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Effects of Particle Size and Field Orientation on the Yield Stress of Magnetostabilized Fluidized Beds

J. M. Valverde,*,† M.A.S. Quintanilla,† and M. J. Espin‡

†Department of Electronics and Electromagnetism and ‡Department of Applied Physics II, University of Seville, Avda Reina Mercedes s/n, Seville 41012, Spain

ABSTRACT: In this work, we present experimental measurements of the yield stress of gas-fluidized beds of magnetizable particles stabilized by an externally imposed magnetic field. Powder samples consist of spherical magnetite particles 35−65 μm in size. The magnetic field is applied in the bubbling regime and the gas velocity is decreased. At a critical gas velocity, particle chains that have formed due to attractive magnetostatic forces become jammed and the bed transits to a solidlike expanded state with a non-negligible yield stress. Our experimental setup allows us for taking measurements of the yield stress of the bed stabilized by a magnetic field oriented either in the vertical or horizontal direction (co-flow and cross-flow field configurations, respectively). In the cross-flow field configuration, the magnetic yield stress is increased with particle size. On the other hand, the magnetic yield stress is decreased in the co-flow field configuration as particle size is increased. This is interpreted as due to the dependence of the interparticle magnetostatic force on the interparticle contact angle with the field, which is, on average, affected by particle size in the jammed bed subjected to small consolidations.

1. INTRODUCTION

Gas-fluidized beds of noncohesive particles are usually unstable. Most of the gas bypasses the bed through large bubbles, which curtails uniform expansion and hampers the gas−solids contact efficiency. This type of behavior is the so-called "Geldart B behavior", according to the Geldart diagram.[1] Bubbling beds can be stabilized by sufficiently strong attractive forces between the particles, which can be induced by externally applied fields, as seen when a magnetic field is imposed on bubbling beds of magnetizable particles.[2,3] Generally, it is observed that application of the magnetic field results in the formation of chainlike particle aggregates that eventually become arrested when the magnetic field strength is strong enough and the gas velocity is decreased below a critical value larger than the minimum fluidization velocity. At the jamming transition, the bed acquires a solidlike stable structure characterized by a non-negligible yield stress.[3] This is an analogous situation to stabilization of bubbling beds of dielectric particles when they are subjected to a strong enough electric field.[4] In this case, and because of the induced electrostatic forces between the polarized particles, chainlike aggregated structures are developed in the so-called electrofluidized bed (EFB) that eventually lead to a transition from bubbling to a solidlike stable behavior. Electrically induced interparticle forces provide the solidlike stable bed with a modulus of elasticity, which was shown to increase linearly with the applied electric field for glass beads fluidized by argon.[5]

Linear stability models were already proposed, along with pioneering observations on beds stabilized by a vertically oriented magnetic field (co-flow field configuration).[2,6] These models were based on a continuum approach founded on the assumptions that all the fields could be averaged on large distances, compared to particle size, and that the medium was inviscid. Short-range attractive forces at the contact between the magnetized particles were neglected. Despite this drastic idealization of magnetofluidized beds (MFBs), suppression of gas bubbles was successfully predicted as being due to the magnetic body force arising from gradients of magnetic susceptibility caused by bulk density perturbations. However, quantitative discrepancies between predicted and experimental results were systematically found, particularly concerning the effect of particle size.[2] Moreover, the stability model predicted that MFBs could not be stabilized by a horizontally oriented magnetic field (cross-flow field configuration) while experimental observations have demonstrated otherwise.[3,7] Interestingly, good agreement was found between model predictions and experimental observations on beds of nonmagnetic particles fluidized by ferrofluids,[8] where interparticle contact forces that had been dismissed by the model assumptions, were, in fact, absent. This suggested that the stabilization mechanism of gas-fluidized beds of magnetized particles was mainly ruled by the nonnegligible induced interparticle contact forces in analogy with EFBs,[4,5] as it was later demonstrated by experimental observations.[3]

In order to better understand the role of interparticle contact forces on magnetic stabilization, measurements on the yield stress of MFBs, as a function of the intervening physical parameters and conditions, would provide useful information to be incorporated on stability models. Experimental measurements on the yield stress and transition to stability of MFBs subjected to a cross-flow magnetic field have been recently reported.[3,9−11] These works showed that field operation mode and particle size distribution can be relevant parameters on magnetic stabilization. Application of the field to a naturally stabilized bed, wherein particles were held in enduring positions

