



# A Hybrid UKF-MGA Algorithm for Finite Element Model Updating of Historical Constructions

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## Abstract

The finite element model (FE) updating is a calibration method that allows minimizing the discrepancies between the numerical and experimental modal parameters. As result, a more accurate FE model is obtained and the structural analysis can represent the real behaviour of the structure. However, it is a high computational cost process. To overcome this issue, alternative techniques have been developed. This study focuses on the use of the unscented Kalman filter (UKF), which is a local optimization algorithm based on statistical estimation of parameters taken into account the measurements. The dome of a real chapel is considered as benchmark structure. A FE model is updated applying two different algorithms: (i) the multi-objective genetic algorithm and (ii) a hybrid unscented Kalman filter-multi-objective genetic algorithm (UKF-MGA). Finally, a discussion of the results will be presented to compare the performance of both algorithms.

**Keywords:** model updating; historical constructions; unscented Kalman filter; multi-objective genetic algorithm.

## 1. Introduction

The structural assessment of historical constructions is an increasingly relevant issue. The most usual approach for the assessment of their structural behaviour is to design a mathematical model (e.g. finite element model). To build the Finite Element (FE) model, the material and geometrical properties are assumed and the boundary conditions implemented may not be the proper. These facts may involve large discrepancies between the numerical results obtained from these mathematical models and the experimental results obtained from field

vibration tests. The model updating may be formulated as an optimization procedure where these discrepancies are minimized. Friswell *et al.* [1] introduced in 1995 the main techniques which may be employed for FE model updating. The procedure consists in selecting one or several physical parameters of the numerical model and changing their values iteratively. For each iteration, the differences between numerical and experimental results (usually modal parameters) are evaluated. In this manner, viewed from the optimization perspective, the parameter identification solved as an inverse problem can be considered as a general minimization problem.

The objective is to find the physical parameters that minimizes the mean square error (objective function) between actual and simulated modal parameters.

Due to the high nonlinearity intrinsic to structural models, the objective functions could present many local minima. To prevent the algorithm to converge to a local minimum, the use of global optimization algorithms must be considered. Genetic algorithm (GA) has been widely used to solve a parameter identification inverse problem. The high computational cost has led to the emergence of hybrid techniques that reduce the convergence time of global optimization algorithms. In this study, a hybrid unscented Kalman filter-multi-objective genetic algorithm (UKF-MGA) is considered. The unscented Kalman filter (UKF) [6] is an algorithm used to estimate the parameters of a nonlinear system subjected to white noise. The parameters updating is based on the estimation of the Gaussian means and covariances of the quantities being estimated, minimizing the variance of the estimation error. The Kalman filter finds the local minimum around the current estimate.

This study focuses on the comparison of the results obtained using both the maximum likelihood approach and a hybrid UKF-MGA algorithm. As benchmark structure, the chapel of the Würzburg Residence (Germany) is considered. This building, whose construction dates from the early eighteenth century, was declared a World Heritage Site by UNESCO in 1981. The updating procedure is based on the actual modal parameters of the structure, obtained through the accelerations recorded during an ambient vibration test. Finally, the number of iterations, the computation time and the differences between the updated and experimental modal parameters are compared for both algorithms.

## 2. FE Model Updating based on the maximum likelihood method

### 2.1 Basics of FE Model Updating

The FE model updating tool aims to design a numerical model which is more adequate to the

real behaviour of the structure [1]. It is an iterative process in which the values of certain pre-selected physical parameters are changed until the optimal solution is reached. The updated FE model, under the maximum likelihood method, is obtained by minimizing the objective functions defined in terms of the relative differences between experimental and numerical modal parameters.

Within the framework of multi-objective optimization, the problem to be solved consists in minimizing the objectives functions defined with regard to the residual of the natural frequencies and the residual of the vibration modes. The expressions to calculate both residuals are:

$$r_{f,j} = \frac{f_{num,j} - f_{exp,j}}{f_{exp,j}}, \quad j = 1, \dots, m_f \quad (1)$$

$$r_{s,j}^2 = \frac{(1 - \sqrt{MAC_j})^2}{MAC_j}, \quad j = 1, \dots, m_f \quad (2)$$

