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Aperiodic dynamic event-triggered control for linear systems: A looped-functional approach

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ABSTRACT

The recent literature on event-triggered control has demonstrated the potential of dynamic periodic event-triggered control. Compared to continuous-time event-triggering rules, the benefit of considering periodic event-triggered control is to avoid the Zeno phenomenon, which refers to the situation when there are an infinite number of updates in a bounded interval of time. The idea of periodic event-triggered control is to trigger the control law only at known allowable periodic sampling instants. In this paper, our objective is to relax the constraint on the periodicity of the allowable sampling instants and to adapt this framework to the dynamic event-triggered control, which has not been considered in the literature, as far as we are aware of. Following the successful efforts to assess the stability of aperiodic sampled-data control, here we propose a generic framework to emulate aperiodic dynamic event-triggered control law for linear systems, for which the allowable sampling instants are not necessarily equidistant. Such an analysis is made possible thanks to the looped-functionals framework, which gives the flexibility to consider the periodic/aperiodic static/dynamic event-triggered control in a single formulation. Finally, the efficiency of the proposed results is illustrated through the study of two academic examples.

1. Introduction

With the rapid development of computer and network technologies, networked control systems (NCSs) have attracted considerable attention. When designing networked control loops, it is essential to consider the limited communication and/or computation capabilities of the system. The event-triggered (ET) strategy offers an effective tool which can balance the available system resources with the closed-loop system performances better than the classical periodic or aperiodic time-triggered control schemes [1,2]. In event-triggered control (ETC) systems, when the control input has to be updated is determined by a triggering rule which is predefined. That means control inputs are updated only when necessary, namely when an event is generated, rather than time-triggered periodically or aperiodically. Thus, as an effective way to balance system performance and limited network resources, ETC systems have attracted increasing interests of researchers in the last decade. Since an ET feedback controller that asymptotically stabilizes the plant is proposed in [3], various analyses and designs for ETC systems have been proposed [3–21].

Most existing ET strategies can be divided into continuous- and discrete-time strategies based on their time sets, and the controlled plant is generally considered to have the same time nature [4]. However, with the increasing popularity of sampled-data systems in NCSs, it is more interesting to find a "sampled-data"-like approach to break the parallels between the continuous-time ET approaches

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and the discrete-time counterparts. This means that a discrete-time ET scheme based on sampled data operates on a continuous-time plant. To name a few, a periodic event-triggered strategy (PETS) in a quadratic form is proposed in [5], and the approaches of impulsive systems, piece-wise linear systems, perturbed linear systems are presented to analyze the stability and the $\mathcal{L}_2$-gain properties of resulting periodic event-triggered control (PETC) systems. More PETC problems for linear and nonlinear systems can be found in [7–11]. Another very popular method for ETC of sampled-data systems is the time-delay system approach [12,13]. By introducing an artificial piece-wise linear delay function, a delay system model can be constructed for the ETC system based on sampled data [12,13]. Nevertheless, almost all the above mentioned results are based on the technique of the Lyapunov–Krasovskii functional and only address related ET problems for periodic samplings without much attention to aperiodic cases.

Sampled-data systems have been widely investigated in recent decades [22,23]. The stability of the system under periodic samplings has been extensively discussed and still attracting a lot of attentions [24]. When the difference between two successive sampling instants is time-varying, several methods are used to analyze system dynamics [25–34]. Discrete time models are adopted in [34] for aperiodic samplings and an input-delay approach based on the Lyapunov–Krasovskii theorem is provided in [25,26]. Small gain theorem is adopted to make some improvement [27] and the impulsive systems method is proposed in [28]. Although thanks to the time-dependent Lyapunov–Krasovskii functionals, the input delay approach and the impulsive system approach can deal with time-varying sampling periods as well as with uncertain systems in a simple way, these sufficient stability conditions are still more conservative than the discrete-time methods. However, due to the exponential term of the transition matrix, it is not an easy task to find an efficient extension of the discrete-time approach to include robustness with respect to the uncertainties of sampling periods and parameters. Another popular way to deal with asynchronous samplings of the system is the looped-functional approach [29,30]. By introducing a looping condition, the equivalence between the discrete-time and continuous-time approaches for the stability analysis is disclosed in [29]. The constraint on the positivity of the Lyapunov functional in continuous-time approaches can be relaxed. Based on the discrete-time Lyapunov theory and the proposed looped-functionals method, the asymptotic and exponential stability analyses for the cases of synchronous, asynchronous, and multiple samplings are discussed in a unified framework [29]. It is worth noting that the looped-functional method provides the missing link between discrete-time approaches and continuous counterparts. The main advantages of this framework with respect to the context of this paper is that the inclusion of additional features such as the triggering variable of a dynamical event-triggered control scheme can be performed in a very elegant manner, without invoking an S-procedure.

In this paper, a novel framework is proposed to extend static and dynamic periodic ETC to the case of aperiodic sampled-data systems. To do so, we introduce a discrete-time dynamic event-triggering rule, which includes several existing ET strategies as its special cases. This rule is formulated in quadratic form, which only uses the sampled data from the system. The stability analysis of the continuous-time ETC system is performed using the looped-functional framework. Sufficient conditions for asymptotic stability of the closed-loop system are derived in terms of LMIs, which can be applied to the cases of periodic and aperiodic samplings for nominal and polytopic uncertain systems.

Notation. Throughout the paper, the sets $\mathbb{N}$, $\mathbb{R}$, $\mathbb{R}^n$, $\mathbb{R}^{n\times n}$ and $\mathbb{S}^n$ denote the sets of non-negative integers, real numbers, n-dimensional vectors, $n\times n$ matrices and symmetric matrices of $\mathbb{R}^{n\times n}$, respectively. The notation $|\cdot|$ and the superscript $\top$ stand for the Euclidean norm and for matrix transposition, respectively. The notation $P>0$ for $P\in\mathbb{S}^n$ means that P is positive definite. For any matrix $A\in\mathbb{R}^{n\times n}$, the notation $\mathrm{He}\{A\}$ refers to $A+A^T$. The symbols I and 0 represent the identity and zero matrices of appropriate dimensions.

2. Problem statement

2.1. System data

Consider the following linear sampled-data system

$$\dot{x}(t) = Ax(t) + Bu(t_k), \quad \forall t \in [t_k, t_{k+1}), \tag{1}$$

where $x(t)\in\mathbb{R}^n$ and $u(t)\in\mathbb{R}^m$ represent the state and the control input vectors, respectively. Matrices A and B, of appropriate dimensions, are assumed to be either constant and known, or possibly uncertain and time-varying. The sampling is assumed to be aperiodic. This means that there exists a minimal and maximal dwell times $\mathcal{T}_1>0$ and $\mathcal{T}_2\geq\mathcal{T}_1$ such that the distance between two successive sampling instants $t_{k+1}-t_k=T_k$ verifies

$$\mathcal{T}_1 \leq T_k \leq \mathcal{T}_2, \quad \forall k \in \mathbb{N}. \tag{2}$$

In this paper, the objective is the design of *aperiodic event-triggered rules* such that a given state feedback control law asymptotically stabilizes the closed-loop system, corresponding to an emulation problem. In order to solve this problem, we will formulate the dynamics of the system using the looped-functional framework, which will be proven to be well suited for this study. To do so, in the following subsections, we define the control law and the event-triggering rule that will be employed hereafter.

2.2. Event-triggered state-feedback control law

In this paper, we are looking for a linear state feedback control gain $K \in \mathbb{R}^{m\times n}$, such that the control input to be implemented is given by

$$u(t_k) = \begin{cases} Kx(t_k), & \text{if the control law is updated,} \\ u(t_{k-1}), & \text{otherwise.} \end{cases} \tag{3}$$

One can understand the previous expression as a 'double' sampling effect. The continuous-time sampling makes that the control input remains constant during the sampling interval $[t_k, t_{k+1})$. Then, a discrete-time sampling is performed depending on whether or not the previous control input has to be updated.

In order to complete the implementation rules of the control law, one has to provide a specific rule to impose a control update. To do so, let κ be a strictly increasing function from $\mathbb{N}$ to $\mathbb{N}$, with condition $\kappa(0) = 0$. This function defines triggering instants, that is, when the control input has to be updated, in a more formal manner. It means that the triggering instants $t_{\kappa(\ell)}$ are all instants of control updates such that

$$u(t_{\kappa(\ell)}) = Kx(t_{\kappa(\ell)}).$$

To determine these $\kappa(\ell)$, static and dynamic event-triggered control algorithms will be provided in the next subsections.

2.2.1. Static event-triggering rule

Let us introduce the *static* event-triggering rule defined as follows

$$t_{\kappa(\ell+1)} = \min_{m\in\mathbb{N}} \left\{ t_{\kappa(\ell)+m} \geq t_{\kappa(\ell)+1} \quad | \quad \varphi\left(x(t_{\kappa(\ell)+m}), x(t_{\kappa(\ell)})\right) \geq 0 \right\}, \tag{4}$$

where $\varphi : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ is called the triggering function to be defined. The only requirement so far, is that the function φ verifies the following constraint:

$$\varphi(x, x) \leq 0, \qquad \forall x \in \mathbb{R}^n. \tag{5}$$

Remark 1. The previous event-triggering rule excludes the Zeno phenomenon since the minimal distance between two successive sampling instants is at least $\mathcal{T}_1 > 0$. It is also important to mention that the event-triggering rule (4) is only evaluated at the aperiodic sampling instants $\{t_k\}_{k\in\mathbb{N}}$. Compared to the usual periodic event-triggering rules introduced in [5,14,15], the assumption on the periodicity of the sampling instants is not required.

