Monte Carlo Simulation Applicable for Predictive Algorithm Analysis in Aerospace

Jorge Bautista-Hernández[1,2(✉)] and María Ángeles Martín-Prats[1]

[1] Electronics Engineering Department, University of Seville, 41092 Seville, Spain
jorbauher@alum.us.es, mmprats@us.es
[2] Aerospace Electrical Engineering Department, Airbus Poland, 02-256 Warsaw, Poland

Abstract. Safety investigations about electrical wiring harness caused by failures in electrical systems establish that origin of these accidents are related to electrical installation. Predictive techniques which mitigate and reduce risk of the occurrence of errors to enhance safety shall be considered. The development of machine learning has evolved towards the creation of innovative predictive algorithms which show high performance in data analysis and making predictions in the context of artificial intelligence. The Monte Carlo approach is used to validate the model performance. In this paper, Monte Carlo simulation was used to evaluate the level of the uncertainty of the selected parameters over 1000 runs. This study analyzes the reliability of the predictive algorithm in order to be implemented as an automatic error predictor in aerospace. The results obtained are within the expected range suggesting that the model used is accurate and reliable.

Keyword: Monte Carlo Simulation · Predictive Algorithms · Sensitivity Analysis · System Reliability · Automatic Error Predictor

1 Introduction

Safety is a pillar in our lives. It has been evaluated in aerospace that major accidents are consequential from human errors which can contribute to up to 80% of the total accidents. Human errors can never be eliminated completely but they can be reduced to the minimum by implementing predictive and automatic algorithms which are focus upon the risk rather than on the error elimination. An analysis of dataset is encouraged to be performed in order to better identify relevant indicators and situations which are vulnerable to create an error in order to implement measures to avoid potential failures.

Human errors in aerospace are considered as a multi-event and can be mainly generated from design (e.g. models errors), manufacturing (incorrect procedures), installation issues (incorrect assembly) and operational errors of the aircraft (miscommunication or poor decisions). Furthermore, some of these errors are likely to generate a hazard. The complex process for error generation needs to be better analyzed in order to show a holistic view of one indicator towards the creation of the error [1]. Based on the aviation authorities investigation the main cause for accident creation was the failures generated in the electrical harness installation. Thus, quantification of the main parameters that

exist in the electrical system of an aircraft and used predictive techniques to predict uncertainties are necessary. Advanced technology such as cyber-physical systems and automation are effective strategies to prevent errors [2].

Cyber-physical systems (CPS) are based on computational and physical elements which can be used to monitor processes in order to prevent errors before they occur. For example, in electrical manufacturing, the risk matrix can be used as an outcome of the predictive algorithm in order to detect in real time any anomalies before they cause a failure. Additionally, automation can be used to prevent errors and reduce the risk of human error creation.

The main goal is to implement an innovative methodology to keep the aerospace industry at the greatest level of safety and potentially analyses its applicability to other disciplines such as energy, health care, transportation or infrastructure. Thus, the research question established in this paper is: What can be the benefit of introducing this novel methodology in the industry?

To answer to this question requires to give evidences to the following assumptions:

- Time assessment during the creation of the manufacturing engineering processes,
- Mitigation errors in the end-to-end process,
- Positive impact in safety in order to keep aviation standards at the highest level.

The quantification of the successful results have generated a decreased of 93% in manufacturing time and 90% in potential errors creation during creation of applicable manufacturing engineering processes.

This specific paper aims to assess reliability and validation for the model system engineering represented in Fig. 1 by using a Monte Carlo simulation. This technique can be used as a proper method to assess reliability for a system engineering. The model in Fig. 1 presents a level of uncertainty in the three input parameters selected in this simulation. The Monte Carlo method will solve this uncertainty by running 1000 samples for each parameter and representing the result using the probabilistic function. This function represents the probability of the possible outcomes values are below a threshold. The aim of the study is to investigate the level of sensitivity of the key parameters used for the prediction to consider them as good indicators [3, 4].The impact of varying these parameters on model with 1000 runs simulation will define the robustness of the model as a tool to be implemented as an auto-failure detector. Thus, the model uncertainties are identify towards a more reliable system in order to improve predictions in the future [3].

The remainder of this paper is as follows. The review of the relationship to connected cyber physical spaces is in Sect. 2, Sect. 3 addresses the research methodology related to the validation model. The main results are presented in Sect. 4. Discussions are in Sect. 5 and finally to summarize in Sect. 6 conclusions and future work.

