Capítulo 15

The logic of semantic and pragmatic strategies in discourse. A linguistic point of view

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Abstract. In a Dialogue System, it is a more complex task to program the semantic understanding modules than the generation modules. This is due to the fact that human agents do not always act rationally in the communicative interaction, they do not obtain all possible inferences from their current knowledge and do not offer all the relevant data to the system in an explicit way when they interact. On the basis of the communicative interaction with a human agent, the system must make a series of inferences to complete given information and manage the communicative intentions of the human speaker. To do this it is necessary to define an underlying logic —better a multimodal logic— that contains different modal operators, including a dynamic epistemic logic and abductive rules that allow to properly contextualize the given information.

15.1 Initial considerations

We conceive the interpretation of a dialogue —and even of a discourse fragment— as a process in which the meaning of each expression is interpreted as

a "cognitive program" that changes the states of information accessible to the addressee as information is being received. In the case of dialogue it is even more evident that the transition from one state of information to another must be governed by rules that define inferential processes in a logic of knowledge (epistemic logic) capable of describing both the properties of the different epistemic states of the involved agents, and the rules of transition from one state to another (changes, processes and results).

Thus, the interpretation of a statement becomes the quest of the best explanation that makes it true or compatible with the previous information known by the agent. In this way it becomes necessary to relate an abductive theory with an epistemic logic in which the possible worlds considered are going to be the successive states of knowledge that can be described while discourse or dialogue progress.

In the specific case of dialogue, this can be conceived as a process of abductive reasoning in which the participants must construct a theory that makes the current utterances coherent with the previous ones and with their own cognitive states, as well as with those cognitive states mutually attributed among the participating epistemic agents. Each intervention in the dialogue can be interpreted as a response to the previous interventions, regardless of whether the constituent utterances are questions or assertive statements —with or without modality— which include epistemic, desiderative, optional, imperative statements, etc. And in the specific case of questions —for both Yes/No-Questions (total interrogatives) and Wh-Questions (partial interrogatives)— the answer may be generated by an abductive process in which the information expressed by the answer is an explanation of the information presupposed by the question[1].

Furthermore, each agent's intervention can be analyzed as a statement of an epistemic state, as a public announcement that modifies the interpretation of previous epistemic states and operates on the subsequent epistemic states of all the participants in the dialogue. We must, therefore, treat the dialogue as a dynamic inferential process in which both abductive and epistemic reasoning are involved, which means defining a dynamic abductive epistemic logic.

Inferences are processes that lead from an *initial epistemic state* to a *final epistemic state* resulting from applying certain well-defined rules to some or all of the previous states, with or without a predetermined order. These "epistemic states" can be considered either as cognitive states or as informative states. In both cases, a certain knowledge is described that can be expressed by means of some type of logical language.

Inferential processes are of interest for reasoning, argumentation, information and communication theory, since they are necessary from the linguistic point of view to explain human communicative behavior and the ability to interpret the communicative intentions of individuals expressed through speech

[1]The extreme case are Why questions, where the answer is clearly and necessarily an explanation of what is being asked.

acts. Typical inferential language processes involved in the interpretation of speech acts are:

- **Lexical and semantic relations**. These relations are definable as implicit relations *stricto sensu*, and include identity (co-implication) and dependence (material implication), both linked to the denotational reference of terms and expressions, as well as to the meanings associated with their denotation (Cruse 1986, Murphy 2003).
- **Presuppositions**, conceived as tacit informative states necessary to be able to establish a semantic link between a set of linguistic expressions and their interpretation (Chierchia 1995, Beaver 2001, Domaneschi 2016).
- **Implications**, conceived as additional cognitive states necessary to be able to interpret a set of linguistic expressions that share a certain contextual link (Sperber & Wilson 1986, Blakemore 1992, Davis 2003, Goodman & Stuhlmüller 2013).

Therefore, in any human communicative act, information should not be understood as a given and closed whole, but rather it should be analyzed as a process or a series of processes in which the different informative states are related to the other informative states that are involved —be they previous or subsequent to the process— and to external informative or cognitive states, to which they are open. The interaction between informative states and cognitive states —explicit or inferred— supposes a dynamic interpretation of the information flow in communication exchange between intelligent agents, human or not.