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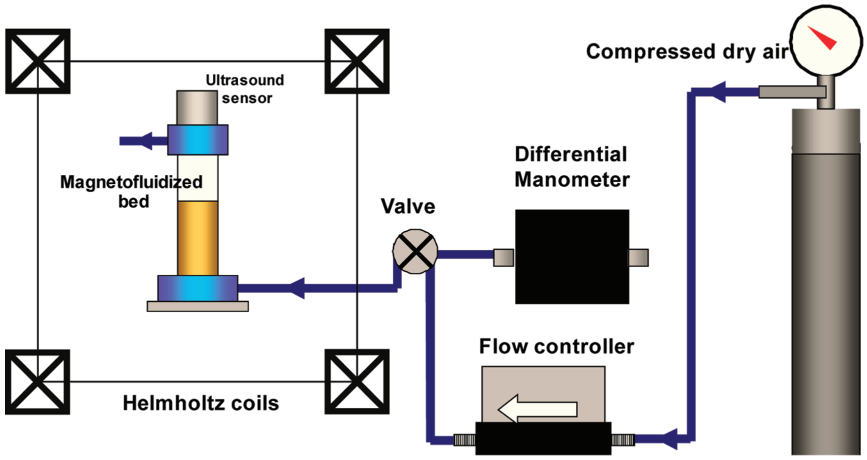

Figure 1. Sketch of the experimental setup used in the magnetofluidization experiments reported in this work. The powder bed is subjected to a controlled gas flow in the vertical direction in the presence of a uniform magnetic field that can be applied either in the vertical or horizontal direction.

and were not allowed rearranging, did not cause an appreciable increment of the yield stress. On the other hand, the yield stress of the magnetically stabilized bed was notably increased if the field was applied in the bubbling regime[3,9] (Magnetization LAST operation mode[12]). In this operation mode, particles were free to move and form chainlike aggregates, because of induced attractive forces. In the case of powders with a high level of particle size polydispersion, the natural aggregation of the fine particles due to van der Waals forces favored the formation of large-scale branched structures in the MFB, which increased the yield stress and favored stabilization at large values of the gas velocity.[10] Further experimental results on the yield stress of magnetically stabilized beds in the cross-flow field configuration and operated in the magnetization LAST mode demonstrated that the yield stress was enhanced, and it increased with the field strength at an appreciably higher rate, as the particle size was increased.[11] It was argued that the yield stress was critically influenced by the microstructure of the stabilized bed subjected to a small consolidation. As the particle size was increased, particles became more closely packed in the jammed bed subjected to a small consolidation. Thus, interparticle contacts became, on average, more closely oriented in the direction of the horizontal field. As a consequence, the attractive magnetic force between the particles—and, therefore, the yield stress—would increase with particle size. Accordingly, the opposite effect should be observed if the field is applied in the co-flow configuration, since, in that case, the average angle of the contact normal with the field in the jammed bed would be increased as particle size is increased. In order to corroborate this argument, we show, in the present paper, new experimental results on the yield stress of magnetically stabilized beds operated in the Magnetization LAST mode as affected by field orientation.

2. EXPERIMENTAL SETUP

Figure 1 is a sketch of the setup used in our experimental work. The magnetic powder sample is held in a vertically oriented cylindrical vessel (2.54 cm internal diameter) and rests on a nonmagnetizable porous plate that acts as gas distributor (5 μm pore size). By means of a series of computer-controlled valves and a mass flow controller, a controlled flow of filtered and dried air is pumped through the powder bed while the gas pressure drop across it is read from a differential pressure transducer. The height of the bed, which gives an average value of the particle volume fraction (ϕ), is measured by means of an ultrasonic sensor placed on top of the vessel. A uniform magnetic field is externally imposed by means of a pair of square Helmholtz coils (50 cm × 50 cm) with each coil consisting of 500 turns of 2-mm-diameter copper wire. The coils can be rotated in order to change the orientation of the field, relative to the gas flow. In the present study, the field has been applied both in the vertical and horizontal directions (co-flow and cross-flow field configurations, respectively). The strength of the field has been fixed to B = 2.6 mT.

3. MATERIALS

The powders tested consisted of spherical magnetite particles of nominal particle size: d_p = 65, 50, and 35 μm with small particle size dispersion (scanning electron microscopy (SEM) pictures can be seen in Figure 2). Particle density was ρ_p = 5060 kg/m^3, as measured by an AccuPyc Model 1330 pycnometer. Magnetic characterization of the powders used in our experiments was made by means of a SQUID magnetometer (SQUID Quantum Design MPMS XL) and also using the L-method (see ref 3 for a detailed description). The initially demagnetized samples behaved linearly in the range of field strengths applied in our MFB experiments with similar bulk magnetic susceptibilities (χ = 3−4). Using the coherent potential approximation and the measured bulk susceptibilities, the particles' magnetic susceptibility was obtained[3] as $\chi_p \approx 11.5$.