2.2.2. Dynamic event-triggering rule

Let us now introduce the following *dynamic* event-triggering rule defined as an extended version of the previous static rule. This dynamic rule is formulated as follows:

$$t_{\kappa(\ell+1)} = \min_{m\in\mathbb{N}} \left\{ t_{\kappa(\ell)+m} \geq t_{\kappa(\ell)+1} \quad | \quad \varphi\left(x(t_{\kappa(\ell)+m}), x(t_{\kappa(\ell)})\right) \geq \rho\eta_{\kappa(\ell)+m} \right\}, \tag{6}$$

where, again, $\varphi : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ is the triggering function to be defined which must also verify (5). The above rule is called a *dynamic* event-triggering rule because it relies on the construction of a new dynamic variable η_k, that is driven by the following discrete-time dynamic equation:

$$\begin{cases} \eta_{k+1} = (\lambda+\rho)\eta_k - \varphi\left(x(t_k), x(t_{\kappa(\ell)})\right), & \forall k \in [\kappa(\ell), \kappa(\ell+1))_{\mathbb{N}}, \quad \forall \ell \in \mathbb{N}, \\ \eta_0 \geq 0, \end{cases} \tag{7}$$

where the parameters λ and ρ are such that $\rho \geq 0$ and $(\lambda + \rho) \in [0, 1)$. Compared to the classical "continuous-time" dynamic variable provided in [6], the dynamic here is written as a discrete-time equation because we are considering a periodic (or aperiodic) event-triggered control. Here, Eq. (7) means that the update of η_k is only performed at the sampling instant t_k. Between two successive sampling instants, the value of η_k remains constant, i.e.,

$$\frac{\mathrm{d}}{\mathrm{d}t}\eta_k = 0, \quad \forall t \in (t_k, t_{k+1}), \quad \forall k \in \mathbb{N}. \tag{8}$$

Remark 2. It is worth mentioning that the static event-triggering rule is retrieved by imposing $\rho = 0$ in (6), whatever the value of λ.

Following classical methods of the continuous-time dynamic ETC design [6,7], one has to guarantee that the variable η_k is non-negative for all k in $\mathbb{N}$. The following lemma addresses this issue.

Lemma 1. *Consider scalar parameters (λ, ρ) such that $\rho \geq 0$ and $(\lambda + \rho) \in [0, 1)$ and assume that $\eta_0 > 0$. Then, the dynamic event-triggered variable is non-negative for any sampling instants t_k, i.e., $\eta_k \geq 0$ for all $k \in \mathbb{N}$.*

Proof. Consider the dynamic of η_k provided in (7) together with the event-triggering rule (6). Assume that $\rho \geq 0$, $(\lambda+\rho) \in [0,1)$ and $\eta_0 > 0$. The proof is then made by recursion.

Initialization ($k=1$): Assume that $\eta_0 > 0$. According to the dynamic Eq. (7) with $k=0$, it holds that

$$\eta_1 = (\lambda+\rho)\eta_0 - \varphi\left(x(t_0), x(t_{\kappa(0)})\right),$$

where obviously $t_{\kappa(0)} = t_0$. Recalling condition (5), it yields

$$\varphi\left(x(t_0), x(t_{\kappa(0)})\right) = \varphi\left(x(t_0), x(t_0)\right) \leq 0.$$

As $(\lambda+\rho) \in [0,1)$, $\eta_0 > 0$ and $\varphi\left(x(t_0), x(t_{\kappa(0)})\right) \leq 0$, it is clear that $\eta_1 \geq 0$, which concludes the initialization.

Recursion ($k \in \mathbb{N}$): Assume that the variable η_k is positive and the associated value of $\kappa(\ell)$, is an integer lower than k. Then, we may face the following two cases.

If the triggering condition is satisfied, that is, $\varphi\left(x(t_k), x(t_{\kappa(\ell)})\right) \geq \rho\eta_k$, then, the value of the control law as well as $\kappa(\ell)$ is updated, i.e., $\kappa(\ell+1) = k$. In this situation, $\varphi\left(x\left(t_{\kappa(\ell)}\right), x(t_{\kappa(\ell)})\right) \leq 0$ still holds since we get the same constraint (5) of the triggering function φ as in the case $k=0$ above. Consequently, $\eta_{k+1} \geq 0$.

If the event-triggered condition is violated, that is, $\varphi\left(x(t_k), x(t_{\kappa(\ell)})\right) \leq \rho\eta_k$, then, according to the discrete-time Eq. (7) it holds

$$\eta_{k+1} = (\lambda+\rho)\eta_k - \varphi\left(x(t_k), x(t_{\kappa(\ell)})\right).$$

Since $\lambda \geq 0$, $\eta_k \geq 0$, it holds

$$\begin{aligned}\eta_{k+1} &= (\lambda+\rho)\eta_k - \varphi\left(x(t_k), x(t_{\kappa(\ell)})\right) \\ &= \underbrace{\lambda\eta_k}_{\geq 0} + \underbrace{\left(\rho\eta_k - \varphi\left(x(t_k), x(t_{\kappa(\ell)})\right)\right)}_{\geq 0}.\end{aligned}$$

Hence, $\eta_{k+1} \geq 0$ holds for all $k \in \mathbb{N}$, which concludes the recursion. □

Remark 3. In the literature on the dynamic event-triggered control [6,7], an additional dynamic variable η is usually defined using a differential equation of the form $\dot{\eta} = -\bar{\lambda}\eta + \bar{\rho}\varphi$ for some positive scalars $\bar{\lambda}$ and $\bar{\rho}$. Here, we use the particular feature of the periodic (or aperiodic) event-triggered algorithm to notice that φ is constant over the sampling interval. Therefore, it is easy to derive the solution of the differential equation that leads to the discrete-time Eq. (7), with particular values of λ and ρ. In this paper, we have adopted a selection of constant parameters λ and ρ for simplicity, noting that it is still possible to use the time-varying but known values of λ and ρ for the sampling interval T_k.

2.2.3. Definition of the triggering function φ

In this paper, we will consider a triggering function that is quadratic with respect to its arguments and given by

$$\varphi\left(x(t_{\kappa(\ell)+m}), x(t_{\kappa(\ell)})\right) = \begin{bmatrix} x\left(t_{\kappa(\ell)+m}\right) \\ x\left(t_{\kappa(\ell)}\right) \end{bmatrix}^\top \Phi \begin{bmatrix} x\left(t_{\kappa(\ell)+m}\right) \\ x\left(t_{\kappa(\ell)}\right) \end{bmatrix}, \quad \forall k \in [\kappa(\ell), \kappa(\ell+1))_{\mathbb{N}}, \forall \ell \in \mathbb{N}, \tag{9}$$

where Φ satisfies

$$\begin{bmatrix} I \\ I \end{bmatrix}^\top \Phi \begin{bmatrix} I \\ I \end{bmatrix} \preceq 0. \tag{10}$$

This above inequality is required to ensure that once an update is performed, the event-triggering condition does not hold any longer until the next update, which ensures $\varphi(x,x) \leq 0$ for all x in $\mathbb{R}^n$.

The intuition behind this condition is the following. If the triggering condition is violated, this necessarily means that inequality $\varphi\left(x(t_k), x(t_{\kappa(\ell)})\right) \geq \rho\eta_k$ holds. As η_k is positive for all k thanks to Lemma 1, this means that $\varphi\left(x(t_k), x(t_{\kappa(\ell)})\right)$ is necessarily positive. Imposing condition (10) implies that, once the memory vector $x(t_{\kappa(\ell)})$ has been updated, $\varphi\left(x(t_k), x(t_{\kappa(\ell+1)})\right) = \varphi\left(x(t_k), x(t_k)\right)$ is negative, which ensures that the triggering condition (before applying the new control input) is not violated anymore. Intuitively, this condition can be interpreted as the avoidance of Zeno and enforce enlarging the interval between two successive control updates. Indeed, by continuity of the systems, the quantity $\varphi(x(t_{\kappa(\ell)+m}), x(t_{\kappa(\ell)}))$, for small values of m is intuitively not so far away from $\varphi(x(t_{\kappa(\ell)}), x(t_{\kappa(\ell)}))$, which is negative. This aims at giving more margin before the next triggering instant.

Remark 4. The previous triggering function (9) that was already considered in papers [16–18] has the benefit of encompassing the rules usually considered in the literature as particular cases. For instance, consider the following selections

- ***Case A:*** Selecting a matrix Φ, which is only required to verify

$$\Phi \quad \text{with} \quad \begin{bmatrix} I \\ I \end{bmatrix}^\top \Phi \begin{bmatrix} I \\ I \end{bmatrix} \preceq 0, \tag{11}$$

  leads to the generic triggering function (9) in this paper.

- ***Case B:*** Selecting a matrix Φ as

$$\Phi=\begin{bmatrix}\Phi_1 & -\Phi_1\\ -\Phi_1 & (1-\alpha)\Phi_1\end{bmatrix}\quad \text{with}\quad \Phi_1>0,\ \alpha>0, \tag{12}$$

leads to the particular triggering function

$$\varphi\left(x(t_{\kappa(\ell)+m}),x(t_{\kappa(\ell)})\right):=\left(x(t_{\kappa(\ell)+m})-x(t_{\kappa(\ell)})\right)^\top\Phi_1\left(x(t_{\kappa(\ell)+m})-x(t_{\kappa(\ell)})\right)-\alpha x^\top(t_{\kappa(\ell)})\Phi_1x(t_{\kappa(\ell)}),$$

which was considered in papers [12,13].

- ***Case C:*** Selecting a matrix Φ as

$$\Phi=\begin{bmatrix}\Phi_2 & -\Phi_2\\ -\Phi_2 & \Phi_2-\Phi_1\end{bmatrix}\quad \text{with}\quad \Phi_1>0,\ \Phi_2>0, \tag{13}$$

leads to the particular triggering function

$$\varphi\left(x(t_{\kappa(\ell)+m}),x(t_{\kappa(\ell)})\right):=\left(x(t_{\kappa(\ell)+m})-x(t_{\kappa(\ell)})\right)^\top\Phi_2\left(x(t_{\kappa(\ell)+m})-x(t_{\kappa(\ell)})\right)-x^\top(t_{\kappa(\ell)})\Phi_1x(t_{\kappa(\ell)}),$$

which was considered, for instance, in papers [20,35].