15.2 A logical-linguistic characterization of dialogue

In order to characterize dialogue, we will focus on the dynamic aspects of information transmission processes (van Benthem 2011), fundamentally on formal aspects of dialogue and dialogical argumentation, as these are the human form of information exchange *par excellence*. The main goals are:

1. To describe strategies for the formalization of dialogue as a sequence of speech acts.
2. To recognize the main problems and difficulties involved in formalizing and interpreting the dialogue.
3. To propose logic models that could solve these problems.

We define dialogue as a sequence of speech acts performed by intelligent agents. These speech acts are conceived as partial discourses that can take two forms: a chain of uninterrupted discourses (monologues) or a sequence of discourses

with multiple speakers that are interrupted and interrogated in an orderly way (colloquy). Both in the monologue and in the colloquy, it is required the participation of at least two intelligent agents (considered as epistemic agents) that should exhibit the following characteristics:

- All the epistemic agents that participate in the dialogue share identical or similar linguistic competences.
- All participating agents change their epistemic states at each intervention.
- Information flow between agents is continuous —interruptions can be treated as noise— and is subject to feedback.

The analysis of the dialogue requires taking into account every speech act as a whole, not only its illocutionary force. Thus, speech as a locutive act requires an analysis that involves speech recognition, identification of the used lexicon and the corresponding parsers. The illocutionary act involves models of interpretation, inferences, implications, contextualization and the attribution of cognitive states to the dialogical agents. Finally, the perlocutionary act requires a proper analysis of *Action Theory* and *Game Theory*, which also play a necessary role in the interpretation models of the dialogue. But, if we focus on the illocutionary force of the speech acts involved in a dialogical argumentation, for example, we find important problems when formalizing them.

First, we are faced with the descriptive fallacy and the need to distinguish between constatative and performative speech acts[2]. Another problem are unhappy speech acts, those that do not achieve their purpose or are not consistent with the interpretation model of the dialogue being handled at the moment. To these ones, indirect speech acts must be added, which at first sight may seem unhappy but do achieve their purpose and, therefore, must be consistent with the model of interpretation being handled. Finally, the ambiguity of some speech acts and the necessary presuppositions and implications for their correct interpretation require them to be formalized in a more expressive language than classical Predicate Logic.

In all these cases, we can assume that the information provided by each speech act involved in the dialogue does not represent a complete informative state, but must be increased through rational —inferential— processes by the epistemic agents who intervene. Or, in other words, the flow of information is incomplete in each informative state of a dialogical communication process, so that it is necessary to establish procedures for the informative enrichment of these states, in accordance with the informative states that constitute the dialogue and the consecutive cognitive states of the participating agents. To this effect, we need to define a formalized metalanguage that is sufficiently expressive and that can also provide interpretation models of the information

[2]The logical treatment of these two types of speech acts cannot be identical, although it is essential to define a formal language capable of representing both of them.

flow that occurs in the communicative acts, as well as complete them through inferential processes suitable for the described linguistic phenomena.

As it was said before, the starting hypothesis is that a discourse fragment is a process in which each expression meaning is interpreted as a "cognitive program" that changes the information states accessible to the addressee as this information arrives to him/her. To interpret each utterance, the listener must find the best explanation that makes it true or compatible with his/her current state of knowledge.

Thus, if we are faced with questions asked to the system by the user, both in those that require a yes/no answer (total interrogatives) and in Wh-Questions (partial interrogatives), the answer can be generated by an abductive process in which the information of that answer is an explanation of the question. This happens because a question can be interpreted as a request for information about the topic of the question. The answer should therefore be an explanation of the informational presuppositions contained in the corresponding interrogative speech act. That is to say, if the DS user asks "Are there places on the first Seville-Madrid flight tomorrow morning?" (total interrogative), the assumption is that there are flights from Seville to Madrid, regardless of whether the specific answer is Yes or No, compared to the question "Are there places on the first Mojave-Gobi Deserts flight tomorrow morning?". And when the user asks "How much does the ticket cost?" (partial question), the assumption is that the ticket has a cost and the system answer will be a contextualization of the information contained in that question —in which it is implicit that there is a demand for obtaining a ticket for the Mojave-Gobi Deserts flight with the cheapest fare.

