Socio-Economic Planning Sciences 89 (2023) 101697

Contents lists available at ScienceDirect

Socio-Economic Planning Sciences

journal homepage: www.elsevier.com/locate/seps

A novel methodology for public management of annual greenhouse gas emissions in the European Union

E. Algaba [a,*], G. Márquez [b], J. Martínez-Lozano [a], J. Sánchez-Soriano [c]

[a] IMUS and Department of Applied Mathematics II, University of Seville, 41092 Sevilla, Spain
[b] Center for Research in Sustainable Chemistry, University of Huelva, 21819 Huelva, Spain
[c] R.I. Center of Operations Research (CIO), Miguel Hernández University, 03202 Elche, Spain

ARTICLE INFO

Keywords:
Public management of greenhouse gas emissions
European Union countries
Sectors and categories
(Multi-issue) bankruptcy models
Allocation rules

ABSTRACT

Global warming and climate change, as a result of greenhouse gas emissions, pose a major threat to the international community; therefore, such emissions must be reduced by moving to clean energy resources. In this paper, we follow an approach based on bankruptcy models using the proportional rule and the proportional run-to-the-bank rule to illustrate a novel allocation protocol for managing annual CO_2 equivalent emissions in European Union countries among a set of sectors included in Annex I to Decision No 406/2009/EC. Unlike the standard bankruptcy model, the current model deals with situations in which agent's claims are multi-dimensional and the issues correspond to greenhouse gases contained in Annex II to Directive 2003/87/EC. Considering that any Member State can limit the greenhouse gas emissions beyond their obligations under the European legislation, being able to establish national greenhouse gas emission reduction objectives in relation to 2005. Two types of situations will be considered to allocate emissions among greenhouse gases, sectors and source categories: when total emissions and removals from activities related to the Land Use, Land-Use Change and Forestry are excluded, we will make a first allocation among sectors and categories to determine the amount of tonnes of CO_2 equivalent that they can emit for each greenhouse gas; and when land-use/forestry activities are taken into account for the distribution of emissions reduction efforts, we will make a second allocation among sectors in order to determine which source categories are involved in each case.

1. Introduction

Global warming and climate change arise a major problem throughout the world and the reduction of greenhouse gas (GHG) emissions is one of the most important aims of environmental policies on a global basis, nowadays (e.g., [1]). The accumulation of greenhouse gases, namely carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), and fluorine compounds such as sulphur hexafluoride (SF_6) and a set of fluorocarbons ($HFCs$ and $PFCs$; see Annex II to Directive 2003/87/EC) in the atmosphere is exceeding the planet's absorption capacity and can cause disastrous effects [2]. Limiting the global cumulative CO_2 between 2000 and 2050 would reduce the risk of exceeding the 2 °C limit above the levels in the pre-industrial era [3]. Thus, GHG emissions must be reduced by moving to clean energy resources. As regards the European Union, the main goal of the Kyoto Protocol to the United Nations Framework Convention on Climate Change (UNFCCC), which was approved by Council Decision 94/69/EC, is to reduce anthropic greenhouse gas emissions at a level which prevents harmful interference with the climate system, in accordance with Decision 2002/358/EC. In this context, Directive 2003/87/EC led to the establishment of a European Community-wide greenhouse gas emission allowance trading scheme ("EU ETS") since 2005. All sectors of the economy should contribute to emission reductions in order to reach the objective of a 20% reduction of GHG emissions by 2020 compared to 1990 levels. This latter norm amended Council Directive 96/61/EC establishing a general framework for contamination, by way of which GHG emissions permits have been issued.

In connection with the effort of each Member State to reduce its annual greenhouse gas emissions, Decision No 406/2009/EC established annually binding national limits for the period from 2013 to 2020. Each Member State shall, in 2020, limit its whole GHG emissions from a series of sectors (excluding Land Use, Land-Use Change and Forestry; [4]) such as Energy, Industrial Processes and Product Use (IPPU), Agriculture and Waste (Annex I to this Decision) at least by the percentage set for that Member State in Annex II (e.g., −20% and −10% for Ireland and Spain, respectively) compared to its GHG emissions

* Corresponding author.
E-mail addresses: ealgaba@us.es (E. Algaba), gonzalo.marquez@diq.uhu.es (G. Márquez).

https://doi.org/10.1016/j.seps.2023.101697
Received 22 February 2023; Received in revised form 12 June 2023; Accepted 26 July 2023
Available online 2 August 2023

in 2005. In the case of Croatia, which accessed to European Union in 2013, this country took on a goal of reducing GHG emissions in 2020 by 20% compared to those in 1990 [5]. Additionally, all Member States shall limit their GHG emissions in each year between 2013 and 2019 along a linear trend line. Specifically, Ireland, Spain and any other Member State with a negative limit under Annex II shall ensure that their maximum allowances in 2013 do not exceed their respective average annual greenhouse gas emissions during 2008, 2009 and 2010.

Nevertheless, Member States can limit the greenhouse gas emissions beyond their obligations under this Decision, with the inclusion or exclusion of total emissions and removals from different land-use/forestry activities (LULUCF) in GHG emissions reduction efforts, in order to meet more stringent national objectives. The approach of annual maximum allowed GHG emissions in terms of tonnes of CO_2 equivalent taken in Decision No 406/2009/EC continues by Regulation (EU) 2018/842, which currently provides the reductions in the European Union emissions trading system and non-ETS sectors compared to emissions in 2005. The methodology to set the annual binding national limits from 2021 to 2030 is very similar to that applied in Decision No 406/2009/EC. Again, each Member State shall, this time in 2030, limit its greenhouse gas emissions at least by a percentage (see Annex I to this Regulation) in relation to 2005 (e.g., −30% and −26% for Ireland and Spain, respectively). Regulation (EU) 2018/842 has as purpose keep the global temperature increase to 1.5 °C above pre-industrial levels under the Paris Agreement [6], which was concluded on behalf of the Union by Council Decision (EU) 2016/1841 and has replaced the Kyoto Protocol beyond 2020.

Nonetheless, global, EU-level and/or national cooperation on issues based on the carbon budget approach [7] has been historically hampered by a disagreement among countries and/or sectors of the economy on a matter of policy [8]: it is evident that we are confronting a conflicting claims problem or situation of bankruptcy [9]. Most of the works in the literature consider only the CO_2 equivalent emissions without disaggregating by the main greenhouse gases and, therefore, the problems can be modeled as bankruptcy problems[1] (see for example, [9,13–15]). However, in this work we go a step further to carry out a disaggregated distribution by the main greenhouse gases and by the different productive sectors that emit these gases into the atmosphere. Therefore, in our study we are considering a more general situation called a multi-issue bankruptcy problem in which the issues are the GHG, and the claimants are the emitting productive sectors. To our knowledge, this disaggregated approach is novel in the greenhouse gas emission abatement literature. Multi-issue bankruptcy problems [16] analyze situations, in which there are a perfect divisible estate, which can be divided between various issues, and a number of agents with claims on these issues such that the total amount of claims is above the estate.[2] There are different approaches in the literature on how the estate should be distributed (see, for instance, [16–21]). In this paper, we apply two different approaches for multi-issue bankruptcy problems based on proportionality, the proportional run-to-the-bank rule [16] and the proportional rule [22,23] to assign allocations of GHG to each sector. The idea of this first rule, as its name suggests, is: the claimants take a run towards the managing authority of the endowment, when each claimant arrives, she can choose the most favorable order to her on the issues. Averaging over all possible orders of arrival, a run-to-the-bank rule is obtained. Moreover, the new arriving claimant takes into account the effect of the chosen order on the issues on the claimants already present. If they lose because of this choice, the new claimant has to compensate them for this. Then, the amount of resource assigned to a certain issue is proportional divided. The idea of the second rule simply consists of applying a proportional distribution among the claimants, either distributing proportionally considering the amount of each issue that an agent claims, as an independent claim of a new claimant and then aggregating, or first distributing proportionality among the issues by considering that claims for the same issue are aggregated into a single claim and then the amount allocated to each issue is proportionality distributed among agents. Both approaches provide the same allocation as proved in [23].

In this work, we center in one of the most important in the literature [24]: the proportional rule; although other rules could, in principle, be used, such as the constrained equal awards rule or the constrained equal loses rule. However, these last two rules have been dropped for the following reasons. The constrained equal awards rule equals the quantities obtained allocating the endowment such that to each claimant is assigned the same amount as long as it is not exceeded the amount claimed by the agent. Therefore, its application could lead to extreme situations, in the sense that either all the reduction in GHG emissions would fall solely on CO2 gas, or only the sectors with the most CO2 emissions would be the ones that would have to make every effort to reduce emissions. This would respond more or less to what is currently being done, but the effort to reduce GHGs should be a shared effort among all gases and by all sectors and source categories, obviously, each one in its fair measure. The constrained equal losses rule equals the quantities lost allocating the endowment such that to each claimant is reduced the same amount as long as it is not exceed the amount claimed by the agent. In this case, we could have the opposite effects to those described above, either we would have to reduce (even completely) all greenhouse gases except CO2 gas, or almost all the effort would fall on those who emit the least GHG, which seems a bit absurd . Therefore, it seems appropriate to use the concept of proportionality in this context, which is the one with the longest tradition of use and whose spirit can be traced back to Aristotle (4th Century BD).

In principle, the analysis of the methodology proposed in this paper could be done for all the countries that make up the European Union, but in this paper we have chosen only two of them, one that represents the countries that are below the committed emissions and another that is above it, Spain and Ireland, respectively. The exceeding of the fixed threshold by Ireland and other few EU countries in 2018 can be explained, at least in part, by the direct relationship between economic and gross domestic product (GDP) growth and GHG emissions [25]; in particular, it can be noted a strong increase in Irish real GDP per capita (9.8%) compared to the value (2.1%) for the European Union averaging in the period 2015–2018 [26]. Of course, other countries could have been chosen using other classification criteria for EU countries, but we think that the criteria used and the choice of countries made are sufficient to adequately illustrate the proposed methodology and its management and economic implications as ecological. Thus, in this paper, knowing that maximum allowed emissions for Ireland were exceeded in 2018 and taking into account that Spain had limited its whole GHG emissions, with or without LULUCF, regardless of the annual limitation imposed by Decision No 406/2009/EC, we focus on the allocation of GHG emissions as CO_2 equivalent taking into account the amount of tonnes that each sector in Ireland and Spain can emit, and how these quantities could be shared among source categories distinguishing each one of the greenhouse gases, which act as issues. We consider a multi-issue bankruptcy problem where the agents (sectors and categories or sub-sectors) claims are the GHG emissions in 2018, and the resource to be shared corresponds to any percent reduction in

[1] O'Neill [10] and Aumann and Maschler [11]. In these situations, an endowment (perfectly divisible) must be allocated among a set of claimants on it but the endowment is not enough to completely satisfy all of them. There exist many different bankruptcy rules to allocate the endowment among the claimants (see [12]).

