Hub Location with Protection under InterHub Link Failures
–Supplementary Material–

Víctor Blanco[a,b], Elena Fernández[c] and Yolanda Hinojosa[d,e]

[a] Institute of Mathematics (IMAG), Universidad de Granada
[b] Dpt. Quant. Methods for Economics & Business, Universidad de Granada
[c] Dpt. Stats. & Operations Research, Universidad de Cádiz
[d] Institute of Mathematics (IMUS), Universidad de Sevilla
[e] Dpt. Applied Economics I, Universidad de Sevilla

1. Computational Experience: Instance generation

We have generated several instances based on the entire CAB, AP and TR datasets with a number of nodes (n) initially ranging in $\{10, 15, 20, 25\}$ for the instances based on the CAB and TR datasets and in $\{10, 20, 25\}$ for the instances based on the AP dataset. Let c'_{kl} be the standard unit transportation costs provided in ORLIB for CAB and AP instances or the travel distances provided for the TR instances.
The unit routing costs for access/distribution arcs ($\overline{c}_{ij}$) and the inter-hub routing costs (c_{kl}) have been obtained as follows. We take the original costs as the unit routing cost through the access and delivery arcs, i.e., $\overline{c}_{ij} = c'_{ij}$. For the routing costs through the inter-hub arcs, we assume that these costs include not only transportation costs but also some additional handling costs at the end nodes of the traversed arcs, associated with the collection (at the entering node) and distribution (at the leaving node) of the routed commodity. Then, we define the unit routing costs through arc $(k, l) \in A$ as:

$$c_{kl} = \alpha(a_k + c'_{kl} + d_l),$$

where:

- $\alpha \in [0, 1]$ is the usual discount factor applied to routing costs through inter-hub arcs due to economies of scale. Three values for the discount factor $\alpha \in \{0.2, 0.5, 0.8\}$ have been considered in our study.
- $a_k \geq 0$ and $d_k \geq 0$ are the unit collection and distribution costs at node k, respectively. Note that applying the discount factor α to these terms implies no loss of generality. Note also that with this choice of costs, in case $k = l$, the unit routing (service) cost through the loop (k, k) reduces to $c_{kk} = \alpha(a_k + d_k)$. In our computational study we define $a_k = d_k = \min\{\min_{j \neq k} c'_{kj}, \min_{j \neq k} c'_{jk}\}$.

As usual in the literature (O'Kelly 1992), we have considered the same fixed costs for all potential hubs $k \in V$, $f_k = 100$ for the CAB dataset, two types of fixed costs (T and L) for the hub nodes provided with the AP dataset, and the fixed costs provided in the TR dataset. Service demand, w_r, $r \in R$, was also taken from the provided datasets.
As considered in the literature (see e.g. Alumur et al. 2009, Calık et al. 2009), the fixed costs for activating hub edges for the CAB and the AP datasets were set:

$$h_{kl} = \begin{cases} 100 \dfrac{c_{kl}/\mathrm{W}_{kl}}{\mathrm{MAXW}} & \text{if } k \neq l, \\ 100 \dfrac{c_{kl}/\bar{\mathrm{W}}}{\mathrm{MAXW}} & \text{if } k = l, \end{cases}$$

where W is the normalized vector of flows, $\bar{\mathrm{W}}$ is the mean of W and $\mathrm{MAXW} = \max\{\frac{c_{ij}}{\mathrm{W}_{ij}} : i, j \in V, \mathrm{W}_{ij} > 0\}$. For TR we used the hub edge activation costs provided in the original dataset.

In formulation (HLPIHLF-λ), we have estimated the costs of backup paths as $\bar{C}^r_{kl} = (1 + \beta)C^r_{kl}$ for two different values of $\beta \in \{0.5, 1\}$. Observe that in this case the expected routing cost simplifies to:

$$\sum_{r \in R} \sum_{(k,l) \in A} (1 + \beta p_{kl}) C^r_{kl} x^r_{kl}. \tag{1}$$

As for the failure probabilities p_{kl}, $\{k, l\} \in E$, we have considered three different scenarios:

RP: Random probabilities. The failure probability of each edge is randomly generated from a uniform distribution, i.e. $p_{kl} \equiv U[0,\rho]$ for all $\{k,l\} \in E$, with $\rho \in \{0.1, 0.3\}$.

