Structural safety assessment of geometrically complex masonry vaults by non~~-~~-linear analysis. The Chapel of the Würzburg Residence (Germany)

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Abstract

This paper addresses the structural safety assessment of the Chapel of the Würzburg Residence (Germany), one of the most important churches of the Central European Baroque. It was declared as World Heritage Site by UNESCO in 1981, one of its most unique and distinctive characteristics is the geometry of its complex vaults. Intersections between vaults are warped and vaults surfaces were built using only one layer of brick masonry. In this work, a nonlinear finite elements (FE) model has been developed and used to assess the structural safety of the building. In order to update the model by identifying the dynamic response of the building, experimental ambient vibration tests have been previously subsequently carried out. Operational ~~m~~Modal ~~a~~Analysis (OMA) has been used to experimentally identify both modal displacements and natural frequencies. The numerical FE model is them adjusted using genetic algorithms until its dynamic response resembles that experimentally observed, thus providing a valid model to further analyse the structural behaviour of the building. After briefly descripting the Chapel, the methodology followed to update the numerical model and the obtained results from a non-linear analysis on this over-complex vaulted structure are the main goals of the paper.

Keywords: Masonry vaults; Ambient vibration test; Historical building; Chapel of the Würzburg Residence; Operational Modal Analysis; FE model updating; Non-linear analysis; Genetic algorithms

1 Introduction

The Würzburg Residence is a large German Baroque construction built during the first half of the 18th Century, see Fig. 1. It was declared a World Heritage Site by UNESCO in 1981. The chapel is integrated into the main building and is located on its south-west corner. This chapel is a very singular construction, mainly due to the configuration of its vaults, with a complex spatial geometry, which includes warped intersections between them, and due to the high level of technology required from a construction point of view. In this sense, it is one of the few examples that can be found in Europe with this level of complexity [1].

Fig. 1 Front of the Würzburg Residence (Germany).

One of the main difficulties that need to be addressed in structural analysis of historical buildings is the level of uncertainty associated with many factors affecting the behaviour of the structure. Aspects like the mechanical properties of the structural materials, the building construction process, the connections between structural parts or the cracking condition of the building may cause important changes between the results obtained from a numerical analysis and those experimentally observed [2]. In this sense, non-destructive techniques appear as useful tools to provide information about the structural behaviour of the building [3,4]. In particular, dynamic properties provided by ambient vibration techniques have proved to be quite well-suited to validate and update numerical models [5].

Operational Modal Analysis (OMA) has consolidated as one of the most adequate methods to estimate the modal parameters of a structure, due to the facts that: (i) it is a non-destructive and non-invasive technique and (ii) it can be performed under service conditions. For these reasons, OMA is currently recognised as a quite convenient technique to dynamically characterise historical buildings, since the use of the stronger external excitation (impact hammers or shakers) required to perform the traditional Experimental Modal Analysis (EMA) is not needed. The interested reader is referred to ~~Reference~~ Ref. [6] for a more involved discussion on the practical and technical differences between OMA and EMA when applied for testing of masonry vaults. Subsequently, modal properties provided by the application of OMA allows the adjustment of numerical models in order to obtain an accurate estimation of the actual behaviour of the structure. In this sense, updated FE models can be used to carry out a structural analysis under existing conditions or further predict the effects of different structural situations that the structure could undergo. In the last decades, many cases of application of ambient vibration tests to update numerical models can be found in historical buildings [7–12]. However, the applications of OMA to assess structural behaviour of specific complex parts of a building, such as domes or vaults, are much more limited [6,13,14].

The main purpose of this study is to evaluate the current structural state of the vaults of the chapel in order to gain further knowledge over such complex structure and estimate its current safety level. A brief description of the chapel, the methodology followed to dynamically characterise the building, the updating process and the results obtained from a nonlinear analysis will be presented in this paper. The paper is organised as follows: Section 2 summarizes the main characteristics of the chapel together with a historical overview and a brief description of its architectural configuration. Section 3 presents the dynamic characterisation of the vaults of the chapel by using the Operational Modal Analysis method. It further describes the initial finite element model developed to estimate the modal parameters of the system. Section 4 is devoted to discuss the updating of the finite element model based on the experimentally obtained modal parameters. In Section 5, the updated finite element model is then used to further analyse the nonlinear behaviour of the vaults when subjected to vertical loads until reaching collapse. Finally, Section 6 draws the main conclusions of this study.

2 Chapel of the Würzburg Residence

2.1 Historical Aspects

The construction of the Würzburg Residence [15] dates from the early eighteenth century when the Schönborn family decided to build a palace to relocate the Episcopate. Balthasar Neumann [16,17] was the main work master for over thirty years. The Chapel exhibits an impressive Baroque decoration, see Fig. 2.

Fig. 2 Chapel of the Würzburg Residence. Altar views.

During the building design process, the chapel was placed at different locations within the whole complex, until Balthasar Neumann, finally, located it at the south-west corner of the Residence (Fig. 3). Johann Dientzenhofer was involved in the design of the chapel [18]. He was a German architect with a wide experience on the design and construction of geometrically complex brick masonry vaults.