[2] These situations arise when, for example, a general budget (estate) has to be divided into budget headings (issues) such as education, health, security, etc. and, at the same time, there are a number of entities, such as regions, cities, etc., that have demands on each of these budget headings. In these cases, it seems more appropriate to take into account the entire disaggregated structure of the problem to obtain an allocation of the budget among the entities.

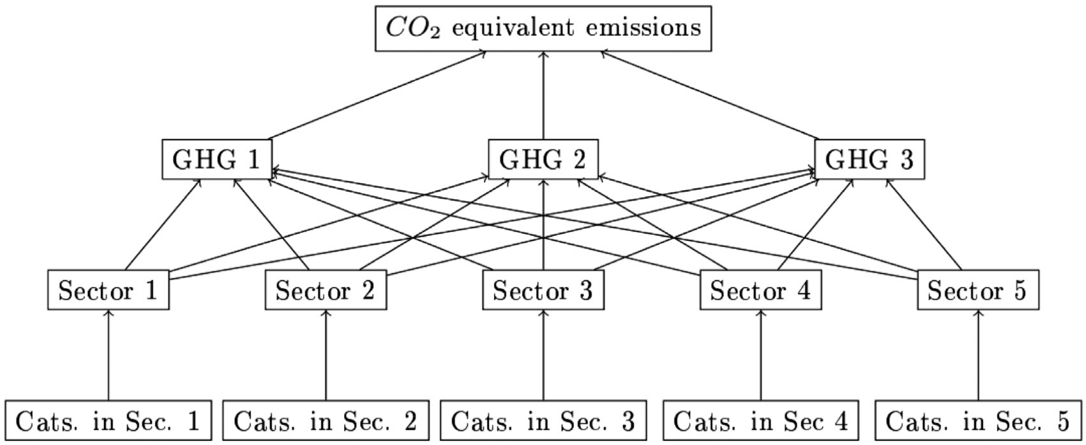

Fig. 1. Example of a four levels hierarchical structure.

relation to Irish and Spanish GHG emissions (without LULUCF) in 2005. These cases are intended to illustrate a new methodology to address in a more comprehensive way the reduction of the main greenhouse gases and what part of the reduction corresponds to each of the productive sectors that emit those gases. This new methodology consists of determining, through a multi-issue bankruptcy model, the emission limits that correspond to each gas within each sector, and once these limits are obtained, apply bankruptcy models to assign emission limits for each of the gases to the categories within each sector. Therefore, we consider a hierarchical structure like in Fig. 1 and to solve the problem of setting emission limits, we successively apply a multi-issue bankruptcy model for the three highest levels and bankruptcy models in the last level, categories. Obviously the bankruptcy rules for multi-issue bankruptcy problems and the bankruptcy rules could be different, but the methodology would not change. In any case, the rules chosen in this work, described in the following section, are based on fairness principles, that are common in distribution problems, such as equality and proportionality.

2. Methodology

Bankruptcy situations [10] study problems where an estate must be divided among several claimants. The problem arises when the estate is not large enough to cover all claims. A bankruptcy situation is a triple (N, E, c) where $N = \{1, 2, \dots, n\}$ is the set of agents, $E > 0$ is the estate to be divided between agents and $c = (c_1, \dots, c_n) \in \mathbb{R}^N_{++}$ is the vector of claims such that $\sum_{i \in N} c_i > E$.

To allocate the estate E among the agents in N, an *allocation rule* for bankruptcy problems is defined as a function f that assigns to every (N, E, c) a unique vector $f(N, E, c) \in \mathbb{R}^N$ such that,

1. $0 \le f_i(N, E, c) \le c_i$, for all $i \in N$,
2. $\sum_{i \in N} f_i(N, E, c) = E$.

So, according to an allocation rule, every agent should receive a non-negative allocation, smaller or equal than his claim, and the entire estate should be distributed completely among the agents.

Probably the best known and most widely used solution concept for bankruptcy situations is the *proportional rule* [24,27]. It distributes awards proportionally to claims, equalizing the ratios between claims and awards. This rule is defined for all bankruptcy situations (N, E, c) as follows

$$P(N, E, c) = \lambda c, \tag{1}$$

where $\lambda = (E / \sum_{i \in N} c_i) \in (0, 1]$.

The second is the *constrained equal-awards (CEA) rule*, one of the most prominent rules in a bankruptcy situation. The underlying idea is that every agent should receive the same amount as long as this does not exceed her claim. As it is made explicit in [11], *"this rule has been adopted as law by most major codifiers, including Maimonides (in his Laws for Lending and Borrowing)* [28]*"*. Formally, for each $i \in N$,

$$CEA_i(N, E, c) = min\{\lambda, c_i\}, \tag{2}$$

where λ satisfies $\sum_{i \in N} min\{\lambda, c_i\} = E$.

This rule can be obtained by an iterative process [11]. First, all agents receive the same amount of estate, which cannot be greater than the smallest claim among the agents. Then, the agents with the smallest claim leave the sharing procedure. Next, a new iteration begins by equally dividing the remaining part of the estate, if any, among the rest of agents, and taking into account to distribute to each agent equal or less than the second smallest claim. Then, the agents claiming the second smallest claim leave the sharing process, and the procedure continues considering the rest of agents until there is not any state left to share.

Calleja et al. [16] extend the bankruptcy problems to incorporate situations in which the available amount of resource must be allocated among several agents, each of whom has several claims related to different issues. Quoting [16, page 731]: *"An issue constitutes a reason on the basis of which the estate is to be divided"*. In this case, a problem also arises when the amount of resource is insufficient to satisfy all claims. These authors named these problems *multi-issue allocation (MIA) situations*.

Formally, a MIA situation is a four-tuple (R, N, E, C) where $R = \{1, \dots, r\}$ is the set of issues, again $N = \{1, 2, \dots, n\}$ is the set of agents and $0 < E < \sum_{j \in R} \sum_{i \in N} c_{ij}$ is the estate to be distributed between agents, but now $C \in \mathbb{R}^{R \times N}_{+}$ is a matrix of claims. Every row in C represents an issue, and every column is an agent. An element $c_{ij} \ge 0$ represents the amount that agent $i \in N$ claims according to issue $j \in R$.

A *multi-issue bankruptcy rule* is a mapping that associates with every (R, N, E, C) a unique matrix $X(R, N, E, C) = (x_{ij})_{i \in R, j \in N} \in \mathbb{R}^{R \times N}$ such that,

1. $0 \le x_{ij} \le c_{ij}$ for all $i \in R, j \in N$,
2. $\sum_{i \in R} \sum_{j \in N} x_{ij} = E$.

The following basic assumption is made: an issue must first be fully distributed before allocating the appropriate amount to the following issue. Therefore, this assumption implies that an order of the issues is needed to allocate the estate. An order of the elements of a finite set F is a bijection $\pi : \{1, \dots, \#F\} \to F$, where $\#F$ denotes the number of elements in F and $\pi(i)$ denotes the element in the ith position in the order π. The set of all orders is denoted by $\prod(F)$. According to this, the set of all issues orders is $\prod(R)$, and τ denotes an order of the issues. Under the above assumption, given an order of distribution of the issues, τ, let $t = max\{t' | \sum_{s=1}^{t'} c_{\tau(s)} \le E\}$, where $c_{\tau(s)} = \sum_{j \in N} c_{\tau(s),j}$. Therefore, the agents' claims for the first t issues in order τ are fully

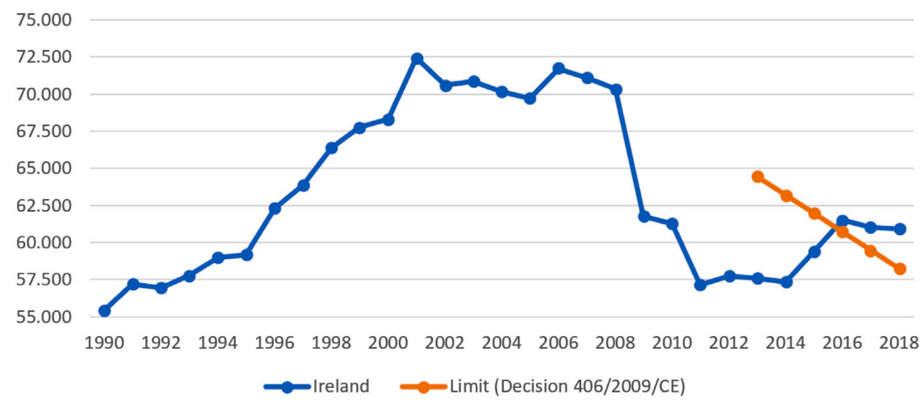

Fig. 2. Annual Irish GHG emissions and limits imposed by the European Union from 1990 to 2018. Data expressed in CO_2 equivalent (kt).

satisfied. What to do with the part of the estate that is left? An alternative is to resort to the proportional allocation, thus the part of the estate that is left, $E' = E - \sum_{s=1}^{t} c_{\tau(s)}$, is divided proportionally to the claims according to the next issue in the order, i.e., issue $\tau(t+1)$. So in total, agent $j \in N$ will be allocated the following amount of the estate

$$f_j^P(\tau) = \sum_{s=1}^{t} c_{\tau(s),j} + \frac{c_{\tau(t+1),j}}{c_{\tau(t+1)}} E'.$$

Therefore, on the basis of these results, a multi-issue allocation solution, called *proportional run-to-the-bank rule* [16] is defined

$$\rho^P = \frac{1}{|N|!} \sum_{\sigma \in \prod(N)} \rho^P(\sigma),$$

where $\prod(N)$ is the set of all agents orders, $\sigma \in \prod(N)$, and $\rho^P(\sigma) \in \mathbb{R}^N$ is defined recursively by

$$\rho^P_{\sigma(p)}(\sigma) = \max_{\tau \in \prod(R)} \left[f^P_{\sigma(p)}(\tau) - \sum_{q=1}^{p-1} (\rho^P_{\sigma(q)}(\sigma) - f^P_{\sigma(q)}(\tau)) \right] \quad (3)$$

for all $p \in \{1, \ldots, n\}$. That is interpreted as a "race" between the agents to obtain the highest profit according to their demands for each issue. The first agent that arrives, $\sigma(1)$, can choose the order, $\tau \in \prod(R)$, in which the issues are dealt with and receives her payoff accordingly. Next, agent $\sigma(2)$ arrives and she is asked to do the same. She will choose that order that maximizes her own payoff minus the corresponding compensation payments $\sigma(1)$ for the difference between her settled payoff $\rho^P_{\sigma(1)}(\sigma)$ and her payoff according to the new order. The same procedure is applied to all subsequent agents, each having to compensate all her predecessors. Calleja et al. [16] proved that this allocation rule coincide with the Shapley value (Shapley 1953) of a suitable game associated with the problem. This fact is relevant because the Shapley value is one of the most outstanding solutions in game theory (see for example, [29,30]). Moreover, the Shapley value is a useful tool to analyze allocation problems in the most general sense [31]. Therefore, the selection of the proportional run-to-the bank rule as an allocation mechanism is justified both from the point of view of game theory and the elementary principles of proportionality and fairness.