Recalling (10), the constraint of Φ is required for all cases of triggering functions mentioned above to guarantee $\varphi(x,x)$ non-positive. The matrix Φ of the triggering rule plays a major role both in the solvability of the LMIs and in the event-triggered mechanism. As it thus appears that the proposed event-triggering rule encompasses the above usual rules with parameters (Φ_1,Φ_2,α). In the next developments, we will present the conditions for finding a matrix Φ as a candidate.

2.2.4. Modeling of the closed-loop system

As is discussed in previous sections, the control law (3) can be rewritten as

$$u(t_k)=\begin{cases}Kx(t_k), & \text{if}\quad \varphi\left(x(t_k),x(t_{\kappa(\ell)})\right)\geq\rho\eta_k,\quad k=\kappa(\ell+1),\\ Kx(t_{\kappa(\ell)}), & \text{if}\quad \varphi\left(x(t_k),x(t_{\kappa(\ell)})\right)\leq\rho\eta_k,\quad k\in[\kappa(\ell),\kappa(\ell+1)).\end{cases} \tag{14}$$

The closed-loop system composed of (1), (6), (14), can be reformulated as

$$\dot{x}(t)=Ax(t)+BKx(t_{\kappa(\ell)}),\quad \forall t\in\left[t_k,t_{k+1}\right),\forall k\in[\kappa(\ell),\kappa(\ell+1)),\ \forall\ell\in\mathbb{N}. \tag{15}$$

The plant (15) can be seen as a particular sampled-data system for which several theoretical tools can be applied to study its stability properties, see the survey paper [24,36]. In this paper, we will focus on the looped functional framework introduced in [29,37] to assess the stability of the closed-loop system.

3. Preliminaries and looped-functional

3.1. Preliminary lemmas

In the next development of the paper, we will use the following lemmas:

Lemma 2 (*Wirtinger-based Inequality [38]*)**.** *For a given matrix $R>0$, the following inequality holds for all continuously differentiable function ω in $[a,b]\to\mathbb{R}^n$:*

$$\int_a^b\dot{\omega}^T(u)R\dot{\omega}(u)\mathrm{d}u\geq\frac{1}{b-a}(\omega(b)-\omega(a))^TR(\omega(b)-\omega(a))+\frac{3}{(b-a)}\tilde{\Omega}^TR\tilde{\Omega}, \tag{16}$$

where $\tilde{\Omega}=\omega(b)+\omega(a)-\frac{2}{b-a}\int_a^b\omega(u)\mathrm{d}u$.

Lemma 3. *For all matrices X, Y and R of appropriate dimensions and positive scalar $\epsilon>0$, the following inequality holds:*

$$-\frac{1}{\epsilon}X^TRX\leq-2X^TY+\epsilon Y^TR^{-1}Y. \tag{17}$$

3.2. Lifting of sampled-data systems trajectories

The looped functional approach [29] is based on the characterization of the system trajectories (1) in a lifted domain [37,39]. Define $\mathbb{K}^n$ as

$$\mathbb{K}^n:=\bigcup_{\mathcal{T}\in[\mathcal{T}_1,\mathcal{T}_2]}C([0,\mathcal{T}]\to\mathbb{R}^n).$$

Then, we therefore view the entire state-trajectory as a sequence of functions $\left\{x(t_k+\tau),\ \tau\in(0,T_k]\right\}_{k\in\mathbb{N}}$ with elements having a unique continuous extension to $[0,T_k]$ defined as

$$\chi_k(\tau):=x(t_k+\tau)\quad \text{with}\quad \chi_k(0)=\lim_{s\downarrow t_k}x(s).$$

Such that the system (1) can be rewritten as

$$\dot{\chi}_k(\tau)=A\chi_k(\tau)+BK\chi_{\kappa(\ell)}(0),\ \forall(\tau,k,\ell)\in\mathbb{H}^n_\kappa(T_k):=[0,T_k]\times[\kappa(\ell),\kappa(\ell+1))\times\mathbb{N}. \tag{18}$$

3.3. Definition and results about looped-functionals

Let us first define looped-functionals following the principles of the paper [37].

Definition 1 (*[37]*)**.** A functional

$$f : [0,\mathcal{T}] \times \mathbb{K}^n \times \mathbb{R}^n \times [\mathcal{T}_1,\mathcal{T}_2] \to \mathbb{R},$$

is said to be a **looped-functional** if

1. the equality
$$f(0,z,z_0,\mathcal{T}) = f(\mathcal{T},z,z_0,\mathcal{T}) \tag{19}$$
holds for all functions $z \in \mathbb{K}^n$, for all vectors $z_0 \in \mathbb{R}^n$ and all $\mathcal{T} \in [\mathcal{T}_1,\mathcal{T}_2]$.
2. it is differentiable with respect to the first variable with the standard definition of the derivative.

The set of all these functionals is denoted by $\mathcal{LF}^n([\mathcal{T}_1,\ \mathcal{T}_2])$.

Note that in the above definition, we have lightly modified the argument of the looped functional to include an additional component z_0, which is a vector in $\mathbb{R}^n$.

The idea of proving the stability of (18) under the proposed ET scheme (6) is to look now for a Lyapunov function $\bar{V}$ such that $\bar{V}$ is monotonically decreasing along the sampling instants. The novelty with respect to previous results on looped-functionals and event-triggered control relies on the particular construction of the Lyapunov function $\bar{V}$. In this paper, $\bar{V}$ is composed of :

(i) the *non-negative* discrete-time variable η_k, which is the additional state that has been built to include a dynamic event-triggering scheme,
(ii) a classical quadratic Lyapunov function $V : \mathbb{R}^n \to \mathbb{R}^+$.

This is formalized through the following functional existence result:

Theorem 1. *For given $0 < \mathcal{T}_1 \le \mathcal{T}_2$ and $\rho \in [0,\lambda) \subset [0,1]$, consider $V : \mathbb{R}^n \to \mathbb{R}_+$ a quadratic form satisfying*

$$\mu_1|x|^2 \le V(x) \le \mu_2|x|^2, \quad \forall x \in \mathbb{R}^n, \tag{20}$$

for some scalars $0 < \mu_1 \le \mu_2$ and the non-negative discrete-time event-triggering variable η_k defined in (7). Assume further that one of the following equivalent statements hold:

1. *The sequence $\{\bar{V}(\chi_k(0),\eta_k) := V(\chi_k(0)) + \eta_k\}_{k\in\mathbb{N}}$ is decreasing along the trajectories of closed-loop system (18) with the dynamic event-triggering rule (6).*
2. *There exists a looped-functional $\mathcal{V} \in \mathcal{LF}^n(\mathcal{T})$ such that the functional $\mathcal{W}_k$ defined as*
$$\mathcal{W}_k(\tau,\chi_k,\chi_{\kappa(\ell)}) \quad := \quad V(\chi_k(\tau)) + \mathcal{V}(\tau,\chi_k,\chi_{\kappa(\ell)},T_k) + \frac{\tau}{T_k}[\eta_{k+1}-\eta_k], \quad \forall(\tau,k,\ell) \in \mathbb{H}^n_\kappa(T_k), \tag{21}$$
is decreasing along the trajectories of the system (18) under the proposed dynamic event-triggering scheme (6), i.e.,
$$\frac{d}{d\tau}\mathcal{W}_k(\tau,\chi_k,\chi_{\kappa(\ell)}) \quad := \quad \frac{d}{d\tau}V(\chi_k(\tau)) + \frac{d}{d\tau}\mathcal{V}(\tau,\chi_k,\chi_{\kappa(\ell)},T_k) + \frac{\eta_{k+1}-\eta_k}{T_k} < 0, \quad \forall(\tau,k,\ell) \in \mathbb{H}^n_\kappa(T_k). \tag{22}$$

Then, the aperiodic sampled-data system (18) under the proposed dynamic event-triggering scheme (6) is asymptotically stable.

Proof. The proof strictly follows the lines of the proof derived in [31]. However a sketch of the proof is provided to better understand that the inclusion of the triggering variable η_k does not imply critical modifications.

Assume that the first statement in Theorem 1 holds. Then, functional $\mathcal{V} = -V(\chi_k(\tau)) + \frac{\tau}{T_k}\Delta V(k)$ verifies the looping condition and the associated functional $\mathcal{W}_k$ straightforwardly verifies condition (22).

Assume now that the second statement in Theorem 1 is satisfied, that is there exists a looped-functional $\mathcal{V}$ such that $\mathcal{W}$ verifies condition (22). Then, integrating $\dot{\mathcal{W}}$ over the interval $[0,T_k]$, we get

$$\begin{aligned}
\mathcal{W}_k(T_k,\chi_k,\chi_{\kappa(\ell)}) - \mathcal{W}_k(0,\chi_k,\chi_{\kappa(\ell)}) &= V(\chi_k(T_k)) + \mathcal{V}(T_k,\chi_k,\chi_{\kappa(\ell)},T_k) + \frac{T_k}{T_k}[\eta_{k+1}-\eta_k] - V(\chi_k(0)) - \mathcal{V}(0,\chi_k,\chi_{\kappa(\ell)},T_k) \\
&= V(\chi_k(T_k)) - V(\chi_k(0)) + (\eta_{k+1}-\eta_k) + \mathcal{V}(T_k,\chi_k,\chi_{\kappa(\ell)},T_k) - \mathcal{V}(0,\chi_k,\chi_{\kappa(\ell)},T_k) \\
&= \Delta\bar{\mathcal{V}}(\chi_k,\eta_k) + \mathcal{V}(T_k,\chi_k,\chi_{\kappa(\ell)},T_k) - \mathcal{V}(0,\chi_k,\chi_{\kappa(\ell)},T_k).
\end{aligned}$$

As $\mathcal{V}$ verifies the looping condition, the previous expression resumes to

$$\mathcal{W}_k(T_k,\chi_k,\chi_{\kappa(\ell)}) - \mathcal{W}_k(0,\chi_k,\chi_{\kappa(\ell)}) = \Delta\bar{\mathcal{V}}(\chi_k,\eta_k) < 0,$$

which is exactly the first statement. Hence, both statement are equivalent.