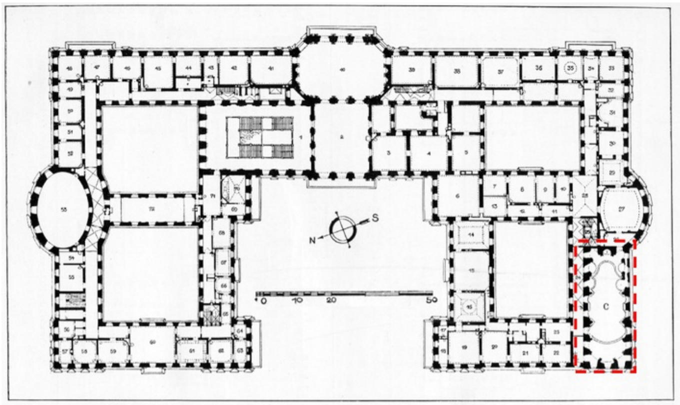

Fig. 3 Würzburg Residence. Location of the chapel at the SW corner.

2.2 Architectural configuration and construction

The architectural configuration of the chapel presents a spatial view with a clear longitudinal character, composed by three main spatial cells connected by two other cells, as illustrates Fig. 4. The walls are particularly slender (15.2 m high with a variable thickness, between 1.20 and 2.00 m) and with a high percentage of voids, greater than 50% of the façade surface.

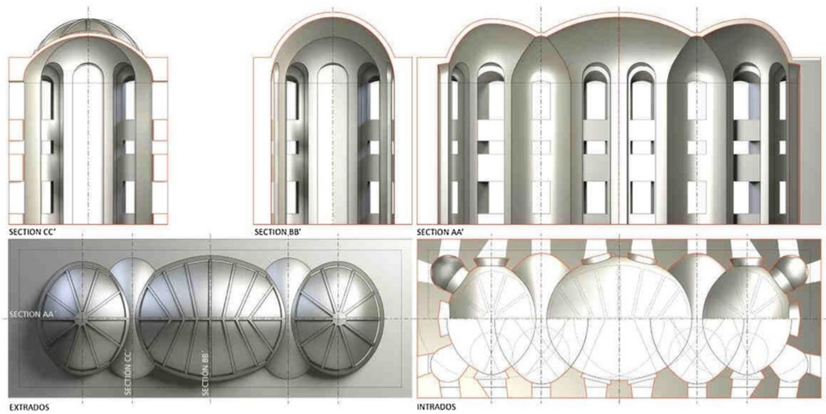

Fig. 4 Study of the trace of the Chapel of the Würzburg Residence.

There are two main reasons why the chapel vaults are unique in their complexity. A first feature, from a geometrical point of view, is that the ceiling is made by five in-plan oval vaults with warped intersections among them (Fig. 4). Each one of these intersections was geometrically determined by the intersection between two cylinders with orthogonal axis and different diameters. These cylindrical shape was essential in the construction process, because intersections were defined first as edges of the vaults boundaries, and then the surface of the vaults was built according to the intersections. A second feature is that the construction technique used was quite refined. Vaults surfaces are made of only one layer of brick masonry (30 cm thick), with neither ribs nor any sort of local reinforcement at warped intersections [15]. For constructive reasons, vaults only increase 15 cm their thickness at their base and at small radial ribs located on selected locations of the three main vaults (Fig. 4 and Fig. 5).

Fig. 5 Chapel of the Würzburg Residence. Vaults extrados.

3 Ambient vibration tests and Operational Modal Analysis method

The dynamic characterization of the Chapel of the Würzburg Residence has been performed by means of ambient vibration tests. These tests were carried out between 14 and 17 of July 2014, with the aim of identifying the natural frequencies, mode shapes and damping ratios of the vaults.

3.1 Initial finite element model

The application of the Operational Modal Analysis usually requires the creation of an initial model (Fig. 6) in order to estimate the natural frequencies and mode shapes and thus determine adequate positions of the accelerometers. This initial FE model was built using Abaqus/CAE 6.13 Software [19]. The model consists mainly of two components, namely walls and vaults, modelled using solid elements. The final model comprises 1,606,908 elements, 329,472 nodes and 988,416 degrees of freedom. Material properties were initially estimated from bibliography [17,20]. For brick masonry of the vaults, the adopted properties were: density, 1980 kg/m^3; Young's Modulus, 1100 MPa; Poisson's ratio, 0.2. Similarly, the following values were considered for the stone masonry: density, 2100 kg/m^3; Young's Modulus, 2200 MPa; Poisson's ratio, 0.2. Furthermore, the weight of the roof over the vaults was taken into account in the FE model, as well as the connections between the chapel and the rest of the Residence Building, by means of spring elements [7,8].

Fig. 6 Initial FE model.

Based on the largest modal displacements obtained from this initial model, the appropriate position for the reference accelerometers is set. Fig. 7 illustrates the first three vibration modes together with their corresponding natural frequencies.

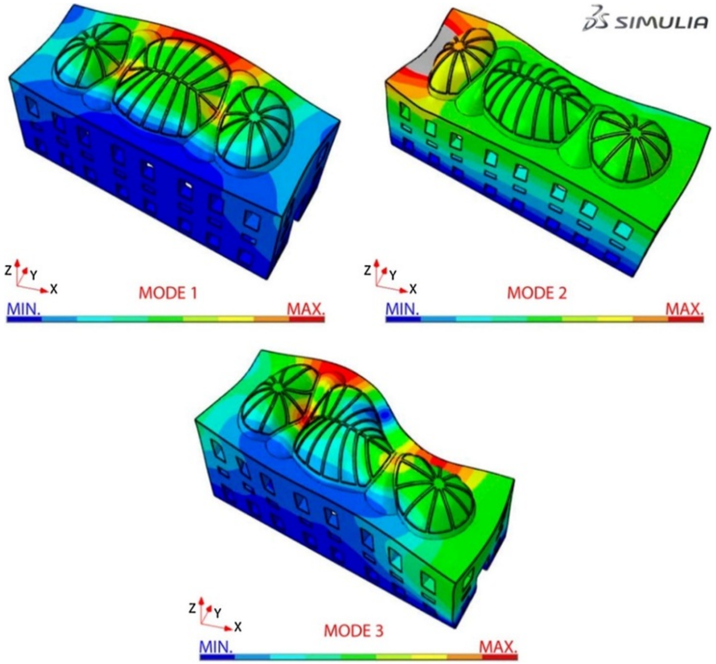

Fig. 7 Initial FE model. Modal displacements of the first three vibration modes (f_1 = 2.59, f_2 = 3.04 and f_3 = 4.15 Hz).