CP: Clustered probabilities. Edges are clustered into three groups, each of them with a different failure probability. For this, each edge $\{k,l\} \in E$ is randomly assigned a failure probability in $p_{kl} \in \{0.1, 0.2, 0.3\}$.

SP: Same probability. All edges have the same failure probability, i.e. $p_{kl} = \rho$, for all $\{k,l\} \in E$ with $\rho \in \{0.1, 0.3\}$.

2. Computational Experience: Computing times with (HLPIHLF-λ)

Table 1 presents the computing times of the first set of experiments with (HLPIHLF-λ). The meaning of the labels in Table 1 is the same as in Table 2 of the paper. Now the results are grouped in three blocks ($\lambda = 2$, $\lambda = 3$ and $\lambda = 4$) one for each of the three considered values of λ. Since the value of the parameter β does not affect the results, the information contained in each row refers to average values of 10 instances (5×2 different values of β) for RP and CP scenarios, and the average value of the two instances (different values of β) for the scenario SP.

			$\lambda=2$					$\lambda=3$					$\lambda=4$				
			RP			SP		RP			SP		RP			SP	
n	α	Data	0.1	0.3	CP	0.1	0.3	0.1	0.3	CP	0.1	0.3	0.1	0.3	CP	0.1	0.3
10	0.2	AP_T	2	2	3	2	2	3	5	5	3	5	3	3	2	3	2
		AP_L	6	3	6	7	6	2	2	2	2	2	3	4	3	3	5
		CAB	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
		TR	1	1	1	1	1	5	4	5	6	2	8	7	6	14	9
	0.5	AP_T	2	2	3	4	2	3	4	4	4	4	3	3	2	2	2
		AP_L	6	3	8	5	3	2	2	2	3	2	3	3	2	2	4
		CAB	0	0	0	0	0	0	0	1	0	0	3	2	2	4	3
		TR	3	1	2	4	2	4	4	4	2	3	8	7	6	6	9
	0.8	AP_T	2	3	3	3	2	3	3	3	4	2	2	2	3	2	2
		AP_L	3	3	5	7	2	2	2	2	2	1	2	3	2	2	3
		CAB	0	0	0	0	0	1	0	1	1	0	2	2	2	2	2
		TR	3	3	3	2	4	4	3	3	4	2	7	8	6	7	6
15	0.2	CAB	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		TR	9	6	8	6	7	22	17	25	40	17	42	26	31	40	24
	0.5	CAB	1	1	1	1	1	1	1	2	2	1	3	4	11	3	1
		TR	31	13	19	26	17	18	23	23	19	18	35	36	31	33	24
	0.8	CAB	1	1	1	1	1	3	2	3	3	3	12	10	12	12	10
		TR	23	18	16	24	15	17	20	17	14	16	26	23	25	25	17
20	0.2	AP_T	56	42	52	58	35	133	93	132	112	88	134	119	95	98	113
		AP_L	95	69	99	179	49	102	137	90	81	105	111	97	98	163	137
		CAB	1	1	1	1	1	1	2	1	1	1	2	2	1	2	1
		TR	25	26	23	17	15	98	66	77	50	121	139	133	127	130	95
	0.5	AP_T	51	55	61	40	42	119	109	134	73	99	114	111	121	201	81
		AP_L	101	60	52	36	51	101	82	71	90	117	81	140	89	130	78
		CAB	2	2	2	1	1	2	2	2	1	1	2	2	4	2	1
		TR	46	43	44	28	70	91	81	58	154	98	198	122	141	116	150
	0.8	AP_T	41	54	49	27	59	106	91	137	112	114	81	146	97	168	105
		AP_L	44	77	41	46	47	74	109	90	78	81	95	112	102	103	105
		CAB	1	2	1	1	1	1	3	1	1	1	27	40	57	21	21
		TR	49	42	46	64	34	101	82	52	132	76	168	144	117	163	239
25	0.2	AP_T	301	242	238	111	144	179	205	161	182	251	481	550	412	402	523
		AP_L	205	246	249	228	105	360	434	356	451	289	628	723	704	537	596
		CAB	4	4	4	4	4	4	4	4	4	4	4	8	4	4	4
		TR	316	173	246	204	304	608	545	557	298	486	1364	1383	1018	1544	922
	0.5	AP_T	187	179	165	108	64	163	177	167	151	150	330	386	277	326	421
		AP_L	178	403	122	329	250	387	341	329	529	338	543	674	490	676	738
		CAB	4	4	4	5	4	4	4	5	4	4	135	220	236	71	11
		TR	458	425	282	410	596	790	749	974	558	1003	1795	1650	1376	1016	2003
	0.8	AP_T	180	121	142	165	40	142	158	141	124	139	266	369	338	303	365
		AP_L	125	87	153	86	130	271	258	296	181	283	589	383	370	414	396
		CAB	3	10	11	4	4	4	8	7	4	4	90	255	150	202	131
		TR	396	421	348	216	499	848	1250	1036	606	928	1870	1593	1599	2062	2323

TABLE 1. Average CPU times for (HLPIHLF-λ).