The proof is then concluded by noting that the inter-sampling trajectories are uniformly bounded by the $\chi_k(0)$ and $\chi_{\kappa(\ell)}$ and $\mathcal{T}_2$ because of the linear differential equation that defines $\chi_k(\tau)$. Therefore, the asymptotic convergence of the discrete sequence $\bar{V}(\chi_k(0),\eta_k)$ to the origin as k tends to infinity ensures the asymptotic convergence of $\bar{V}(\chi_k(\tau),\eta_k)$ to the origin as k tends to infinity, for any $\tau \in [0,\mathcal{T}_2]$. □

4. Main results

In this section, we will first present the main result of this paper, dealing with the emulation of aperiodic dynamic ETC. Then, several more or less direct corollaries will be derived to address relevant particular cases.

4.1. Aperiodic dynamic event-triggered control

Theorem 2. *Let $0 < \mathcal{T}_1 \leq \mathcal{T}_2$ be two positive scalars and consider the scalar parameters (λ, ρ, η_0) such that $\rho \in [0, \lambda) \subset [0, 1]$ and $\eta_0 > 0$. Assume that there exist matrices $P = P^\top \succ 0$, $R = R^\top \succ 0$, $S = S^\top$, $\Phi = \Phi^\top$, the controller gain K, and matrices Q, M of appropriate dimensions such that the following LMIs are satisfied*

$$\begin{bmatrix} \Omega_1\left(\mathcal{T}_i, \frac{1-\lambda}{\rho\mathcal{T}_2}, A, B, M\right) & \mathcal{T}_i M^\top \\ * & -\mathcal{T}_i\tilde{R} \end{bmatrix} \prec 0, \quad \Omega_1\left(\mathcal{T}_i, \frac{1-\lambda}{\rho\mathcal{T}_2}, A, B, M\right) + \mathcal{T}_i\Omega_2(A,B) \prec 0, \quad \begin{bmatrix} I \\ I \end{bmatrix}^\top \Phi \begin{bmatrix} I \\ I \end{bmatrix} \leq 0, \tag{23}$$

for $i = 1, 2$, where

$$\begin{aligned} \Omega_1\left(\mathcal{T}_i, \frac{1-\lambda}{\rho\mathcal{T}_2}, A, B, M\right) &= \mathrm{He}\left(G_2^\top(A,B)PG_1 - G_3^\top QH_1 - G_w^\top M\right) - \mathcal{T}_i F_1^\top S F_1 - \frac{1-\lambda}{\rho\mathcal{T}_2} G_\varphi^\top \Phi G_\varphi, \\ \Omega_2(A,B) &= G_2^\top(A,B)RG_2(A,B) + 2F_1^\top SF_1 + \mathrm{He}\left(F_2^\top SF_1 + G_2^\top(A,B)QH_1 + G_3^\top QH_2(A,B)\right), \end{aligned} \tag{24}$$

and

$$\begin{aligned} G_1 &= \begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix}, \quad G_2(A,B) = \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}, \quad G_3 = \begin{bmatrix} I & -I & 0 & 0 \end{bmatrix}, \\ G_\varphi &= \begin{bmatrix} 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix}, \quad H_1 = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix}, \quad H_2(A,B) = \begin{bmatrix} A & 0 & BK & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\ F_1 &= \begin{bmatrix} 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ I & 0 & 0 & -I \end{bmatrix}, \quad G_w = \begin{bmatrix} I & -I & 0 & 0 \\ I & I & 0 & -2I \end{bmatrix}, \\ \tilde{R} &= \begin{bmatrix} R & 0 \\ 0 & 3R \end{bmatrix}. \end{aligned} \tag{25}$$

Then, the system (18) with the aperiodic dynamic event-triggered control (6) is asymptotically stable.

Proof. Consider the Lyapunov function $\bar{V}(x) = V(x) + \eta_k$, where $V(x) = x^\top Px$ for $\forall x \in \mathbb{R}^n$ and η_k is the discrete-time dynamic event-triggered variable governed by (7). Let us introduce an appropriate looped functional for this problem. The proposed functional finds its origin in paper [38] with a few modifications to account for the memory variable $\chi_{\kappa(\ell)}$. It is defined as follows

$$\mathcal{V}(\tau, \chi_k, \chi_{\kappa(\ell)}, T_k) = 2(T_k - \tau)(\chi_k(\tau) - \chi_k(0))^\top Q\zeta_k(\tau) + (T_k - \tau)\tau\rho_k^\top(\tau)S\rho_k(\tau) + (T_k - \tau)\int_0^\tau \dot{\chi}_k{}^\top(s)R\dot{\chi}_k(s)\mathrm{d}s,$$

where

$$\zeta_k = \begin{bmatrix} \chi_k(\tau) \\ \chi_k(0) \\ \chi_{\kappa(\ell)}(0) \end{bmatrix}, \qquad \rho_k = \begin{bmatrix} \chi_k(0) \\ \chi_{\kappa(\ell)}(0) \\ \frac{1}{\tau}\int_0^\tau \chi_k(s)\mathrm{d}s \end{bmatrix}.$$

Note that the argument '(τ)' has been deliberately omitted in these notations to ease the reading of the latter developments. It is easy to verify that the functional $\mathcal{V}$ verifies the conditions of the looped-functional, since $\mathcal{V}(0, \chi_k, \chi_{\kappa(\ell)}, T_k) = \mathcal{V}(T_k, \chi_k, \chi_{\kappa(\ell)}, T_k) = 0$. Therefore, following the statement of Theorem 1, the stability of the closed-loop system can be assessed by studying the constructed functional

$$\mathcal{W}(\tau, \chi_k, \chi_{\kappa(\ell)}, T_k) = V(\chi_k(\tau)) + \mathcal{V}(\tau, \chi_k, \chi_{\kappa(\ell)}, T_k) + \frac{\tau}{T_k}(\eta_{k+1} - \eta_k). \tag{26}$$

Differentiation of $\mathcal{W}$ along the trajectories of the system leads to

$$\begin{aligned} \dot{\mathcal{W}}(\tau, \chi_k, \chi_{\kappa(\ell)}, T_k) = &\ 2\chi_k^\top(\tau)P\dot{\chi}_k(\tau) + (T_k - 2\tau)\rho_k^\top S\rho_k + 2\tau(T_k - \tau)\dot{\rho}_k^\top S\rho_k \\ &- 2(\chi_k(\tau) - \chi_k(0))^\top Q\zeta_k + 2(T_k - \tau)\dot{\chi}_k^\top(\tau)Q\zeta_k + 2(T_k - \tau)(\chi_k(\tau) - \chi_k(0))^\top Q\dot{\zeta}_k \\ &+ (T_k - \tau)\dot{\chi}_k^\top(\tau)R\dot{\chi}_k(\tau) - \int_0^\tau \dot{\chi}_k{}^\top(s)R\dot{\chi}_k(s)\mathrm{d}s + \frac{\eta_{k+1} - \eta_k}{T_k}. \end{aligned}$$

Recalling (7), for $\forall k \in [\kappa(\ell), \kappa(\ell+1))_{\mathbb{N}}$, $\forall \ell \in \mathbb{N}$, no sampled data is triggered so that $\rho\eta_k \geq \varphi(\chi_k(0), \chi_{\kappa(\ell)}(0))$. Further considering $\lambda \in [0, 1)$, it yields

$$\frac{\eta_{k+1} - \eta_k}{T_k} = \frac{\lambda + \rho - 1}{T_k}\eta_k - \frac{1}{T_k}\varphi(\chi_k(0), \chi_{\kappa(\ell)}(0)) \tag{27}$$

$$\leq \frac{\lambda + \rho - 1}{\rho T_k}\varphi(\chi_k(0), \chi_{\kappa(\ell)}(0)) - \frac{1}{T_k}\varphi(\chi_k(0), \chi_{\kappa(\ell)}(0)) \tag{28}$$

$$\leq -\left(\frac{1-\lambda}{\rho T_k}\right)\varphi(\chi_k(0),\chi_{\kappa(\ell)}(0)) \tag{29}$$

$$\leq -\left(\frac{1-\lambda}{\rho \mathcal{T}_2}\right)\varphi(\chi_k(0),\chi_{\kappa(\ell)}(0)). \tag{30}$$

Furthermore, based on Lemmas 2 and 3, the following inequality holds for any matrix M of appropriate dimensions

$$-\int_0^\tau \dot{\chi}_k^\top(s)R\dot{\chi}_k(s)\mathrm{d}s \leq \tau\xi_k^\top(\tau)M^\top\begin{bmatrix} R & 0\\ 0 & 3R\end{bmatrix}^{-1}M\xi_k(\tau) - 2\begin{bmatrix}\chi_k(\tau)-\chi_k(0)\\ \chi_k(\tau)+\chi_k(0)-\frac{2}{\tau}\int_0^\tau \chi_k(s)\mathrm{d}s\end{bmatrix}^\top M\xi_k(\tau), \tag{31}$$

where we define the augmented vector $\xi_k(\tau)$ as follows

$$\xi_k = \begin{bmatrix}\chi_k^\top(\tau) & \chi_k^\top(0) & \chi_{\kappa(\ell)}^\top(0) & \frac{1}{\tau}\int_0^\tau \chi_k^\top(s)\mathrm{d}s\end{bmatrix}^\top, \tag{32}$$

and by noting that

$$\begin{aligned} &\chi_k(\tau) = G_1\xi_k, \quad \dot{\chi}_k(\tau) = G_2(A,B)\xi_k, \quad \chi_k(\tau)-\chi_k(0) = G_3\xi_k, \quad \begin{bmatrix}\chi_k(0)\\ \chi_{\kappa(\ell)}(0)\end{bmatrix} = G_\varphi\xi_k,\\ &\zeta_k = H_1\xi_k, \quad \dot{\zeta}_k = H_2(A,B)\xi_k, \quad \rho_k = F_1\xi_k, \quad \dot{\rho}_k = \frac{1}{\tau}F_2\xi_k. \end{aligned} \tag{33}$$