3.2 Ambient vibration tests

Following the results obtained from the initial FE model, Fig. 8 shows a schematic representation of the sensors arrangement. The set-up consists of a total of 51 measuring points, which is a rather dense mesh. It is also noted that, due to the boundary conditions imposed by its integration into a large complex, the structure is not symmetrical. All of the measuring points were set in the three principal directions, in order to capture the global vibration modes in the longitudinal, lateral and vertical direction of the vaults. Since only eight accelerometers were available for the testing process and two of them (placed at points 17–19, Fig. 8) were kept fixed for reference, a series of twenty-five set-ups were necessary to cover all measuring points. In each one of these set-ups, accelerations were recorded with a sampling rate of 100 Hz and a sampling time of 12 min. These assumptions ensure that natural frequencies in the range from 1 to 50 Hz could be properly recorded. However, in order to perform a more accurate analysis and to take into consideration that the expected natural frequencies are below 10 Hz, a decimation factor of 5 was previously applied.

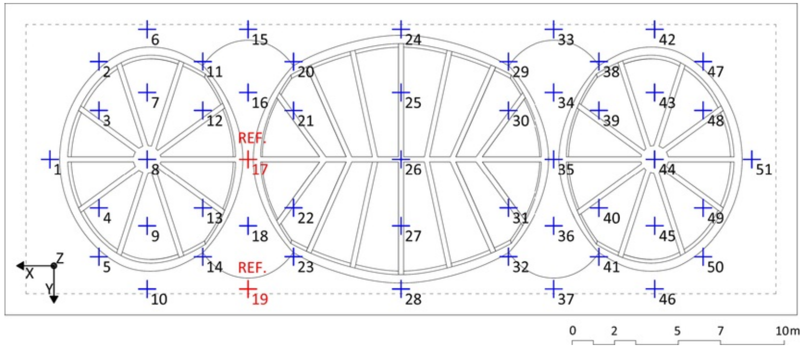

Fig. 8 Accelerometer locations and directions (plan view of the chapel). Ref. accelerometers in red.

Excitations during ambient vibration tests were associated with environmental loads. The equipment used for these tests was composed by eight uniaxial force balance accelerometers with a bandwidth ranging from 0.01 to 200 Hz, a dynamic range of 140 dB, a sensitivity of 10 V/g and 0.35 kg of weight (model ES-U2). These accelerometers were connected via eight 40 m long cables to a twelve-channel data acquisition system with a 24-bit ADC, provided with *anti*-alias filters (model GRANITE). The equipment is manufactured by the company KINEMETRICS (Fig. 9).

Fig. 9 Measurement equipment.

3.3 Operational Modal Analysis

In situ obtained data were processed using two different identification methods: one of them in frequency domain, the Enhanced Frequency Domain Decomposition (EFDD) technique [21]; and the other in the time domain, the Stochastic Subspace Identification (SSI) method [22]. Both methods are implemented in the software Artemis [23] (Fig. 10).

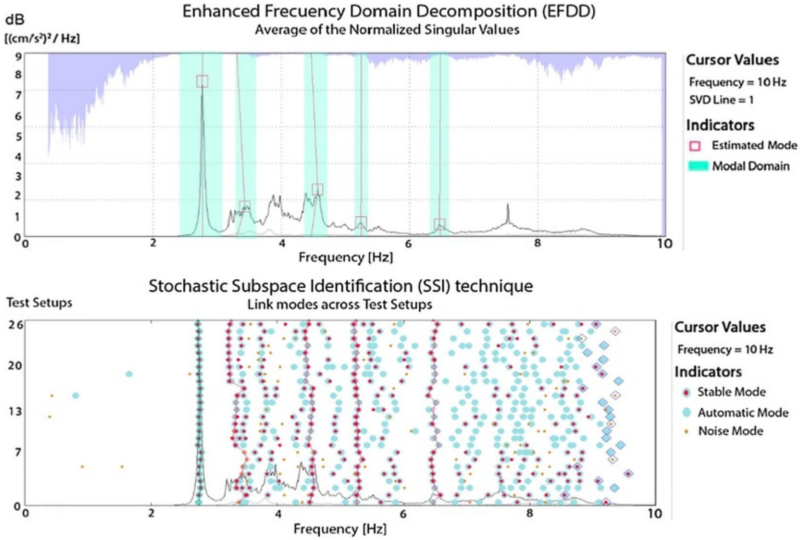

Fig. 10 Ambient tests set up (EFDD and SSI).