where  $f_{exp,j}$  is the experimental natural frequency  $j$ ,  $f_{num,j}$  is the numerical natural frequency  $j$  and  $MAC_j$  is the Modal Assurance Criterion. The MAC is a parameter used to assess the correlation between the numerical and experimental natural frequency  $j$  and is calculated by the following expression:

$$MAC_j = \frac{(\phi_{num,j}^T \cdot \phi_{exp,j})^2}{(\phi_{num,j}^T \cdot \phi_{num,j}) \cdot (\phi_{exp,j}^T \cdot \phi_{exp,j})} \quad (3)$$

being  $\phi_{num}$  and  $\phi_{exp}$  the numerical and experimental vibration modes, respectively. Both residuals are used to define the objective functions. Therefore, the minimization problem may be defined as follows:

$$\min f_1(\theta) = \min \frac{1}{2} \left[ \sum_{j=1}^{m_f} r_{f,j}^2(\theta) \right]^{\frac{1}{2}} \quad (4)$$

$$\min f_2(\theta) = \min \frac{1}{2} \left[ \sum_{j=1}^{m_f} r_{s,j}^2(\theta) \right]^{\frac{1}{2}} \quad (5)$$

where  $f_1(\theta)$  and  $f_2(\theta)$  are the objective functions related to the natural frequencies and vibration modes, respectively,  $\theta$  is the possible solution vector containing the updated physical parameters of the model and  $m_f$  is the number of vibration modes considered.

## 2.2 Multi-objective Genetic Algorithm (MGA)

The multi-objective global optimization algorithm (MGA) employed in this study is the nature-inspired NSGA-II (Non-Dominated Sorting Genetic Algorithm) algorithm [3]. This algorithm consists of the followings steps. First, the initialization consists in creating an initial random population (solutions) and the assessment of the objective functions. The value of the objective functions are then classified to distinguish between dominated and non-dominated (Pareto front) solutions. Second, the stochastic selection function selects several classified solutions and generate a new population by using the crossover and the mutation functions. The former allows creating a new solution from two previous ones and the latter is used to randomly modify the value of one parameter of the new solution in order to search on new areas of the domain. Finally, the objective functions are evaluated for the new generation and the non-dominated solutions are obtained. The two last steps are repeated until a stop criterion is reached. As result, the Pareto's optimal front is build where each point represents a possible solution.

## 3. FE Model Updating considering the hybrid UKF-MGA algorithm

The Kalman filter [2] is an estimator that consists in estimating the unknown variables by considering the measurements which contain statistical white noise. The mathematical representation of a general parameter identification problem solved by using the Kalman filter is based on the state-space model:

$$\boldsymbol{\theta}_k = \boldsymbol{\theta}_{k-1} + \boldsymbol{w}_{k-1} \quad (6)$$

$$\boldsymbol{z}_k = \boldsymbol{h}(\boldsymbol{\theta}_k, \boldsymbol{x}_k) + \boldsymbol{v}_k \quad (7)$$

where  $\boldsymbol{\theta}$  is the vector containing the model parameters and  $\boldsymbol{z}$  extracts the modelling outputs of the modelling function  $\boldsymbol{h}(\cdot)$ . The estimation process noise and the modelling uncertainty are taken into account in the vectors  $\boldsymbol{w}$  and  $\boldsymbol{v}$ , respectively. Both terms are assumed to be white Gaussian noise with zero-mean and covariance matrices  $\boldsymbol{Q}$  and  $\boldsymbol{R}$ , respectively. The matrix  $\boldsymbol{R}$  is the sum of measurements noise and modelling noise:  $\boldsymbol{R} = \boldsymbol{R}^{meas} + \boldsymbol{R}^{model}$  [4]. Nevertheless, the modelling uncertainty is neglected in this study as the same model is considered for each iteration of the simulation.

An extension of the Kalman filter to nonlinear systems, the so-called extended Kalman filter (EKF) was developed [5]. The EKF requires linearization (Taylor series expansion) of Equation (7). In case  $\boldsymbol{h}$  is a numerical model, there is no analytical solution of the Jacobian of the function. Instead, a numerical method such as the finite difference algorithm must be used to approximate the Jacobian.

