpful in reducing the SNS. When an inquired customer, $n_l \in \mathcal{L}$, grants the continuity of supply of its feeding SS, all the nodes whose failure causes the tripping of any PD located upstream n_l can be excluded from SNS. Otherwise, if its supply is interrupted, the SNS can be reduced to just those nodes capable of tripping PDs located upstream n_l. The algebraic closed-form expression for $\mathcal{F}_\Sigma^{post}$, in terms of $\mathcal{F}_\Sigma^{prev}$, n_l, and the answer to the query on its feeding status, is as follows:

$$F_\Sigma^{post} = \begin{cases} F_\Sigma^{prev} \circ \left([S^t \cdot C]_{lx}\right)^t & \text{if } n_l \text{ is interrupted} \\ F_\Sigma^{prev} \circ \left(N^t - [S^t \cdot C]_{lx}\right)^t & \text{if not interrupted} \end{cases} \tag{12}$$

According to (11) and (12), the proposed n_l for the inquiry should be chosen among the indices with maximum positive value in the vector (13),

$$L \circ \left[\left(S^t \cdot C\right) \cdot \hat{r}\right] \circ \left[\left(N^t \cdot \hat{r}\right) \cdot N - \left(S^t \cdot C\right) \cdot \hat{r}\right] \tag{13}$$

where $\hat{r} = r \circ F_\Sigma^{prev}$ is the failure rate of the nodes in $\mathcal{F}_\Sigma^{prev}$, since any answer of the proposed query yields the same value in (11) due to complementarity in the possible sets $\mathcal{F}_\Sigma^{post}$ in (12). Note that if (13) is an empty vector, no customer inquiries provide useful information to reduce the SNS.

In order to illustrate an application, let us consider the example network in case the PD in n_4 does not have (or has temporarily lost) its reclosing capability, the only signal received comes from a FPD ($\Pi = \emptyset$), and a flat failure rate r is considered. Table VI shows the two possible signaling sets, that never satisfy (6), leading to the permanent interruption after tripping of PD at n_4. The fourth column shows $\mathcal{F}_\Sigma^{prev}$, while the last column shows $\mathcal{F}_\Sigma^{post}$ after processing the response of customers fed by the SS at n_{22}, selected according to (13).

TABLE VI
SNS REDUCTION BY CUSTOMER INQUIRIES

Π	Δ_d	Δ_u	$\mathcal{F}_\Sigma^{prev}$	SS selected (feed)	$\mathcal{F}_\Sigma^{post}$
-	6	-	7,8,23,24,25	22 (off)	7,8,23
				22 (on)	24,25
-	-	6	5,13,14, 20,21,22	22 (off)	5,20,21,22
				22 (on)	13,14

2) Selection of switching devices: When the SNS is partially downstream a switching device, $n_w \in \mathcal{W}$, then maneuvers on that switching device, or test node [34], allow reducing the SNS. The success or not of the restoration attempt after opening n_w makes it possible to deduce whether the faulted element is located in the shaded network section (nodes downstream n_w) or in the energized section [36]. The algebraic expression for $\mathcal{F}_\Sigma^{post}$ in terms of $\mathcal{F}_\Sigma^{prev}$, n_w and the switching maneuver success is as follows:

$$F_\Sigma^{post} = \begin{cases} (F_\Sigma^{prev}) \circ \left([S-I]_{wx}\right)^t & \text{it tryout succeeds} \\ (F_\Sigma^{prev}) \circ \left(N^t - [S-I]_{wx}\right)^t & \text{if tryout fails} \end{cases} \tag{14}$$

According to (11) and (14), and similarly to (13), the proposed node, n_w, for switching, will be chosen among the indices with maximum positive value in the vector:

$$W \circ \left[(S-I) \cdot \hat{r}\right] \circ \left[\left(N^t \cdot \hat{r}\right) \cdot N - (S-I) \cdot \hat{r}\right] \tag{15}$$

Two SNSs in Table IV will be used to illustrate this procedure. Table VII shows those sets, as $\mathcal{F}_{\Sigma}^{prev}$, in the first column. The second column shows the sequence of test nodes chosen according to (15) and the restoration attempt results. The last column shows the $\mathcal{F}_{\Sigma}^{post}$ finally obtained after each sequence.