Fig. 10 shows that almost all identified modal frequencies – using the SSI technique – exhibit slight changes between the different test set-ups. This fact may be explained provided that the measurements were performed during three consecutive summer days, with similar but not equal environmental conditions. Several research studies have recently proved that different temperature or humidity conditions can slightly modify the dynamic properties of a masonry structure [23–26]. Bearing in mind this consideration, the modal frequencies, damping ratios and mode shapes were determined by applying both methods – EFDD and SSI – and were later correlated using the Modal Assurance Criterion (MAC) [27] between both sets of results, in order to assess the accuracy of the obtained mode shapes:

$$MAC_{j,k} = \frac{\left(\phi_j^T \cdot \phi_k\right)^2}{\left(\phi_j^T \cdot \phi_j\right) \cdot \left(\phi_k^T \cdot \phi_k\right)} \quad \textbf{(1)}$$

where ϕ_k and ϕ_j are the two modes to be compared and T denotes the transpose.

Data processing results, including the standard deviation of modal frequencies and damping ratios, are presented in Table 1.

Table 1 Results OMA: natural frequencies (f), damping ratios (ξ) and standard deviation (Std).

	SSI				EFDD				MAC
	f (Hz)	Std. f	ξ(%)	Std. ξ	f (Hz)	Std. f	ξ(%)	Std.ξ	
Mode 1	2.77	0.01	1.12	0.40	2.77 (0,0%)	0.01	0.89 (26%)	0.28	0.99
Mode 2	3.28	0.05	1.50	0.68	3.30 (0,6%)	0.37	0.92 (63%)	1.24	0.87
Mode 3	4.52	0.05	2.36	0.66	4.46 (1,4%)	0.20	1.79 (32%)	0.91	0.88
Mode 4	5.26	0.03	1.24	0.48	5.25 (0,2%)	0.02	0.88 (61%)	0.28	0.90
Mode 5	6.48	0.04	2.04	0.95	6.48 (0,0%)	0.03	1.11 (84%)	0.46	0.82

The percentage within parenthesis indicates the relative difference between SSI and EFDD results (SSI results as reference).

As Table 1 shows, the ambient vibration tests allowed to accurately identify the first five vibration modes in a frequency range up to 10 Hz. The frequencies were identified with relative differences lower than 1.5%, taking the results of the SSI method as reference. The results for the damping ratio show as higher variability (up to 84%), with average modal damping ratios of the building being 1.65% and 1.18% for SSI and EFDD techniques, respectively. This result is not surprising and higher excitation seems to be required in order to obtain reliable measures of damping, which is not needed for static structural analyses, as done later in this paper. With respect to mode shapes, MAC values were always higher than 0.80, which indicate a good correlation between both methods. The second mode shows a longitudinal translation mode, while the other modes correspond to bending modes of the vaults (Fig. 11).

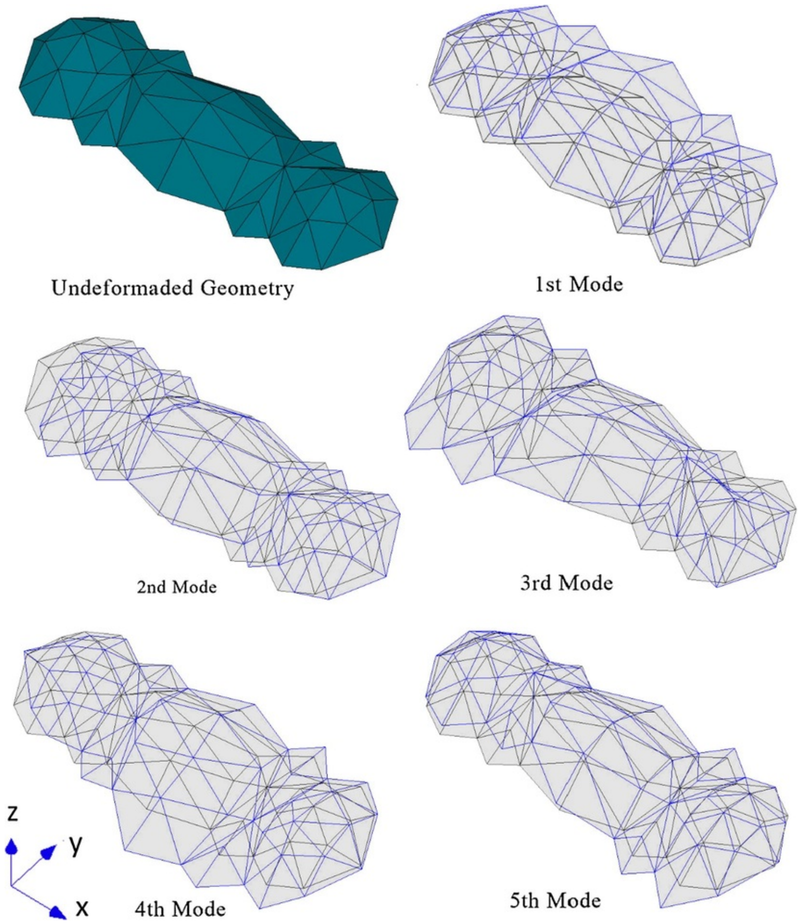

Fig. 11 Mode shapes associated with the experimental results (SSI).

4 Model updating

Standard FE models of historical buildings usually include uncertainties because of parameters such as unknown material properties, poorly known boundary conditions, existing damage, complex internal composition of structural elements and modelling approximations. Therefore, calibrating the model with the aid of experimental information becomes necessary in order to appropriately model the actual structural behaviour. For our purposes, results obtained from ambient vibration tests will be used to update and improve the initial FE model.