To overcome this issue, the unscented Kalman filter (UKF) was proposed by Julier *et al.* [6]. The UKF is an estimator for nonlinear systems where no linearization of the modelling function  $\boldsymbol{h}$  is needed. This algorithm uses the unscented transform technique to select a group of sample points (sigma points) which are propagated through the nonlinear functions to calculate the mean and the covariance of the estimated parameters [7]. It implies that the second order Gaussian approximation of the estimates is preserved. In this study, the genetic algorithm hybridized with UKF is considered due to the advantages of this algorithm over the EKF. A detailed description of the algorithm for parameter estimation problems is given below.

As a Kalman filter, the UKF consists of two steps: (i), the prediction and; (ii) the correction (or update). The former consists in assessing the sigma points and the prior estimation error, modelling error and the cross covariance. The latter involves computing Kalman's Gain matrix ( $\boldsymbol{K}$ ) and correcting the prior estimates of the parameters and the covariance, based on the measurements  $\boldsymbol{z}^{obs}$ .

The sigma points are  $2n + 1$  distributed points in the vicinity of the current estimate. They are defined from the prior estimation error covariance [8]:

$$\mathcal{S} = \eta \sqrt{\mathbf{P}_{k-1}} \quad (8)$$

$$(\chi_{k-1})_0 = \hat{\boldsymbol{\theta}}_{k-1} \quad (9)$$

$$(\chi_{k-1})_i = \hat{\boldsymbol{\theta}}_{k-1} + \mathcal{S}_i, \quad i = 1, 2, \dots, n \quad (10)$$

$$(\chi_{k-1})_{i+n} = \hat{\boldsymbol{\theta}}_{k-1} - \mathcal{S}_i, \quad i = 1, 2, \dots, n \quad (11)$$

being  $n$  the number of parameters,  $\eta$  is the scaling parameter computed as  $\eta = \sqrt{n + \lambda}$ ,  $\mathbf{P}$  is the estimation error covariance and  $\boldsymbol{\chi}$  are the  $2n + 1$  sigma points. The sigma points are multiplied by a weight, defined as:

$$W_0 = \frac{\lambda}{n + \lambda} \quad (12)$$

$$W_i = W_{i+n} = \frac{1}{2(n + \lambda)}, \quad i = 1, 2, \dots, n \quad (13)$$

This algorithm is integrated into the MGA. Specifically, the UKF searches around the candidate values of the parameters proposed by the genetic algorithm. Once the value of the parameters  $\hat{\boldsymbol{\theta}}_{k|k}$  are obtained, the objective functions are assessed by minimising the mean square error between numerical and experimental modal parameters. The genetic algorithm uses these values of the objective functions to propose a new possible solution (vector of parameters). The algorithm is shown below.

**Initial Step:**

$$\hat{\boldsymbol{\theta}}_0 = \boldsymbol{\theta}_{prior}$$

$$\mathbf{P}_0^m = \mathbf{P}_{prior}$$

**Main loop:** for  $k = 1: N_{UKF}$  (number of iterations of the UKF)

*Prediction step:*

$$(\chi_{k-1})_i$$

$$(\chi_{k|k-1})_i = (\chi_{k-1})_i$$

$$\hat{\boldsymbol{\theta}}_{k|k-1} = \sum_0^{2n} W_i \cdot (\chi_{k|k-1})_i$$

$$\mathbf{P}_{k|k-1}^\theta = \sum_0^{2n} \left( W_i [(\chi_{k|k-1})_i - \hat{\boldsymbol{\theta}}_{k|k-1}] \cdot [(\chi_{k|k-1})_i - \hat{\boldsymbol{\theta}}_{k|k-1}]^T \right) + \mathbf{Q}$$

$$(\mathcal{Z}_{k|k-1})_i = \mathbf{h}((\chi_{k|k-1})_i)$$

$$\hat{\mathbf{z}}_{k|k-1} = \sum_0^{2n} W_i \cdot (\mathcal{Z}_{k|k-1})_i$$

$$\mathbf{P}_{k|k-1}^z = \sum_0^{2n} \left( W_i [(\mathcal{Z}_{k|k-1})_i - \hat{\mathbf{z}}_{k|k-1}] \cdot [(\mathcal{Z}_{k|k-1})_i - \hat{\mathbf{z}}_{k|k-1}]^T \right) + \mathbf{R}$$

$$\mathbf{P}_{k|k-1}^{\theta z} = \sum_0^{2n} \left( W_i [(\chi_{k|k-1})_i - \hat{\boldsymbol{\theta}}_{k|k-1}] \cdot [(\mathcal{Z}_{k|k-1})_i - \hat{\mathbf{z}}_{k|k-1}]^T \right)$$

