Quantum Transport in Nonuniform Magnetic Fields: Aharonov-Bohm Ring as a Spin Switch

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We study spin-dependent magnetoconductance in mesoscopic rings subject to an inhomogeneous in-plane magnetic field. We show that the polarization direction of transmitted spin-polarized electrons can be controlled via an additional magnetic flux such that spin flips are induced at half a flux quantum. This quantum interference effect is independent of the strength of the nonuniform field applied. We give an analytical explanation for one-dimensional rings and numerical results for corresponding ballistic microstructures.

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Recent experimental progress [1] in creating spin-polarized charge carriers in semiconductors indicates the principle ability to perform spin electronics [2] based on nonmagnetic semiconductor devices. This widens the field of usual magnetoelectronics in metals and opens up the intriguing program of combining the rich physics of spin-polarized particles with all the advantages of semiconductor fabrication and technology, e.g., precise design of nanoelectronic devices with controllable charge carrier densities and optoelectronical applications. Besides, the spin relaxation times involved can be rather long; coherence of spin states can be maintained up to scales of more than 100 μm [3]. Hence coherent control and quantum transport of spin states in semiconductor heterojunctions or quantum dots are attracting increasing interest [4], also in view of proposed future applications including spin transistors [5], filters [6], and scalable devices for quantum information processing [7], to name only a few.

In nonmagnetic semiconductors the coupling of the carrier spin to an applied magnetic field can be used to control the spin degree of freedom. In this respect, nonuniform magnetic fields whose direction varies on mesoscopic length scales (textured fields) are of particular interest. Besides the usual Zeeman spin splitting, they give rise to a variety of additional effects absent in conventional charge quantum transport.

In the limit of a strong magnetic field the electron spin can adiabatically follow the spatially varying field direction, and the spin wave function acquires a geometrical or Berry phase [8]. In mesoscopic physics, Berry phases were first theoretically studied for one-dimensional (1D) rings [9,10]. They are also expected to give rise to clear signatures in the magnetoconductance of two-dimensional (2D) ballistic microstructures [11]. Nonuniform magnetic fields on mesoscopic scales have been realized in semiconductors, for instance, by placing micromagnets [12,13] or ferromagnetic stripes [14] above or into the plane of a 2D electron gas in high-mobility semiconductor heterostructures. However, although magnetic inhomogeneities of up to 1 T have been reported [15], it is difficult to experimentally reach the truly adiabatic regime. The coupling of the carrier spins to more realistic moderate inhomogeneous fields generally leads to nonadiabatic, spin-flip processes counteracting geometrical phases. Hence, despite various experimental efforts [13,16] a clear-cut demonstration of Berry phases in mesoscopic transport remains an experimental challenge.

In this Letter we study *nonadiabatic,* spin-dependent coherent transport through ballistic mesoscopic rings in the presence of textured fields. This enables us, on the one hand, to quantitatively investigate for *unpolarized* electrons the relevant conditions necessary to observe geometrical phases or their nonadiabatic generalizations, Aharonov-Anandan phases [17]. On the other hand, we show for *spin-polarized* charge carriers how to use inhomogeneous fields to induce spin flips in a controlled way. For Aharonov-Bohm (AB) ring geometries (with in-plane nonuniform field) coupled symmetrically to two leads we demonstrate that the spin direction of polarized particles transversing the rings can be tuned and even reversed by applying an additional small control field. This quantum effect exists irrespective of adiabaticity.

We consider symmetric 1D and 2D ballistic rings with two attached leads as shown in Fig. 1. For the inhomogeneous magnetic field we assume a circular configuration $\vec{B}_{\rm i}(\vec{r}) = B_{\rm i}(r)\hat{\varphi} = (a/r)\hat{\varphi}$ (in polar coordinates) centered around the inner disk of the microstructure [18]. The Hamiltonian for noninteracting electrons with effective mass m^* and spin given by the Pauli matrix vector $\vec{\sigma}$ reads, in the presence of a magnetic field $\vec{B} = \vec{\nabla} \times \vec{A}$,

$$H = \frac{1}{2m^*}\left[\vec{p} + \frac{e}{c}\vec{A}(\vec{r})\right]^2 + V(\vec{r}) + \mu\vec{B}\cdot\vec{\sigma}\,. \quad (1)$$