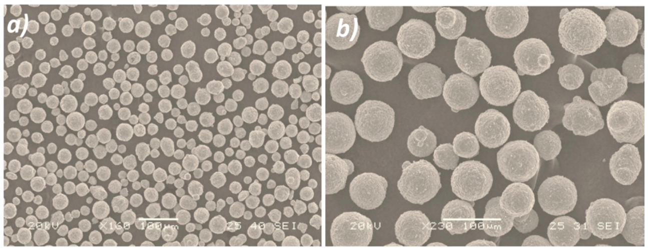

Figure 2. Scanning electron microscopy (SEM) pictures of magnetite powders used with nominal particle sizes of (a) 35 μm and (b) 65 μm.

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4. EXPERIMENTAL PROCEDURE

The experimental procedure generally followed in our experiments to measure the yield stress of stabilized beds of cohesive powders is schematized in Figure 3. The bed is first driven to the bubbling regime by imposing a large gas velocity v_b, at which the gas pressure drop Δp is near the powder weight per unit area W. Once the bubbling bed has reached a stationary state in which it has lost memory of its previous history, the gas velocity is slowly decreased. As the gas velocity is decreased, Δp falls below W at a critical gas velocity v_c, indicating that part of the powder becomes then sustained by permanent interparticle contacts in a solidlike stable state. This is the jamming transition, delimiting the unstable bubbling regime and the solidlike stable regime, in which the bed is stabilized by interparticle attractive forces. As the gas velocity is further decreased below the jamming transition, Δp decreases below W. At a gas velocity $v_0 < v_c$, there is a consolidation stress on the bed at its bottom σ_c, which is given by $\sigma_c = W - \Delta p(v_0)$ if wall effects are neglected. Wall effects can be neglected if the bed height is similar or smaller than the bed diameter.[13] For this reason, we use, in our experiments, shallow beds with a typical height of ∼2 cm. Figure 3 shows the procedure employed to measure the tensile yield stress corresponding to this state of consolidation. Once the bed is stabilized at a given gas velocity $v_0 < v_c$, and in order to put the bed under tension, it is now subjected to a slowly increasing gas velocity. For small increments of the gas velocity, the solidlike structure remains stable and Δp, which is just due to frictional resistance, increases linearly as v_g is increased in accordance with Darcy's law for the passage of a fluid flow through a porous solid in the low-Reynolds-number regime. Δp balances W at the point of minimum fluidization velocity v_{mf}. Further increment of the gas velocity subjects the bed to a tensile stress $\sigma = \Delta p - W$, which is maximum at the bottom of the bed.[13] As the tension builds up, there comes a point at which the bed breaks, which is detected by an abrupt drop of Δp, because of fracture of the bed. The condition for tensile yield is met first at the bottom of the bed, where the first fracture is accordingly observed. The tensile yield stress σ_t needed to break the powder is thus

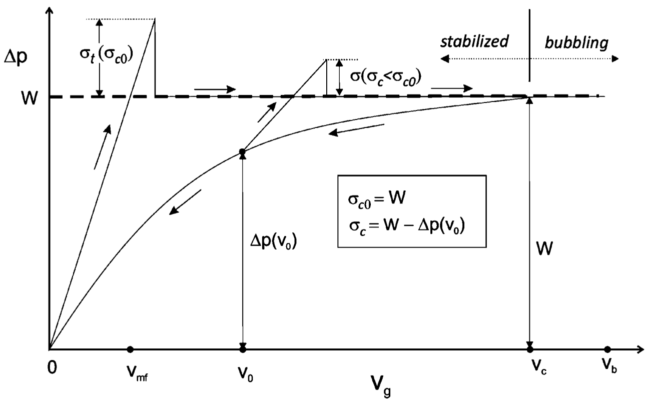

Figure 3. Schematic representation of the experimental procedure followed to measure the tensile yield stress of the settled bed (subjected to a consolidation stress $\sigma_c = \sigma_{c0} = W$) and of the bed stabilized at a gas velocity $v_0 < v_c$ (subjected to a consolidation stress $\sigma_c = W - \Delta p$).