TABLE VII
SNS REDUCTION BY SWITCHING DEVICES

$\mathcal{F}_{\Sigma}^{prev}$	Sequence of SD (tryout failure)	$\mathcal{F}_{\Sigma}^{post}$
2,3,4,9,10,11,12, 15,16,17,18,19	9(yes)	2,3,4,9
	9(no)-17(no)	18,19
	9(no)-17(yes)-11(no)	12,15,16,17
	9(no)-17(yes)-11(yes)	10,11
5,20,21,22	20 (yes)	5,20
	20 (no)	21,22

VI. Real-life case study

A semi rural MV feeder, delivering electricity to 2,205 customers through 77 SS, is used as case study. The actual feeder, totaling 54.5 km in length, has been slightly modified to better illustrate the application of the proposed methodology under any evidence of fault condition. Figure 6 shows the one-line diagram of the feeder, comprising 285 nodes, which is directly built from the existing geographic information system (GIS); however, for the sake of clarity, the lengths of branches in the diagram do not reflect the real distances. Following the notation of Figure 1, the SS and PD nodes are shown with triangles and squares, respectively, while the symbol '/' is adopted for switches. The components where failures will be assumed are shown with nearby 'spark' symbols. Network elements to be identified in the subsequent analysis are labelled with a number.

In addition to the customary PD at the head of the feeder, seven more PDs are located downstream, four of them fuses at nodes $\{n_{110}, n_{158}, n_{258}, n_{264}\}$. The feeder layout first runs through a rural area, and then feeds an urban settlement downstream the PD at n_{162}, with neither reclosing nor signaling capability. The urban area comprises 30% of the total feeder length and 80% of the customers. The PD at n_{36} does count with reclosing capability, as it is located approximately at half of the total feeder length and is intended to clear faults originating mainly from an overhead layout; however, it is not equipped with signaling capability. Finally, the PD at n_{13}, without reclosing or signaling capabilities, clears the faults within an area of agricultural activity.

An oriented FPD at n_{179} provides directional fault signaling in case of failure of any feeder component, while it additionally procures useful information about the fault location within the urban area. It is worth noting that a DG photovoltaic plant is placed downstream of the FPD, at n_{195}.

Several examples of faulty conditions on this feeder are presented below in order to illustrate how the proposed methodology helps the DMS to first determine the supply continuity status, as described in Section V-A, and, secondly, to isolate the faulted zones, as described in Section V-C. These examples use statistical failure rates from [11].

In what follows, $\mathcal{C}(i)$ will refer to the set of nodes whose failure is cleared by the PD at node n_i, its respective vector array being the i-th row of matrix C. Table VIII shows, for every set $\mathcal{C}(i)$, the number of customers and secondary substations affected, the total line length and the computed value of failures per year. The network components involved in the different $\mathcal{C}(i)$ sets have been colored in Figure 6 for a better visualization.

TABLE VIII
SUBSETS OF NODES WHOSE FAILURE IS CLEARED BY THE SAME PD

Subset	Customers (units)	SS (units)	Length (km)	Failures per year
$\mathcal{C}(1)$	63	12	12.69	2.55
$\mathcal{C}(13)$	62	12	12.11	2.34
$\mathcal{C}(36)$	286	20	11.95	2.30
$\mathcal{C}(110)$	3	1	0.06	0.01
$\mathcal{C}(158)$	21	1	0.72	0.15
$\mathcal{C}(162)$	1755	29	16.36	3.02
$\mathcal{C}(258)$	1	1	0.26	0.03
$\mathcal{C}(264)$	14	1	0.31	0.06

A. Signaled fault cases

1) Failure at n_{55}: In case of failure at node n_{55}, a notification is received from the PD at the head of the feeder, $\Pi = \{n_1\}$, as well as from the FPD (n_{179}) if the DG is operative, the latter signal being not useful to generate additional knowledge. If the reclosing of the PD is not successful, a signal will be received after the preset number of reconnection attempts and, afterwards, the faulted status affecting all the customers in the feeder could be concluded. This way, the SNS contains all the elements in $\mathcal{C}(1)$.