In both cases, the answer is expected to be relevant and congruent with the information implicit in the assertive statements that serve as the basis for constructing the interrogative speech acts, namely: "There are flights from Mojave Desert to Gobi Desert" and "The ticket has a cost". Likewise, each intervention can be analyzed as a declaration of an epistemic state, as a public announcement that modifies the interpretation of previous epistemic states and operates on the subsequent epistemic states of all the participants in the dialogue. The notion "epistemic state" includes the different states of information through which the system flows —if we understood it as the explicit and retrievable information that appears in the speech acts that make up a particular dialogue fragment and that can be stored in the system memory used by the Dialogue Manager— and also includes the cognitive states attributable to the epistemic agents that intervene in the dialogue —that is, the implicit information that can be inferred from the knowledge that the user or the machine have about a given subject, even though this has not been made explicit in the dialogue. Therefore, *epistemic states* would be defined as the product (or, even, the union, in the case of highly conscious agents) of the whole information contained in the successive informative states in which the dialogue could be structured, as well as in the user's and the system's cognitive states.

For this reason, we propose to treat dialogue as a dynamic inferential process

in which both abductive and epistemic reasoning are involved, which implies defining a dynamic epistemic logic with abduction to be implemented in our Dialogue System. Dynamic Epistemic Logic (DEL) allows us to represent not only the epistemic states —whether knowledge or belief— of a group of agents in different formats, such as semantic information, or information based on evidence and/or reasons, but also the different epistemic actions —diverse types of observations, inferences, communication— that affect them. And we can also explore the relationship between these logical calculus and reasoning systems on non-omniscient rational agents or on multiagent systems protocols, as well as extend DEL to abductive logic by revising the classical model of abduction (Nepomuceno & al. 2017) and the development of new abduction techniques, with the aim of applying this extension in a SD, so we will also be able to add in the future tools from the theory of argumentation that take into account speech acts (pragma-dialectics) as well as from different logics based on semantic games such as dialogics, for example (van Eemeren & Grootendorst 2004; Redmond & Fontaine 2011; Barés & Fontaine 2017).

15.3 Abductive inferences and semantic enrichment of discourse

In the communicative interaction between two intelligent agents we often deal with speech acts or discourse fragments in which information is not complete. And yet they are part of a "successful" dialogue between humans. This is due to the fact that human agents use inferential mechanisms that allow them to enrich semantically their discourse in order to be able to interpret it beyond the literal information that has been encoded.

Noise is one of the main sources of incomplete information in speech, whether it is dialogic or not. We understand noise as the occasional interruption of the flow of information. This interruption may be caused by external interferences to the communicative interaction between agents or by the occurrence of incomplete speech acts. Thus, for example, an interrupted —voluntarily or involuntarily— speech act or dialogue, or a text in which some words or fragments have been deleted or crossed out, remain interpretable if we try to reconstruct the missing information by means of inferences.

Another source of incomplete information are the not fully referential expressions that could appear in speech acts. These are expressions that highly develop the structural linguistic property of *efficiency*, defined by Jon Barwise and John Perry (1983). For instance:

1. Spatial and temporal deixis, which can only be referenced in relation to the situation described by the speech act in which they appear, compelling the rational agent to complete his or her state of information with external knowledge.

2. Pronominal anaphoras, which require variable instantiation inferential

processes that allow to attribute reference to them within or outside the discourse.

3. Indefinite or general noun phrases, which require an analysis of quantification and the quantificational scope of noun determiners within the discourse domain for a correct interpretation.

Ambiguity is another linguistic phenomenon that results in incomplete information. Ambiguity can be lexical —caused, mainly, either by the existence of homonymous and polysemic words, or by metaphorical or metonymic semantic shifts— or structural. In the case of lexical ambiguity, the agent interpreting a speech act is forced to choose among all the possible interpretations the one that is most appropriate for the subsequent informative state to be consistent with previously established cognitive states. In the case of structural ambiguity, the same occurs with the different possible interpretations of sentences, being necessary to reconstruct the context in which they make sense.