3. Computational Experience: Managerial Insight

Figure 1 shows the percent contribution to the objective function value of the different types of costs: routing costs, fixed costs for activating hubs (Hubs_Costs) and fixed costs for activating inter-hub edges (Links_Costs).

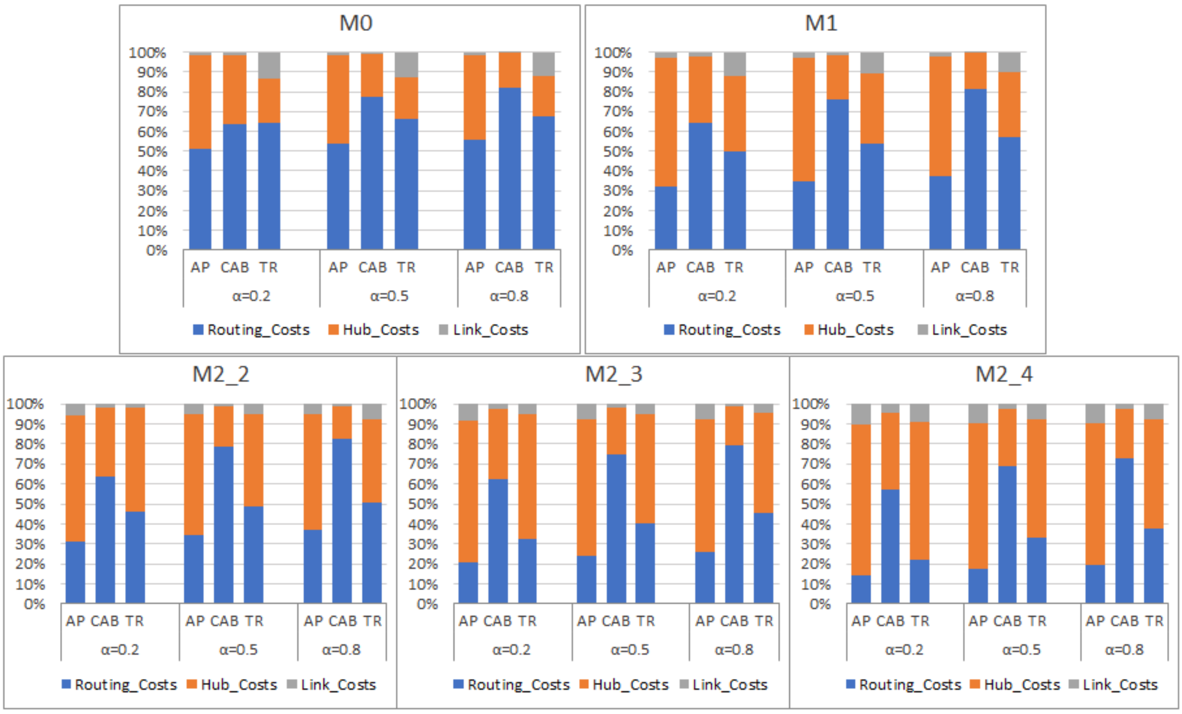

FIGURE 1. Percent contribution to the objective function of the different types of costs

Table 2 gives average values for the number of hubs (#H), inter-hub edges including loops (#Lk), and loops (#Lp) activated in the optimal backbone networks for the different models. .
The distribution of the indices I_1 and I_2 for the different models and the different values of α, are shown in Figures 2, 3, and 4 for instances, CAB, AP, and TR. The height of each bar in the plots represents the percentage of instances reaching the value of the index indicated in the x-axis.