The potential $V(\vec{r})$ defines the confinement of the ballistic conductor. In our case the vector potential has two contributions, $\vec{A} = \vec{A}_0 + \vec{A}_{\rm i}$. The term $\vec{A}_{\rm i}(\vec{r})$ generates the inhomogeneous field $\vec{B}_{\rm i}(\vec{r})$ and $\vec{A}_0$ represents a (weak) perpendicular uniform field $\vec{B}_0$ or an AB flux ϕ to be used as an additional tunable parameter to study the magnetoconductance. In Eq. (1), $\mu = g^*\mu_{\rm B}/2 = g^*e\hbar/(4m_0c)$ where $\mu_{\rm B}$ is the Bohr magneton, m_0 is the bare electron mass, g^* is the effective gyromagnetic ratio, and $e > 0$.

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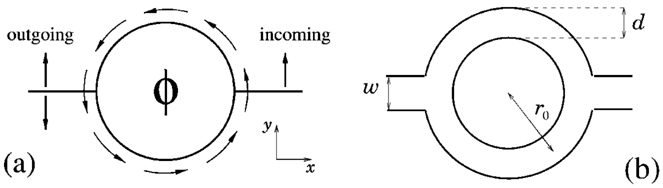

FIG. 1. Geometries of ballistic microstructures used in the quantum calculations of the spin-dependent conductance for a circular (in-plane) magnetic field texture plus a magnetic flux ϕ. Spin directions are defined with respect to the y axis.

We compute the spin-dependent conductance $G(E, B_i, \phi)$ for two-terminal quantum transport through the microstructures using the Landauer formula. We focus on the case where the two leads of width w support only one open channel [19]. The spin-dependent conductance then reads, for zero temperature,

$$G(E, B_{\mathrm{i}}, \phi) = \frac{e^2}{h}(|t^{\uparrow\uparrow}|^2 + |t^{\downarrow\downarrow}|^2 + |t^{\downarrow\uparrow}|^2 + |t^{\uparrow\downarrow}|^2). \quad (2)$$

We define the spin direction with respect to the y axis in Fig. 1. The transmission coefficients $T^{\downarrow\uparrow} = |t^{\downarrow\uparrow}|^2$ ($T^{\uparrow\downarrow} = |t^{\uparrow\downarrow}|^2$) describe transitions between an incoming state from the right with spin up (down) to an outgoing state to the left with spin down (up). They vanish for $B_{\mathrm{i}} = 0$. In the opposite, adiabatic limit of a strong magnetic field, the magnetic moment associated with the electron spin traveling around the ring stays (anti)aligned with the local inhomogeneous field. Hence, for the field geometry in Fig. 1(a) an incoming spin-up state is then converted into a spin-down state upon transmission through the ring, and vice versa. In the strong-field limit, $T^{\uparrow\uparrow} = |t^{\uparrow\uparrow}|^2 = 0$ and $T^{\downarrow\downarrow} = |t^{\downarrow\downarrow}|^2 = 0$.

For the experimentally relevant, intermediate case of moderate magnetic fields one must solve coupled equations for the spin states to account for spin flips. We calculate the four spin-dependent transmission amplitudes by projecting the corresponding Green function matrix of the system onto the transverse mode spinors (of incoming and outgoing states) in the leads. We obtain the Green functions for the Hamiltonian (1) numerically after generalizing the recursive Green function method for spinless particles to the case with spin. This requires one to replace the on-site and hopping energies in a tight-binding approach by 2×2 spin matrices [20].