Finally, we may be faced with a fragment of discourse or a dialogue whose interpretation domain has not been explicitly defined, either partially or totally, or that lacks context or is inserted in an incomplete context —what may be due to the mentioned phenomena of noise, ambiguity or the occurrence of expressions that cannot be directly referenced in the discourse—. In this case we have again an incomplete information that must be completed by means of rational inference processes.

All these phenomena, that we have identified as sources of incomplete information in discourse and dialogue, give rise to the same need in human communication: inventing —in the sense of finding by means of rational processes— the reference or the framework of interpretation in which an incomplete state of information reaches a meaning that is consistent with the knowledge an epistemic agent handles about a topic. For this purpose, we postulate that abductive reasoning is much more adequate than induction or deduction, so the linguist's task consists in analyzing all these phenomena from the standpoint of the construction of an explicative context that gives consistency to the informative context in which the speech acts that constitute a dialogue take place.

Since Aristotle, abduction is understood as a type of reasoning different from induction, and both are clearly distinguished from deduction (*Prior Analytics* II 23–25). But the one who established the actual logical nature of abduction was Charles S. Peirce. In his essay "On the Natural Classification of Arguments" (CP 2.461–516), Peirce tells how he became convinced that abduction —also called *hypothesis* in other places (CP 2.619–644), or even *retroduction* (CP 6.469–470)— is a form of logical argumentation different from induction and deduction, and that this is already found in Aristotle's treatment of syllogistic reasoning. For Peirce, abduction is a type of reasoning that consists of constructing a syllogism in which the conclusion and the minor premise are inverted, so that the latter is concluded from the major premise and the conclusion from an apodictic syllogism. His subsequent reflections on this type

of reasoning led him to define abduction as the process of forming explanatory hypotheses (CP 5.172), from which an observed fact can be derived that would otherwise be unexplained[3]:

> "The surprising fact, C, is observed. But if A were true, C would be a matter of course. Hence, there is reason to suspect that A is true." (CP 5.189)

Regardless of whether for Peirce the explanatory hypotheses are rationally discovered by means of abductive inferential schemes or whether these schemes rather serve to justify their adoption —as proposed by Harry Frankfurt (1958)—, or whether the concept is univocal or not in the different stages of Peirce's thought —see Guy Deutscher's review of the use of the notion of abduction in linguistics (Deutscher 2002)—, abductive reasoning has been developed in the last two decades as a well defined logic (or class of logics) starting from the intuitions of the American philosopher, but without the inconvenience of the ambiguity of his successive definitions (Magnani 2001; Gabbay & Woods 2005; Aliseda 2006, 2014; Nepomuceno & al. 2014).

The requirements that an argument must fulfill in order to be considered an abductive reasoning were proposed by Jaakko Hintikka on the basis of the so-called *Kapitan Theses* (Kapitan 1997) in an influential article for the further development of the logic of abduction (Hintikka 1998):

1. Abduction must be —or must include, at least— an inferential process (Inferential Thesis).

2. Abduction should allow the generation of new explanatory hypotheses and the selection of the best ones (Thesis of Purpose).

3. Abduction should include all operations by which new theories are generated (Comprehension Thesis).

4. Abduction must be irreducible to both deduction and induction (Autonomy Thesis).

To define an abductive logic that satisfies such requirements we will use the classical AKM model (Aliseda 2006; Kowalski 1979; Kuipers 1999; Kakas, Kowalski & Toni 1993; Magnani 2001; Meheus, Verhoeven, van Dyck & Provijn 2002), and we will also consider the modification of the AKM model by Dov Gabbay and John Woods (2005), Woods (2013) or Magnani (2009, 2017), that treats abduction as a problem of preservation or mitigation of ignorance (Bertolotti & al. 2016) —that is, the conclusion of any abductive reasoning is no more than the mitigation of ignorance since no new knowledge is reached, because it remains being a hypothesis.

[3]For this reason, abductive logic is also known as logic of explanatory reasoning or even as logic of discovery.

There are several models for abduction and different types and classifications (Park 2015; Schurz 2008). However, we will start from the following basic concepts that are common to all of them.