Obviously, inquiring sampled clients about their supply status offers no useful information to reduce the SNS, as (13) results in a null vector. According to the vector calculated with (15), the first switching device to be maneuvered will be at n_{24}, then the second at n_{48} and, finally, the third at n_{68}. This way, the SNS is reduced from those nodes in the initial 12.7 km feeder section to only the set of nodes downstream n_{48} but not downstream of n_{68}, containing a line length of just 0.8 km.

2) Failure at n_{269}: The signaling received by the DMS in this case is $\Delta_d = \{n_{179}\}$ with $\Pi = \emptyset$. According to (5), $\mathcal{T}_\Sigma = \{n_{162}, n_{258}\}$ and, as (6) is not fulfilled, the faulted status is concluded. In order to reduce the SNS, (13) is evaluated, from which it is concluded that asking any customer in $\mathcal{C}(162)$ gives the same information, namely their continuity of supply. Consequently, $\mathcal{T}_\Sigma$ reduces to $\{n_{258}\}$.

3) Failure at n_{205}: The starting situation is the same as in the previous case, but in this case the inquired client informs of its supply being interrupted, and therefore $\mathcal{T}_\Sigma = \{n_{162}\}$. The SNS contains all the nodes downstream the FPD n_{179}, except those downstream the fuse at n_{258}. Expression (15) provides the switching sequence n_{200}, n_{225} and n_{224} to reduce the SNS length from 7.4 km to 1.4 km.

4) Spurious failure at n_{137}: The DMS receives upstream oriented signaling from the FPD, $\Delta_u = \{n_{179}\}$ with $\Pi = \emptyset$, so according to (5), $\mathcal{T}_\Sigma = \{n_{13}, n_{36}, n_{110}, n_{158}, n_{162}, n_{264}\}$.

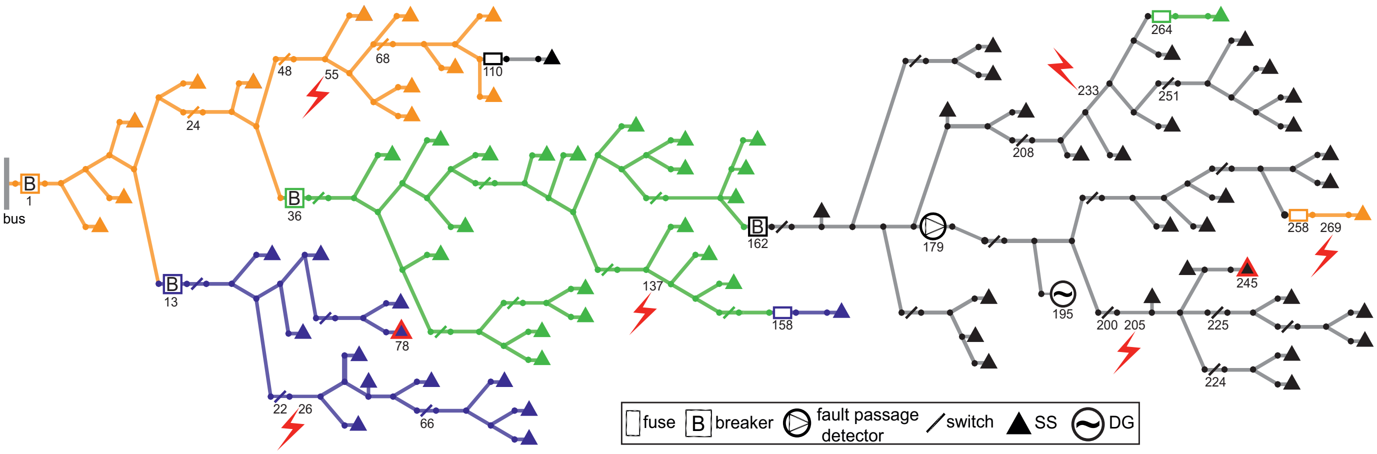

Fig. 6. One-line diagram of the real-life case study

As the PD at n_{36} has reclosing capability, (6) is satisfied, and since (7) is also verified, the supply status cannot be ascertained in the signaling analysis module (Figure 4), so the customer inquiry module must be launched, with $\mathcal{U} = \emptyset$. As the failure condition is spurious, the reclosing attempt has been successful; however, at this point, the DMS is still uncertain about a possible interruption affecting 97% of customers, or not interruption at all.