*Correction (update) step:*

$$\begin{aligned} \mathbf{K}_k &= \mathbf{P}_{k|k-1}^{\theta z} (\mathbf{P}_{k|k-1}^z)^{-1} \\ \hat{\boldsymbol{\theta}}_k &= \hat{\boldsymbol{\theta}}_{k|k-1} + \mathbf{K}_k (\mathbf{z}^{obs} - \hat{\mathbf{z}}_{k|k-1}) \\ \mathbf{P}_k^m &= \mathbf{P}_{k|k-1}^m - \mathbf{K}_k \mathbf{P}_{k|k-1}^z \mathbf{K}_k^T \\ &end \end{aligned}$$

#### 4. Model updating of a real historical construction

As benchmark structure, the domes of the chapel integrated in the Würzburg Residence (Germany) is considered. The domes are masonry surfaces of bricks grown in a single lawyer. The bricks are 30 cm thick, and at the base of the dome, they are reinforced by increasing the thickness until 45 cm. The reinforcement is also presented in the radial ribs reaching a section of 45x45 cm. In the construction process of the domes, the nerves are built alongside the rest of the sheet.

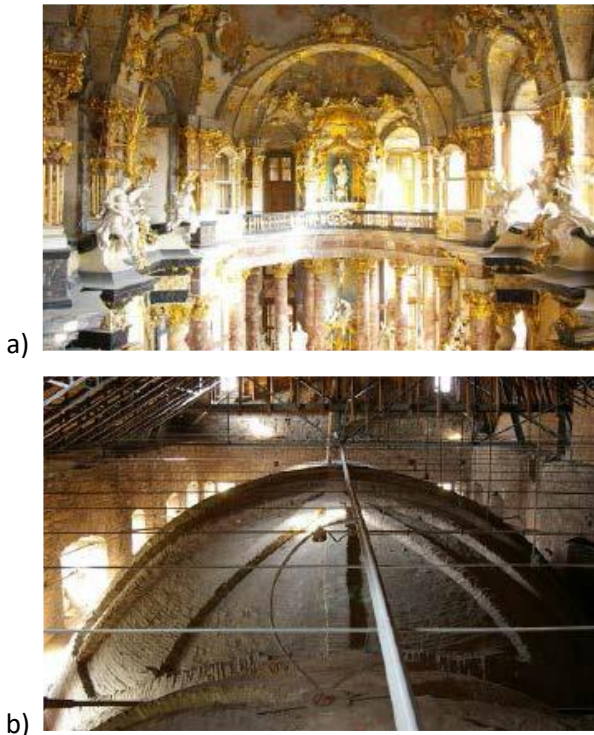


Figure 1. a) Interior of the chapel and b) exterior of the dome of the chapel of the Würzburg Residence

##### 4.1 Preliminary Numerical Model

The preliminary FE model was built using the Ansys software [9]. The structural model of the

dome was defined by considering 3D shell elements (SHELL63) which have 4 nodes per element. The effect of the lateral walls of the chapel on the dome was established, in a simplified manner, by means of two 1D spring elements (COMBIN14) whose stiffness represents the bending rigidity of the wall in the direction under consideration. The following mechanical properties of the material have been considered: Young's modulus,  $E$  (GPa) = 2; Poisson's ratio,  $\nu$  = 0.2; density,  $\rho$  (kg/m<sup>3</sup>) = 1700 and horizontal stiffness of the walls,  $k_h$  = 1000 kN/m. The latter was determined by considering that these elements were made with the same material than the rest and assimilating their behaviour to a cantilever. To adequately model the behaviour of the walls, four different zones were assumed (a wall for each lateral dome and two walls for the main dome) (see Figure 2). The numerical modal analysis led to the three natural frequencies shown in Table 1.