In addition, the following identity holds:

$$\begin{bmatrix}\chi_k(\tau)-\chi_k(0)\\ \chi_k(\tau)+\chi_k(0)-\frac{2}{\tau}\int_0^\tau \chi_k(s)\mathrm{d}s\end{bmatrix} = G_w\xi_k.$$

As mentioned above all, an upper bound of the derivative of $\mathcal{W}$ expression is given by

$$\dot{\mathcal{W}}(\tau,\chi_k,\chi_{\kappa(\ell)}) \leq \xi_k^\top(\tau)\left(\Omega_1\left(\mathcal{T}_i,\frac{1-\lambda}{\rho\mathcal{T}_2},A,B,M\right) + \tau M^\top\begin{bmatrix}R & 0\\ 0 & 3R\end{bmatrix}^{-1}M + (T_k-\tau)\Omega_2(A,B)\right)\xi_k(\tau), \tag{34}$$

where matrices Ω_1 and Ω_2 have been defined in (24). Therefore, the derivative of functional $\mathcal{W}$ is negative definite if the following condition holds:

$$\Omega_1\left(\mathcal{T}_i,\frac{1-\lambda}{\rho\mathcal{T}_2},A,B,M\right) + \tau M^\top\begin{bmatrix}R & 0\\ 0 & 3R\end{bmatrix}^{-1}M + (T_k-\tau)\Omega_2(A,B) \prec 0, \quad \forall\tau\in[0,T_k].$$

Since the previous condition is convex with respect to $\tau\in[0,T_k]$, an equivalent condition writes

$$\begin{bmatrix}\Omega_1\left(T_k,\frac{1-\lambda}{\rho\mathcal{T}_2},A,B,M\right) & T_kM^\top\\ * & -T_k\tilde{R}\end{bmatrix} \prec 0, \qquad \Omega_1\left(T_k,\frac{1-\lambda}{\rho\mathcal{T}_2},A,B,M\right) + T_k\Omega_2(A,B) \prec 0. \tag{35}$$

We use a convexity argument with respect to the inter-sampling time T_k that belongs to the interval $[\mathcal{T}_1,\mathcal{T}_2]$ to conclude that Theorem 2 ensures $\dot{\mathcal{W}}(\tau,\chi_k,\chi_{\kappa(\ell)},T_k) < 0$, which guarantees the stability of the closed-loop system with the event-triggering rule (6). □

The above theorem presents a stability condition for closed-loop systems under dynamic aperiodic event-triggered control (6). This theorem is based on the use of the Wirtinger-based inequality, which can be further refined by using a less conservative integral inequality such as Bessel's inequality but this is not the objective of this paper.

Similarly, the number of decision variables involved in the previous stability condition is a crucial issue when dealing with LMIs. Here Theorem 2 introduce an LMI condition whose complexity is measured by the total number of decision variables ($18.5n^2+3.5n$) and the dimension of the condition is $11n\times 11n$. Altogether, the complexity of the problem is reasonable for systems of dimension lower than $n=20$. While the parameters of the Lyapunov function and of the looped-functional must remain free to allow flexibility, it has been shown in [40] that particular cases of the slack variable M, with fewer decision variables, can be considered to reduce the complexity of the LMI. These particular cases refer, for instance, to the (parameter dependent) reciprocally convexity combination lemma but, again, this is not the objective of this paper.

If one is dealing with systems of large dimension, it is possible to reduce the complexity by removing the last components of ξ_k (e.g $\frac{1}{\tau}\int_0^\tau \chi_k^\top(s)\mathrm{d}s$), which will reduce the number of decision variables as well as the dimension of the LMI, of course at the price of introducing conservatism.

4.2. Relevant corollaries

It is important to stress that particular cases can be stated to demonstrate the generality of our proposed theorem. The first one is for the case of periodic dynamic event-triggered control. The second deals with aperiodic static event-triggered control. The last one presents an extension to uncertain plants. The other possible combinations, for instance the uncertain case with periodic sampling and static rule, can be easily stated and are therefore omitted.

4.2.1. Periodic event-triggered control

The following corollary addresses the case of periodic event-triggered control, i.e., when $\mathcal{T}_1 = \mathcal{T}_2 = \mathcal{T} > 0$.

Corollary 1. *Let $\mathcal{T}$ be a strictly positive scalar and consider scalar parameters (λ, ρ, η_0) such that $\rho \in [0, \lambda) \subset [0, 1]$ and $\eta_0 > 0$. Assume that there exist matrices $P = P^\top \succ 0$, $R = R^\top \succ 0$, $S = S^\top$, $\Phi = \Phi^\top$, the controller gain K and matrices Q, M of appropriate dimensions such that the conditions (23) are satisfied with $\mathcal{T}_1 = \mathcal{T}_2 = \mathcal{T}$, that is*

$$\begin{bmatrix} \Omega_1\left(\mathcal{T}, \frac{1-\lambda}{\rho\mathcal{T}}, A, B, M\right) & \mathcal{T} M^\top \\ * & -\mathcal{T}\tilde{R} \end{bmatrix} \prec 0, \quad \Omega_1\left(\mathcal{T}, \frac{1-\lambda}{\rho\mathcal{T}}, A, B, M\right) + \mathcal{T}\Omega_2(A, B) \prec 0, \quad \begin{bmatrix} I \\ I \end{bmatrix}^\top \Phi \begin{bmatrix} I \\ I \end{bmatrix} \preceq 0. \tag{36}$$

Then the system (18) with the periodic dynamic event-triggered control (6) is asymptotically stable.

Proof. The proof is derived by simply imposing $\mathcal{T}_1 = \mathcal{T}_2 = \mathcal{T}$, which makes the sampling periodic. □

4.2.2. Static event-triggered control

Following Remark 2, the following corollary is stated to address the case of aperiodic static event-triggered control.

Corollary 2. *Let $0 < \mathcal{T}_1 \leq \mathcal{T}_2$ be two positive scalars and consider scalar parameters $(\lambda, \rho) = (1 - \epsilon, 0)$, with an arbitrarily small $\epsilon > 0$ and $\eta_0 > 0$. Assume that there exist matrices $P = P^\top \succ 0$, $R = R^\top \succ 0$, $S = S^\top$, $\Phi = \Phi^\top$, the controller gain K, and matrices Q, M of appropriate dimensions such that the following LMIs are satisfied*

$$\begin{bmatrix} \Omega_1\left(\mathcal{T}_i, \frac{1}{\mathcal{T}_2}, A, B, M\right) & \mathcal{T}_i M^\top \\ * & -\mathcal{T}_i\tilde{R} \end{bmatrix} \prec 0, \quad \Omega_1\left(\mathcal{T}_i, \frac{1}{\mathcal{T}_2}, A, B, M\right) + \mathcal{T}_i\Omega_2(A, B) \prec 0, \quad \begin{bmatrix} I \\ I \end{bmatrix}^\top \Phi \begin{bmatrix} I \\ I \end{bmatrix} \preceq 0, \tag{37}$$

for $i = 1, 2$, where Ω_1 and Ω_2 are given in (24).

Then, system (18) with the aperiodic static event-triggered control (4) is asymptotically stable.

Proof. The proof is performed by selecting the particular case of the dynamic event-triggered variable η_k in (7) with $\lambda = 1 - \epsilon$ and the event-triggering rule (6) with $\rho = 0$. It is worth noting that selecting $\lambda = 1 - \epsilon$ is mandatory to guarantee that variable η_k converges to zero even though the computation of this variable is not required to compute the control law in the static case but is still required for the proof. These selections of (λ, ρ) lead us to the definition of $\bar{\Omega}_1(\mathcal{T}_i)$ in (24), by eliminating η_k in (27). □

4.2.3. Uncertain and time-varying systems

Consider now that A and B are uncertain and/or time varying. A possible way to represent such uncertain systems is to include A and B in a polytope of $J \in \mathbb{N}$ vertices, which is defined by some positive scalars $\alpha_j \geq 0$, $j = 1, \ldots, J$ such that $\sum_{j=1}^{J} \alpha_j = 1$ and

$$[A \ \ BK] = \sum_{j=1}^{J} \alpha_j [A_j \ \ B_j K]. \tag{38}$$

An extension of Theorem 2 to this class of uncertain systems is proposed below.

Corollary 3. *Let $0 < \mathcal{T}_1 \leq \mathcal{T}_2$ be two positive scalars and consider scalar parameters (λ, ρ, η_0) such that $\rho \in [0, \lambda) \subset [0, 1]$ and $\eta_0 > 0$. Assume that there exist matrices $P = P^\top \succ 0$, $R = R^\top \succ 0$, $S = S^\top$, $\Phi = \Phi^\top$, the controller gain K, and matrices Q, M_j with $j = 1, \ldots, J$ of appropriate dimensions such that the following LMIs are satisfied*

$$\begin{bmatrix} \Omega_1\left(\mathcal{T}_i, \frac{1-\lambda}{\rho\mathcal{T}_2}, A_j, B_j, M_j\right) & \mathcal{T}_i M_j^\top \\ * & -\mathcal{T}_i\tilde{R} \end{bmatrix} \prec 0, \quad \Omega_1\left(\mathcal{T}_i, \frac{1-\lambda}{\rho\mathcal{T}_2}, A_j, B_j, M_j\right) + \mathcal{T}_i\Omega_2(A_j, B_j) \prec 0, \quad \begin{bmatrix} I \\ I \end{bmatrix}^\top \Phi \begin{bmatrix} I \\ I \end{bmatrix} \preceq 0, \tag{39}$$

for $i = 1, 2$, and $j = 1, \ldots, J$, where Ω_1 and Ω_2 are given in (24).