In this case, the FE model updating based on the obtained experimental dynamic properties of The Chapel was performed by means of iterative methods [28]; that is, model updating arises from changes applied on some well-defined structural physical parameters selected by the users. For this purpose, a sensitive study was carried out in order to identify the physical parameters of the structure that have a stronger influence on its dynamic behaviour. These parameters were the Young's modulus of the masonry brick (E_b) and the stone (E_s), the springs stiffness that simulate the connection with the palace (S_p) and the inertial mass associated to the rest of the building (I_p) (Fig. 12). The selection of these last two parameters was essential to perform the calibration process. It is important to remember that the Chapel modelled is included within a larger complex, whose influence on the modal behaviour of the chapel has to be considered. The selection of more parameters would unnecessarily complicate the physical interpretation of the updating process.

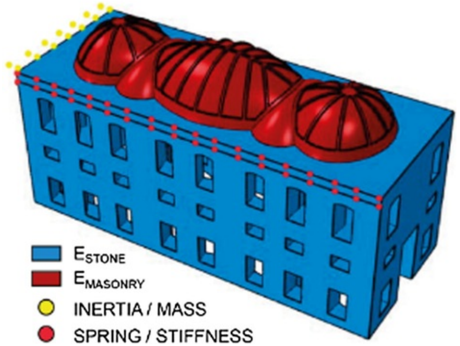

Fig. 12 Finite element model and group of material/element types considered.

Given the high quality of the experimentally detected first four vibration modes, they have been chosen to carry out the updating process. Only natural frequencies values were selected, due to the lower reliability of the identified mode shapes in comparison with the measured natural frequencies. Therefore, considering four identified natural frequencies, four residual components were adjusted and minimised during model updating. The updating process has been performed via an optimization algorithm, namely genetic algorithms [29], as implemented in Matlab software [30], according to the following fitness value $I(\theta)$:

$$I(\theta) = \frac{1}{2} \cdot \sum_{j=1}^{m} w_j \cdot \left[z_{NUM,j}(\theta) - z_{EXP,j} \right]^2 = \frac{1}{2} \sum_{j=1}^{m} w_j \cdot r_j(\theta)^2 \quad \textbf{(2)}$$

where $z_{NUM,j}(\theta)$ are the frequencies obtained from the numerical model, which are related to the physical parameters of the model, θ (elastic moduli of masonry brick and stone, springs stiffness and inertial mass), which are the object of the adjustment. The variables $z_{EXP,j}$ represent the same frequencies obtained from experimental data, specifically, the values obtained from the EFDD method. The differences between the experimental and numerical parameters are denoted as residues, $r_j(\theta)$. A weight variable w_j could be established for each residue to take into account the different reliability of the identified modal parameters. In our case, $w_j = 1_{(j=1-4)}$ is adopted.

Several iterative methods have been proposed in the literature to update FE models in structural dynamics. For a more involved discussion on different alternatives, the interested reader is referred to the book by Marwala [31]. For our purposes, genetic algorithms are selected to conduct the optimization, as they exhibit a higher probability to converge to a global optimal solution than gradient-based methods. Genetic algorithms [29] aim at finding such optimal solution among a population of possible solutions by applying the principles of evolutionary biology (crossover, mutation, selection and reproduction) to computer science. Genetic algorithms have proven successful when applied to complex optimization problems and, in particular, when applied to FE updating (see, e.g., Marwala [31] and references therein).

Fig. 13 illustrates the summary of the updating process and shows its importance for obtaining reliable models that replicate the actual response of the structures. Based on the properties of the initial finite element model, and establishing a range of values for each updating parameter (Table 2) and the updating objective, the calibration process started. In each iteration, a population of 1000 vectors was created that, using the genetic algorithm rules (as implemented in Matlab software), minimized the objective function (2). Such calibration process terminated when the difference between the mean value (blue points, Fig. 13) and the best value (black points, Fig. 13) of the population was less than 1×10^{-4}. Table 2 shows the considered lower and upper bounds for the updating parameters and their corresponding initial and updated values. As it can be seen, updated values differ up to 30% with respect to values defined in the initial model.

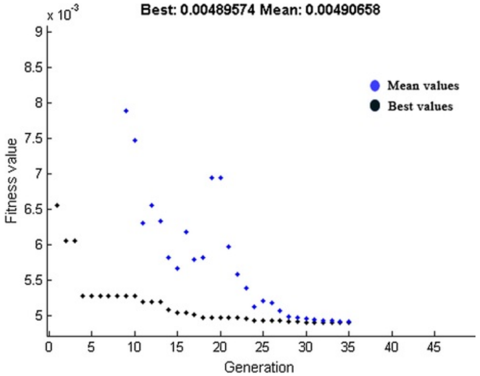

Fig. 13 Genetic algorithm. Fitness value $J(\theta)$ versus Generation. Blue point: Mean values of the objective function of all the population of the corresponding generation. Black point: Best values result of an individual of the population.

Table 2 Summary of the results of the FE model updating process: parameters.

Updating parameter	Initial value	Optimizing interval of values		updated value
		Lower bound	Upper bound	
E_b (MPa)	1100	700	2000	921
E_s (MPa)	2200	1500	3500	2710
S_p (kN/m)	9000	6000	12,000	10,800
I_p (kg)	4000	2000	6000	4680

Table 3 and Fig. 14 summarize the results following the updating process and confirm the high correspondence of the results between the calibrated model and those obtained from ambient vibration tests. Table 3 shows that the updated frequencies are close to the experimental ones, differing less than 2% while exhibiting MAC values trial range from 0.82 (mode 5) to 0.99 (mode 1) for the five considered vibration modes.

Table 3 Comparison of frequencies (Hz) experimentally (f_{EFDD}) and analytically (f_{FEM}) obtained.