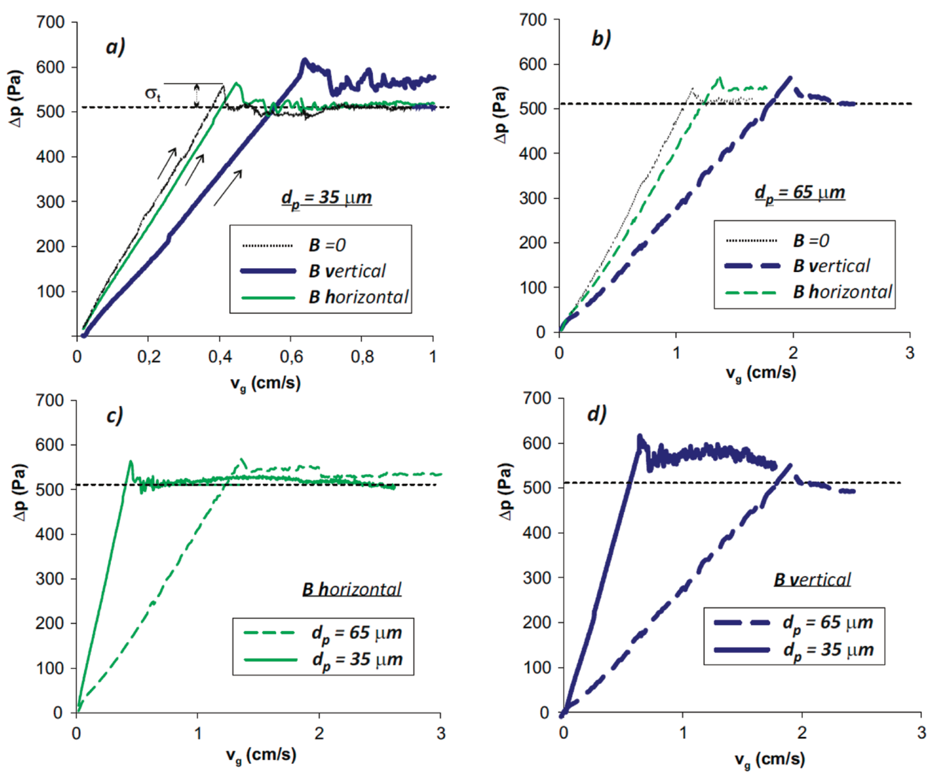

Figure 4. Gas pressure drop across the bed as the gas velocity is increased for beds stabilized in the absence of magnetic field and in the presence of magnetic fields applied either in the vertical or horizontal direction. In panel (a), the particle size is 35 μm and results are shown for a vertical field, horizontal field, and zero field. In panel (b), the particle size is 65 μm and results are shown for a vertical field, horizontal field, and zero field. In panel (c), the field is horizontal and results are shown for particle sizes of 35 and 65 μm. In panel (d), the field is vertical and results are shown for particle sizes of 35 and 65 μm. The horizontal dashed lines indicate the value of the powder weight per unit area (W = 510 Pa).