According to the vector obtained with expression (9), the first customer to be inquired can be any customer in $\mathcal{C}(264)$, that will report its continuity of supply. This new information reduces $\mathcal{T}_\Sigma$ to $\{n_{13}, n_{110}, n_{158}\}$.

Further inquiring is required to reach the normal status conclusion. This way, by cycling in the customer inquiry module (Figure 5), the next customer to be asked can be anyone in $\mathcal{C}(13)$, followed by anyone in $\mathcal{C}(158)$ and finally, any of those in $\mathcal{C}(110)$.

B. Customers call cases

When the DG is in operation, the lack of signaling implies continuity of supply. However, in case the PV plant is not operative (e.g., at nights), no signaling would be compatible with the tripping of the PDs at $\{n_{13}, n_{36}, n_{110}, n_{158}, n_{162}, n_{264}\}$. The following examples illustrate the latter condition:

1) Failure at n_{26}, interruption complaint by customer at n_{78}: After the customer's call, the DMS begins the identification procedure with $\Delta = \Pi = \emptyset$ and $\Lambda_{off} = \{n_{78}\}$. By applying (5), $\mathcal{T}_\Sigma = \{n_{13}\}$ is obtained. In this situation, the customer inquiry module is launched with $\mathcal{U} = \emptyset$, finding out that the faulted status must be confirmed by further inquiring customers in $\mathcal{C}(13)$, according to (9). Finally, the switching sequence to reduce the initial SNS, totaling a length of 12.1 km, is obtained from (15). It starts with the switch at n_{22} and is followed by the one at n_{66}. This way, the SNS line-length is reduced to 3.2 km.

2) Failure at n_{233}, interruption complaint by customer at n_{245}: In this case, the initial calculation of (5) results in $\mathcal{T}_\Sigma = \{n_{36}, n_{162}\}$. Expression (9) proposes that any customer in $\mathcal{C}(162)$ should be inquired, which confirms the faulted status.

Later on, expression (13) proposes to ask any customer in $\mathcal{C}(36)$, discarding any interruption in these nodes; and finally, in accordance to (15), first switching n_{208} and afterwards n_{251}, reduces the SNS line-length to 2.5 km.

C. Computational complexity

Handling real networks with the proposed methodology entails the use of sparse structures. The successor matrix S for this case study, as shown in Figure 7, has $n^{1.6}$ non-zero elements, where n is the size of S.

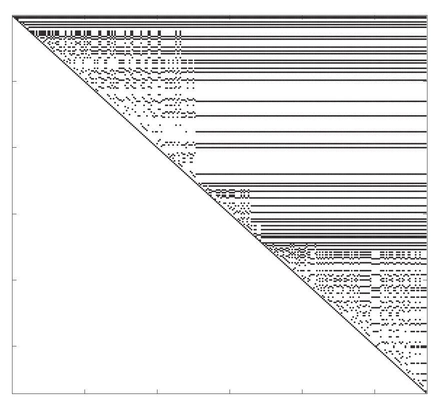

Fig. 7. Visualization of matrix S for the 285-node case study

By far the most demanding computational step in the proposed framework is the calculation of matrix S. Note however that this step can be performed off-line, by applying very efficient graph search algorithms. Anyway, even if S is obtained by a brute-force approach, such as algebraically inverting the sparse incidence matrix, it takes less than 5 milliseconds on a 3.20-GHz, Intel® Core™ i5-4460-based laptop, for the above example with 285 nodes. On this computer, each of the failure examples discussed above took less than 1 millisecond for all calculations performed online.

VII. Conclusions

This paper has presented an algebraic framework to systematically deal with the stream of information reaching the DMS during faulty conditions. Signals arriving from field devices as well as the information obtained from customer calls are managed using a matrix-based procedure allowing, first, to determine the supply status of the distribution grid exclusively using algebraic expressions; and second, to assist the DMS operator in decision making for identification and isolation of the affected network sections. The different procedures discussed in the paper are illustrated, first, by means of a tutorial example, and then through several simulated faults on a real-life network, showing the potential of the proposed methodology to be implemented as a useful tool for real-time outage management. The flexibility of the matrix-based procedure opens the door to address specific challenges, such as taking into account the cost-benefit balance of the different actions to consider.