2   Relationship to Connected Cyber Physical Spaces

The motivation to implement this novel and innovative methodology based on the proposed automatic and predictive algorithm in the electrical manufacturing processes is fundamental in order to keep aerospace safety at the greatest level. The advanced technologies such as Big Data and the increasing system complexity together with the necessity that the data needs to be fast analyzed to provide the best solution, enable the creation of a Cyber Space Model to respond to this necessity [2]. Cyber-Physical systems provide feedback in real time and present very good adaptive and predictive capacity. The latest research in the aerospace framework shows these systems not only have a positive impact on the aviation safety but also enhancing efficiency, integration and autonomy of the next generation aerospace systems.

The solution approach proposal based on this hybrid and predictive algorithm contributes to connect Cyber Physical Spaces through the use of machine learning techniques. The automatic manufacturing processes and predictive tasks enable engineering systems to execute activities independently with minimum human actions. Therefore the mitigation error is guaranteed. Additionally, the assessment in real time of the error creation by using the risk matrix not only decrease the probability of create a failure, but it also ensures the correct decision provided by the automatic system. This situation generates a positive impact in the electrical harness built process. Thus, automation process and predictive tasks are references towards connected Cyber Physical Spaces.

This emerging and innovative methodology convergences on the new technologies used in the new dynamic manufacturing industry. The required multi-interaction to satisfy the demand of systems complexity establish better collaboration between academic engineering disciplines and industry since cyber-physical systems also requires more elements to be inter-connected and easy adaptability to this new technology and future applications.

3   Methods

3.1   The Algorithm Overview

The innovative procedure using predictive algorithms has been developed in aerospace not only to mitigate the errors but also to predict the error creation in the electrical manufacturing engineering processes in the aerospace context by using innovative machine learning techniques. The Fig. 1 represents the algorithm structure and is based on the following elements.

The risk matrix mechanism that assess the probability of error creation in each specific harness. It determinates five main categories from 'very low' to 'very high' probability of error creation [5]. The automation tool is developed to avoid manual tasks during creation of the engineering documentation. Thus, it mitigates creation of an error during the engineering process. The dendrogram is using the hierarchical agglomerative method which creates groups with similar objects within the data set. The clusters data provides information about the critical groups which require special attention. The logistic regression estimates the parameters by establishing relationships between the input data and the outcomes according to a mathematic criterion. The confusion matrix will

define the accuracy of the algorithm by table of contingency. Finally, the computation time will be the minimum necessary to define the optimal number of iterations using the method of gradient descent after regression logistic is applied to the categorical variable.
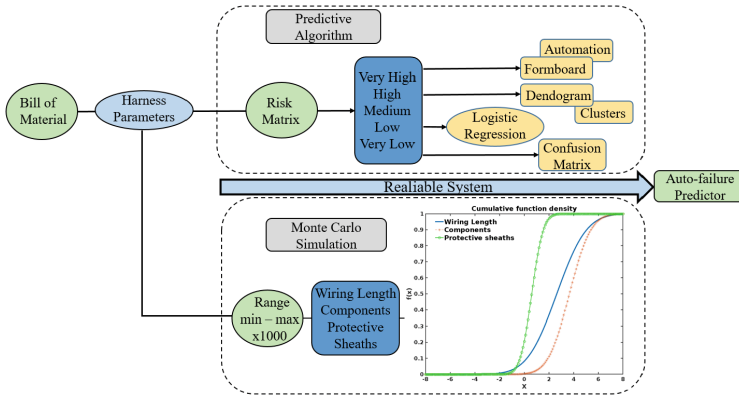

Fig. 1. Diagram representation of the predictive algorithm and Monte Carlo simulation

3.2 The Input Parameters

The first step in the creation of the predictive algorithm is to define the parameters. These are based on the electrical configuration of the harness to be manufactured and eventually installed on the aircraft. These are the number of zones (Z), number of wires (H) and number of electrical components (N) which define the risk matrix ϕ. Such criteria defines each harness category based on these parameters. The risk matrix function defines the probability of creating an error during the manufacturing process of electrical harness as part of the outcomes from predictive algorithms. The risk matrix as a function can be expressed as follows:

$$\phi = \phi(Z, H, N) \tag{1}$$

Scores assigned to each parameter will be evaluated on a scale from 1 to 5, being 1 the simplest geometry and 5 the most complex. A detailed description of the model has been recently proposed by Bautista Hernández and Martín Prats assessing its performance towards introduction in aerospace applications [6].