We first study how adiabaticity is approached in mesoscopic rings by considering the spin-dependent transmission of unpolarized electrons in the full crossover regime between $B_{\mathrm{i}} = 0$ and the adiabatic limit. The appearance of geometrical phases requires an adiabatic separation of time scales: For 1D rings of radius r_0 the Larmor frequency of spin precession, $\omega_{\mathrm{s}} = 2\mu B/\hbar$, must be large compared to the frequency $\omega = v_{\mathrm{F}}/r_0$ of orbital motion with Fermi velocity v_{F} around the ring [10]. In the adiabatic limit a geometric phase $\gamma^{\uparrow(\downarrow)}$ is acquired during a round-trip. For the in-plane field considered, $\gamma^{\uparrow(\downarrow)} = \pi$ giving rise to a geometric flux $-\phi_0/2$ with $\phi_0 = hc/e$. With an AB flux ϕ it adds up to an effective flux $\phi - \phi_0/2$. This causes a shift in the AB magnetooscillations of $T^{\uparrow\downarrow}$ and $T^{\downarrow\uparrow}$ such that the overall transmission $T(\phi = 0) = 0$ [11], since also $T^{\uparrow\uparrow}$ and $T^{\downarrow\downarrow}$ tend to zero; see above. The condition for adiabaticity can be written as

$$q \equiv \frac{\omega}{\omega_{\mathrm{s}}} = \frac{k_{\mathrm{F}} r_0}{g^*(m^*/m_0)(\pi r_0^2 B/\phi_0)} \ll 1, \quad (3)$$

with $k_{\mathrm{F}} = m^* v_{\mathrm{F}}/\hbar$. For 2D rings of width d and mean radius r_0 the angular ($\hat{\varphi}$) component of $\vec{k}_{\mathrm{F}}$ is relevant for adiabaticity. For the mth propagating mode in a 2D ring, q in Eq. (3) is then replaced by the rescaled parameter $q_\varphi \equiv q\sqrt{1 - [m/(k_{\mathrm{F}} d/\pi)]^2}$ (provided that $d/r_0 \ll 1$).

To show how adiabaticity is approached we consider transport through a ring with one open channel, $m = 1$. The solid line in Fig. 2 depicts the numerically obtained average transmission $\langle T(E, \phi = 0)\rangle_E$ as a function of $1/q_\varphi$ for the quasi-1D ring of Fig. 1(b) ($d/r_0 = 0.25$) at $\phi = 0$. The average is taken over an energy interval (between the first and the second open channel) at fixed q_φ to smooth out energy-dependent oscillations. With increasing $1/q_\varphi$ the transmission $\langle T(E, \phi = 0)\rangle_E$ tends to zero which is a clear signature of the geometrical phase as discussed above. The overall decay is Lorentzian, $\sim(1 + q_\varphi^{-2})^{-1}$ (dotted line in Fig. 2). This curve and the dashed line, which agrees well with the numerical result, are obtained in an independent transfer matrix approach for a 1D ring [Fig. 1(a), $q_\varphi \equiv q$] to be discussed below.

In our calculations $\langle k_{\mathrm{F}} r_0\rangle \simeq 15$. In a typical experimental setup, $k_{\mathrm{F}} r_0 = 2\pi r_0/\lambda_F \simeq 60$ for $r_0 \simeq 500$ nm and $1/q \simeq 0.07B$[T] and $0.86B$[T] for GaAs and InAs. However, despite the relatively large fields necessary for satisfying Eq. (3) for q, the scaling factor entering into q_φ allows one to reach adiabaticity for considerably lower