A logic is an ordered pair $\langle L, \models \rangle$, where L is a language —for instance, classical Propositional Logic or classical Predicate Logic with the usual logical functors or others, with or without modal, epistemic, temporal, operators, etc.— and $\models$ is a logical consequence relation that can be classical or not.

Given a language L and a formula φ, if φ is found in L then we say $\varphi \in L$. If Γ is a set of formulas found in L then $\Gamma \subseteq L$. If φ is reached by an inferential process from the set of formulas Γ, then we say that $\Gamma \models \varphi$. Otherwise we say that $\Gamma \not\models \varphi$.

Given a theory Θ, a fact φ and a logical system $\vdash$, we say that $\langle \Theta, \varphi, \vdash \rangle$ is an abductive problem if and only if (iff) it is not the case that $\Theta \vdash \varphi$. Generally speaking, abductive solutions to an abductive problem can be found in one of two ways:

1. Extending the theory (this is, properly speaking, the AKM proposal). This can be done in different ways, although the most satisfactory one seems to be the reformulation of the abductive style of "explanation and consistency", proposed by Aliseda (2006: 74). According to this proposal, the statement α is an abductive solution iff it checks the following requirements:
   (a) $\Theta, \alpha \vdash \varphi$
   (b) α is consistent with Θ
   (c) α does not logically imply φ
2. Changing the logic (a different case, but compatible with AKM). The new logic $\vdash^*$ will be an abductive solution iff $\Theta \vdash^* \varphi$.

This second way of finding abductive solutions to abductive problems has been called *structural abduction* (Keiff 2007: 199–201), as opposed to the first one, which can be called *classical abduction*. We could even postulate a third type of abduction applicable to Predicate Logic Languages, consisting on the existential presupposition of one or more individuals expanding the discourse domain of the theory, so that we could have the following types of abduction involved in discourse semantic enrichment:

- **Classical abduction**: a formula (proposition, sentence) is added to a theory, so that either the "surprising observation" is deduced from that theory enriched with the mentioned formula in the case of novelties —we will call this procedure *expansion of the theory*—, or a belief revision is carried out in the case of anomalies —by means of theory contraction and the subsequent expansion.
- **Structural abduction**: a new logic is described by redefining the logical consequence relation $\vdash$, so that the "surprising observation" is inferred

from the current information or from previous knowledge handled by the epistemic agent after changing the inferential mechanism. The idea is that changes do not occur only at the propositional or sentence level, but that the abductive inference is in turn an abduction from the logical structure we are using.

- **Existential abduction**: the existence of one or more individuals is postulated to enrich the domain in which a "surprising observation" is interpreted or evaluated. This kind of abduction is called by Magnani "creative abduction" (Magnani 2001, 2009). What is interesting about this type of abduction is the introduction of new elements into the universe of discourse that can be useful in a dialogic interaction —or, in other words, we introduce new participants that solve interpretation problems raised in dialogue.

15.4 Epistemic inferential processes in the interpretation of linguistic utterances occuring in information flow

For all these reasons, we must apply all the different types of abductive reasoning mentioned above to those modal logics that enable us to express the knowledge contained in the successive epistemic states that constitute a dialogue, considering it as a process of transmission and interpretation of information and communicative intentions —that is, both in the informative states of the DS that arise after different speech acts that are uttered (public announcements), and in the cognitive states of the epistemic agents that participate in daialogue (the human user as well as the machine).

A Dynamic Epistemic Logic (DEL) makes it possible to describe both the epistemic states of the agents and the actions affected by those states. The paradigmatic example is the logic of public announcements (van Ditmarsch et al. 2008), which allows us to describe the effects of announcements that are heard by all agents. A formula such as $[\chi]\,\psi$ express that, after χ is announced, ψ is fulfilled. Thus, it is possible to describe situations such as $[\chi]\,K_a\psi$ —after χ is announced, the agent a knows that ψ is true—. The logics that explore announcements in which only some of the agents involved receive the information are equally representative, because they allow to express situations of private (non-common) knowledge —for instance, the agent a privately receives the result of a coin toss and this agent knows that the result is heads; on the other hand, another agent b does not know whether the result was heads or tails, but he/she knows that the first agent knows what happened—.