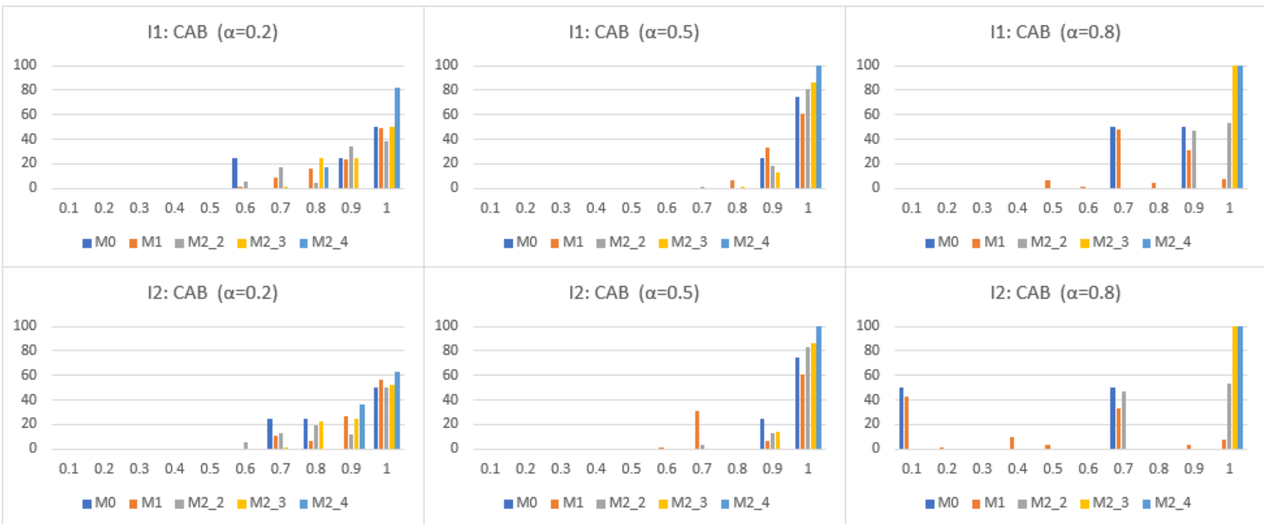

FIGURE 2. Backbone network density for CAB-based instances

n	α	Data	M0 # H	M0 # Lk	M0 # Lp	M1 # H	M1 # Lk	M1 # Lp	M2_2 # H	M2_2 # Lk	M2_2 # Lp	M2_3 # H	M2_3 # Lk	M2_3 # Lp	M2_4 # H	M2_4 # Lk	M2_4 # Lp
10	0.2	AP	1.00	1.00	1.00	2.00	2.00	2.00	2.00	3.00	2.00	3.00	6.00	3.00	4.00	10.00	4.00
		CAB	2.00	3.00	2.00	2.94	5.47	2.59	2.53	4.12	2.12	3.00	6.00	3.00	4.00	10.00	4.00
		TR	1.00	1.00	1.00	2.00	2.00	1.00	3.00	3.06	0.06	4.00	6.00	0.00	5.00	10.00	0.00
	0.5	AP	1.00	1.00	1.00	2.00	2.00	2.00	2.00	3.00	2.00	3.00	6.00	3.00	4.00	10.00	4.00
		CAB	2.00	3.00	2.00	2.35	3.71	2.35	2.03	3.09	2.03	3.00	6.00	3.00	4.00	10.00	4.00
		TR	1.00	1.00	1.00	2.00	2.00	1.00	3.00	3.00	0.00	4.00	6.00	0.00	5.00	10.00	0.00
	0.8	AP	1.00	1.00	1.00	2.00	2.00	2.00	2.00	3.00	2.00	3.00	6.00	3.00	4.00	10.00	4.00
		CAB	2.00	2.00	2.00	2.29	2.41	2.29	2.00	3.00	2.00	3.00	6.00	3.00	4.00	10.00	4.00
		TR	1.00	1.00	1.00	2.00	2.00	1.00	3.00	3.65	0.65	4.00	6.12	0.12	5.00	10.00	0.00
15	0.2	CAB	4.00	6.00	2.00	4.18	8.06	2.94	4.24	7.56	2.71	4.21	8.59	2.88	4.71	11.41	3.29
		TR	1.00	1.00	1.00	2.00	2.00	1.00	3.00	3.53	0.53	4.00	6.00	0.00	5.00	10.00	0.00
	0.5	CAB	2.00	3.00	2.00	2.71	4.47	2.71	2.15	3.29	2.03	3.03	6.06	3.00	4.00	10.00	4.00
		TR	1.00	1.00	1.00	2.00	2.00	1.00	3.00	3.12	0.12	4.00	6.00	0.00	5.00	10.00	0.00
	0.8	CAB	2.00	2.00	2.00	2.18	2.65	2.18	2.00	3.00	2.00	3.00	6.00	3.00	4.00	10.00	4.00
		TR	1.00	1.00	1.00	2.00	2.00	1.00	3.00	3.41	0.41	4.00	6.00	0.00	5.00	10.00	0.00