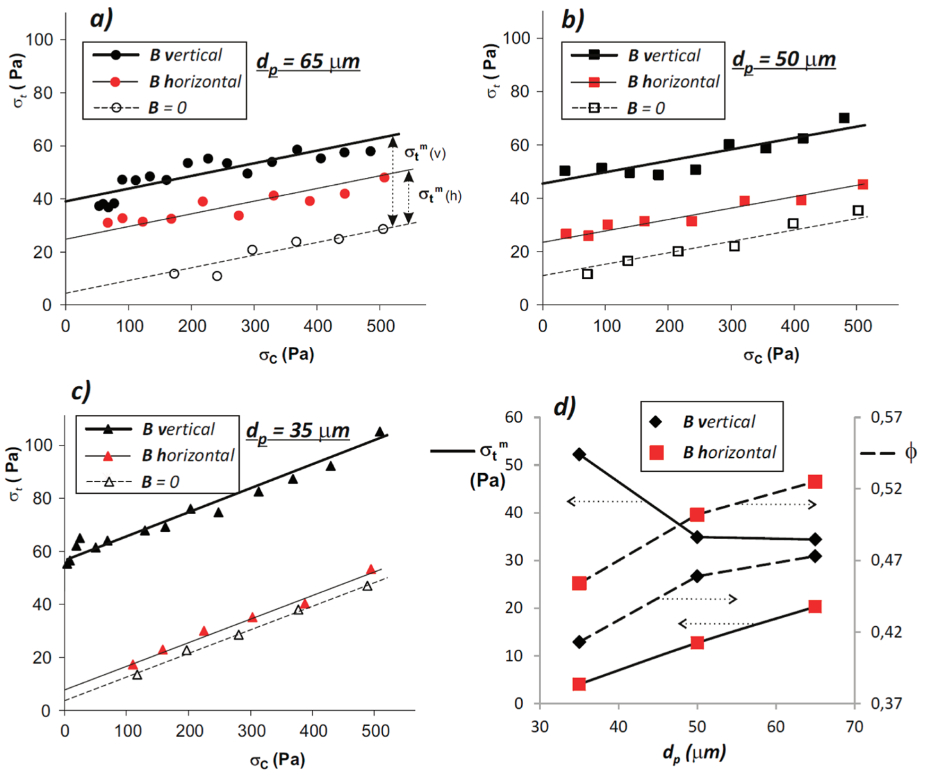

Figure 5. Yield stress measured as a function of the consolidation stress for beds stabilized in the absence of magnetic field and in the presence of a magnetic field applied either in the vertical or horizontal direction (as indicated) for different particle sizes ((a) 65 μm, (b) 50 μm, and (c) 35 μm). Panel (d) shows the magnetic yield stress (obtained as illustrated in panel (a)) for beds stabilized by vertical and horizontal magnetic fields (left axis) as well as the particle volume fraction for $\sigma_c = W$, as a function of particle size (right axis).

obtained as the maximum value of σ, which is located just before the breaking point.

The procedure described above can be used to measure the tensile yield stress of the stabilized bed subjected to small consolidation stresses σ_c between 0 and W, as shown in Figure 3. Note that the rate of increase of Δp with the gas velocity before the bed fractures is decreased as σ_c is decreased, since the stabilized bed at smaller σ_c has a smaller particle volume fraction ϕ. As ϕ is decreased, the average number of contacts per particle and the yield stress are also decreased.

A similar experimental procedure has been employed to obtain the tensile yield stress of stabilized beds of the magnetizable powders used in our study as affected by an externally applied magnetic field of fixed strength ($B = 2.6$ mT), which was oriented either in the horizontal or vertical direction (cross-flow and co-flow field configurations, respectively). All the experimental runs were started by subjecting the bed to a large gas flow in order to drive it into a bubbling state in the absence of the magnetic field. The field then was applied and was kept fixed, such that the initial gas flow was large enough to maintain a bubbling state, even after the magnetic field was applied.

5. EXPERIMENTAL RESULTS

Examples of pressure drop curves obtained to measure the tensile yield stress are shown in Figure 4. Figures 4a and 4b illustrate the effect of the magnetic field on the slope of the linear regime and on the yield stress. The slope of the linear regime, which is inversely proportional to the bed permeability to the gas flow, is decreased when the field is present, since magnetic stabilization gives rise to beds with smaller particle volume fractions. Note, however, in Figures 4a and 4b that this effect is not marked in the case of the cross-flow field configuration. Yet, it is quite important in the case of the co-flow field configuration, indicating that expansion of the stabilized bed is significantly enhanced in the co-flow field configuration, compared to the cross-flow field configuration.

Figures 4a and 4b show that the yield stress is enhanced by the magnetic field. Yet, the role of field orientation on yield stress enhancement depends on particle size. This is seen in Figures 4c and 4d, where pressure drop curves for a given field configuration are plotted for different particle sizes. As can be observed, the yield stress is enhanced for the powder with smaller particle size when the field is applied in the co-flow field configuration. Contrarily, the yield stress is increased in the cross-flow field configuration for the powder with larger particle size (further experimental results using this configuration have been already reported[11]). These results indicate that there is a close link between the roles of particle size and field orientation on the yield stress of magnetically stabilized beds.

In order to assess the yield stress just due to the induced magnetostatic forces between the particles σ_t^m, it must be taken into account that the overall yield stress measured σ_t also contains the contribution from the yield stress due to nonmagnetic attractive forces between the particles (σ_t^0). In the absence of humidity, nonmagnetic attractive forces arise mainly from the universal van der Waals interaction.[13] In Figure 5, we plot the yield stress measured for the beds stabilized in the presence of a magnetic field and in the absence of magnetic field as a function of the consolidation stress (σ_c). As it is well reported in the literature, we see that σ_t^0 increases with σ_c, which is generally due to the increase of the average number of

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contacts per particle as the structure is consolidated and also to plastic deformation of asperities at interparticle contacts.[13]