3. Results and discussion

The database for Global Atmospheric Research provides emission time series until 2018 for greenhouse gases for all European Union (EU) countries. As shown in Appendix, it can be observed that GHG emissions from Spain and the majority of EU Member States, with the exception of Ireland and few other countries, were below the limits imposed on them by the European Union between 2016 and 2018 according to Decision No 406/2009/CE [32].

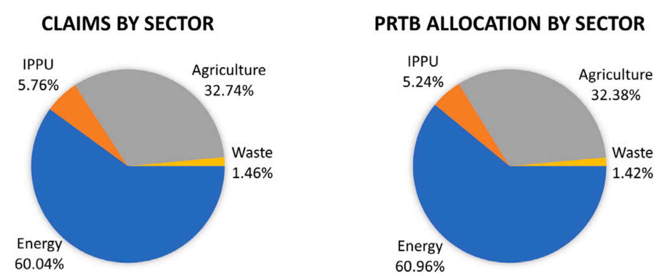

Fig. 3. Pie charts showing relative Irish GHG emissions by sector if bankruptcy is ignored or considered.

3.1. Case study: Irish GHG emissions

Table 1 shows the distribution of Irish emissions in terms of tonnes of CO_2 equivalent among the sectors, source categories and types of greenhouse gases in 2018. GHG emissions for that year can be considered as normal in contrast to 2020, the year the coronavirus disease 2019 (Covid-19) dominated. We consider four claimants (Energy, IPPU, Agriculture and Waste sectors) to which we assigned the maximum amounts of CO_2 equivalent that they could have emitted in 2018.

As shown in Fig. 2, whole Irish GHG emissions exceeded in 2018 the limits imposed on them by the European Union until 2020 according to Decision No 406/2009/CE. Regardless of flexibilities by means of banking and transfer, among others, we make the distribution among Energy, IPPU, Agriculture and Waste, leading to a multi-issue bankruptcy case where these four sectors jointly claim higher GHG emissions than the maximum allowance for Ireland in 2018, in which we apply the proportional and proportional run-to-the-bank (PRTB) rules, defined by Eq. (1) and recursively by (3), respectively. As mentioned above, GHG emissions from the sectors (the claimants) were referred to as the claims, and the amount they could emit once the above rule is applied would be the allocations. Irish LULUCF activities were not taken into account for the allocation in accordance with Decision No 406/2009/EC.

Data for the Irish bankruptcy case are shown in Table 2. In this case, we can note here the reduction percents or cuts to be achieved by any sector (without LULUCF) with respect to what was claimed by each one (Table 3). When the PRTB rule is used, IPPU would have had to achieve a 13.05% cut in its emissions (Table 3) and it would have been the most affected sector in situation of bankruptcy. By contrast, Energy was the least affected sector in such situation (emissions reduction near 3%). Also, as it can be seen in Fig. 3, the relative Irish GHG emissions by sector, except Energy, are reduced in the situation of bankruptcy. By contrast, all the sectors would have to achieve a 4.42% cut in its emissions using the proportional rule (Table 3).

Table 1
Irish total and individual GHG emissions by sector and category in 2018.

GHG emitting sources and sinks	CO_2	CH_4	N_2O	HFCs	PFCs	SF_6	Total
	Equivalent CO_2 (kt)						
Energy	**35 961.53**	**257.84**	**365.39**				**36 584.76**
Energy industries	10 398.45	10.32	141.64				10 550.41
Manufacturing industries and construction	4714.94	10.88	16.57				4742.39
Transport	12 083.84	10.55	130.33				12 224.72
Other sub-sectors	8763.21	145.88	76.85				8985.94
Non-specific mobile emissions	0.88						0.88
Solid fuels (fugitive emissions)	0.01	18.65					18.65
Oil and natural gas (fugitive emissions)	0.20	61.56					61.76
Industrial processes and product use	**2273.13**		**42.99**	**1100.36**	**49.86**	**40.92**	**3507.16**
Mineral industry	2094.51						2094.51
Chemical industry	0.10		0.01				0.11
Metal production	0.01						0.01
Non-energy products from fuels and solvent use	178.51						178.51
Product uses as substitutes for ODS[a]				1100.36	49.86		1150.22
Other product manufacture and use			42.98			40.92	83.90
Agriculture	**546.21**	**12 970.14**	**6436.79**				**19 953.14**
Enteric fermentation		11 543.21					11 543.21
Manure management		1426.89	542.86				1969.75
Rice cultivation		0.01					0.01
Agricultural soils			5893.92				5893.92
Field burning of agricultural residues		0.03	0.01				0.04
Liming	457.45						457.45
Urea application	88.76						88.76
Waste	**23.48**	**758.18**	**108.56**				**890.22**
Solid waste disposal		692.71					692.71
Biological treatment of solid waste		14.95	10.69				25.64
Incineration and open burning of waste	23.48	0.17	0.25				23.90
Wastewater treatment and discharge		50.25	97.62				147.87
Other sources		0.10					0.10
Land Use, Land-Use Change and Forestry	**3433.25**	**459.88**	**470.97**				**4364.10**
Forest land	−3822.73	81.69	181.11				−3559.93
Cropland	−160.49	0.02	0.01				−160.46
Grasslands	6630.54	256.33	81.44				6968.31
Wetlands	1509.29	121.84	24.75				1655.88
Settlements	94.07		140.80				234.87
Other land	8.23		41.59				49.82
Harvested wood products	−825.66						−825.66
Other			1.27				1.27
Total CO_2 equivalent emissions (without LULUCF)	**38 804.35**	**13 986.16**	**6953.73**	**1100.36**	**49.86**	**40.92**	**60 935.38**
Total CO_2 equivalent emissions (with LULUCF)	**42 237.40**	**14 446.04**	**7424.70**	**1100.36**	**49.86**	**40.92**	**65 299.48**

[a]ODS = Ozone Depleting Substances.

Table 2
Sectoral maximum allowed GHG emissions (kt of CO2 equivalent) for Ireland in 2018.

Sectors	Claims	Allocation	
		PRTB rule	Prop. rule
Energy	36 584.76	35 502.32	34 968.29
IPPU	3507.16	3049.33	3352.20
Agriculture	19 953.14	18 861.39	19 071.52
Waste	890.22	829.95	850.89
Total (reduction)	60 935.38	58 242.99 (2692.39)	

Moreover, another important aspect of this research work lies in the need to take into account all the greenhouse gases contained in Annex II to Directive 2003/87/EC. In 2018, the four referred sectors emitted CO_2 and N_2O in Ireland, while IPPU was the only one that did not emit CH_4 (Table 1). After applying both cuts to each sector with respect to what was claimed by each one in 2018, and once obtained the limits of tonnes of CO_2 equivalent that each sector could have emitted that year in Ireland, these amounts could have been distributed proportionally among the individual greenhouse gases emitted by each sector (Table 3), by using the proportional rule. Focusing on the individual greenhouse gases in the situation of bankruptcy (see Table 3), CO_2 represented most of the Irish GHG emissions from Energy and IPPU in 2018, while N_2O and CH_4 were the major individual greenhouse gases emitted by Agriculture and Waste. Thus, it is worth noting that Energy would have claimed about 98% of the maximum amount of CO_2 that could be emitted in the situation of bankruptcy (Table 3). It can be also highlighted that Agriculture accounted for about 92% of the total amount of CH_4 and N_2O emitted by the four mentioned sectors in the situation of bankruptcy (Table 3). In 2018, the main CH_4 and N_2O emissions were observed for the source categories named as Enteric Fermentation and Agricultural Soils, respectively (Table 1).

Furthermore, we made an allocation among categories to determine the maximum allowed quantities of CO_2 equivalent that they could have emitted for each individual greenhouse gas in 2018. The proportional rule has been used for the allocation of GHG emissions from the four referred sectors among their categories in the situation of bankruptcy. When comparing claims on each category (Table 1) and corresponding sub-sectoral maximum allowed GHG emissions obtained after using the proportional and PRTB rules to allocate the emissions to sectors (Table 4), the Energy categories would have been the only ones less affected by a reduction in CO_2, CH_4 and N_2O emissions if using the PRTB rule, and vice versa if the proportional rule is used.

3.2. *Case study: Spanish GHG emissions*

Table 5 shows the distribution of Spanish emissions in terms of tonnes of CO_2 equivalent among the sectors, source categories and types of greenhouse gases in 2018. GHG emissions for 2018 or 2019 should be also taken rather than 2020 data, which is also available,

Table 3
Sectoral maximum allowances for Irish GHG emissions (kt of CO2 equivalent) by gas in 2018.

Energy				IPPU			
Gas	Claims	Allocation		Gas	Claims	Allocation	
		PRTB	Prop.			PRTB	Prop.
CO_2	35 961.53	34 897.53	34 372.59	CO_2	2273.04	1976.31	2172.69
CH_4	257.84	250.21	246.45	CH_4			
N_2O	365.39	354.58	349.25	N_2O	42.98	37.37	41.09
HFCs				HFCs	1100.36	956.72	1051.74
PFCs				PFCs	49.86	43.35	47.66
SF_6				SF_6	40.92	35.58	39.11
Total	36 584.76	35 502.32	34 968.29	Total	3507.16	3049.33	3352.20
(reduction)		(2.96%)	(4.42%)	(reduction)		(13.05%)	(4.42%)
Agriculture				**Waste**			
Gas	Claims	Allocation		Gas	Claims	Allocation	
		PRTB	Prop.			PRTB	Prop.
CO_2	546.21	516.32	522.08	CO_2	23.48	21.89	22.44
CH_4	12 970.14	12 260.47	12 397.06	CH_4	758.18	706.85	724.68
N_2O	6436.79	6084.60	6152.38	N_2O	108.56	101.21	103.76
HFCs				HFCs			
PFCs				PFCs			
SF_6				SF_6			
Total	19 953.14	18 861.39	19 071.52	Total	890.22	829.95	850.89
(reduction)		(5.47%)	(4.42%)	(reduction)		(6.77%)	(4.42%)

due to this latter year is defined as anomalous since COVID-19 reached Europe at the beginning of 2020 and a lockdown was imposed across the country for that year, limiting human mobility to decrease the people infected by SARS-CoV-2. Consequently, abnormally low Spanish GHG emissions were observed throughout a confinement period (16th March–10th May 2020) followed by a relaxation period between 11th May and 31st July characterized by a mild reduction in mobility restrictions [33]. Obviously, the claimants are the same as in the case of Ireland.