20	0.2	AP	1.00	1.00	1.00	2.00	2.26	1.50	2.00	3.00	2.00	3.00	6.00	3.00	4.00	10.00	4.00
		CAB	5.00	13.00	5.00	4.29	10.88	4.24	4.76	12.12	4.59	4.76	12.29	4.74	4.74	12.94	4.74
		TR	1.00	1.00	1.00	2.00	2.00	1.00	3.00	3.00	0.00	4.00	6.00	0.00	5.00	10.00	0.00
	0.5	AP	1.00	1.00	1.00	2.00	2.21	2.00	2.00	3.00	2.00	3.00	6.00	3.00	4.00	10.00	4.00
		CAB	4.00	9.00	4.00	3.76	8.94	3.76	3.76	8.47	3.76	3.76	8.53	3.76	4.00	10.00	4.00
		TR	1.00	1.00	1.00	2.00	2.00	1.00	3.00	3.41	0.41	4.00	6.06	0.06	5.00	10.00	0.00
	0.8	AP	1.00	1.00	1.00	2.00	2.12	2.00	2.00	3.00	2.00	3.00	6.00	3.00	4.00	10.00	4.00
		CAB	3.00	5.00	3.00	3.18	5.41	3.18	2.97	4.94	2.97	3.00	6.00	3.00	4.00	10.00	4.00
		TR	1.00	1.00	1.00	2.00	2.00	1.00	3.00	3.76	0.76	4.00	6.06	0.06	5.00	10.06	0.06
25	0.2	AP	1.00	1.00	1.00	2.00	2.18	1.82	2.00	3.00	2.00	3.00	6.00	3.00	4.00	10.00	4.00
		CAB	4.00	10.00	4.00	3.75	8.56	3.75	4.03	9.88	4.03	4.03	9.88	4.03	4.03	10.12	4.03
		TR	1.00	1.00	1.00	2.06	2.06	0.94	3.00	3.76	0.76	4.00	6.00	0.00	5.00	10.00	0.00
	0.5	AP	1.00	1.00	1.00	2.00	2.18	1.94	2.00	3.00	2.00	3.00	6.00	3.00	4.00	10.00	4.00
		CAB	3.00	6.00	3.00	3.62	7.62	3.62	3.00	5.94	3.00	3.00	6.00	3.00	4.00	10.00	4.00
		TR	1.00	1.00	1.00	2.00	2.00	1.00	3.00	4.00	1.00	4.00	6.65	0.65	5.00	11.00	1.00
	0.8	AP	1.00	1.00	1.00	2.00	2.15	1.97	2.00	3.00	2.00	3.00	6.00	3.00	4.00	10.00	4.00
		CAB	3.00	5.00	3.00	3.46	6.15	3.46	2.94	4.91	2.94	3.00	6.00	3.00	4.00	10.00	4.00
		TR	1.00	1.00	1.00	2.00	2.00	1.00	3.00	4.00	1.00	4.00	7.00	1.00	5.00	11.00	1.00