Modes	f_{EFDD} (Hz)	$f_{(Initial\ FEM)}$ (Hz)	$f_{(FEM\ UPDATED)}$ (Hz)	MAC value ($_{EFDD\text{-}FEM\ UPDATED}$)
Mode 1	2.77	2.59 (6.5%)	2.79 (0.71%)	0.99
Mode 2	3.30	3.04 (7.9%)	3.29 (0.30%)	0.96
Mode 3	4.46	4.15 (6.9%)	4.43 (0.67%)	0.93
Mode 4	5.25	4.65 (11.4%)	5.15 (1.90%)	0.88
Mode 5	6.48	5.54 (14.5%)	6.36 (1.85%)	0.82

The percentage within parenthesis indicates the relative error taking as reference the results of the EFDD method.

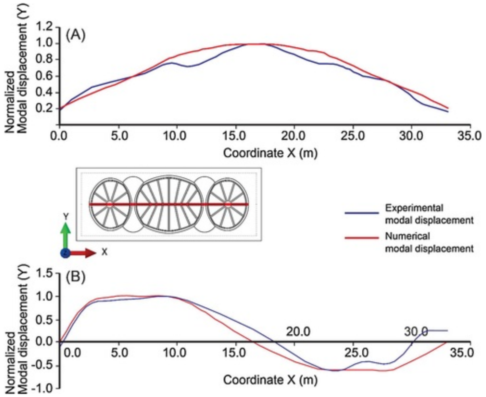

Fig. 14 Example of correlation between the numerical and experimental vibration modes. (A) Updated versus experimental first mode; (B) Updated versus experimental third mode.

The conducted updating process directly focused on minimizing the differences between the numerical FE model predictions and the observed experimental results, thus solving the main uncertainties of the initial FE model. Other authors propose an alternative approach to verify and validate the numerical model, by addressing the uncertainty quantification of the different variables involved in the overall process, including those associated to the variability in experimental measurements [32].

5 Non-linear structural analysis

5.1 Constitutive law for masonry and load case

Once the numerical model has been calibrated, it will be further used to assess the structural safety of such a singular building. To this end, structural materials are modelled using non-linear constitutive models. The model is characterised by its linear behaviour in the elastic regime and elastic plastic damageable behaviour in the nonlinear range, taking into account the difference of compressive and tensile behaviour.

The model assumes that a combined damage-plasticity model, as shown in Fig. 15, characterizes the uniaxial tensile and compressive responses of the material. Under uniaxial compression, stress–strain relationship is linear until the value of the maximum stress in compression, f_c, is reached. Then, the response is assumed to follow a strain softening regime. Under uniaxial tension, the stress–strain response is also linear elastic up to the value of the maximum stress, f_t, corresponding to the onset of micro-cracking in the material. After reaching the value of failure stress, the formation of micro-cracks is represented with a softening stress–strain relationship.

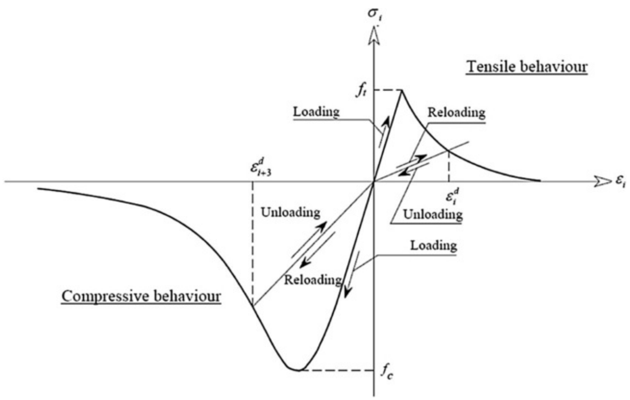

Fig. 15 Model response to uniaxial loading in tension and compression.

On the other hand, the post-failure behaviour for direct straining is modelled with tension softening, which allows to define the strain softening behaviour for cracked masonry. In this case, the tension softening is specified by applying a fracture energy cracking criterion. Hillerborg defines the energy required to open a unit area of crack, G_f, as a material parameter, using quasi-brittle fracture concepts [33]. With this approach the material's quasi-brittle behaviour is characterized by a stress-displacement response rather than a stress-strain response. Under tension, a specimen will crack across some sections. After it has been pulled apart sufficiently for most of the stress to be removed (so that the undamaged elastic strain is small), its length will be determined primarily by the opening at the crack. The fracture energy, G_f, can be specified directly as a material property; in this case, the failure stress, f_t, is defined. This model assumes a linear loss of strength after cracking, as shown in Fig. 16. Details about the extension of the above introduced damage model to three-dimensional multiaxial conditions and the definition of failure surfaces can be found in [34]. Table 4 shows the material properties adopted for our subsequent self-weight analysis.

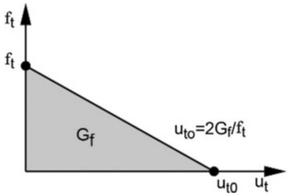

Fig. 16 Post-failure stress in tension. Fracture energy curve.

Table 4 Material properties adopted in the analysis.

Material	Density (kg/m³)	E (MPa)	υ	f_c (MPa)	f_t (MPa)	G_f (N/m²/m)
Brick masonry	1980	921	0.20	1.00	0.10	18
Stone masonry	2100	2710	0.20	2.80	0.28	18

Only vertical loads will be next considered in order to assess structural safety. For our purposes, two different analyses are carried out: the first one considers the self-weight of the structure to examine the building. In its current state, no relevant cracks are observed in the structure, which should be replicated in this analysis. The second one considers the increase of these vertical loads by means of a gravity factor, which is increased until reaching the collapse of the structure, in order to obtain the safety factor of the structure to vertical loads. The structural analysis was carried out using Abaqus/CAE 6.13 Software [19].