Appendix: Matrix algebra

- $[S-I]_{ij}=1$ when $n_j \in N$ is downstream $n_i \in N$.
- $[S^t-I]_{ij}=1$ when $n_j \in N$ is upstream $n_i \in N$.
- $[S\cdot(S-I)]_{ij}$ is the graph distance between $n_j \in N$ and its upstream node $n_i \in N$.
- $[(S^t-I)\cdot X]_i$ is the number of nodes in $\mathcal{X}$ upstream $n_i \in \mathcal{N}$.
- $[(S-I)\cdot X]_i$ is the number of nodes in $\mathcal{X}$ downstream $n_i \in \mathcal{N}$.
- $\text{nnz}\left([S(\mathcal{X},\mathcal{Y})]_{ix}\right)$ is the number of nodes in $\mathcal{X}$ downstream $n_i \in \mathcal{Y}$.
- The expression

$$\left[S(\mathcal{X},\mathcal{Y})\cdot(S-I))\right]_{ij} \tag{16}$$

provides the number of nodes in $\mathcal{Y}$ which are contained in the path between $n_i \in \mathcal{X}$ and $n_j \in \mathcal{N}$ (including n_i but not n_j).

References

[1] J. Casazza and F. Delea, *Understanding Electric Power Systems. An Overview of the Technology, the Marketplace, and Government Regulation*. Wiley-IEEE Press, 2003.

[2] I. Waseem, M. Pipattanasomporn, and S. Rahman, "Reliability benefits of distributed generation as a backup source," *Power & Energy Society General Meeting, 2009. PES '09. IEEE*, 2009.

[3] P. Sritakaew, A. Sangswang, and K. Kirtikara, "On the reliability improvement of distribution systems using pv grid-connected system," *ECTI Transactions on Electrical Eng., Electronics, and Communications Vol.5, No.1*, 2007.

[4] N. Balijepalli, S. Venkata, and R. Christie, "Modeling and analysis of distribution reliability indices," *IEEE Transactions on Power Delivery, 19*, Oct 2004.

[5] P. Costa and M. Matos, "Assessing the contribution of microgrids to the reliability of distribution networks," *Electric Power Systems Research, Elsevier*, 2008.

[6] J. Chen and C. Chen, "A neural network approach to evaluate distribution system reliability," *Electric Power Systems Research, Elsevier*, 1992.

[7] M. Mohammoud, N. Ahmed, and P. Sazali, "Reliability evaluation of distribution power systems based on artificial neural network techniques," *Journal of Electrical and Computer Engineering*, 2011.

[8] A. Chowdhury and D. Koval, *Power Distribution System Reliability: Practical Methods and Applications*. Wiley-IEEE Press, 2009.

[9] S. Jamali and H. Shateri, "Optimal siting of recloser and sectionalizers to reduce non-distributed energy," in *Transmission and Distribution Conference and Exhibition: Asia and Pacific, IEEE/PES*, 2005.

[10] W. Li, P. Wang, Z. Li, and Y. Liu, "Reliability evaluation of complex radial distribution systems considering restoration sequence and network constraints," *IEEE Transactions on Power Delivery, Vol. 19, No. 2*, 2004.

[11] J. Rosendo, A. Gomez Exposito, G. Tevar, and M. Rodriguez, "Evaluation and improvement of supply reliability indices for distribution networks," *Transmission and Distribution Conference and Exposition, Chicago*, 2008.

[12] H. Zhenga, Y. Chenga, B. Goub, D. Frankc, A. Bernc, and W. Mustonc, "Impact of automatic switches on power distribution system reliability," *Electric Power Systems Research. Elsevier*, Oct 2011.

[13] R. Billinton and P. Wang, "Reliability-network-equivalent approach to distribution-system-reliability evaluation," *IEE Proceedings - Generation, Transmission and Distribution, Vol. 145, No. 2*, 1998.

[14] J. E. Romero and A. Vargas, "Inference of operative configuration of distribution networks using fuzzy logic techniques. part i: Real-time model," *IEEE Transactions on Power Systems, vol.20, N.3, August*, 2005.