The compressibility of fine powders increases as the granular Bond number (defined as the ratio of interparticle attractive force to particle weight) is increased,[14] which implies that the rate of increase of the average number of contacts per particle with σ_c is increased as the particle size is decreased. Accordingly, we see in Figures 5a, 5b, and 5c that $\partial\sigma_t/\partial\sigma_c$ is increased as particle size is decreased. Furthermore, it can be noticed that $\partial\sigma_t/\partial\sigma_c$ remains unchanged in the presence of the magnetic field, which indicates that the magnetic yield stress ($\sigma_t^m = \sigma_t - \sigma_t^0$) is approximately independent of σ_c in the range of consolidation stresses tested. Using the data plotted in Figures 5a, 5b, and 5c, a magnetic yield stress σ_t^m can be thus calculated independently of the consolidation stress of the stabilized bed. The calculated values of σ_t^m are plotted in Figure 5d as a function of particle size, showing that σ_t^m increases with particle size for the cross-flow field configuration, whereas it decreases with particle size if the field is applied in the co-flow field configuration. Data on the particle volume fraction ϕ of the beds in the presence of the magnetic field are plotted also in Figure 5d. As previously indicated, ϕ is decreased as particle size is decreased. Moreover, the co-flow field configuration yields relatively smaller values of ϕ, since the bed is stabilized in states of higher expansion.

6. VISUALIZATION OF THE MAGNETOFLUIDIZED BED STRUCTURE

Visualization of particle arrangement in the MFB bed, as affected by the presence of the magnetic field, was made using two different techniques. In one method, a card of adhesive tape was approached to the free surface of the MFB.[15] Then, on carefully withdrawing the card, the layer of powder that had adhered to the tape was observed via optical microscopy. Images taken from the vicinity of the free surface allowed us to look for the formation of structures by the elutriated particles. Typical pictures obtained in this way are shown in Figures 6a and 6b. The second technique used was noninvasive and served to observe the fluidized bed surface in situ. In this technique, images of the MFB were acquired using a binocular magnifying lens coupled to a CCD camera focused on the free surface. Examples of photograms obtained in this way are shown in Figures 6c and 6d.

Pictures in Figure 6 clearly illustrate the formation of chainlike aggregates, which are due to the induced magnetostatic forces of attraction between the particles. In the cross-flow field configuration, chains are, on average, preferentially oriented, forming an angle of $\theta_c \approx 60°$ with the horizontal field. This value can be approximately predicted by means of an unconfined chain model, based on the balance between the gas shear force on the chains, which acts on the vertical direction, and the attractive interparticle magnetostatic force, which has its largest value when the chain is oriented along the direction of the field. Because of this balance, the length of the chains is limited to a size which, according to the unconfined chain model, scales proportionally to $d_p^2B^2$ (see ref 3 for further details on the unconfined chain model). In the case of the co-flow configuration, both gas shear and magnetostatic forces favor the formation of vertical chains of theoretically unlimited size. Therefore, stabilization would occur in states of relatively higher expansion (i.e., lower particle volume fraction), as seen experimentally.

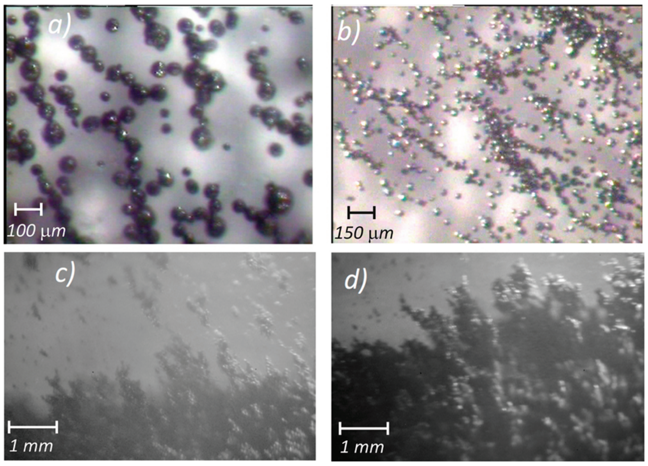

Figure 6. Images of MFBs of magnetite beads showing chainlike aggregates elutriated from the free surface ((a) d_p = 65 μm and (b) d_p = 35 μm) and the free surface of the MFB ((c, d) d_p = 35 μm). The gas flows in the vertical direction and the applied magnetic field is horizontal.