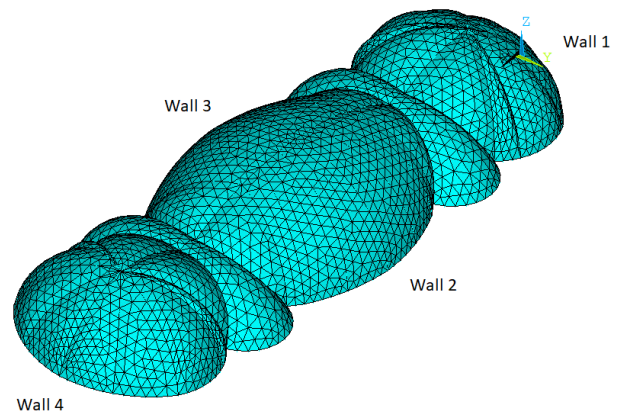


Figure 2. FE model of the dome of the chapel

##### 4.2 Ambient Vibration Test and Operational Modal Analysis

An ambient vibration test was performed in order to experimentally identify the modal parameters of the structure. To this effect, the accelerations



were recorded during 10 minutes by eight uniaxial accelerometers at 51 instrumented points. The signals were processed using the time domain SSI algorithm [10], implemented in the software ARTEMIS Modal [11]. This whole procedure has been detailed in [12]. The three identified natural frequencies are shown in Table 1. Besides, the relative difference between the numerical and experimental natural frequencies and the MAC are calculated for each vibration mode.

Table 1. Numerical and experimental natural frequencies

Vibration Mode	$f_{num}$ [Hz]	$f_{exp}$ [Hz]	$\Delta f$ [%]	MAC
1 (lateral)	3.362	2.764	21.635	0.809
2 (longitudinal)	3.431	3.231	6.190	0.877
3 (lateral)	3.988	4.015	-0.672	0.648

From the results in Table 1, it can be noticed that despite the use of a very detailed FE model, the natural and experimental mode shapes are not in good agreement as the errors between them are high. A FE model updating process must be conducted to guarantee that the numerical model represents more accurately the actual behaviour of the dome.

### 4.3 Finite Element Model Updating

This process was implemented in the software programme MATLAB [13]. As global optimization algorithm, the MGA was used. This study focuses on the comparison of the two previous described techniques to find the global minimum of the objective functions: (i), the maximum likelihood method (MLM) based on MGA and; (ii), the hybrid UKF-MGA algorithm.

The parameters selected for the model updating process are the same for both problems. A sensitivity study was performed to select the physical parameters that influence more the dynamic behaviour. This influence was taken into account on the basis of the modal strain energy. The seven parameters chosen were the followings: the Young's modulus of the dome, the Young's modulus of the ribs, the Young's modulus

of the starting wall, the longitudinal stiffness of the walls 1 and 4, the lateral stiffness of the walls 2 and 3, the lateral stiffness of the walls 1 and 4 and the longitudinal stiffness of the walls 2 and 3. The search range of the parameters were: (i) for the Young's moduli, [1–4] GPa and; (ii) for the stiffness, [200-2200] kN/m.

The objective functions were defined as expressed in Equations (4) and (5).

#### 4.3.1 MLM-MGA method

The value of the updated parameters is shown in Table 2.

Table 2. Updated physical parameters (MLM-MGA)

Parameter	Updated value
$E$ dome [Pa]	1.164E+9
$E$ ribs [Pa]	1.468E+9
$E$ starting wall [Pa]	1.319E+9
$k_x$ walls 1 and 4 [N/m]	7.531E+5
$k_y$ walls 2 and 3 [N/m]	3.941E+5
$k_y$ walls 1 and 4 [N/m]	1.729E+6
$k_x$ walls 2 and 3 [N/m]	9.793E+5

The modal analysis of the updated FE model led to the three natural frequencies given in Table 3. The improvement of the relative differences between natural frequencies (except the third vibration mode) and the increase of MAC ratios exhibit the good performance of the updating procedure.

Table 3. Natural frequencies of the updated FE model (MLM-MGA)

Vibration Mode	$f_{num,up}$ [Hz]	$f_{exp}$ [Hz]	$\Delta f$ [%]	MAC
1	2.725	2.764	-1.411	0.861
2	3.237	3.231	-0.186	0.921
3	4.188	4.015	4.299	0.728