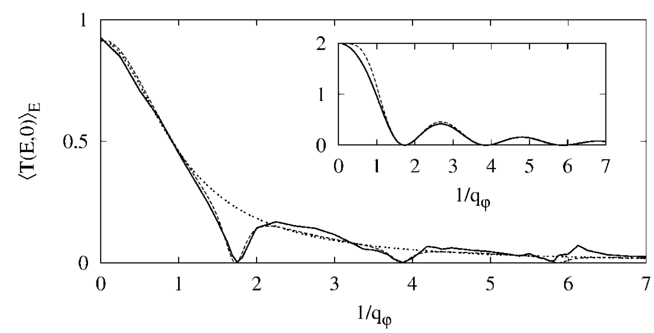

FIG. 2. Energy-averaged quantum transmission as a function of the adiabaticity parameter q_φ (see text) for unpolarized electrons through rings with circular in-plane magnetic field (as in Fig. 1) and zero flux. The solid line represents numerical results for the geometry in Fig. 1(b). The dashed curve shows results from a corresponding transfer-matrix approach for a 1D ring [Fig. 1(a); see text]. The dotted line shows an overall Lorentzian dependence $0.916/(1 + q_\varphi^{-2})$. Inset: 1D approximate [Eq. (4), dashed line] and full (solid line) results for strongly coupled leads ($\epsilon = 0.5$). In all curves $\langle T(E, \phi = 0)\rangle_E \to 0$ for $q_\varphi \to 0$ owing to geometrical phases.

field strengths. This is achieved by reducing either the width of quasi-1D rings or k_F via the electron density [21].

In the following we study how the *spin-dependent* transmission changes as a function of an additional flux $\phi = \pi r_0^2 B_0$ with $B_0 \ll B_i$. Our main results are summarized in Fig. 3 showing $\langle T(E, \phi)\rangle_E$ for three scaled strengths $q_\varphi \approx 20$, 1.4, 0.25 of the inhomogeneous field. We consider up-polarized, incoming spins; equivalent results are obtained for spin-down states. In the weak-field limit, Fig. 3(a), $\langle T^{\downarrow\uparrow}\rangle$ (dotted line) is close to zero, and the total transmission (solid line) shows usual AB oscillations predominantly given by $\langle T^{\uparrow\uparrow}\rangle$ (dashed line). The behavior is reversed in the adiabatic limit, panel (c), where $\langle T^{\downarrow\uparrow}\rangle$ exhibits AB oscillations, shifted by $\phi_0/2$ due to the geometrical phase as discussed above.

Panel (b) shows the general case of an intermediate field. With increasing flux the polarization of transmitted electrons changes continuously. Most interestingly, $\langle T^{\downarrow\uparrow}\rangle = 0$ at $\phi = 0$, while $\langle T^{\uparrow\uparrow}\rangle = 0$ for $\phi = \phi_0/2$. For zero flux an ensemble of spin-polarized charge carriers is transmitted always keeping the spin direction, while for $\phi = \phi_0/2$ the transmitted electrons just reverse their spin direction. In other words, by tuning the flux from 0 to $\phi_0/2$, one can reverse the polarization of transmitted particles in a controlled way. Hence, the AB ring plus the rotationally symmetric magnetic field acts as a tunable spin switch, *independent* of the field strength $B_i > 0$, which determines only the size of the spin-reversed current. Alternatively, for a fixed flux $0 < \phi < \phi_0/2$ (vertical dotted line in Fig. 3) the spin polarization is reversed upon going from the nonadiabatic to the adiabatic regime, while the total transmission remains nearly constant.