5.2 Analysis of results

5.2.1 Service condition

For the first self-weight analysis, the total weight of the chapel is around 63,700 kN. The average compressive stress obtained at the base is 0.5 MPa and the maximum compressive stress is about 1.0 MPa at the bottom southwest corner of the building. With regard to compressive stresses, materials would remain in the elastic range considering that the maximum stresses at the base of the chapel are around 34% of the compressive strength. However, there are some areas where the maximum tensile stresses are close to reaching 0.09 MPa (Fig. 17), which is the value of the failure stress in tension. These values are located in very specific areas of the upper surface of the domes and the openings of the outer walls. Although the safety factor is low, these results confirm the absence of cracks in the Chapel of the Würzburg Residence.

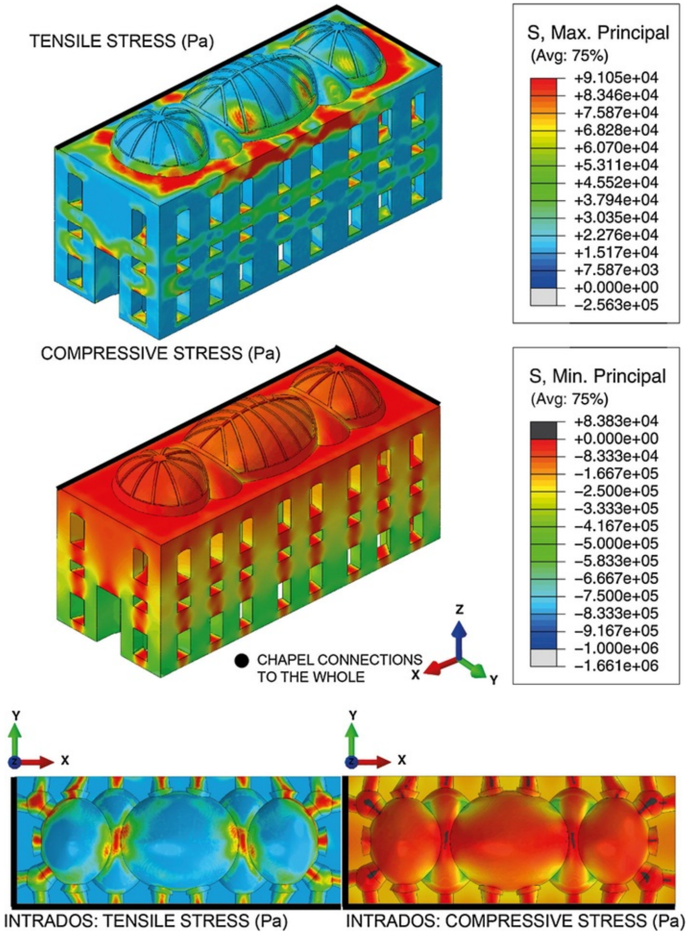

Fig. 17 Principal stresses (expressed in Pa). Self-weight.

5.2.2 Structural safety assessment

Once validated the FE model, a non-linear analysis was performed increasing the gravity factor until the collapse of the structure. The maximum gravity multiplier factor obtained was 4.84. Fig. 18 shows the evolution of the cracks until the collapse of the structure (4.84 g).The structural behaviour predicted comes from the superposition of two main factors. The first one is caused by the horizontal action of the vaults, while the second one is due to the deformation of the walls at the southwest corner of the chapel. The crack pattern due to vaults horizontal action becomes symmetrical and visible for a gravity factor of 2 g. There are cracks that are located at the top of the transversal vaults and follow the Y direction (Fig. 18). Cracks on main lateral vaults can also be observed (Fig. 18. Load: 2 g). The beginning of the cracking effects caused by the deformation that the south-west corner experiments maybe observed for a gravity factor of 3.7 g and its evolution is very fast, with an important increase of the cracking state at the closest vault to this corner (Fig. 18. Load: 4.84 g).