A further interesting observation illustrated in Figure 6 regards the general presence of nonlinear aggregates for the smaller particle size powder due to its natural higher cohesiveness. When the universal van der Waals force becomes much larger than particle weight, natural aggregates may form large-scale branched structures when magnetized by an external field, which may represent an additional contribution to the increase of the yield stress of the co-flow field-stabilized bed. Magnetic aggregation coupled with natural aggregation was observed to play a remarkable role on a fine powder where most of particles were naturally aggregated.[10]

7. DISCUSSION

Let us analyze the experimental results obtained in more detail. A typical value of the universal van der Waals force of attraction between uncompressed fine particles (σ_c = 0) is f_{vdW} = 10 nN.[13] The yield stress σ_t^0 arising from the existence of an interparticle attractive force f can be estimated by means of Rumpf's averaging equation as $\sigma_t^0 \approx f\zeta\phi/(\pi d_p^2)$, where ζ is the average number of contacts per particle that can be related to the particle volume fraction ϕ by the equation[13] $\zeta = (\pi/2)(1 - \phi)^{-3/2}$. Using the data measured for the naturally stabilized beds at the transition to stability ($\sigma_c \cong 0$), the calculated yield stress is σ_t^0 = 3 Pa for d_p = 35 μm (ϕ = 0.38 and v_0 = 0.5 cm/s at the transition to stability), which is in agreement with our experimental results. When the field is applied, the attractive force between the particles is increased by the addition of the anisotropic magnetic force (f_m):

$$f_m = f_m^0\left(\frac{d_p}{r}\right)^4[(2f_\parallel \cos^2\theta - f_\perp \sin^2\theta)\hat{u}_r + f_\Gamma \sin 2\theta\hat{u}_\theta] \tag{1}$$

Here, r is the distance between the centers of the two spheres, $f_m^0 = [(3/16)\mu_f]\pi d_p^2\beta^2B^2$, where $\beta = (\mu_p - \mu_f)/(\mu_p + 2\mu_f)$, μ_p is the particle's magnetic permeability, μ_f is the fluid permeability ($\mu_f \approx \mu_0 = 4\pi \times 10^{-7}$ H/m in our case), θ is the angle that forms the line between the centers of the two spheres and the field direction, and the terms f_i are the so-called force coefficients. These coefficients can be obtained in terms of the multipole moments[16] and depend on the magnetic permeabilities and the distance between the particles. In the dipolar limit, it is $f_{\parallel} = f_{\perp} = f_{\Gamma} = 1$. The main effect of increasing the magnetic permeability of the particles is a significant increase of $f_{\parallel}$, which greatly enhances the attractive component of the force. By implementing the multipolar expansion method described by Clercx and Bossis,[16] we have recalculated[3] the force coefficients for two spheres in contact, as a function of the relative permeability $\alpha = 1 + \chi_p = \mu_p/\mu_0$. In the range $1 < \alpha < 100$, the results can be well-fitted by the equations

$$f_{\parallel} = 0.0122\alpha^2 + 0.5935\alpha + 0.3992$$

$$f_{\perp} = 0.5563 + \frac{0.6524}{0.4714 + \alpha}$$

$$f_{\Gamma} = 0.933 + \frac{1.609\alpha}{18.89 + \alpha}$$

In the case under study ($\alpha = 12.5$), it is $f_{\parallel} = 9.724$, $f_{\perp} = 0.607$, and $f_{\Gamma} = 1.574$. Therefore, the dipole approximation clearly is not admissible to analyze stabilization by magnetostatic forces.

Because of the significant increase of $f_{\parallel}$ with the magnetic susceptibility, it is explainable that a MFB can be stabilized, even by a horizontal field, because of the induced attractive forces between chained particles forming a large angle with the field, which gives rise to a finite tensile yield stress in the horizontal plane, as measured in our experiments. Obviously, in the case of a vertical field, the orientation of the chains favored by both the magnetostatic interaction and the gas flow drag is the vertical direction. This yields relatively stronger attractive forces between the vertically chained particles, larger yield stresses in the horizontal plane, and stabilization in states of higher expansion.

Using eq 1 and the calculated force coefficients, it can be estimated that two chained particles in contact ($r = d_p$) forming an angle $\theta_c = 60°$ with a horizontal field of strength $B = 2.6$ mT (Figure 6) are attracted by a force $f_m \approx 10$ nN for $d_p = 35$ μm, which is similar to the van der Waals force. Accordingly, the increment of the yield stress measured for $d_p = 35$ μm is small when the bed is stabilized in the presence of a horizontal field (see Figures 5c and 5d). On the other hand, if the field is applied in the vertical direction, f_m is increased up to 50 nN (for two particles in contact coaligned with the field). As seen in Figures 5c and 5d, the magnetic yield stress is appreciably increased for $d_p = 35$ μm, in agreement with the increase of the interparticle magnetic force. However, since f_m scales proportionally to d_p^2 and $\sigma_t^m \approx f_m/d_p^2$, the magnetic yield stress should be independent of particle size, which contradicts the experimental results obtained in our work.