Table 2. Average number of open hubs, edges and loops

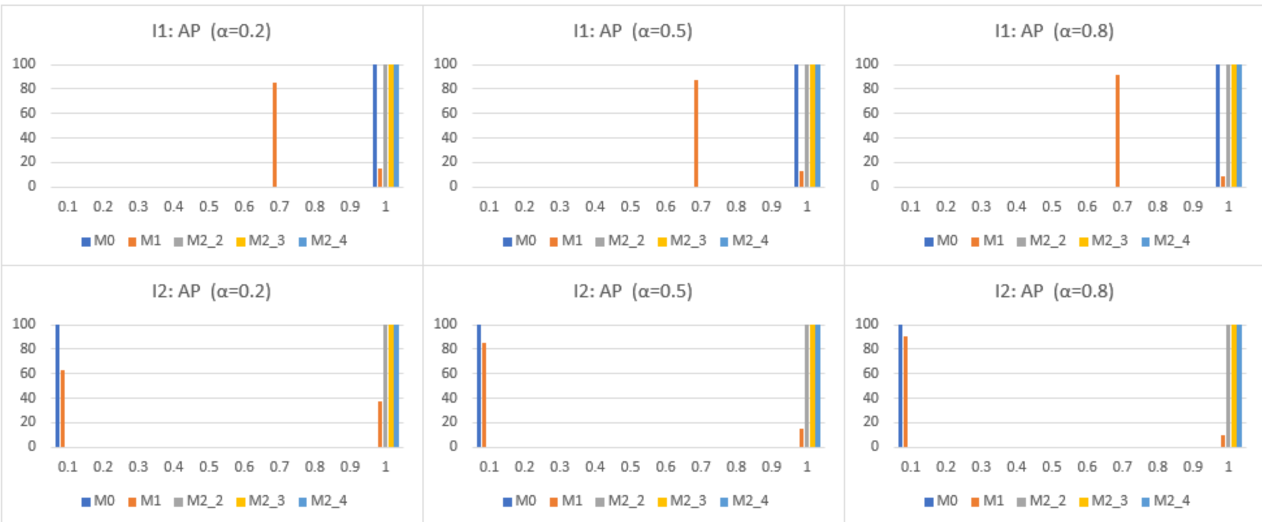

Figure 3. Backbone network density for AP-based instances

4. The price of robustness: Additional results

Table 3 gives, for each of the models M1, M2_2, M2_3, and M2_4, the average percent deviation of the activated hubs and inter-hub edges fixed costs with respect those of M0. For the sake of simplicity, in this table we only show the results for the TR dataset, although the behavior of the CAB and AP datasets is similar.

Figure 5 reports, for all the instances and all the scenarios, the average percentage of after-failure networks for which the demand can no longer be routed. There we draw light blue bars to represent the results for model $M0$, dark blue for $M1$, orange for $M2\_2$, gray for$M2_3$, and yellow for $M2\_4$.

Figure 6 shows the average behavior of $\Phi(q)$ cost disaggregated by failure scenario (FS1, FS2, FS3, and FS4). Each line represents $\Phi(q)$, as a function of the parameter q, for each of the *after-failure* networks produced by the simulations constructed with the five different models (M0, M1, M2_2, M2_3, and M2_4).