The Spanish LULUCF sector has not been initially taken into account for the allocation in accordance with Decision No 406/2009/EC. The negative amounts (removals) observed for this latter sector, which means that it absorbs a quantity of CO_2, will be distributed using the proportional and PRTB rules. As shown in Fig. 4, like most countries of the European Union (see Appendix), whole Spanish GHG emissions were in 2018 below the limits imposed on them by the European Union. Nevertheless, given that any Member State can limit the greenhouse gas emissions beyond its obligations under the European legislation, Spain can establish annual binding limits of its GHG emissions in order to meet more stringent national objectives, so we investigated four cases based on different emission reduction targets (26, 30, 34 and 38%) in relation to 2005. As mentioned above, we first made the distribution among Energy, IPPU, Agriculture and Waste, without emissions and removals from LULUCF activities, leading to four situations of bankruptcy where these sectors jointly would have claimed higher GHG emissions than maximum allowances in 2018, in which we can apply the proportionality and PRTB rules, defined by Eq. (1) and recursively by Eq. (3), respectively.

Data for the above-mentioned bankruptcy cases are shown in Table 6. Noteworthy is that as the reduction target was higher, the Spanish objectives would have been more stringent and the greater the efforts that sectors would have made to reduce their GHG emissions in 2018. In the four bankruptcy cases when the PRTB rule is used, we may note here the reduction percents or cuts to be achieved by any sector (without LULUCF) with respect to what was claimed by each one (Table 7). Except for the 26% reduction target where IPPU would have had to achieve a 7.77% cut in its emissions (Table 7), Agriculture and Waste would have been the most affected sectors by bankruptcy cases. Thus, the 38% target for reducing Spanish GHG emissions in 2018 would have led to emissions from the two latter sectors of approximately half of the tonnes of CO_2 equivalent claimed by them both in that year (Table 7). By contrast, for each of the four emission reduction targets in relation to 2005, Energy was the least affected sector by any bankruptcy case. When excluding LULUCF, it is observed that as the reduction target was higher (Fig. 5), the relative Spanish GHG emissions from Waste and Agriculture sectors were lower, as opposed to Energy; while the relative GHG emissions from IPPU were reduced below 8% for the lowest reduction targets and to 8.2% for the highest reduction targets (see Fig. 5). By contrast, all the sectors would have to achieve the same cut in its emissions using the proportional rule in each bankruptcy case (Table 7).

In regards to the individual greenhouse gases, during 2018 the four referred sectors emitted CH_4 and N_2O in Spain, while IPPU was the only one that emitted fluororinated greenhouse gases and Waste was the unique sector that did not emit CO_2 (Table 5). After applying the reduction percents or cuts to each sector using the proportionality and PRTB rules (without LULUCF) with respect to what was claimed by each one in 2018, and once obtained for each reduction target the limits of tonnes of CO_2 equivalent that each sector could have emitted that year in Spain, these quantities could have been distributed proportionally among the individual greenhouse gases emitted by each sector (Table 7), by using the proportional rule.

Focusing on the individual greenhouse gases in the bankruptcy cases if excluding LULUCF, as shown in Table 7, CO_2 represented the majority of the Spanish GHG emissions from Energy and IPPU in 2018; while CH_4 and N_2O were the main individual greenhouse gases emitted by Agriculture and Waste. Thus, it can be noted that Energy would have claimed more than 90% of the maximum amount of CO_2 that could be emitted in each of the four bankruptcy scenarios (Table 7). It is also noteworthy that Agriculture accounted for around 60% and 75% of the total quantity of CH_4 and N_2O emitted by the four referred sectors in each of the four bankruptcy scenarios (Table 7). As in the Irish case, CH_4 and N_2O, respectively, were mostly emitted from the source categories named as Enteric Fermentation and Agricultural Soils in 2018 (Table 5).

Last, if excluding LULUCF, we make an allocation among categories to determine the maximum allowed amounts of tonnes of CO_2 equivalent that they could have emitted for each individual greenhouse gas in 2018. For this purpose, the proportional rule has been used for the allocation of GHG emissions from the four referred sectors among source categories in the bankruptcy case for the most stringent Spanish emission reduction target (38%) related to 2005. In a comparison of the claims on each category (Table 5) and corresponding sub-sectoral maximum allowed GHG emissions obtained using the proportional and

Table 4
Maximum allowed GHG emissions by gas and category for Ireland in 2018. Values from the application of the PRTB rule to allocate the emissions to sectors in cursive.

Sectors and categories	CO_2	CH_4	N_2O	HFCs	PFCs	SF_6	Total
	Equivalent CO_2 (kt)						
Energy							
Energy industries	*10 090.78* 9939.00	*10.01* 9.86	*137.45* 135.38				*10 238.24* 10 084.25
Manufacturing industries and construction	*4575.43* 4506.61	*10.56* 10.40	*16.08* 15.84				*4602.07* 4532.85
Transport	*11 726.30* 11 549.92	*10.24* 10.08	*126.47* 124.57				*11 863.01* 11 684.58
Other sub-sectors	*8503.92* 8376.01	*141.56* 139.43	*74.58* 73.45				*8720.06* 8588.90
Non-specific mobile emissions	*0.88* 0.84						*0.88* 0.84
Solid fuels (fugitive emissions)	*0.01* 0.01	*18.10* 17.83					*18.11* 17.83
Oil and natural gas (fugitive emissions)	*0.20* 0.19	*59.74* 58.84					*59.94* 59.03
Industrial processes and product use							
Mineral industry	*1821.08* 2001.97						*1821.08* 2001.97
Chemical industry	*0.10* 0.11		*0.01* 0.01				*0.11* 0.12
Metal production	*0.01* 0.01						*0.01* 0.01
Non-energy products from fuels and solvent use	*155.21* 170.62						*155.21* 170.62
Product uses as substitutes for ODS				*956.72* 1051.74	*43.35* 47.66		*1000.07* 1099.40
Other product manufacture and use			*37.36* 41.08			*35.58* 39.11	*72.94* 80.19
Agriculture							
Enteric fermentation		*10 911.61* 11 033.18					*10 911.61* 11 033.18
Manure management		*1348.82* 1363.84	*513.16* 518.87				*1861.98* 1882.72
Rice cultivation		*0.01* 0.01					*0.01* 0.01
Agricultural soils			*5571.43* 5633.50				*5571.43* 5633.50
Field burning of agricultural residues		*0.03* 0.03	*0.01* 0.01				*0.04* 0.04
Liming	*432.42* 437.24						*432.42* 437.24
Urea application	*83.90* 84.84						*83.90* 84.84
Waste							
Solid waste disposal		*645.81* 662.10					*645.81* 662.10
Biological treatment of solid waste		*13.94* 14.29	*9.97* 10.22				*23.91* 24.51
Incineration and open burning of waste	*21.89* 22.44	*0.16* 0.16	*0.23* 0.24				*22.28* 22.84
Wastewater treatment and discharge		*46.85* 48.03	*91.01* 93.31				*137.86* 141.34
Other sources		*0.09* 0.10					*0.09* 0.10

PRTB rules to allocate the emissions to sectors (Table 8), the Energy categories would have been again the only ones less affected by a reduction in CO_2, CH_4 and N_2O emissions if using the PRTB rule, and vice versa if the proportional rule is used.

With respect to the second type of situations under study, for each of the cases based on different emission reduction targets (26, 30, 34 and 38%) in relation to 2005, we may take into account the sum of total net removals and total net emissions from the LULUCF sector for 2018 (approximately −38 Mt of CO_2 equivalent; see Table 5). The proportional and PRTB rules have been used for the allocation of GHG emissions among the four referred sectors for each reduction target in case that emissions and removals from LULUCF activities in 2018 were included or not. As shown in Table 9, inclusion of LULUCF only led to bankruptcy cases for the 34% and 38% reduction targets compared

Table 5
Spanish total and individual GHG emissions by sector and category in 2018.

GHG emitting sources and sinks	CO_2	CH_4	N_2O	HFCs	PFCs	SF_6	Total
	Equivalent CO_2 (kt)						
Energy	**248 829.03**	**2585.40**	**1969.70**				**253 384.03**
Energy industries	71 485.93	239.62	510.59				72 236.14
Manufacturing industries and construction	45 255.31	950.87	202.33				46 408.51
Transport	89 214.72	91.69	962.36				90 268.77
Other sub-sectors	38 634.68	1045.61	290.50				39 970.79
Non-specific mobile emissions	447.31	0.30	3.92				451.53
Solid fuels (fugitive emissions)	7.40	75.41					82.81
Oil and natural gas (fugitive emissions)	3783.68	181.90					3965.58
Industrial processes and product use	**20 326.64**	**134.56**	**830.17**	**6107.72**	**130.42**	**226.90**	**27 756.39**
Mineral industry	12 656.82						12 656.82
Chemical industry	3685.93	112.04	425.11				4223.08
Metal production	3125.09	22.52	0.06		123.80		3271.47
Non-energy products from fuels and solvent use	858.80						858.80
Product uses as substitutes for ODS				6107.72	6.62		6114.34
Other product manufacture and use			405.02			226.90	631.92
Agriculture	**498.60**	**24 916.12**	**14 229.09**				**39 643.81**
Enteric fermentation		17 668.90					17 668.90
Manure management		6794.92	1906.37				8701.29
Rice cultivation		433.19					433.19
Agricultural soils			12 316.83				12 316.83
Field burning of agricultural residues		19.11	5.89				25.00
Liming	25.82						25.82
Urea application	472.78						472.78
Waste		**12 086.39**	**1384.56**				**13 470.95**
Solid waste disposal		9930.88					9930.88
Biological treatment of solid waste		380.21	256.70				636.91
Incineration and open burning of waste		321.33	326.32				647.65
Wastewater treatment and discharge		1453.17	801.54				2254.71
Other sources		0.80					0.80
Land Use, Land-Use Change and Forestry	**−38 549.55**	**164.66**	**288.68**				**−38 096.21**
Forest land	−33 740.83	151.18	154.60				−33 435.05
Cropland	−3713.57	7.10	61.59				−3644.88
Grassland	41.81	6.38	8.52				56.71
Wetlands	53.80						53.80
Settlements	1235.00		57.04				1292.04
Other land	21.78		1.91				23.69
Harvested wood products	−2447.54						−2447.54
Other			5.02				5.02
Total CO_2 equivalent emissions (without LULUCF)	**269 654.27**	**39722.47**	**18 413.52**	**6107.72**	**130.42**	**226.90**	**334 255.30**
Total CO_2 equivalent emissions (with LULUCF)	**231 104.72**	**39 887.13**	**18702.20**	**6107.72**	**130.42**	**226.90**	**296 159.09**