The baseline parameters for the Monte Carlo simulation will be based on the following relevant metrics, p_1 for wiring length, p_2 for number of electrical components and p_3 for the protective sheath quantities present in the 'bill of material' on each harness for a military aircraft. The Monte Carlo function determinates the performance of the model for implementation purposes. The generic function for Monte Carlo simulation λ, is expressed as follows:

$$\lambda = \lambda(p_1, p_2, p_3) \tag{2}$$

The C295 aircraft dataset with a total of 221 electrical harness to fully define the electrical system of this aircraft. In total in this case study, 157 harness considered are presenting the following parameters, 18523.95 m of the total wiring length, 21200.55 electrical components and 250.84 m of protective sheath length. The baseline run Monte Carlo simulation was executed 1000 times across a range between maximum and minimum values of meters of wiring length (0.29–2374.79), units of number of components (4–2243) and meters of protective sheath (0.06–22.59).

3.3 Sensitivity Analysis

The sensitivity analysis was carried out to understand the correlation between the input parameters and indicate the importance for the outcomes. Additionally, this analysis evaluates the model performance by considering the response of the input variables parameters after simulation [7]. The high correlation between the variables indicates that increasing of their values will enhance outcomes values. Thus, the probability of creating an error will be also higher [8].

The outcomes presented were analyzed to understand the impact of the maximum and minimum values within the dataset suggesting that the model performs well. In this medium-light size aircraft the predicted outcomes after 1000 runs were within the range of expected values.

The box-plots performed for the chosen parameters represented in Fig. 2 aim to analyze the data set and demonstrate the spread of numerical data through their quartiles follows one of the known distributions probabilistic. The Fig. 2 shows the dataset distribution associated to each parameter defined by the generic function Monte Carlo in a logarithmic scale.

In the first case related to the p_1, the first quartile $Q_1 = e^{0.9} = 2.45$ m marks one quarter (25%) of the ordered dataset and the value 1.5 IQR (-) below the first quartile is 1.5 IQR (-) $= e^{0.1} = 1.01$ m.

The maximum value in the dataset is 2374 m and the value for 1.5 IQR (+) $= e^{6.3} = 544.21$ m above the $Q_3 = e^{3.2} = 24.52$ m, which marks three quarter (75%) of the ordered dataset. The maximum value is above the 1.5 IQR (+), so in this case the maximum is an outlier which may indicate the measurements are not in the center of the data.

In the second case related to the p_2, the first quartile $Q_1 = e^{2.4} = 11.02$ components and the value 1.5 IQR (-) $= e^{1.4} = 4.05$ components.

The maximum value of the dataset is 2243 components. In this case, this maximum value is above the 1.5 IQR (+) $= e^{6.9} = 991$ components showing outliers.

In the third case related to the p_3, the first quartile $Q_1 = e^{0.1} = 1.1$ m and the 1.5 IQR (+) $= e^{2.3} = 9.97$ m below the maximum value of this parameter in the dataset which is 22.59 m.

The analytical results estimate the input data for the three parameters selected can be fitted to a normal distribution.
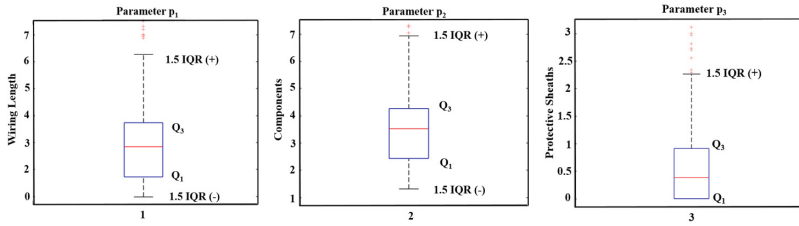
Fig. 2. Box-plots representation for input metrics p_1, p_2, p_3 on the Monte Carlo function λ

3.4 Monte Carlo Simulation

The Monte Carlo simulation is a method to assess reliability of an engineering system. The simulation uses as input parameters the p_1, p_2, p_3 values. These values are randomly assigned 1000 times variating within the range of the maximum and minimum values defined from the entire dataset in a logarithmic scale shown in the Table 1.

This simulation is used to obtain outcomes estimation from a set of stochastic trial from proper definition quantities [4].