Then the system (18) with uncertain matrices (38) and the aperiodic dynamic event-triggered control (6) is asymptotically stable.

Proof. To perform the proof, we first need to introduce the flexibility in the matrix M to account for the uncertain case. Let us define a time-varying or uncertainty-dependent matrix $M = \sum_{j=1}^{J} \alpha_j M_j$, where the α_j are the weights of the polytopic representation and where the M_j are J matrices of the same size as M. The proof is performed by noting that conditions (23) of Theorem 2 are convex with respect to A and B but also with respect to M. Consequently, it holds that

$$\begin{bmatrix} \Omega_1\left(\mathcal{T}_i, \frac{1-\lambda}{\rho\mathcal{T}_2}, \sum_{j=1}^{J} \alpha_j A_j, \sum_{j=1}^{J} \alpha_j B_j, \sum_{j=1}^{J} \alpha_j M_j\right) & \mathcal{T}_i \sum_{j=1}^{J} \alpha_j M_j^\top \\ * & -\mathcal{T}_i\tilde{R} \end{bmatrix} \leq \sum_{j=1}^{J} \alpha_j \begin{bmatrix} \Omega_1\left(\mathcal{T}_i, \frac{1-\lambda}{\rho\mathcal{T}_2}, A_j, B_j, M_j\right) & \mathcal{T}_i M_j^\top \\ * & -\mathcal{T}_i\tilde{R} \end{bmatrix},$$

and the same calculations hold for the second inequality of (23), which concludes the proof. □

Table 1
Table showing the numerical values of the ET matrixes Φ for periodic static/dynamic ETC obtained by solving the optimization problem 1 for Example 1.

Periodic sampling	Case A	Case B	Case C
Φ_{sta}	$\begin{bmatrix} 0.0000 & 0.4934 & -0.0536 & -0.1644 \\ 0.4934 & 0.1723 & -0.2174 & -0.6665 \\ -0.0536 & -0.2174 & 0.0392 & 0.1203 \\ -0.1644 & -0.6665 & 0.1203 & 0.3690 \end{bmatrix}$	$\begin{bmatrix} 0.0835 & 0.2561 & -0.0835 & -0.2561 \\ 0.2561 & 0.7854 & -0.2561 & -0.7854 \\ -0.0835 & -0.2561 & 0.0668 & 0.2049 \\ -0.2561 & -0.7854 & 0.2049 & 0.6283 \end{bmatrix}$	$\begin{bmatrix} 0.0880 & 0.2698 & -0.0880 & -0.2698 \\ 0.2698 & 0.8273 & -0.2698 & -0.8273 \\ -0.0880 & -0.2698 & 0.0582 & 0.1783 \\ -0.2698 & -0.8273 & 0.1783 & 0.5468 \end{bmatrix}$
Φ_{dyn}	$\begin{bmatrix} 0.0000 & 0.2467 & -0.0268 & -0.0822 \\ 0.2467 & 0.0861 & -0.1087 & -0.3333 \\ -0.0268 & -0.1087 & 0.0196 & 0.0602 \\ -0.0822 & -0.3333 & 0.0602 & 0.1845 \end{bmatrix}$	$\begin{bmatrix} 0.0418 & 0.1281 & -0.0418 & -0.1281 \\ 0.1281 & 0.3929 & -0.1281 & -0.3929 \\ -0.0418 & -0.1281 & 0.0334 & 0.1025 \\ -0.1281 & -0.3929 & 0.1025 & 0.3143 \end{bmatrix}$	$\begin{bmatrix} 0.0440 & 0.1349 & -0.0440 & -0.1349 \\ 0.1349 & 0.4136 & -0.1349 & -0.4136 \\ -0.0440 & -0.1349 & 0.0291 & 0.0892 \\ -0.1349 & -0.4136 & 0.0892 & 0.2734 \end{bmatrix}$

5. Optimization

Theorem 2 and its corollaries provide stability conditions for the closed-loop system and the existence of a dynamic or static event triggering rule. However, at this stage, there is no optimization process to select the 'best' event-triggering rule. In this section, the objective is to provide a possible method to optimize the selection of the matrix Φ to reduce the number of control updates.

To do so, let us first note that the solutions of the LMI conditions are scalable in the sense that if there exists a solution $\mathcal{S}_1 := \{P, R, S, Q, M, \Phi\}$ to the LMIs, then for any positive scalar $\mu > 0$, $\mathcal{S}_\mu := \{\mu P, \mu R, \mu S, \mu Q, \mu M, \mu\Phi\}$ is also a solution to the same problem. Therefore, when optimizing the solution, using for instance the minimization criteria, it will be necessary to introduce an additional constraint to ensure the well-conditioning of the solution. Among the possible ways to do include an optimization process, presented in [18,20], was to minimize $\mathrm{Trace}(\Phi)$ to enforce the reduction of the number of control updates. Therefore, a possible optimization process to be included to the LMI condition is the one presented in the following statement:

Optimization 1. *Consider the following optimization problem*

$$\begin{array}{ll} \min\limits_{P,R,S,Q,M,\Phi,} & \mathrm{Trace}\quad \Phi \\ \textit{subject to} \quad (23) & \textit{and} \quad P_0 \preceq P \preceq P_1 \end{array} \tag{40}$$

For given matrices $P_0 \succ 0, P_1 \succ 0$.

Remark 5. The previous optimization problem aims at minimizing the trace of Φ, as a similar optimization problem that has already been employed in [18,20]. This optimization problem can be adapted to the various conditions in Section 4.2 via replacing conditions (23) by (36), (37), or (39) to deal with the problems of periodic dynamic event-triggered control, aperiodic static event-triggered control or aperiodic dynamic event-triggered control for uncertain systems, respectively.

Remark 6. In this optimization problem, constraint $P_0 \preceq P \preceq P_1$ was included to avoid the conditioning problem in optimization. Indeed, as the solutions of the LMIs are scalable, i.e. if $\mathcal{S}$ is a solution then $\mathcal{S}_\mu$ is also solution for any positive scalars $\mu > 0$, a minimization process without such additional constraint could simply make μ goes to zero to solve the problem. By imposing $P_0 \preceq P \preceq P_1$, the scaling parameter μ cannot tend to zero and the minimization becomes more efficient, with acceptable values of decision variables such as Φ.

In practice, one can simply choose $P_0 = I, P_1 = 10P_0$ to find the appropriate solution.

6. Numerical application and evaluation

6.1. Study of the case of a nominal system

Example 1. Consider the sampled-data system borrowed from [25] with following constant and known matrices:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, \quad BK = \begin{bmatrix} 0 \\ -0.1 \end{bmatrix} \begin{bmatrix} 3.75 \\ 11.5 \end{bmatrix}^\top. \tag{41}$$

For the above nominal system, the parameters in the proposed dynamic equation of the variable η_k are chosen as $\lambda = 0.6$, $\rho = 0.2$, $\eta_0 = 2$ and set $\alpha = 0.2$ in (12). The initial condition is selected as $x(0) = \left[\begin{smallmatrix} 5 \\ 0 \end{smallmatrix}\right]$. It should be remarked that the static ET rule can be retrieved by setting $\rho = 0$ whatever the value of $(\lambda, \eta_0, \alpha)$. Then, let us define the following indexes:

- the *average release interval length* given by

$$\bar{h} = \frac{\text{Operation time}}{\text{Triggered times}}$$

- the *average transmission rate* given by

$$f = \frac{\text{Average number of data transmission}}{\text{Average number of sampling}}$$

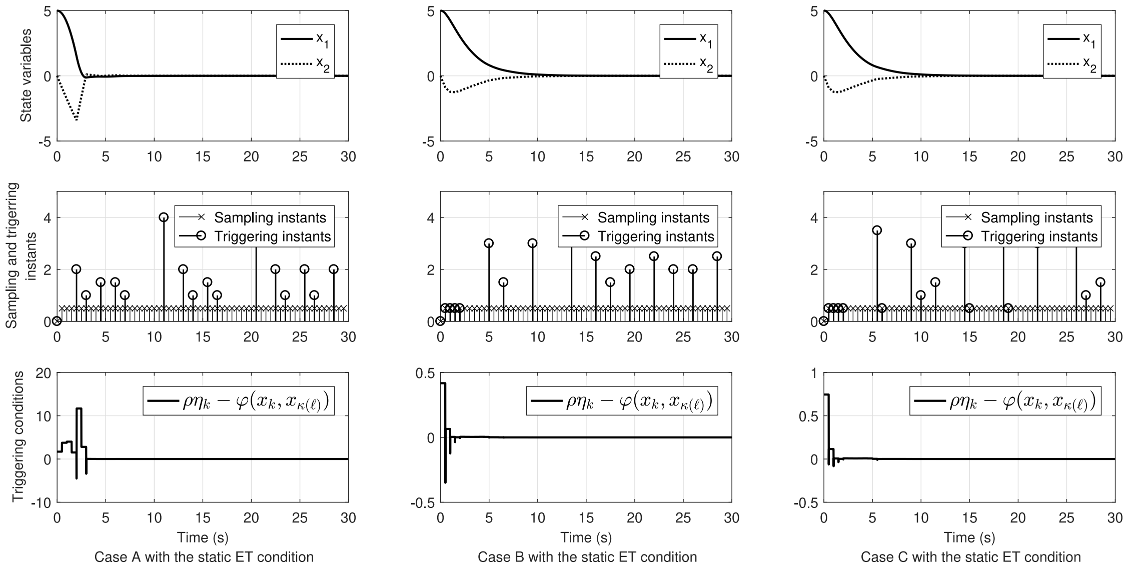


Fig. 1. The simulation of the system (18) with static ETC for the periodic sampling.