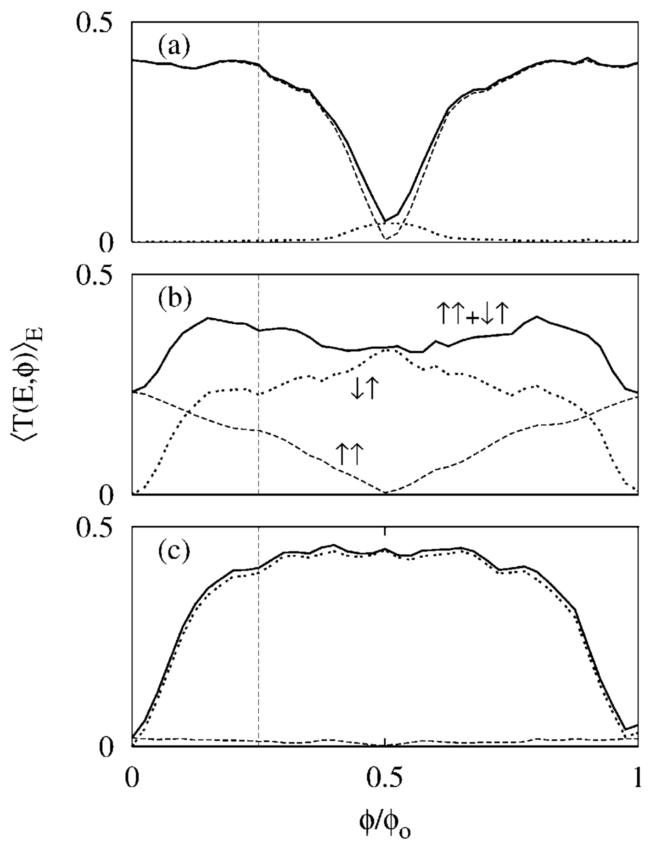

FIG. 3. Averaged transmission for up-polarized incoming electrons (see Fig. 1) through a quasi-1D ring as function of a flux $\phi = \pi r_0^2 B_0$ in the presence of a circular in-plane field $B_i \gg B_0$ of increasing strength: (a) weak, (b) moderate, and (c) strong. The overall transmission (solid line) is split into its components $\langle T^{\uparrow\uparrow}\rangle$ (dashed line) and $\langle T^{\downarrow\uparrow}\rangle$ (dotted line). Note the change in the polarization upon tuning the flux and the spin-switch mechanism at $\phi = \phi_0/2$.

This mechanism for changing the spin direction neither relies on the spin coupling to the control field B_0 nor on the Zeeman splitting often exploited in spin filters. It is a pure quantum interference effect which exists also for the nonaveraged transmission at a given energy.

In the following we give an analytical explanation for the numerically observed effects (Figs. 2 and 3). To this end we consider the model of a 1D AB ring coupled to 1D leads, Fig. 1(a), and extend the transfer matrix approach for spinless particles [22] to the case with spin. We follow the method outlined in [23] but consider fluxes instead of probabilities to work with unitary transfer matrices.

The eigenstates of the Hamiltonian (1), which are analytically obtained for a ballistic 1D ring [10], are necessary for implementing the transfer matrix algorithm. They read $\Psi_{n,s} = \exp(in\varphi) \otimes \psi_n^s(\varphi)$, where the first factor describes the motion along the ring and the second refers to the spin state $s = \uparrow, \downarrow$ [with respect to the vertical (z) axis]. The Zeeman term causes a slight difference in the kinetic energy of spin-$\uparrow$ and spin-$\downarrow$ electrons traveling clockwise or counterclockwise around the ring so that we must distinguish four possible n: $n_j^\uparrow, n_j^\downarrow$ $(j = 1, 2)$. They are given by $n' \equiv n + \phi/\phi_0$, where the n' are the solutions of the equation [20] $\tilde{E}_F = n'^4 + 2n'^3 + (1 - 2\tilde{E}_F)n'^2 - 2(\tilde{E}_F + \tilde{\mu}B\cos\alpha)n' + \tilde{E}_F^2 - \tilde{\mu}B\cos\alpha - (\tilde{\mu}B)^2$. Here, $\tilde{E}_F = (2m^* r_0^2/\hbar^2)\ E_F$ is the scaled Fermi energy, $\tilde{\mu} = (2m^* r_0^2/\hbar^2)\mu$, and α is the tilt angle of the textured magnetic field with respect to the z axis. In the general, nonadiabatic case four angles $\gamma_j^\uparrow, \gamma_j^\downarrow \le \alpha$ take the role of α and characterize the spin eigenstates which read

$$\psi_{n_j}^\uparrow = \begin{pmatrix} \cos\frac{\gamma_j^\uparrow}{2} \\ \pm ie^{i\varphi}\sin\frac{\gamma_j^\uparrow}{2} \end{pmatrix}, \qquad \psi_{n_j}^\downarrow = \begin{pmatrix} \sin\frac{\gamma_j^\downarrow}{2} \\ \mp ie^{i\varphi'}\cos\frac{\gamma_j^\downarrow}{2} \end{pmatrix},$$

with $\cot\gamma_j = \pm[\cot\alpha + (2n_j' + 1)/(2\tilde{\mu}B\sin\alpha)]$. In the adiabatic limit $\gamma \to \alpha$. The transfer matrices in the generalized basis enter into the transmission formulas which generally require a numerical evaluation.