#### 4.3.2 Hybrid UKF-MGA algorithm

In the implementation of the UKF-MGA the following parameters were chosen [14]: initial estimation error covariance,  $\mathbf{P}_0^m = \text{diag}\left(\left(\frac{u_b - l_b}{100}\right)^2\right)$ ; covariance matrix of process noise,  $\mathbf{Q} = 0.005\mathbf{P}_0$ ; the covariance matrix of measurements noise is taken from the OMA,  $\mathbf{R} = 0.001 \cdot \mathbf{I}(r, r)$ , with  $\mathbf{I}(r, r)$  the identity matrix having dimensions equal to the size of the measurements data,  $r$  and  $\lambda = 0.0001$ . The measurements vector,  $\mathbf{z}^{obs}$ , was defined taking into account both the experimental natural frequencies and the mode shapes. The latter was implemented in terms of the MAC, considering the unit for the three vibration modes. In this manner, the difference  $\mathbf{z}^{obs} - \hat{\mathbf{z}}$  represent the error between observed and numerical modal parameters. The updating process gave as a result the model parameters presented in Table 4.

Table 4. Updated physical parameters (UKF-MGA)

Parameter	Updated value
$E$ dome [Pa]	2.848E+9
$E$ ribs [Pa]	3.424E+9
$E$ starting wall [Pa]	2.778E+9
$k_x$ walls 1 and 4 [N/m]	6.418E+5
$k_y$ walls 2 and 3 [N/m]	2.371E+5
$k_y$ walls 1 and 4 [N/m]	1.862E+6
$k_x$ walls 2 and 3 [N/m]	9.728E+5

The natural frequencies obtained after this optimization are shown in Table 5. This algorithm also conducts to a good adjustment of the numerical and experimental modal parameters.

Table 5. Natural frequencies of the updated FE model (UKF-MGA)

Vibration Mode	$f_{num,up}$ [Hz]	$f_{exp}$ [Hz]	$\Delta f$ [%]	MAC
1	2.741	2.764	-0.832	0.859
2	3.207	3.231	-0.743	0.928
3	4.423	4.015	10.161	0.695

#### 4.4 Discussion of the results

A discussion of the performance of both algorithms, GA and GA-UKF, is presented. The comparison is developed in terms of two key criteria: regarding the relative differences between numerical and experimental natural frequencies and MAC ratios and with respect to the number of iterations and computation time.

First, from Table 3 and Table 5, it can be observed that the relative differences,  $\Delta f$ , are smaller for the MLM-GA (except the first vibration mode). In addition, the MAC ratios are larger for this algorithm (except for the second vibration mode).

Second, the number of iterations and the computational cost is compared (see Table 6). The algorithms were run in a 3.60 GHz processor with a 29.3 RAM. For the MLM-GA, the number of iterations of the multi-objective genetic algorithm was established in 100 and a population of 200 solutions (vector of possible parameters) was considered. The time to complete the procedure was around 47000 seconds. However, for the UKF-MGA, the number of iterations of the multi-objective genetic algorithm was set to 20, the initial population was made up of 10 vectors and 5 iterations of the unscented Kalman filter were considered. The reduced number of iterations and population of the genetic algorithm is due to the multiple ( $2n + 1$ , being  $n$  the number of parameters) evaluations of the objective functions for each iteration of the UKF. The duration of the UKF-MGA is around 16000 seconds. This is 34% less than the time of the GA.

Based on the two criteria, the UKF-MGA represents a useful tool to perform a model updating process, as the computational cost is considerably reduced and the results, although worst, are similar to those obtained with the MLM-MGA.

Table 6. Comparison of the MLM-MGA and UKF-MGA algorithms

	$N$ (MGA)	Pop.	$N$ (UKF)	Time [s]
<b>MLM-MGA</b>	100	200	-	47710
<b>UKF-MGA</b>	20	10	5	16068

## 5. Conclusions

The usual way of conducting a FE model updating is to carry out an optimization process. To this effect, optimization algorithms are used, being the evolutive algorithms, such as the GA, the most common. The high computational cost of the GA has guided to authors to employ alternative techniques. A multi-objective GA hybridized with UKF algorithm is proposed in this study. In order to analyse the performance of both algorithms, a comparative study has been carried out. It has been elaborated via the FE model updating of a real chapel, placed in Würzburg (Germany). The results of the comparison have shown that UKF-MGA is able to obtain similar results to those of the MLM-MGA. Moreover, the UKF-MGA lasted around 16000 seconds while the MLM-MGA completed the process in around 47000 seconds. The hybridized tool used in this study can be considered as an efficient technique to conduct a FE model updating to obtain a more accurate numerical model of the real structure.

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