Table 2
Table showing the numerical values of the ET matrixes $\boldsymbol{\Phi}$ for aperiodic static/dynamic ETC obtained by solving the optimization problem 1 for Example 1.

Aperiodic sampling	Case A	Case B	Case C
$\boldsymbol{\Phi}_{sta}$	$\begin{bmatrix} 0.1348 & 0.8099 & -0.1588 & -0.4870 \\ 0.8099 & 0.5784 & -0.4605 & -1.4123 \\ -0.1588 & -0.4605 & 0.1255 & 0.3848 \\ -0.4870 & -1.4123 & 0.3848 & 1.1800 \end{bmatrix}$	$\begin{bmatrix} 0.1905 & 0.5840 & -0.1905 & -0.5840 \\ 0.5840 & 1.7911 & -0.5840 & -1.7911 \\ -0.1905 & -0.5840 & 0.1524 & 0.4672 \\ -0.5840 & -1.7911 & 0.4672 & 1.4328 \end{bmatrix}$	$\begin{bmatrix} 0.1588 & 0.4869 & -0.1588 & -0.4869 \\ 0.4869 & 1.4934 & -0.4869 & -1.4934 \\ -0.1588 & -0.4869 & 0.1584 & 0.4854 \\ -0.4869 & -1.4934 & 0.4854 & 1.4885 \end{bmatrix}$
$\boldsymbol{\Phi}_{dyn}$	$\begin{bmatrix} 0.0674 & 0.4050 & -0.0794 & -0.2435 \\ 0.4050 & 0.2892 & -0.2303 & -0.7061 \\ -0.0794 & -0.2303 & 0.0627 & 0.1924 \\ -0.2435 & -0.7061 & 0.1924 & 0.5900 \end{bmatrix}$	$\begin{bmatrix} 0.0952 & 0.2920 & -0.0952 & -0.2920 \\ 0.2920 & 0.8956 & -0.2920 & -0.8956 \\ -0.0952 & -0.2920 & 0.0762 & 0.2336 \\ -0.2920 & -0.8956 & 0.2336 & 0.7165 \end{bmatrix}$	$\begin{bmatrix} 0.0794 & 0.2435 & -0.0794 & -0.2435 \\ 0.2435 & 0.7467 & -0.2435 & -0.7467 \\ -0.0794 & -0.2435 & 0.0792 & 0.2427 \\ -0.2435 & -0.7467 & 0.2427 & 0.7443 \end{bmatrix}$

In the following, we will perform several simulations for the periodic and aperiodic samplings, respectively. In order to show the generality and effectiveness of our proposed ETC design, all cases of the proposed ET rule will be considered. It should be noted again that as mentioned in Remark 4, **Case A** represents the generic case of the proposed ET rule with a generic $\boldsymbol{\Phi}$ as in (11). **Case B** and **Case C** represent some existing ET rules in other work [12,13,20,35], which can be obtained by selecting a special $\boldsymbol{\Phi}$ as in (12) and (13). They can be seen as special cases of **Case A** and we will give simulations for all them. Furthermore, both static and dynamic ETC will be considered, respectively.

6.1.1. Periodic sampling

For the periodic sampling with constant period $h = 0.5s$, define $\boldsymbol{\Phi}_{sta}$ and $\boldsymbol{\Phi}_{dyn}$ representing the solutions of the static and dynamic ET conditions, respectively. The corresponding results of the three cases mentioned above can be obtained by solving the LMIs (36) and are given in Table 1.

Then with the same sequence of sampling instants and the same constant initial conditions, using the solutions in the table, we get simulation results of the static and dynamic ETC systems in Fig. 1 and Fig. 2, respectively. The trajectories of the system, the sampling and triggering instants, and the value of the corresponding triggering functions for all three cases are included in one figure. It is shown from the trajectories of the system that the stability of the system is guaranteed with our static and dynamic ETC design in the proposed looped-functional framework. It can be found from the sampling and triggering instants that our ETC design is very beneficial in reducing the amount of transmitted data. Our proposed ETC design provides an effective and more general solution to cover some existing periodic ET rules. However, the dynamic ET condition does not seem to work better than the static one for periodic sampling.

6.1.2. Aperiodic sampling

For the aperiodic sampling, the sampling period varying in time belongs to $[\mathcal{T}_1, \mathcal{T}_2] = [0.1, 0.8]$. The other parameters are the same as before. By solving the LMIs (23) and (37), we can obtain ET matrices $\boldsymbol{\Phi}_{sta}$ and $\boldsymbol{\Phi}_{dyn}$ of the proposed ET rule in Table 2. Under these solutions, the trajectories of the system, the sampling and triggering instants, and the value of the corresponding triggering functions of each case are illustrated in Fig. 3 for the static ETC system.

Fig. 4 shows the results for the dynamic ETC system. From the figures, we can see that for the aperiodic sampling, the trajectories of the system still convergence fast with our static and dynamic ETC design. The network resources can be effectively saved due to the obvious decrease of the control updates. The results show that our design also provides an effective and general solution to

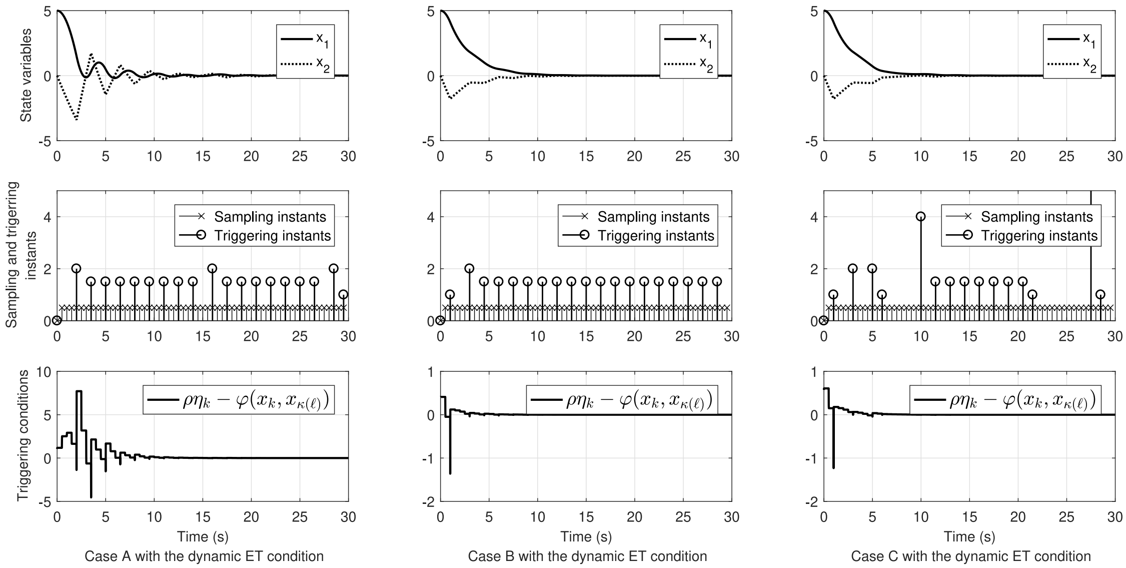


Fig. 2. The simulation of the system (18) with dynamic ETC for the periodic sampling.

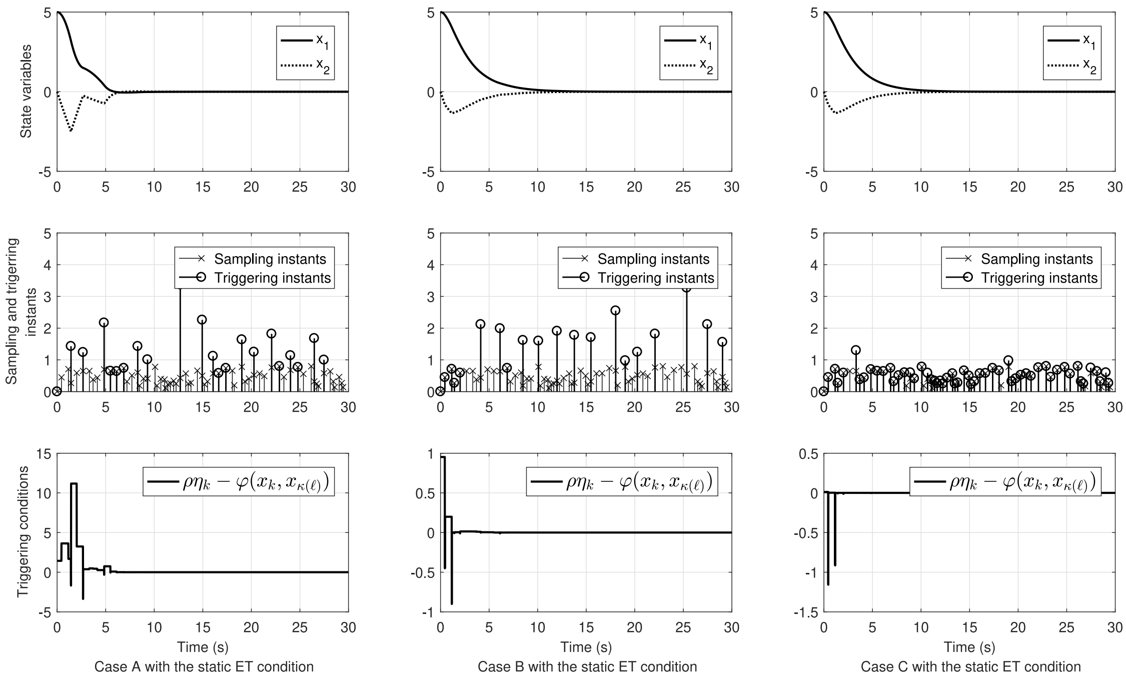


Fig. 3. The simulation of the system (18) with static ETC for the aperiodic sampling.

the aperiodic ETC problem. Moreover, it seems that the dynamic ET condition performs better than the static one in reducing the transmission as is shown in the figures.