For an in-plane field $(\alpha = \pi/2)$ without additional flux $(n = n')$ the equations above simplify considerably and we find $n_1^{\uparrow(\downarrow)} = -(n_2^{\downarrow(\uparrow)} + 1)$. Though being involved, all expressions leading to the transmission can be handled analytically. The transmission depends strongly on the coupling at the junctions between the ring and the leads. It is given by a parameter ϵ [22] where $\epsilon = 0\,(0.5)$ describes zero (strongest) coupling. Adjusting ϵ the analytical model allows us to estimate the effective coupling to the leads in ballistic rings used in the numerical calculations above. The dashed line in Fig. 2 ($\epsilon = 0.316$) fits well with the numerical result (solid line). For $\epsilon = 0.5$ and $\phi = 0$, an approximate analytical expression for $\langle T(E,0)\rangle_E$ can be given in compact form, if we replace the energy averages over rapidly oscillating angular functions involved by their mean. We find, leaving the details to [20],

$$\langle T(E,\phi = 0)\rangle_E \simeq 16\,\frac{\cos^2\bar{\gamma}_1 \sin^2(\Delta n\pi/2)}{4 + \cos^4\bar{\gamma}_1[1 - \cos(\Delta n\pi)]^2}\,. \tag{4}$$

$\Delta n \equiv n_1^{\uparrow} - n_1^{\downarrow}$ and $\bar{\gamma}_1 \equiv (1/2)(\gamma_1^{\uparrow} + \gamma_1^{\downarrow})$ can be expressed through q_φ as $\Delta n = (1 + q_\varphi^{-2})^{1/2}$ and $\cos\bar{\gamma}_1 = (1 + q_\varphi^{-2})^{-1/2}$. The inset in Fig. 2 shows the result (4) (dotted line) compared to the exact 1D result (solid line) for $\epsilon = 0.5$.

All the general features of $\langle T(E,0)\rangle_E$ in Fig. 2 are well described by Eq. (4). Owing to destructive interference, the transmission vanishes at points where Δn is an even integer corresponding to $1/q_\varphi = \sqrt{3}, \sqrt{15}, \ldots$. Equation (4) gives a complicated overall decay factor for $\langle T(E,0)\rangle_E$ which reduces to the Lorentzian $\cos^2\bar{\gamma}_1 = 1/(1 + q_\varphi^{-2})$ in the limit $\epsilon \to 0$. Already for $\epsilon < 0.4$ this is a good approximation for the overall crossover from the diabatic to the adiabatic regime (dotted line in Fig. 2).

Within the 1D model we further reproduce the flux dependence for spin-dependent transport, Fig. 3, and find an analytical proof [20] for the spin-switch effect discussed above. The transmission coefficient $T^{\uparrow\uparrow}$ vanishes completely at $\phi = \phi_0/2$, if the magnetic field to which the spins couple has no component perpendicular to the plane of the ring. Furthermore, we find numerically that the spin-switch effect does not occur only in quasi-1D rings but also in doubly connected structures with more than one open mode and arbitrary shape, as long as reflection symmetry with respect to the horizontal axis is preserved [20]. However, the effect requires single-channel leads [19]. We further note that rings with Rashba (spin-orbit) interaction [24] (yielding an effective in-plane magnetic field in the presence of a vertical electric field) might lead to a similar spin-switch effect.

To summarize, we have studied *nonadiabatic* spin transport through ring geometries in inhomogeneous magnetic fields. We obtain, numerically and analytically, the explicit dependence of the transmission on the scaled field strength q_φ, which acts as an adiabaticity parameter, elucidating the role of geometrical phases in quantum transport and possible experimental realizations. For in-plane fields and symmetric ballistic structures we demonstrate how an additional small flux ϕ can be used to control spin flips and to tune the polarization of transmitted electrons. This quantum mechanism does not require adiabaticity. In combination with a spin detector such a device may be used to control spin polarized current, similar to the spin field-effect transistor of Ref. [5]. Whether related effects may prevail in diffusive ferromagnetic conductors remains as a further interesting problem.

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