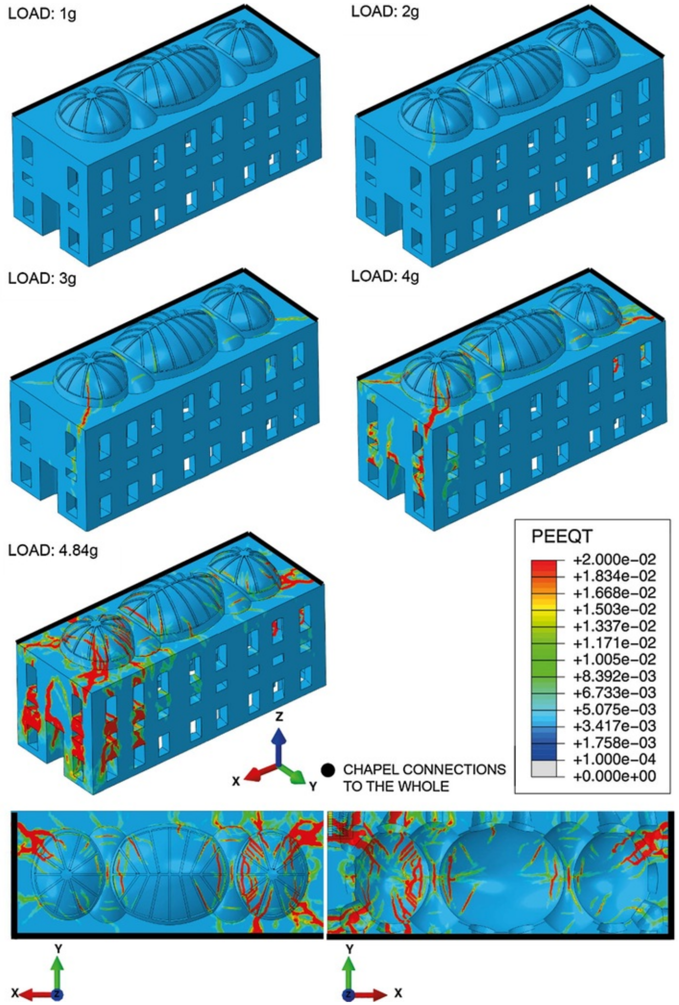

Fig. 18 Crack pattern at different steps: 1 g, 2 g, 3 g, 4 g and 4.84 g.

Therefore, the structure collapse would be caused by the low tensile capability of the masonry. Compressive levels are low both on vaults and walls. Their values are about 1.5 to 2.8 MPa, which are lower than the considered limiting value. The maximum compressive stresses are located in the closest areas to the vaults intersections. Finally, the structural safety of the building is verified against an additional vertical load whose value does not exceed 3.84 times the weight of the structure. Fig. 19 shows the structural response of two representative points of the vaults by means of their load-displacement curves. It should be noted that the non-linear response starts with a load factor really close to the unity.

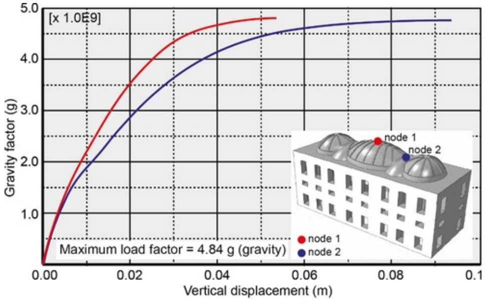

Fig. 19 Non-linear-analysis. Capacity curves.

6 Conclusions

The aim of this research is to estimate the structural safety factor of the vaults of the Chapel of the Residence of Würzburg (Germany). This chapel presents singularities and a level of geometrical complexity that make it unique. The vaults are made of a thin brick masonry (only one 30 cm thick layer) and have been generated by intersecting a series of oval in plant vaults. Their geometrical complexity is mainly due to the warped intersections among the main vaults and the secondary ones, generated as the result of intersecting two cylinders with different diameters. These intersections were built demonstrating a high technical ability in the development of brick masonry vaults.

The use of advanced techniques of structural analysis has been essential to study this construction and to accurately assess its structural behaviour and its structural safety factor. In particular, Operational Modal Analysis has been used to experimentally determine The Chapel modal properties from ambient vibration tests, in order to calibrate the FE model using genetic algorithms. The resulting updated FE model has been subsequently used to perform a non-linear analysis to estimate the ultimate load-carrying capacity of the structure. Five natural frequencies and their associated vibration modes have been successfully identified, with MAC values ranging between 0.82 and 0.99 for the EFDD and SSI results (see Table 1).

Thanks to the identified natural frequencies, the FEM could be updated using four updating parameters, two of them being material properties and the two others defined with the aim of including the connection with the rest of the palace. The adjustment was carried out using in the FE model of The Chapel the effect of genetic algorithms as optimization technique. The dynamic response of the numerical model has been successfully adjusted to the experimental results, with the frequency differences below 2% for the first four vibration modes and obtaining MAC values between 0.88 and 0.99 (see Table 3).

The updated FE model was then first used to analyse the structural behaviour of The Chapel under self-weight service conditions, confirming – as expected – that the vaults have an adequate structural response. The current structural state does not present evident cracks, as predicted by the non-linear FE model.

Subsequently, the FE model was employed to estimate the collapse load of the structure: as the load 1 g is increased until 2 g, 3 g, etc., it can be observed how the two transversal vaults are the weakest elements, presenting greater stress concentrations, mainly due to their geometrical configuration, with a lower rise to span ratio of about 0.35 compared with the rest of the vaults, with a ratio of about 0.50. In this way, tensile stresses are observed at the keystone and the base of the vaults. Furthermore, there are compressive stresses concentrations along the intersections between the vaults, due to their geometrical configuration which implies greater stiffness.

On the other hand, after analysing the results obtained for the ultimate load (load: 4,84 g), the weakness of the façade walls can be clearly identified as the main cause of collapse. Because of their slenderness (between 7 and 11), their large void to surface ratio (about 50%) and their great thickness variability, at collapse there are disconnections of the wall at lintel level of all the windows of the south-west corner. This disconnections imply that the wall structural behaviour is more similar to that found in separated pilasters. This behaviour of the façade walls cause the greatest level of cracking on the vaults.

Last but not least, the stress distribution does not follow the typical pattern for vaulted constructions, due mainly to both complexity of the vaults and the flexibility of the walls. This confirms that using advanced techniques based on non-destructive testing techniques and finite element modelling is necessary to properly assess the actual structural behaviour of complex historical constructions.

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Highlights

- Geometrically complex brick masonry vaults have been analysed to assess their structural safety.
- ~~Ambient vibration tests and~~ ~~operational~~ Operational ~~modal~~ Modal ~~analysis~~ Analysis technique is~~are~~ used to dynamically characterise ~~historical~~ masonry vaults.
- Genetic algorithms are used for numerical models updating.
- Updated FE model is used to perform a nonlinear analysis.

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