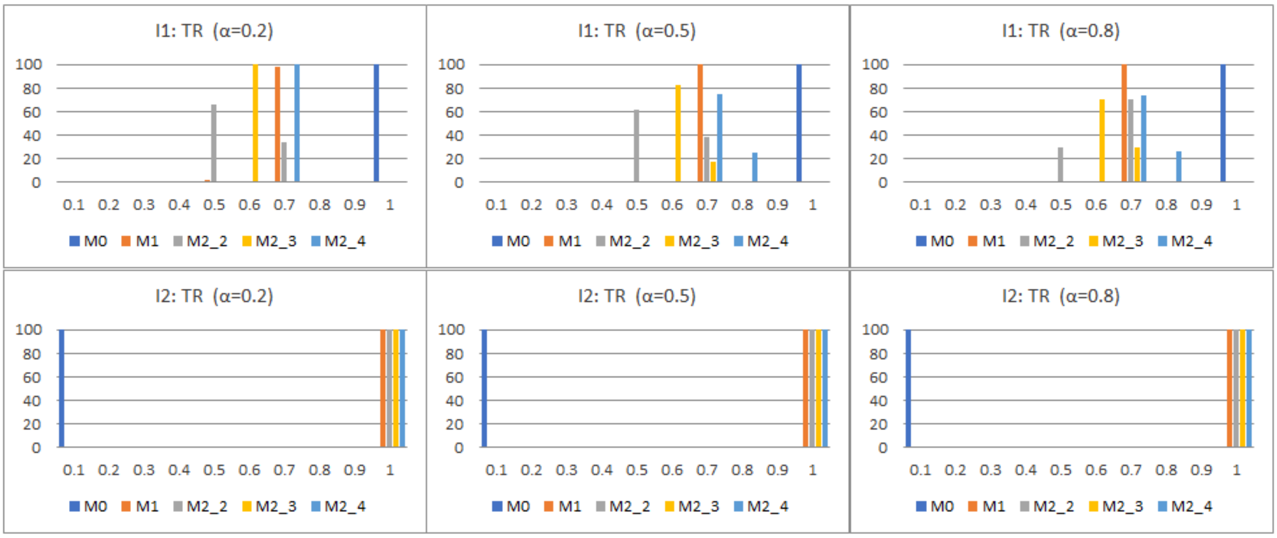

FIGURE 4. Backbone network density for TR-based instances

n	α	M1	M_2	M2_3	M2_4
10	0.2	68.57%	136.86%	235.11%	361.15%
	0.5	68.57%	134.70%	235.11%	360.23%
	0.8	68.57%	148.34%	240.17%	358.39%
15	0.2	68.57%	147.33%	233.01%	358.39%
	0.5	68.57%	134.45%	233.01%	358.39%
	0.8	68.57%	143.17%	234.23%	358.39%
20	0.2	61.52%	98.44%	191.38%	302.97%
	0.5	61.93%	113.60%	184.87%	279.68%
	0.8	61.52%	126.21%	184.87%	280.18%
25	0.2	65.80%	139.29%	231.72%	347.47%
	0.5	61.52%	134.65%	223.84%	314.85%
	0.8	61.52%	134.52%	219.54%	314.85%

TABLE 3. Average percent deviation of activated hubs and inter-hub edges fixed costs relative to those of M0.

References

O'Kelly ME (1992) Hub facility location with fixed costs. *Papers in Regional Science* 71(3):293–306, ISSN 1435-5957, URL `http://dx.doi.org/10.1007/bf01434269`.

Alumur SA, Kara BY, Karasan OE (2009) The design of single allocation incomplete hub networks. *Transportation Research Part B: Methodological* 43(10):936–951.

Calık H, Alumur SA, Kara BY, Karasan OE (2009) A tabu-search based heuristic for the hub covering problem over incomplete hub networks. *Computers & Operations Research* 36(12):3088–3096.

IMAG, UNIVERSIDAD DE GRANADA, SPAIN.
Email address: `vblanco@ugr.es`

DEPARTMENT OF STATISTICS AND OPERATIONS RESEARCH, UNIVERSIDAD DE CÁDIZ, SPAIN.
Email address: `elena.fernandez@uca.es`