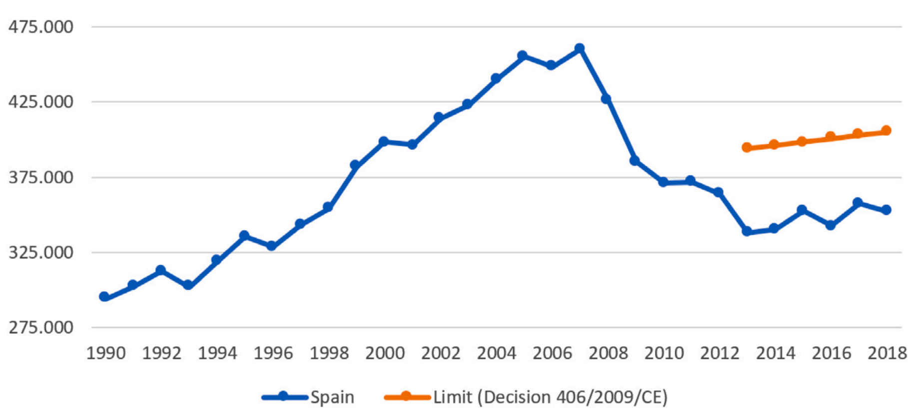

Fig. 4. Annual Spanish GHG emissions and limits imposed by the European Union from 1990 to 2018. Data expressed in CO_2 equivalent (kt).

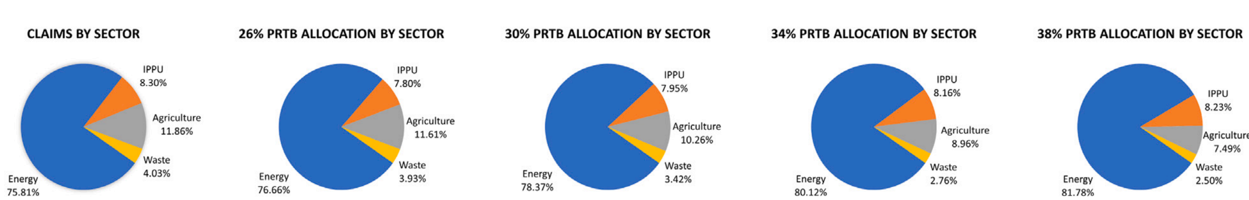

Fig. 5. Pie charts showing relative Spanish GHG emissions by sector (without LULUCF) in each of the bankruptcy cases based on the four reduction targets in comparison with non-bankruptcy.

Table 6
Sectoral maximum allowed GHG emissions (kt of CO2 equivalent) for Spain in 2018 if excluding LULUCF.

Sectors	Claims	Allocation							
		26%		30%		34%		38%	
		PRTB	Prop.	PRTB	Prop.	PRTB	Prop.	PRTB	Prop.
Energy	253 384.00	251 549.08	248 752.01	243 272.69	235 305.93	234 485.61	221 859.84	224 831.36	208 413.84
IPPU	27 756.30	25 601.67	27 248.94	24 665.72	25 776.02	23 883.12	24 303.10	22 626.46	22 830.19
Agriculture	39 643.80	38 105.42	38 919.05	31 842.52	36 815.31	26 230.94	34 711.58	20 603.71	32 607.85
Waste	13 471.00	12 888.69	13 224.77	10 626.37	12 509.91	8070.07	11 795.06	6870.66	11 080.21
Total (reduction)	334 255.00	328 144.90 (6110.10)		310 407.30 (23 847.70)		292 669.70 (41 585.30)		274 932.20 (59 322.80)	

Table 7
Sectoral maximum allowances for Spanish GHG emissions (kt of CO2 equivalent) by gas in 2018 if excluding LULUCF.

Energy

Gas	Claims	Allocation							
		26%		30%		34%		38%	
		PRTB	Prop.	PRTB	Prop.	PRTB	Prop.	PRTB	Prop.
CO_2	248 829.00	247 026.97	244 280.25	238 899.36	231 075.88	230 270.25	217 871.52	220 789.55	204 667.22
CH_4	2585.40	2566.68	2538.10	2482.23	2400.91	2392.57	2263.71	2294.06	2126.52
N_2O	1969.70	1955.435	1933.66	1891.10	1829.14	1822.79	1724.62	1747.74	1620.09
HFCs									
PFCs									
SF_6									
Total (reduction)	253 384.00	251 549.08 (0.72%)	248 752.01 (1.83%)	243 272.69 (3.99%)	235 305.93 (7.13%)	234 485.61 (7.46%)	221 859.84 (12.44%)	224 831.36 (11.27%)	208 413.84 (17.75%)

IPPU

Gas	Claims	Allocation							
		26%		30%		34%		38%	
		PRTB	Prop.	PRTB	Prop.	PRTB	Prop.	PRTB	Prop.
CO_2	20 326.60	18 748.71	19 955.06	18 063.29	18 876.40	17 490.18	17 797.75	16 569.90	16 719.10
CH_4	134.50	124.06	132.10	119.52	124.96	115.73	117.82	109.64	110.68
N_2O	830.20	765.75	815.02	737.76	770.96	714.35	726.91	676.76	682.85
HFCs	6107.70	5633.58	5996.07	5427.63	5671.95	5255.42	5347.84	4978.89	5023.73
PFCs	130.40	120.28	128.05	115.88	121.13	112.20	114.21	106.30	107.29
SF_6	226.90	209.29	222.73	201.64	210.69	195.24	198.65	184.97	186.61
Total (reduction)	27 756.30	25 601.67 (7.76%)	27 248.94 (1.83%)	24 665.72 (11.13%)	25 776.02 (7.13%)	23 883.12 (13.95%)	24 303.10 (12.44%)	22 626.46 (18.48%)	22 830.19 (17.75%)

Agriculture

Gas	Claims	Allocation							
		26%		30%		34%		38%	
		PRTB	Prop.	PRTB	Prop.	PRTB	Prop.	PRTB	Prop.
CO_2	498.60	479.25	489.49	400.48	463.04	329.91	436.58	259.13	410.12
CH_4	24 916.10	23 949.23	24 460.62	20 013.00	23 138.42	16 486.13	21 816.22	12 949.42	20 494.03
N_2O	14 229.10	13 676.94	13 968.93	11 429.04	13 213.86	9414.91	12 458.78	7395.16	11 703.70
HFCs									
PFCs									
SF_6									
Total (reduction)	39 643.80	38 105.42 (3.88%)	38 919.05 (1.83%)	31 842.52 (19.68%)	36 815.31 (7.13%)	26 230.94 (33.83%)	34 711.58 (12.44%)	20 603.71 (48.03%)	32 607.85 (17.75%)

Waste

Gas	Claims	Allocation							
		26%		30%		34%		38%	
		PRTB	Prop.	PRTB	Prop.	PRTB	Prop.	PRTB	Prop.
CO_2									
CH_4	12 086.40	11 563.94	11 865.45	9534.15	11 224.08	7240.60	10 582.70	6164.47	9941.33
N_2O	1384.60	1324.75	1359.31	1092.22	1285.84	829.47	1212.36	706.19	1138.88
HFCs									
PFCs									
SF_6									
Total (reduction)	13 471.00	12 888.69 (4.32%)	13 224.77 (1.83%)	10 626.37 (21.12%)	12 509.91 (7.13%)	8070.07 (40.09%)	11 795.06 (12.44%)	6870.66 (49.00%)	11 080.21 (17.75%)

to 2005. These outcomes are consistent with the fact that differences between total GHG claims and maximum allowed emissions in Spain (without LULUCF; see Table 6), in terms of tones of CO_2 equivalent, would not exceed the total removals (c.a. 38 Mt of CO_2 equivalent in 2018) observed for the LULUCF sector in cases in which the reduction targets were 26% and 30% compared to 2005, in contrast to those cases corresponding to the 34% and 38% reduction targets. As it can be seen in Fig. 6, when the PRTB rule is used, noteworthy is also that the

Table 8
Maximum allowed GHG emissions (without LULUCF) by gas and category for Spain in 2018. Values from the application of the PRTB rule first in cursive.

Sectors and sub-sectors	CO_2	CH_4	N_2O	HFCs	PFCs	SF_6	Total
	Equivalent CO_2 (kt)						
Energy							
Energy industries	*63 430.53* 58 798.76	*212.60* 197.06	*453.00* 419.93				*64 096.13* 59 415.75
Manufacturing industries and construction	*40 155.66* 37 223.43	*843.75* 782.14	*179.51* 166.40				*41 178.92* 38 171.97
Transport	*79 161.45* 73 381.03	*81.37* 75.43	*853.99* 791.63				*80 096.81* 74 248.09
Other sub-sectors	*34 281.11* 31 777.92	*927.77* 860.06	*257.78* 238.93				*35 466.66* 32 876.91
Non-specific mobile emissions	*396.90* 367.92	*0.26* 0.25	*3.46* 3.22				*400.62* 371.39
Solid fuels (fugitive emissions)	*6.57* 6.05	*66.90* 62.02					*73.47* 68.08
Oil and natural gas (fugitive emissions)	*3357.33* 3112.14	*161.40* 149.59					*3518.73* 3261.75
Industrial processes and product use							
Mineral industry	*10 317.64* 10 410.56						*10 317.64* 10 410.56
Chemical industry	*3004.67* 3031.72	*91.30* 92.16	*346.54* 349.66				*3442.51* 3473.53
Metal production	*2547.52* 2570.48	*18.34* 18.52	*0.05* 0.05		*100.84* 101.81		*2666.75* 2690.76
Non-energy products from fuels and solvent use	*700.08* 706.35						*700.08* 706.35
Product uses as substitutes for ODS				*4978.89* 5023.73	*5.46* 5.48		*4984.35* 5029.22
Other product manufacture and use			*330.16* 333.14			*184.97* 186.61	*515.13* 519.72
Agriculture							
Enteric fermentation		*9182.90* 14 533.05					*9182.90* 14 533.05
Manure management		*3531.45* 5588.97	*990.80* 1568.02				*4522.25* 7156.99
Rice cultivation		*225.14* 356.32					*225.14* 356.32
Agricultural soils			*6401.30* 10 130.83				*6401.30* 10 130.83
Field burning of agricultural residues		*9.93* 15.69	*3.07* 4.85				*13.00* 20.54
Liming	*13.41* 21.19						*13.41* 21.19
Urea application	*245.72* 388.92						*245.72* *388.92*
Waste							
Solid waste disposal		*5065.09* 8168.40					*5065.09* 8168.40
Biological treatment of solid waste		*193.91* 312.73	*130.93* 211.17				*324.84* 523.91
Incineration and open burning of waste		*163.87* 264.29	*166.44* 268.42				*330.31* 532.71
Wastewater treatment and discharge		*741.18* 1195.26	*408.82* 659.29				*1150.00* 1854.55
Other sources		*0.41* 0.65					*0.41* 0.65

relative Spanish GHG emissions from the Energy sector was increased in the bankruptcy cases for the 34% and 38% reduction targets (with LULUCF); while the relative GHG emissions from the rest of the sectors were reduced for the 34% and 38% reduction targets (see Fig. 6). By contrast, the relative GHG emissions from all the sectors did not vary using the proportional rule in both bankruptcy cases.