Table 1. Minimum, maximum, mean and standard deviation values of the input parameters used in Monte Carlo simulation

Description	parameter	min	max	mean	std	units
Wiring length	p_1	0.1	7.77	2.46	1.90	m
Number of components	p_2	1.38	7.71	3.66	1.36	–
Protective sheath	p_3	0.1	3.11	0.44	0.74	m

Such statistical distributions are used within the range of the lower and the higher end value for each parameter to estimate the outcomes across a Monte Carlo simulation with 1000 runs. Probabilistic distributions are considered for probability calculations within the dataset. The assignment for calculation of the probability in each individual run was carried out using the normal function from the random Python xlwings library within the given interval. As the input dataset variable X is log-normally distributed then the input values were transformed to a distribution $Y = \ln(X)$ for representation [9].

The simulation is defined by generation of a stochastic trial to each of the input parameters on each of 157 electrical harness on a C295 military aircraft following a probabilistic function normally distributed. After simulation using random combination of the input variables the output is calculated for each parameter. Finally, the outcomes are represented with the probabilistic function distribution and the cumulative function of density.

3.5 Method of Gradient Descent

The regression logistic used to predict the categorical variable needs 144 iterations until the algorithm converges. This situation requires a high computation time. The method

of gradient descent can be used to find the optimal solutions by proper adjustment of the parameters which minimize the function in order to reach the global minimum faster and the convergence of algorithm [10].

Being $f : \Omega \subset R^n \rightarrow R$, this method can find $\theta \subset \Omega /f(\theta)$ min. The Eqs. 3, 4 and 5 define the best solution path in order to find the optimal solution.

$$\theta \in \arg\min \ f(\theta) \tag{3}$$

$$\theta_{t+1} = \theta_t - \delta_t \nabla f(\theta_t) \tag{4}$$

$$\nabla f(\theta_t) = (\partial f / \partial x_1(\theta), \ldots, \partial f / \partial x_n(\theta)) \tag{5}$$

δ_t is the learning rate which determinates the number of iterations until the algorithm converges. The convenient value of δ guarantees minimize the number of iterations improving the computation time.

The use of this method allows the algorithm to perform faster. Thus, the number of iterations decrease to 41 generating significantly benefit in the computation time.

4   Results

The tendency of each parameter presents a linear relation between the parameters chosen for the Monte Carlo simulation (length, number of electrical components and protection sheaths). The correlation of these parameters is quantified by the correlation matrix shown in table 2. A positive value of the coefficient means an increase of this parameter necessary involves an increase in the other parameter. The strongest correlation occurs when the absolute value of the coefficient is as close as possible to 1. From the correlation matrix we observed that p_1 and p_3, as well as parameters p_2 with p_3 have lower positive correlation coefficients, but still indicate some degree of correlation between these variables.

Table 2. Correlation matrix for the input parameters used for the Monte Carlo simulation

Description	Parameter	Length	Components	Protection
Wiring length	p_1	1.0000000	0.928046	0.470757
Number of components	p_2	0.928046	1.0000000	0.471076
Protective sheath	p_3	0.470757	0.471076	1.0000000

The Monte Carlo simulation has performed 1000 runs and the output parameters as a result of the new predicted values for the wiring length, numbers of electrical components and protective sheath are normally distributed. The cumulative distribution function shows the probability that each parameter value stays below specific threshold.

4.1 First Output - Wiring Length

The Eqs. (6–7) quantify through the probabilistic distribution function the prediction of the outputs in a new harness. As a first output of the new parameter for the wiring length, the p_1 will be between 2.01 and 3.00 m with a probability of occurrence between 50 and 90% (Fig. 3).

$$P(0.5) \rightarrow X_1 = e^{0.7} = 2.01 \, \text{m} \tag{6}$$

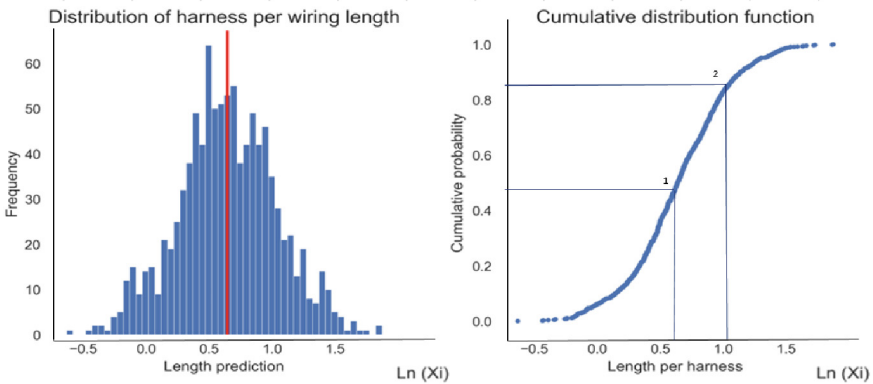$$P(0.9) \rightarrow X_2 = e^{1.1} = 3.00 \, \text{m} \tag{7}$$