But DEL can also describe the effect of other epistemic actions, such as belief revision (van Benthem 2007) or certain forms of abductive reasoning (Nepomuceno & al. 2017). It is even capable, under appropriate models, of describing deductive inference acts (Velázquez 2013) and actions by which an

agent combines the evidence at his or her disposal (van Benthem & Pacuit 2011).

In general, the set Φ of formulas of DEL is constructed following this rule:

$$\Phi ::= \bot \mid p \mid \neg\chi \mid \chi \to \psi \mid K_a\chi \mid C\chi \mid [\alpha]\,\psi$$

where $\bot$ is a propositional constant (the false proposition) and p is a propositional variable. In the epistemic aspect, $K_a\chi$ means that the epistemic agent a knows that χ, while $C\chi$ is common knowledge among all the participating agents that χ is true. In the dynamic aspect, $[\alpha]\,\psi$ expresses that after the epistemic action α, the proposition ψ is true. The use of the other logical connectives and epistemic operators can be defined from those that have been presented[4].

Thus, following Hans van Ditmarsch & al. (2008), we will adopt *Kripke Models* for the semantics of DEL, in which possible worlds correspond to descriptions of epistemic states. An accessibility relation between possible worlds is added, defined for each epistemic agent that takes part in the discourse or dialogue, as well as an interpretation function that assigns truth values to the propositions in each epistemic state:

$$M = \langle S, R, I\rangle$$

where S is a set of states, R is a function that establishes for each agent $a \in A$ a relation of accessibility between states such that $R_a \subseteq S \times S$, and I is a function that assigns a value to all the propositions of the language L such that $I(p) \subseteq S$ defines the set of states in which p is true, for every proposition $p \in P$.

All the formulas of DEL are interpreted in a pair $\langle M, s\rangle$, so that:

1. $M, s \nvDash \bot$
2. $M, s \models p$ iff $s \in I(p)$
3. $M, s \models \neg p$ iff $M, s \nvDash p$
4. $M, s \models \chi \to \psi$ iff $M, s \nvDash \chi$ or $M, s \models \psi$
5. $M, s \models K_a\chi$ iff for every state s' such that sR_as', $M, s' \models \chi$
6. $M, s \models [\chi]\,\psi$ iff when $M, s \models \chi$ then $M_{|\chi}, s \models \psi$ —where $M_{|\chi} = \langle S', R', I'\rangle$, such that $S' = [\![\chi]\!]_M ::= \{s \in S \mid M, s \models \chi\}$, $R'_a = R_a \cap ([\![\chi]\!]_M \times [\![\chi]\!]_M)$ and $I'(p) = I(p) \cap [\![\chi]\!]_M$

The accessibility relation R will be defined by virtue of the different types of epistemic agents we are dealing with. Basically, we will consider that we are dealing with agents capable of transmitting —in discourse or dialogue—

[4]This logic can be extended, as needed, to a predicate logic and new logical connectives or modal, epistemic, temporal operators, etc. can be added.

certain knowledge based on evidence. For this purpose, we will use an Euclidean definition of the accessibility relation —that fulfills the properties of reflexivity, symmetry and transitivity—, compatible with an S5 system of epistemic logic. This system assumes that the epistemic agents that intervene in a dialogue, for example, are perfectly logical and aware of what they know as well as of what they ignore. It is not the best system of epistemic logic for certain situations of information exchange, but it can be interesting to work with it before entering the evaluation of other possible systems.

The different epistemic states that succeed each other when information is being exchanged —which may or may not have the form of a dialogue— are therefore partial descriptions of a more general state of affairs, (i.e.: partial descriptions of a possible world). These partial descriptions constitute the context in which the expressions that make up the exchange of information must be interpreted. This conception of information flow in dialogue allows to use logical decision procedures based on *Hintikka sets* —which are conceived precisely as partial descriptions of possible worlds depending on the *consistency test* (Hintikka 1969: 153-154)— in order to build the successive contextual models in which the utterances of the dialogue are interpreted (Salguero 2014).