Through its focus on the individual greenhouse gases in the bankruptcy cases if including LULUCF (Table 10), predictably, Energy would have again claimed near 100% of the maximum amount of CO_2 that could be emitted in any bankruptcy case, while Agriculture accounted for most of CH_4 and N_2O emitted by the four referred sectors. It can be also observed that Spanish maximum allowed emissions for

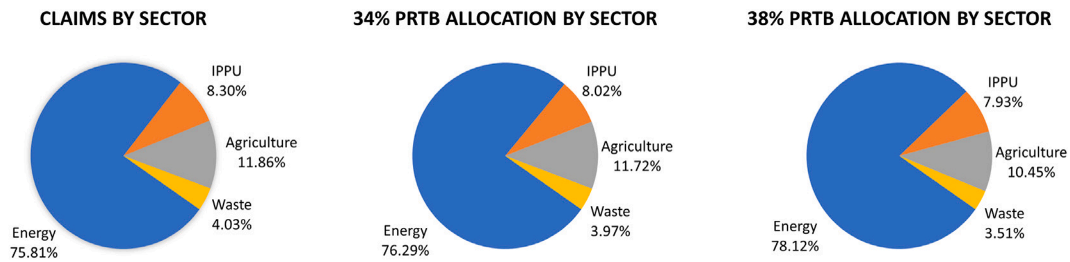

Fig. 6. Pie charts showing relative Spanish GHG emissions by sector (with LULUCF) in bankruptcy cases based on the 34% and 38% reduction targets, as well as in the non-bankruptcy cases based on the 26% and 30% reduction targets.

Table 9
Sectoral maximum allowed GHG emissions (kt of CO2 equivalent) for Spain in 2018 if including LULUCF.

Sectors	Claims	Allocation			
		34%		38%	
		PRTB	Prop.	PRTB	Prop.
Energy	253 384.00	252 336.13	250 738.88	244 526.36	237 292.87
IPPU	27 756.30	26 526.01	27 466.59	24 809.94	25 993.68
Agriculture	39 643.80	38 765.26	39 229.91	32 707.84	37 126.19
Waste	13 471.00	13 138.50	13 330.40	10 984.27	12 615.55
Total (reduction)	334 255.00	330 765.91 (3489.09)		313 028.41 (21 226.59)	

all the sectors would be higher than those corresponding to bankruptcy cases based on the 34% and 38% reduction targets if excluding LULUCF (see Table 7), as was also to be expected.

Finally, when LULUCF activities were taken into account for the distribution of emissions reduction efforts, we also make an allocation among categories to determine the maximum allowances of CO_2 equivalent that they could have emitted for each individual greenhouse gas in 2018. The proportional rule has been again used for the allocation of GHG emissions from the four referred sectors among source categories in the bankruptcy case for the most stringent Spanish emission reduction target (38%) related to 2005. Table 11 shows differences when undertaking a comparison of the claims on each category (Table 5) and corresponding sub-sectoral maximum allowances of GHG emissions obtained using the proportional and PRTB rules to allocate the emissions to sectors (Table 11), the Energy categories would have been the only ones less affected by a reduction in CO_2, CH_4 and N_2O emissions if using the PRTB rule, and vice versa if the proportionality rule is used, as mentioned previously.

3.3. Economic insights

Comparing Irish GHG emissions with Spanish ones before and after applying the model based on the proportional run-to-the-bank rule, the most noteworthy is that Irish and Spanish emissions per capita are about 12 and 7 tons of CO_2 equivalent, respectively, estimating the Irish and Spanish populations at 4.9 and 46.8 people in 2018. Regarding the relative contribution of each sector to national GHG emissions in 2018, the order in Ireland and Spain is the same (Energy > Agriculture > IPPU > Waste). However, it can be observed clear differences between Ireland and Spain when comparing contributions of sectors such as Energy (60 and 75%) and Agriculture (32 and 12%), while the IPPU and Waste contributions show small differences in both countries (6–8% and 2–5%). Finally, the results obtained applying the PRTB rule indicate that Energy is the only sector whose relative contribution increased slightly in Ireland and Spain (see Figs. 3 and 5).

With respect to the dependence of national GHG emissions on macroeconomic indicators such as GDP per capita, trade in goods and services (exports and imports), inflation rate, and unemployment in

Table 10
Sectoral maximum allowances for Spanish GHG emissions (kt of CO2 equivalent) by gas in 2018 if including LULUCF.

Energy					
Gas	Claims	Allocation			
		34%		38%	
		PRTB	Prop.	PRTB	Prop.
CO_2	248 829.00	247 799.94	246 231.40	240 130.56	233 027.11
CH_4	2585.40	2574.67	2558.37	2494.99	2421.18
N_2O	1969.70	1961.52	1949.10	1900.81	1844.58
HFCs					
PFCs					
SF_6					
Total (reduction)	253 384.00	252 336.13 (0.41%)	250 738.88 (1.04%)	244 526.36 (3.50%)	237 292.87 (6.35%)
IPPU					
Gas	Claims	Allocation			
		34%		38%	
		PRTB	Prop.	PRTB	Prop.
CO_2	20 326.60	19 425.64	20 114.45	18 168.92	19 035.80
CH_4	134.50	128.60	133.16	120.28	126.01
N_2O	830.20	793.39	821.53	742.07	777.47
HFCs	6107.70	5836.99	6043.96	5459.37	5719.85
PFCs	130.40	124.65	129.07	116.59	122.15
SF_6	226.90	216.82	224.51	202.79	212.47
Total (reduction)	27 756.30	26 526.01 (4.43%)	27 466.59 (1.04%)	24 809.94 (10.62%)	25 993.68 (6.35%)
Agriculture					
Gas	Claims	Allocation			
		34%		38%	
		PRTB	Prop.	PRTB	Prop.
CO_2	498.60	487.56	493.40	411.37	466.95
CH_4	24 916.10	24 363.96	24 656.00	20 556.88	23 333.81
N_2O	14 229.10	13 913.74	14 080.51	11 739.59	13 325.43
HFCs					
PFCs					
SF_6					
Total (reduction)	39 643.80	38 765.26 (2.22%)	39 229.91 (1.04%)	32 707.84 (17.50%)	37 126.19 (6.35%)
Waste					
Gas	Claims	Allocation			
		34%		38%	
		PRTB	Prop.	PRTB	Prop.
CO_2					
CH_4	12 086.40	11 788.05	11 960.23	9855.25	11 318.85
N_2O	1384.60	1350.45	1370.17	1129.02	1296.69
HFCs					
PFCs					
SF_6					
Total (reduction)	13 471.00	13 138.50 (2.47%)	13 330.40 (1.04%)	10 984.27 (18.46%)	12 615.55 (6.35%)

Table 11
Maximum allowed GHG emissions (with LULUCF) by gas and category for Spain in 2018. Values from the application of the PRTB rule first in cursive.

Sectors and sub-sectors	CO_2	CH_4	N_2O	HFCs	PFCs	SF_6	Total
	Equivalent CO_2 (kt)						
Energy							
Energy industries	*68 987.01* 66 946.26	*231.21* 224.37	*492.69* 478.11				*69 710.91* 67 648.74
Manufacturing industries and construction	*43 673.25* 42 381.33	*917.66* 890.51	*195.23* 189.46				*44 786.15* 43 461.30
Transport	*86 095.99* 83 549.13	*88.50* 85.88	*928.80* 901.32				*87 113.30* 84 536.34
Other sub-sectors	*37 284.18* 36 181.25	*1009.09* 979.24	*280.33* 272.04				*38 573.59* 37 432.52
Non-specific mobile emissions	*431.67* 418.90	*0.29* 0.28	*3.78* 3.67				*435.75* 422.86
Solid fuels (fugitive emissions)	*7.10* 6.89	*72.77* 70.62					*79.87* 77.51
Oil and natural gas (fugitive emissions)	*3651.38* 3543.37	*175.51* 170.32					*3826.92* 3713.72
Industrial processes and product use							
Mineral industry	*11 313.32* 11 853.11						*11 313.32* 11 853.11
Chemical industry	*3294.62* 3451.81	*100.15* 104.92	*379.98* 398.11				*3774.74* 3954.85
Metal production	*2793.38* 2926.66	*20.13* 21.09	*0.05* 0.06		*110.64* 115.92		*2924.09* 3063.60
Non-energy products from fuels and solvent use	*767.60* 804.22						*767.60* 804.22
Product uses as substitutes for ODS				*5459.37* 5719.85	*5.95* 6.24		*5465.34* 5726.10
Other product manufacture and use			*362.03* 379.30			*202.79* 212.47	*564.79* 591.74
Agriculture							
Enteric fermentation		*14 577.62* 16 546.84					*14 577.62* 16 546.84
Manure management		*5606.11* 6363.41	*1572.83* 1785.29				*7178.94* 8148.71
Rice cultivation		*357.41* 405.69					*357.41* 405.69
Agricultural soils			*10 161.90* 11 534.62				*10 161.90* 11 534.62
Field burning of agricultural residues		*15.74* 17.86	*4.86* 5.52				*20.60* 23.38
Liming	*21.26* 24.13						*21.26* 24.13
Urea application	*390.12* 442.81						*390.12* 442.81
Waste							
Solid waste disposal		*8097.68* 9300.27					*8097.68* 9300.27
Biological treatment of solid waste		*310.02* 356.07	*209.34* 240.43				*519.37* 596.50
Incineration and open burning of waste		*262.00* 300.91	*266.10* 305.62				*528.10* 606.52
Wastewater treatment and discharge		*1184.91* 1360.88	*653.58* 750.64				*1838.49* 2111.52
Other sources		*0.64* 0.74					*0.64* 0.74

Europe, literature has shown a positive relationship between increased economic activity and production, measured by GDP per capita (Irish and Spanish values about 79 000 and 30 000 USD/hab in 2018) or the ratio of exports plus imports over GDP (trade openness of 217.1 and 67.6% in Ireland and Spain, respectively, in 2018), and GHG emissions (e.g., [34,35]); whereas the rates of inflation (Ireland and Spain values for 2018 were 0.49 and 1.68%) and unemployment (Irish and Spanish rates of 4.95 and 15.25% in 2018) have a negative effect on GHG emissions because the reduction of both parameters indicates economic growth (e.g., [36,37]).