However, it is not intuitive enough to show the efficiency of our proposed ETC design. To make a deep analysis explicitly, for each case of our ETC design, we calculate the evaluation indexes $\bar{h}$ and f which have been defined before. Select initial conditions as $x(0) = 10\begin{bmatrix} cos(2\pi\theta/N) \\ sin(2\pi\theta/N) \end{bmatrix}$, where θ takes the integer values between 1 and N and $N = 50$. With the same sequence of sampling instants, we do $N = 50$ times of simulations to get the average value of $\bar{h}$ and f.

The results for the three cases of the ET rule are given in Table 3. From Table 3, we can make a quantitative analysis that for the settings of the given parameters, our design provides a better solution, namely **Case A**, for the periodic/aperiodic and static/dynamic ETC problem to include some existing ET rules, namely **Case B** and **Case C**. Moreover, the dynamic ET condition performs better

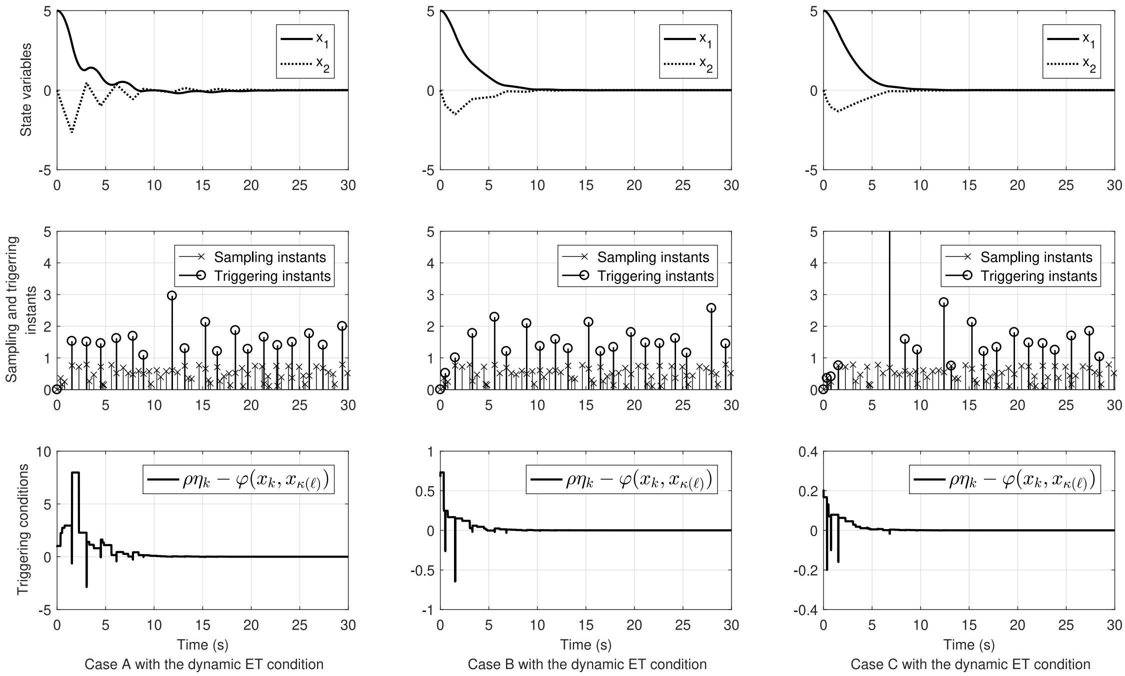


Fig. 4. The simulation of the system (18) with dynamic ETC for the aperiodic sampling.

Table 3
Evaluation indexes for the proposed ET rule.

Periodic sampling	$\bar{h}$		f	
$h = 0.5$	Static	Dynamic	Static	Dynamic
Case A	1.8094	1.9557	27.63%	25.57%
Case B	1.6760	1.7943	29.83%	27.87%
Case C	1.9685	1.6873	25.40%	29.63%
Aperiodic sampling	$\bar{h}$		f	
$[\mathcal{T}_1, \mathcal{T}_2] = [0.1, 0.8]$	Static	Dynamic	Static	Dynamic
Case A	1.4272	1.7483	31.19%	25.51%
Case B	1.6026	1.6556	27.77%	26.93%
Case C	0.5196	1.5385	85.67%	28.98%

than the static one in most cases except **Case C** of the periodic sampling. However, it should be noted that the results could be influenced by many factors, such as the initial state of the system, the parameters of the dynamic condition, the selection of the sampling period, etc. How to make our proposed ET rule work more effectively by selecting the 'best' values of λ and ρ is still an open problem.

6.2. Study of the case of an uncertain system

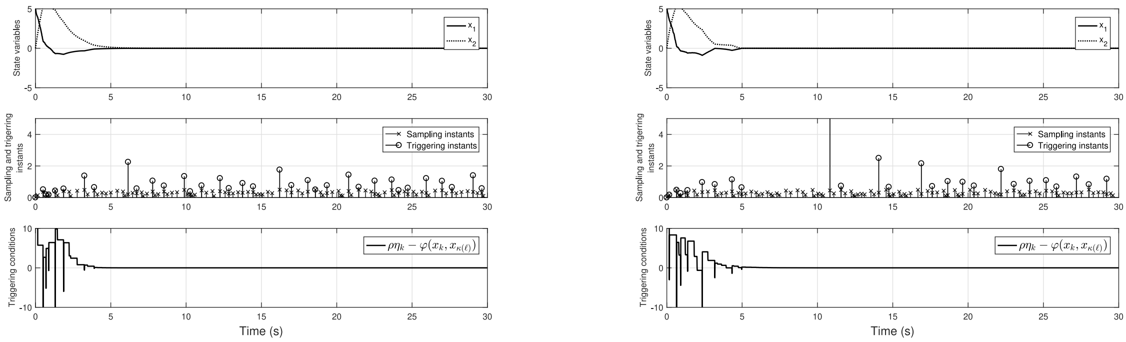
Example 2. Next, consider the uncertain system taken from [29]:

$$A = \begin{bmatrix} 1 & 0.5 \\ g_1 & -1 \end{bmatrix}, \quad BK = \begin{bmatrix} 1 + g_2 \\ -1 \end{bmatrix} \begin{bmatrix} -2.688 \\ -0.664 \end{bmatrix}^T. \tag{42}$$

where $|g_1| \leq 0.1$ and $|g_2| \leq 0.3$. For the aperiodic sampling $[\mathcal{T}_1, \mathcal{T}_2] = [0.1, 0.5]$, the other parameters are the same as those of Example 1. By solving the (39), solutions of the proposed main E-T rule, namely **Case A** of (4) and (6), are obtained as follows:

$$\Phi_{sta} = \begin{bmatrix} 47.0291 & 11.4746 & -46.0302 & -11.3718 \\ 11.4746 & 2.2019 & -10.3407 & -2.5573 \\ -46.0302 & -10.3407 & 43.7257 & 10.7982 \\ -11.3718 & -2.5573 & 10.7982 & 2.6722 \end{bmatrix}, \quad \Phi_{dyn} = \begin{bmatrix} 11.7574 & 2.8687 & -11.5076 & -2.8430 \\ 2.8687 & 0.5506 & -2.5852 & -0.6394 \\ -11.5076 & -2.5852 & 10.9316 & 2.6995 \\ -2.8430 & -0.6394 & 2.6995 & 0.6681 \end{bmatrix}. \tag{43}$$

Then, the system trajectories, the sampling and triggering instants, and the triggering functions of the static and dynamic ETC systems for **Case A** are given in Fig. 5. From the figure we can see that, in our proposed novel framework for the aperiodic ETC,



(a) The static ETC (b) The dynamic ETC

Fig. 5. The simulation of the system (18) for **Case A** of the aperiodic static (a) and dynamic (b) ETC.

Table 4
Evaluation indexes for **Case A** of the proposed ET rule.

Aperiodic sampling	Static ETC	Dynamic ETC
$\bar{h}$	0.8528	1.0267
f	35.22%	30.58%

a good balance between the system performance and resources is obtained for linear systems with polytopic uncertainties, which shows the generality and efficiency of our design. Furthermore, the evaluation indexes are given in Table 4. The values of $\bar{h}$ and f demonstrate that the dynamic ET condition usually works better than the static one.

7. Conclusion

In this manuscript, we have extended the concept of periodic dynamic event-triggered control for linear sampled-data systems to encompass the aperiodic case. To do so, we introduced a discrete-time dynamic triggering rule and provide a stability analysis of the closed-loop system within a novel looped-functional framework. Leveraging the lifting technique and the proposed looped functionals, we have established a unified framework to derive asymptotic stability conditions for periodic/aperiodic and static/dynamic event-triggered control systems, expressed in terms of a set of feasible Linear Matrix Inequalities. The efficacy of the proposed design is demonstrated through simulation results. Future endeavors will explore the extension of our design methodology to discrete-time sampled-data systems, nonlinear systems, and similar domains.

CRediT authorship contribution statement

Yihao Xu: Writing – review & editing, Numerical application. **Alexandre Seuret:** Supervision, Writing – review & editing, Numerical application. **Kun Liu:** Review & editing. **Senchun Chai:** Review & editing.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Alexandre Seuret reports financial support was provided by University of Seville Systems Engineering and Automation. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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