A *Hintikka set* μ is a set of formulas of the language L that satisfies a series of conditions (Hintikka 1969):

1. $\bot \notin \mu$
2. For every $\alpha \in L$, if $\alpha \in \mu$ then $\neg\alpha \notin \mu$
3. For every $\alpha, \beta \in L$, if $\alpha \rightarrow \beta \in \mu$ then $\alpha \notin \mu$ or $\beta \in \mu$
4. If $\alpha(a) \in \mu$ and $(a = b) \in \mu$ then $\alpha(b) \in \mu$ —where a and b are individual constants such that $a, b \in D$
5. If $\exists x\alpha(x) \in \mu$ then $\alpha(a) \in \mu$ for at least one individual a such that $a \in D(\mu)$ —that is, for at least one individual constant that appear in the formulas of μ
6. If $\forall x\alpha(x) \in \mu$ then $\alpha(a_1)...\alpha(a_n) \in \mu$ for all the individuals $a_1...a_n$ such that $\{a_1...a_n\} = D(\mu)$ —that is, for all the individual constants that appear in the formulas of μ
7. If $\Diamond\alpha \in \mu$ then there is at least a *Hintikka set* ν such that $\mu R\nu$ and $\alpha \in \nu$
8. If $\Box\alpha \in \mu$ then $\alpha \in \nu$ for every *Hintikka set* ν such that $\mu R\nu$

This definition of *Hintikka sets* turns them into partial descriptions of a possible world, the referent of a coherent set of sentences of a certain language that cannot be increased with new sentences that make it inconsistent. By relating different *Hintikka sets* through the accessibility relation R, we can compare a variety of consistent sets of sentences that represent different partial descriptions of states —epistemic states, for example— and make the interpretation

of the sentences depend on their acceptability in a *Hintikka set* or not; that is to say: on the set of sentences in which a certain expression is interpreted and on all those other accessible sets of sentences, in case the sentence we are interpreting is somehow modalized (epistemically, temporally, deontologically, etc.).

The procedure of *semantic tableaux* offers interpretation models for the formulas that constitute a *Hintikka set*, and also the domain in which the individual variables obtain their reference through *individuation functions* that connect individuals through different states or situations (Salguero 2014, p. 111–115). In this way, it is possible to get an algebraic model that relates the references of different anaphoric variables (pronouns, for instance) and noun phrases that appear in discourse, even in the process that leads from one epistemic state to another. Therefore, we can treat the *Hintikka sets* that are related in such a model as the contextual frameworks in which each new expression appearing in an informative process must be semantically evaluated. For this, it is possible to define a *Context Logic* L_c that will act as a metallogic for the corresponding epistemic logic. We will base this logic on the proposal of Dynamic Context Logic made by Guillaume Aucher et al (2009):

$$\phi ::= \varphi \in MFOL \mid \neg\varphi \mid \varphi \rightarrow \psi \mid [\mu]\,\varphi$$

where MFOL is the First Order Modal Logic and $[\mu]\,\varphi$ means that φ is true in the context defined by the *Hintikka set* μ. For this operation on contexts, we also define the inverse operation $\langle\mu\rangle\,\varphi := \neg\,[\mu]\,\neg\varphi$, which can be read as "in the context defined by the *Hintikka set* μ, the formula φ is possible" —i.e.: its addition does not make the description of the world represented by μ inconsistent.

We can define the semantics of L_c in the following way. Given a contextual model $M = \langle W, R, I\rangle$, in which W is a non-empty set of *Hintikka sets* that describe certain states or situations, R is a relation such that for every *Hintikka set* μ, $R(\mu) \subseteq W$ —i.e.: a context is a set of accessible states— and I is an interpretation of the formulas of L_c so that $I(\varphi) \subseteq W$ —that is, the interpretation of a formula represents the set of states that satisfy it—, for any $w \in W$ it holds that:

1. For every $\varphi \in \mu$, $M, w \models_c \varphi$ iff $w \in I(\varphi)$
2. $M, w \models_c \neg\varphi$ iff $M, w \nvDash_c \varphi$
3. $M, w \models_c \varphi \rightarrow \psi$ iff $M, w \nvDash_c \varphi$ or $M, w \models_c \psi$
4. For any *Hintikka set* μ, $M, w \models_c [\mu]\,\varphi$ iff $M, w \models_c \varphi$ and for every $w' \in R(\mu)$, $M, w' \models_c \varphi$