4. Conclusions

In this study, we propose a novel methodology to allocate annual GHG emissions in EU Member States among sectors and source categories using bankruptcy rules. We consider the cases of Ireland and Spain, two representative countries claiming for amounts of emissions in 2018 that would be higher than the limits imposed on them by the European Union and more stringent national regulations, respectively. These are conflicting claims problems that require discussion on which division rules should be applied. As a result, we show how bankruptcy techniques emerge as a very useful and suitable tool to allocate annual GHG emissions. While the proportional and proportional run-to-the-bank rules were applied to allocate GHG emissions by sector, only the proportional rule was used in order to distribute the maximum allowances of tonnes of CO_2 equivalent by sector among the individual greenhouse gases emitted by each sector. Finally, the proportional rule also appeared to be useful for the allocation of GHG emissions from the sectors among their source categories in situations of bankruptcy. When comparing the results obtained using the proportional and proportional bank-to-the-bank rules, it is worth noting that the Energy categories would have been less affected by a reduction in CO_2, CH_4 and N_2O emissions if using the PRTB rule to allocate the GHG emissions to sectors, contrary to what happened to the rest of categories.

The proposed methodology allows greater control by the public administration in charge of managing the reduction of greenhouse gases since it differentiates the reductions by gases, sectors and source categories. This implies that the economic impact and the environmental impact can be better measured since emission reductions are made at a more microscopic level than in the more general case of simply seeking to reduce greenhouse gas emissions in terms of CO2 equivalent emissions. Although it is true that transfers between claims and claimants are possible in real life, so their actual implementation could be difficult, it is no less true that better control of differentiated gas emissions is possible today and, moreover, the reduction of emissions should be an effort made by each one according to their emissions and possibilities, which leads to an approach closer to the one presented in this paper. Thus, one of the advantages of this approach is that the economic impact of the reductions can be controlled in a more disaggregated way, by sectors and source categories, and, therefore, would facilitate the design of public economic policies to soften their impact through compensation and incentive systems, which would also allow the transfer between claims and claimants but introducing fair mechanisms among all the agents involved.

Finally, in this paper the principle of proportionality has been used as part of the methodology to allocate greenhouse gas emissions, but others could have been applied. However, as we have already commented, egalitarian rules, both in terms of awards and losses, would be ruled out. It only remains to ask what properties, apart from the proportionality principle itself, make the rules used in this paper interesting. Calleja et al. [16] prove that the proportional run-to-the-bank rule belongs to the core of an associated game which makes the allocation obtained with this rule coalitionally stable. Furthermore, this rule is characterized using a consistency principle similar to that of O'Neill [10]. On the other hand, the proportional rule for multi-issue bankruptcy problems is characterized by using two properties related to the non-advantageous transfer of claims, i.e., the allocation does not change with ex ante transfers between claims and claimants [38].

We would like to emphasize some of the implications of the properties mentioned in the context of the allocation of emissions of different GHGs between sectors and categories of a country. The fact that an allocation is coalitionally stable, it is particularly interesting, since it guarantees that there exists no affected part that may complain individually of the allocation, nor any group of affected (which could form a kind of lobby) that may collectively complain about what is received in the distribution of total emissions. On the other hand, O'Neill's idea refers to the desirable fact that applying the distribution mechanism to the particular problem and applying it to some specific subproblems and adding the results of these subproblems should produce the same allocation. Therefore, the mechanism could be applied to smaller specific subproblems and then add the results. Finally, the property of non-advantageous transfer of claims properties underlines that the final allocation does depend on neither the number of groups of greenhouse gases nor the number of sectors and categories considered but only on the aggregate claims of each greenhouse gas and on the particular claims of each affected part independently of the number of groups into which they are distributed.

CRediT authorship contribution statement

E. Algaba: Conceptualization, Investigation, Writing – review & editing, Supervision. **G. Márquez:** Methodology, Validation, Writing – original draft. **J. Martínez-Lozano:** Software, Formal analysis, Data curation, Visualization. **J. Sánchez-Soriano:** Investigation, Resources, Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgments

This work is part of the R&D&I project grant PGC2018-097965-B-I00, funded by MCIN/AEI/10.13039/ 501100011033/ and by "ERDF A way of making Europe"/EU. The authors are grateful for this financial support. Joaquín Sánchez-Soriano also acknowledges financial suport from the Generalitat Valenciana under the project PROMETEO/2021/063. We would also like to thank the leading editor and the two anonymous referees for their constructive comments in order to improve an earlier version of this manuscript.

Appendix

Annual national GHG emissions (in blue) and limits imposed (orange color) by the European Union from 2013 to 2018 (see Fig. A.1).

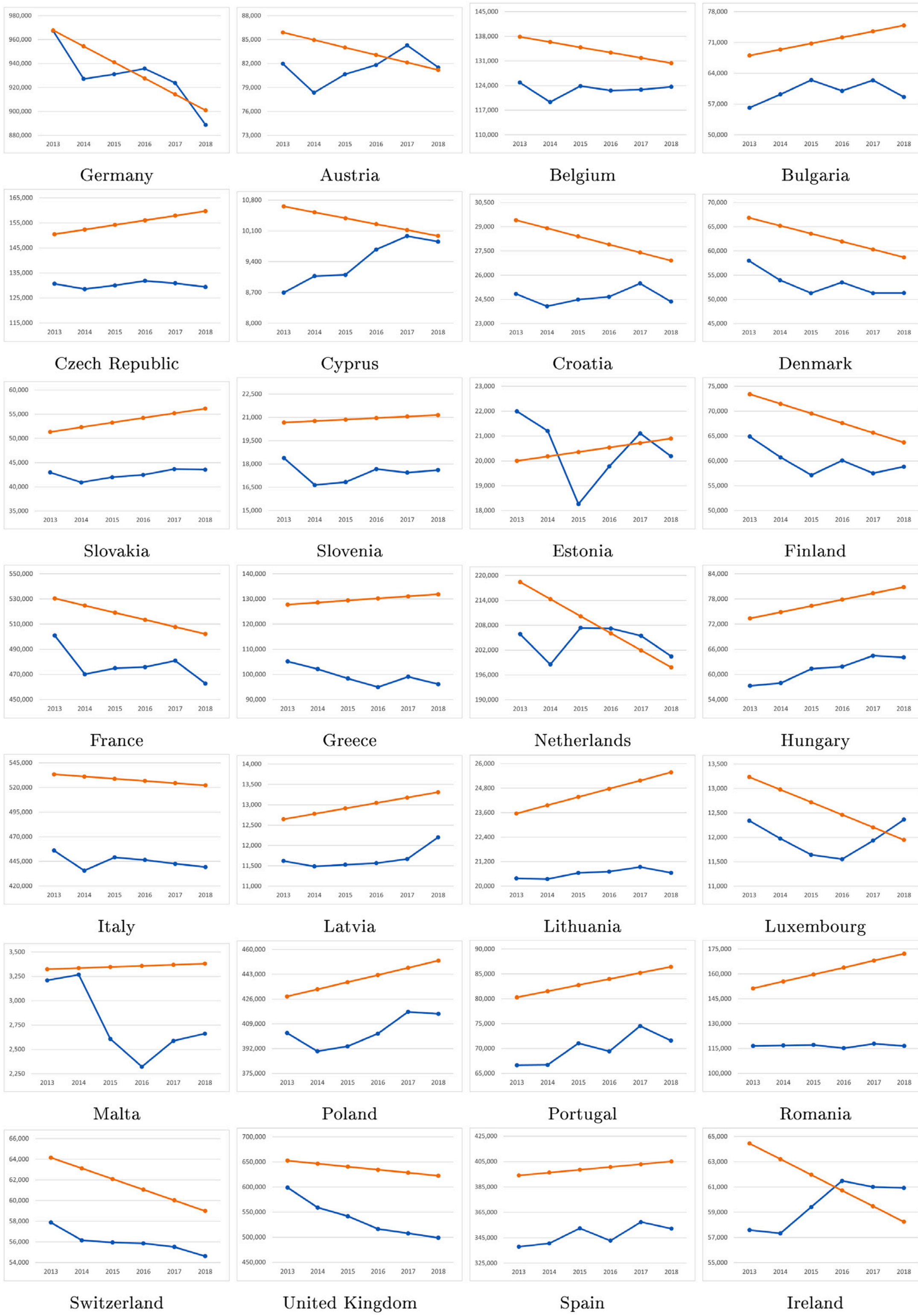

Fig. A.1. Annual national GHG emissions (in blue) and limits (in orange) from 2013 to 2018.

References

[1] Torkayesh AE, Alizadeh R, Soltanisehat L, Torkayesh SE, Lund PD. A comparative assessment of air quality across European countries using an integrated decision support model. Socio-Econ Plan Sci 2022;81:101198. http://dx.doi.org/10.1016/j.seps.2021.101198.

[2] Tuckett R. Greenhouse gases and the emerging climate emergency. In: Letcher TM, editor. Climate change – Observed impacts on planet earth. 3rd ed.. Amsterdam: Elsevier; 2021, p. 19–45. http://dx.doi.org/10.1016/B978-0-12-821575-3.00002-5.

[3] Meinshausen M, Meinshausen N, Hare W, Raper SB, Frieler K, Knutti R, Frame D, Allen M. Greenhouse-gas emission targets for limiting global warming to 2 °C. Nature 2009;458:1158–62. http://dx.doi.org/10.1038/nature08017.