One has to bear in mind that the derivation of the Rumpf equation for the yield stress, as a function of the interparticle contact force, assumes a homogeneous and isotropic distribution of contact forces, which is not valid in the case on the magnetostatic interaction. As inferred from eq 1, the interparticle magnetostatic force is strongly dependent on the orientation of the normal to the contact between particles relative to the field direction, which gives rise to anisotropic chainlike aggregation. In the fluidized bed, chains become oriented, on average, at an angle θ_c in the cross-flow field configuration and they become preferentially aligned along the vertical field lines in the co-flow field configuration. Following an argument used for similar systems such as magnetorheological fluids (MRFs) and electrorheological fluids (ERFs),[17] it can be argued that the magnetic yield stress of a jammed bed of chained particles should scale as $\sigma_t^m \approx f_m N/S$, where N/S is the number of particle chains per unit surface, which should be proportional to ϕ/d_p^2. Thus, σ_t^m should be independent of particle size, even for anisotropically distributed contact forces. Yet, in close analogy with our results, experimental measurements on MRFs and ERFs show that the size of the particles can strongly affect the yield stress, which is still a subject of controversy.[17,18] From a simple dimensional analysis, Rosensweig also concluded that the magnetic yield stress of stabilized MFBs should not depend on particle size.[2] Since the magnetostatic contact force between particles depends basically on particle magnetization M_p and particle size, and the coordination number depends on ϕ, a functional relationship $\sigma_t^m = f(\mu_0, M_p, d_p, \phi)$ could be postulated. By assuming a power-law relationship, the dimensional analysis led to $\sigma_t^m = f(\phi)\mu_0 M_p^2$, where $f(\phi)$ is a function that is dependent on the type of microstructure.[2] Nevertheless, our measurements of the particle volume fraction ϕ (Figure 5d) indicate that the bed structure is critically affected by particle size when the bed is consolidated. As the particle size is increased, particles in the stabilized bed subjected to a small consolidation stress would pack more closely (ϕ is increased). This would affect the average angle between the field and the interparticle contact normal in the consolidated structure and, therefore, would have an influence on the average interparticle magnetostatic force.

The increase of ϕ with particle size (Figure 5d) implies that interparticle contacts become, on average, more closely oriented to the horizontal direction as the particle size is increased. As the angle with the horizontal decreases, on average, higher magnetostatic forces of attraction between the particles would be expected in the case of a horizontal field and thus the magnetic yield stress should increase with particle size, as seen experimentally. On the other hand, attractive forces would be, on average, smaller as the particle size is increased in the case of a vertical field, since the average angle of the contact normal with the field would increase in this case as the particle size is increased. As a result, it would be expected that the magnetic yield stress decreases with particle size as observed in the experiments reported in the present paper on the co-flow field configuration.

6. CONCLUSIONS

The transition to stability in gas−magnetofluidized beds, as the gas velocity is decreased in the presence of a magnetic field, is physically determined by the formation of chainlike particle aggregates, because of short-ranged magnetostatic attractive forces. As the gas flow supply to the bubbling bed is decreased, particle chains become eventually arrested and a permanent solid network spans the entire system, which stabilizes the structure. Interparticle attractive forces induced by the magnetic field confer the magnetically stabilized bed with a magnetic yield stress. In our work, we have shown that the magnetic yield stress of the stabilized bed subjected to small consolidations is critically affected by particle size and field orientation. A co-flow field favors the formation of vertically oriented chains and stabilizes the bed in states of relatively higher expansion. On the other hand, chains in a cross-flow field are oriented along a preferential angle, depending on the balance between the vertical gas flow drag and the attractive magnetostatic force, which is maximum in the horizontal direction. As a consequence, the magnetic yield stress is relatively larger in beds stabilized by a co-flow field. With regard to the influence of particle size, since stabilized beds subjected to a consolidation stress become more closely packed as particle size is increased, the contact angle between particles becomes, on average, closer to the horizontal direction. Thus, it is plausible that the yield stress is increased as particle size is increased in the cross-flow field configuration, but it is decreased as particle size is increased in the co-flow field configuration as seen experimentally.

■ AUTHOR INFORMATION

Corresponding Author

*E-mail: jmillan@us.es.

Notes

The authors declare no competing financial interest.

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