Therefore, to say that the proposition expressed by the formula φ is verified in a context is equivalent to saying that both, the current informative or cognitive state and all the states accessible to them in the context, satisfy φ:

$$M, w \models_c [\mu]\,\varphi \text{ iff } R(\mu) \subseteq I(\varphi)$$

Similarly, saying that φ is verified in some informative or cognitive state of a context is the same as saying that there is at least one state in the context in which φ is possible:

$$M, w \models_c \langle\mu\rangle\,\varphi \text{ iff } R(\mu) \cap I(\varphi) \neq \emptyset$$

From all of the above, it follows that $\models_c [\mu]\,\varphi \rightarrow \langle\mu\rangle\,\varphi$, which matches the axiom $[D]$ of Standard Deontic Logic (Salguero 1991, pp. 62–66), and implies that the relation R is defined by the property of seriality $\forall\mu\exists\nu(\mu\Re\nu)$ and not by that of euclidianeity, as in the case of the Dynamic Epistemic Logic (DEL) previously defined. This can have interesting semantic consequences for Context Logic and the interpretation of the successive expressions that appear in a dialogue, since the states that are immediately accessible from an initial informative state inherit certain characteristics such as the domain, but do not donate them "backwards" or "laterally" to other accesible states. Therefore, this logic is a variable domain non-Kripkean logic and not a fixed domain or a nested domains logic —as in the case of normal Kripkean logics—, what seems logical from the point of view of the flow of information, since new referents may appear as the process of transmission of information progresses, without these referents necessarily influencing the interpretation of the utterances appeared in the preceding states or in successive states not immediately accessible (Salguero 2014, p. 111).

From here we can define a logical abductive calculus for contexts that will enable us to deduce certain necessary properties of the epistemic states in which the successive utterances of a discourse or a dialogue are interpreted. An Abductive Context Logic (ACL) is defined as follows:

1. $MFOL \subset ACL$
2. $[\Omega]\,\varphi \rightarrow \varphi$ —where $[\Omega]$ is a global operator such that $R(\Omega) = W$
3. If $\vdash_c \varphi \rightarrow \psi$ and $\vdash_c \varphi$ then $\vdash_c \psi$ (*Modus Ponens*)
4. If $\vdash_c \varphi$ then $\vdash_c [\Omega]\,\varphi$ (*Necesitation Rule*)
5. For every *Hintikka set* μ, if $\vdash_c [\Omega]\,(\varphi \rightarrow \psi)$ and $\vdash_c [\mu]\,\psi$ then $\vdash_c \langle\mu\rangle\,\varphi$ (*Peirce Rule* or *Abduction Rule*)

That is to say, all MFOL axioms and theorems are in ACL, which, unlike MFOL, is an abductive logic (*Peirce Rule*), but closed with respect to the classic MFOL rules of *Modus Ponens* and *Necesitation*. In this logic, abduction allows us to infer the possibility of adding a formula —that represents a sentence— in a given context if that formula implies in the model another one that necessarily appears in the context. This means that in a given context we can refer to individuals that appear in other contexts of the model as long as there are certain relationships between the formulas, which allows us to apply the individuation functions between contexts (Salguero 2014, p. 117).

As can be seen, in this approach abduction goes beyond a simple intuitive reasoning scheme and can be formalized using a logic with a modal basis — an epistemic basis, for example— and a dynamic or processual interpretation. We are sure that this type of logics can be very useful in developing formal models of meaning applicable to Dialogue Systems Managers. For this purpose, we will have to go deeper into the different Dynamic Epistemic Logics —with or without abduction—, and into the Abductive Context Logic and its applications as a metallogic for the former.

Acknowledgements

This paper is based on a research project carried out for the last two years by the author in collaboration with Ángel Nepomuceno. Some of its most important aspects are therefore due to Professor Nepomuceno and the other members of our research group. I would therefore like to thank each and every one of the members of the Research Group on Logic, Language and Information of the University of Seville (GILLIUS) for their support and ideas for the development of this work.

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