[4] Ellison D, Lundblad M, Petersson H. Reforming the EU approach to LULUCF and the climate policy framework. Environ Sci Policy 2014;40:1–15. http://dx.doi.org/10.1016/j.envsci.2014.03.004.

[5] Boromisa A. Strategical decisions for energy future of Croatia. Zagreb: Institute for International Relations; 2011, https://irmo.hr/publications/hrvatska-eu-koristi-troskovi-integriranja/.

[6] Warren R, Hope C, Gernaat DEHJ, van Vuuren DP, Jenkins K. Global and regional aggregate damages associated with global warming of 1.5 to 4 °C above pre-industrial levels. Clim Change 2021;168:24. http://dx.doi.org/10.1007/s10584-021-03198-7.

[7] Beccherle J, Tirole J. Regional initiatives and the cost of delaying binding climate change agreements. J Public Econ 2011;95:1339–48. http://dx.doi.org/10.1016/j.jpubeco.2011.04.007.

[8] Duro JA, Teixid-Figueras J. World polarization in carbon emissions, potential conflict and groups: An updated revision. Energy Policy 2014;74(C):425–32. http://dx.doi.org/10.1016/j.enpol.2014.08.020.

[9] Duro JA, Giménez-Gómez JM, Vilella C. The allocation of CO_2 emissions as a claims problem. Energy Econ 2020;104652. http://dx.doi.org/10.1016/j.eneco.2019.104652.

[10] O'Neill B. A problem of rights arbitration from the Talmud. Math Soc Sci 1982;2:345–71. http://dx.doi.org/10.1016/0165-4896(82)90029-4.

[11] Aumann RJ, Maschler M. Game theoretic analysis of a bankruptcy problem from the Talmud. J Econ Theory 1985;36:195–213. http://dx.doi.org/10.1016/0022-0531(85)90102-4.

[12] Thomson W. How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Econometric society monographs, Cambridge, UK: Cambridge University Press; 2019, http://dx.doi.org/10.1017/9781108161107.

[13] Giménez-Gómez JM, Teixidó-Figueras J, Vilella C. The global carbon budget: a conflicting claims problem. Clim Change 2016;136:693–703. http://dx.doi.org/10.1007/s10584-016-1633-1.

[14] Gutiérrez E, Llorca N, Sánchez-Soriano J, Mosquera M. Sustainable allocation of greenhouse gas emission permits for firms with Leontief technologies. Eur J Oper Res 2018;269:5–15. http://dx.doi.org/10.1016/j.ejor.2017.10.011.

[15] Trabelsi R, Moretti S, Krichen S. Using bankruptcy rules to allocate CO_2 emission permits. In: Lecture notes of the institute for computer sciences, social-informatics and telecommunications engineering. LNICST, vol. 277, 2019, p. 82–92. http://dx.doi.org/10.1007/978-3-030-16989-3_6.

[16] Calleja P, Borm P, Hendrickx R. Multi-issue allocation situations. Eur J Oper Res 2005;164:730–47. http://dx.doi.org/10.1016/j.ejor.2003.10.042.

[17] Borm P, Carpente L, Casas-Méndez B, Hendrickx R. The constrained equal awards rule for bankruptcy problems with a priori unions. Ann Oper Res 2005;137:211–27. http://dx.doi.org/10.1007/s10479-005-2257-4.

[18] González-Alcón C, Borm P, Hendrickx R. A composite run-to-the-bank rule for multi-issue allocation situations. Math Methods Oper Res 2007;65:339–52. http://dx.doi.org/10.1007/s00186-006-0123-z.

[19] Izquierdo JM, Timoner P. Constrained multi-issue rationing problems. UB Economics Working Papers 2016/347, 2016, http://dx.doi.org/10.2139/ssrn.2841642.

[20] Acosta RK, Algaba E, Sánchez-Soriano J. Multi-issue bankruptcy problems with crossed claims. Ann Oper Res 2022;318:749–72. http://dx.doi.org/10.1007/s10479-021-04470-w.

[21] Acosta-Vega RK, Algaba E, Sánchez-Soriano J. Design of water quality policies based on proportionality in multi-issue problems with crossed claims. European J Oper Res 2023;311:777–88. http://dx.doi.org/10.1016/j.ejor.2023.05.029.

[22] Ju BG, Miyagawa E, Sakai T. Non-manipulable division rules in claim problems and generalizations. J Econom Theory 2007;132:1–26. http://dx.doi.org/10.1016/j.jet.2005.08.003.

[23] Moreno-Ternero J. The proportional rule for multi-issue bankruptcy problems. Econ Bull 2009;29(1):474–81.

[24] Herrero C, Villar A. The three musketeers: Four classical solutions to bankruptcy problems. Math Soc Sci 2001;42:307–28. http://dx.doi.org/10.1016/S0165-4896(01)00075-0.

[25] Hou J, Deng X, Han Springer C, Teng F. A global analysis of CO_2 and non-CO_2 GHG emissions embodied in trade with Belt and Road Initiative countries. Ecosyst Health Sustain 2020;6:1761888. http://dx.doi.org/10.1080/20964129.2020.1761888.

[26] Yang C. Analysis of the relationship between the economy and greenhouse gas emissions in the European Union. EU (BSc Thesis), Barcelona: Polytechnic University of Catalonia; 2020, p. 64.

[27] Aristotle. Nicomachean ethics. 1908, Translated by W. D. Ross. http://classics.mit.edu/Aristotle/nicomachaen.html.

[28] Maimonides M. Book of judgements. New York; Jerusalem: Moznaim Publishing Corporation, [1135-1204], (translated by Rabbi Elihahu Touger, 2000).

[29] Roth A. The Shapley value: Essays in honor of Lloyd S. Shapley. Cambridge, UK: Cambridge University Press; 1988, http://dx.doi.org/10.1017/CBO9780511528446.

[30] Algaba E, Fragnelli V, Sánchez-Soriano J. Handbook of the Shapley value. Boca Raton, USA: CRC Press, Taylor & Francis Group; 2019, p. 16–28. http://dx.doi.org/10.1080/00401706.2020.1744904.

[31] Algaba E, Fragnelli V, Sánchez-Soriano J. The Shapley value, a paradigm of fairness. In: Algaba E, Fragnelli V, Sánchez-Soriano J, editors. Handbook of the Shapley value. Boca Raton, USA: CRC Press, Taylor & Francis Group; 2019, http://dx.doi.org/10.1080/00401706.2020.1744904.

[32] Crippa M, Oreggioni G, Guizzardi D, Muntean M, Schaaf E, Lo Vullo E, Solazzo E, Monforti-Ferrario F, Olivier J, Vignati E. Fossil CO_2 and GHG emissions of all world countries. Luxembourg: Publications Office of the European Union; 2019, EUR 29849 EN JRC117610.

[33] Querol X, Massagué J, Alastuey A, Moreno T, Gangoiti G, Mantilla E, Duéguez JJ, Escudero M, Monfort E, Pérez García-Pando C, Petetin H, Jorba O, Vázquez V, de la Rosa J, Campos A, Muñoz M, Monge S, Hervás M, Javato R, Cornide MJ. Lessons from the COVID-19 air pollution decrease in Spain: now what? Sci Total Environ 2021;779:146380. http://dx.doi.org/10.1016/j.scitotenv.2021.146380.

[34] Al-Mulali U, Sheau-Ting L. Econometric analysis of trade, exports, imports, energy consumption and CO_2 emission in six regions. Renew Sustain Energy Rev 2014;33:484–98. http://dx.doi.org/10.1016/j.rser.2014.02.010.

[35] Sun H, Samuel CA, Amissah JCK, Taghizadeh-Hesary F, Mensah IA. Non-linear nexus between CO_2 emissions and economic growth: A comparison of OECD and B & R countries. Energy 2020;212:118637. http://dx.doi.org/10.1016/j.energy.2020.118637.

[36] Ronaghi M, Reed M, Saghaian S. The impact of economic factors and governance on greenhouse gas emission. Environ Econ Policy Stud 2020;22:153–72. http://dx.doi.org/10.1007/s10018-019-00250-w.

[37] Liu YQ, Feng C. The effects of nurturing pressure and unemployment on carbon emissions: Cross-country evidence. Environ Sci Pollut Res 2022;29:52013–32. http://dx.doi.org/10.1007/s11356-022-19515-1.

[38] Bergantiños G, Lorenzo L, Lorenzo-Freire S. A characterization of the proportional rule in multiissue allocation situations. Oper Res Lett 2010;38:17–9. http://dx.doi.org/10.1016/j.orl.2009.10.003.

Encarnación Algaba is Full Professor in the Department of Applied Mathematics II at the University of Seville and research member in the Mathematics Research Institute of Seville University (IMUS). She also received her Ph.D. degree in Mathematical Sciences from the University of Seville. Her main research line is about Cooperative Games on Combinatorial Structures and their applications, with relevant contributions in the area. She has organized diverse Streams of International Conferences and workshops on Game Theory. She has been invited speaker and guest researcher at various leading universities. She is the main editor of the Handbook of the Shapley value.

Gonzalo Márquez received a B.Sc. in Chemistry (1991) and a Ph.D. in Chemical Engineering (2000) at the University of Seville. Afterwards he continued his professional career at the University of Huelva, and he was promoted later until the current position of Full Professor (2021). Prof. Márquez's work is devoted to the geochemistry of petroleum, coal, and the atmospheric aerosols. He is also co-author of more than fifty articles published in international journals and two books on the aforementioned subject. Since 2015, he has introduced different methodologies for the analysis of organic atmospheric pollutants in the Associated Unit CSIC "Atmospheric Pollution".

Jesús Martínez Lozano is a mathematician and researcher currently working at the University of Seville. He received his undergraduate degree in Mathematics from the University of Almería in 2019. Jesús is now pursuing a master's degree in Big Data and Artificial Intelligence at the University of Malaga, where he is gaining valuable skills and expertise in the rapidly evolving field of data science. His passion for mathematics and data analysis led him to specialize in game theory, and he is currently focused on studying problems related to bankruptcy in game theory.

Joaquín Sánchez-Soriano received his BSc. in Mathematics with Honors (1991) and PhD. in Mathematics (1998) from the University of Murcia. At present he is (Full) Professor of the Department of Statistics, Mathematics and Computer Science and senior researcher of the Research Institute "Center of Operations Research (CIO)", both at the UMH. He has more than 100 scientific papers published in journals, books and proceedings. His research interests are focused on game theory and its applications, scarce resource management, bankruptcy problems, attribution problems, cost/profit sharing in joint logistic projects, optimization of complex systems, and statistical applications to health and economic sciences.