[15] A. Barabadi, "Reliability model selection and validation using weibull probability plot - a case study," *Electric Power Systems Research, Elsevier*, 2013.

[16] R. Brown and J. Ochoa, "Distribution system reliability: Default data and model validation," *IEEE Transactions on Power Systems, Vol. 13, No. 2*, 1998.

[17] M. Saha, J. Izykowski, and E. Rosolowski, "Fault location on power networks," *Springer-Verlag London Limited, London*, 2009.

[18] W. Kersting, "System modeling and analysis," *CRC Press, Taylor&Francis Group, Boca Raton, FL, USA*, Jul 2000.

[19] J. Haakana, J. Lassila, T. Kaipia, and J. Partanen, "Comparison of reliability indices from the perspective of network automation devices," *IEEE Transactions on Power Delivery, vol. 25, N. 3, july*, 2010.

[20] R. Stillman, "Modeling failure data of overhead distribution system," *IEEE Transactions on Power Delivery, 15 (4)*, Oct 2000.

[21] R. E. Brown, *Electric Power Distribution Reliability*. ABB Inc. Raleigh, North Carolina, 2002.

[22] Z. Boor and S. Hosseini, "Optimal placement of dg to improve the reliability of distribution systems considering time varying loads using genetic algorithm," *Majlesi Journal of Electrical Engineering*, 2013.

[23] A. Pregelj, M. Begovic, A. Rohatgi, and D. Novosel, "On optimization of reliability of distributed generation-enhanced feeders," *Proceedings of the 36th Annual Hawaii International Conference on System Sciences*, 2003.

[24] D. Smith, *Reliability, Maintainability and Risk: Practical Methods for Engineers*. Elsevier, 2011.

[25] J. Bouford, "The need to segment abnormal events from the calculation of reliability indices," *IEEE Full Use Guide for Electric Distribution Reliability Indices, IEEE Standard*, 2001.

[26] A. Moradi, M. Fotuhi-Firuzabad, and Rashidi-Nejad, "A reliability cost/worth approach to determine optimum switching placement in distribution systems," *IEEE/PES Transmission and Distribution Conference and Exhibition, Asia and Pacific, Dalian, China*, 2005.

[27] P. Anderson, *Power System Protection*. IEEE Press Power EngineeringSeries, 1999.

[28] N. R. Kulkarni, "A review on autorecloser control methodology for improved distribution reliability," *International Journal of Scientific and Research Publications, Volume 4, Issue 4, April*, 2014.

[29] Y. Chollot, J.-M. Biasse, and A. Malot, "Feeder automation improves medium voltage network operating efficiency," *CIRED Seminar 2008: SmartGrids for Distribution, Frankfurt*, 2008.

[30] A. Shahsavari, M. Farajollahi, E. Stewart, C. Roberts, F. Megala, L. Alvarez, E. Cortez, and H. Mohsenian-Rad, "Autopsy on active distribution networks: A data-driven fault analysis using micro-pmu data," in *2017 North American Power Symposium (NAPS)*, Sept 2017.

[31] G. Celli, E. Ghiani, F. Pilo, and G. Soma, "Reliability assessment in smart distribution networks," *Electric Power Systems Research, Elsevier*, 2013.

[32] Y. Liao, "A nove method for locating faults on distribution systems," *Electric Power Systems Research, Elsevier*, 2014.

[33] J. A. Resh, "The inverse of a nonsingular submatrix of an incidence matrix," vol. 10, no. 1, pp. 131–132, Mar 1963.

[34] G. Li, H. Wu, and F. Wang, "Bayesian network approach based on fault isolation for power system fault diagnosis," *2014 International Conference on Power System Technology (POWERCON 2014) Chengdu*, 2014.

[35] C. Wasson, *System Engineering Analysis, Design, and Development: Concepts, Principles, and Practices, 2nd Edition*. Wiley, 2015.

[36] S. Kazemi, M. Lehtonen, and M. Fotuhi-Firuzabad, "Impacts of fault diagnosis schemes on distribution system reliability," *IEEE Transactions on Smart Grid, vol. 3, N. 2, june*, 2012.