IMUS, UNIVERSIDAD DE SEVILLA, SPAIN.
Email address: `yhinojos@us.es`

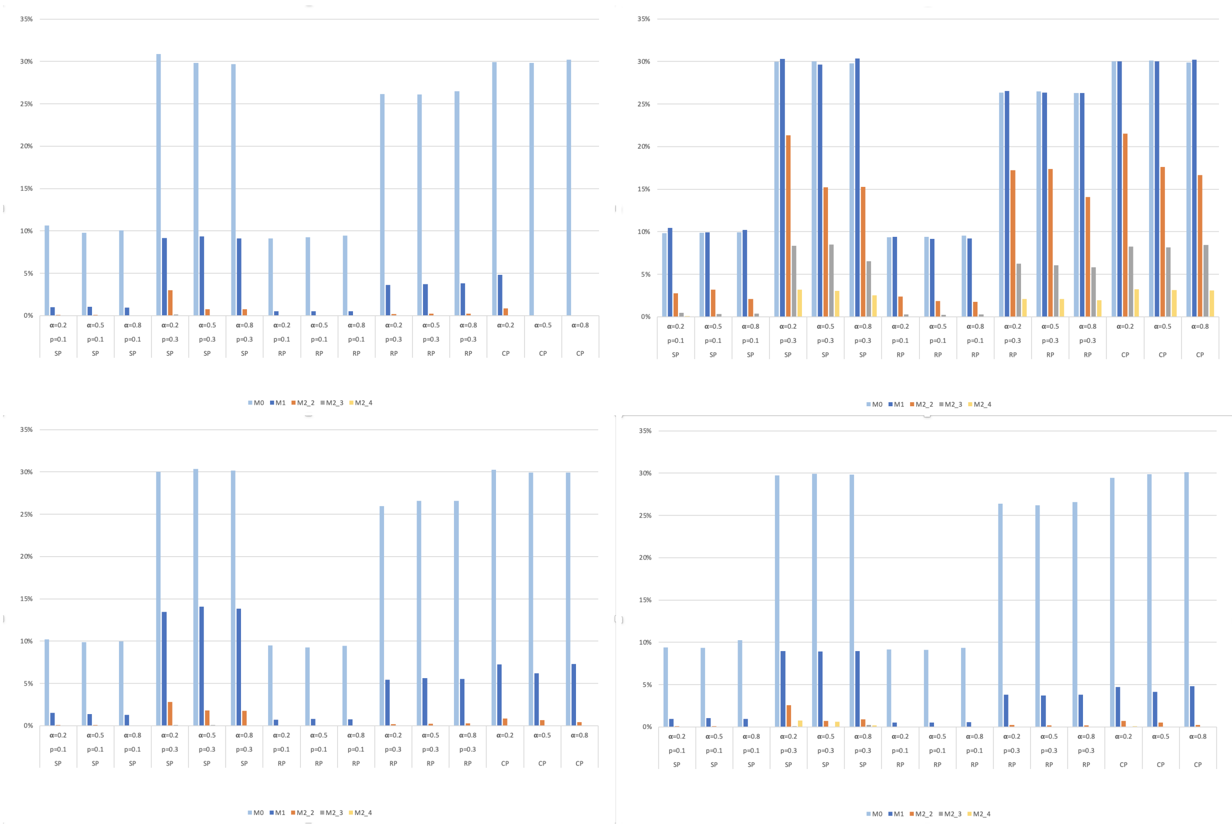

FIGURE 5. Average percentage of *after-failure* networks for which the demand can no longer be routed (from top left to bottom right: FS1, FS2, FS3, FS4). Light blue bars represent $M0$, dark blue $M1$, orange $M2\_2$, gray $M2\_3$ and yellow $M2\_4$.

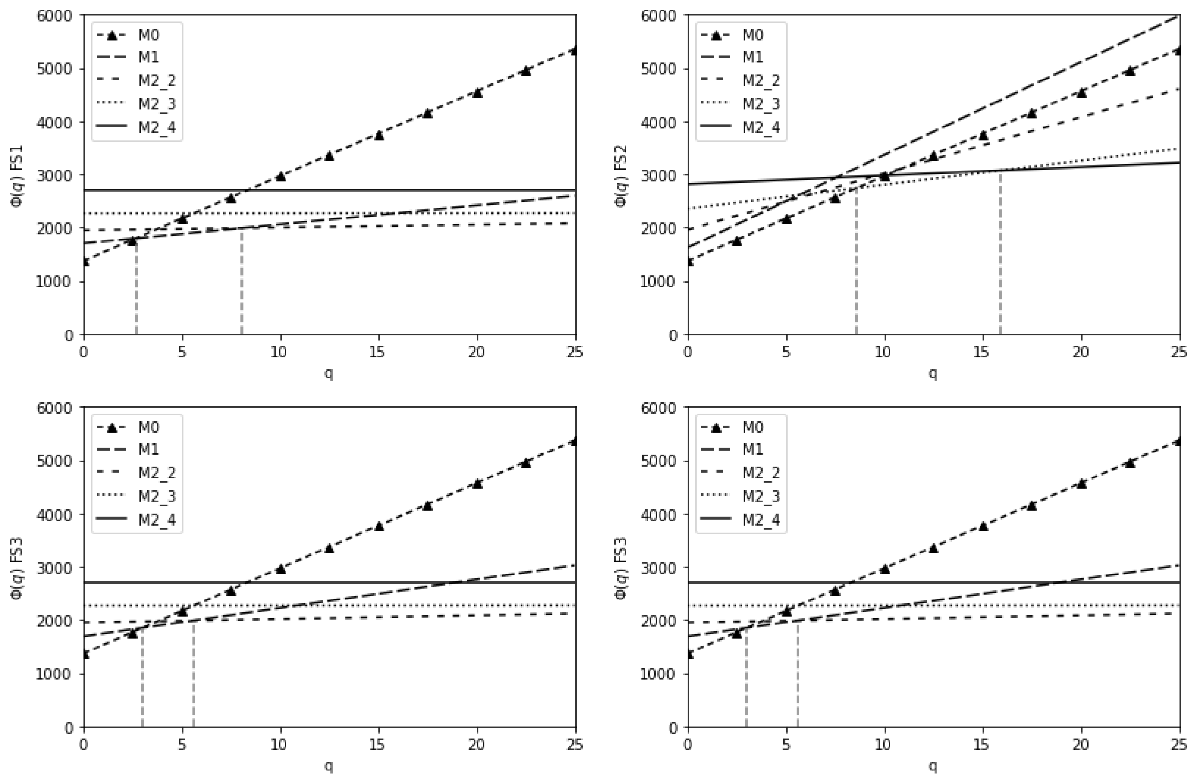

FIGURE 6. Routing costs on the after-failure backbone network as a function of the parameter q for each failure scenario.