Fig. 3. Results for p_1 outcomes on the wiring length prediction

4.2 Second Output – Number of Components

The Eqs. (8–9) quantify through the probabilistic distribution function the prediction of the outputs in a new harness. As a second output of the new parameter for the number of components, the p_2 will stay between 45 and 245 units with a probability of occurrence between 50 and 90% (Fig. 4).
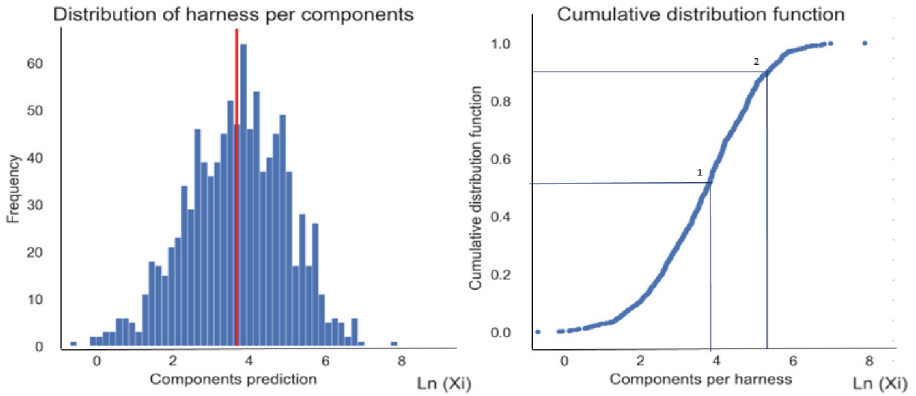
$$P(0.5) \rightarrow X_1 = e^{3.8} = 45 \, \text{units} \tag{8}$$

$$P(0.9) \rightarrow X_2 = e^{5.5} = 245 \, \text{units} \tag{9}$$

4.3 Third Output – Protective Textile Sheaths

The Eqs. (8–9) quantify through the probabilistic distribution function the prediction of the outputs in a new harness. As a third output of the new parameter protective sheath
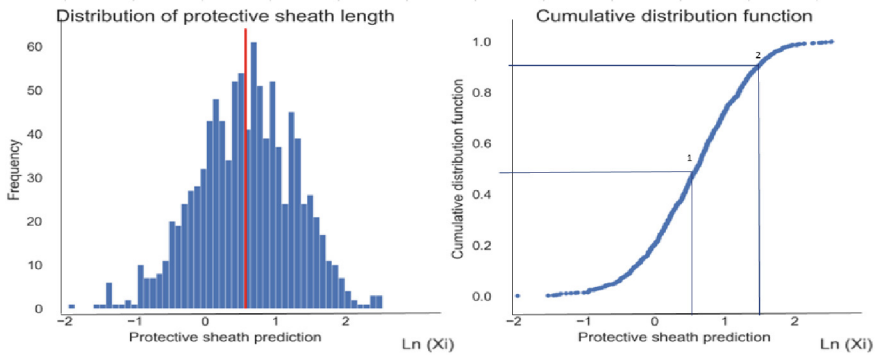
Fig. 4. Results for p_2 outcomes on the number of electrical components prediction

length, the p_3 in a new harness will be between 1.49 and 4.48 m with a probability of occurrence between 50 and 90% (Fig. 5).

$$P(0.5) \rightarrow X_1 = e^{0.4} = 1.49\,\text{m} \tag{10}$$

$$P(0.9) \rightarrow X_2 = e^{1.5} = 4.48\,\text{m} \tag{11}$$

Fig. 5. Results for p_3 outcomes on the protective sheath prediction

The reliability assessment at each parameter level is needed to calculate the probability that a new component does not exceed a limit defined through the cumulative distribution function [11].

The previous figures refer to the Monte Carlo simulation represent the random values of the parameters which are variating from maximum to the minimum within the three parameters. The distribution function shows the outcomes predicted are normally distributed. The y-axis shows the frequencies where a parameter appears and the x-axis the value of that parameter at that frequency. The cumulative distribution function estimates

the probability of the new output. The x-axis represents the value of the parameter and the y-axis represents the probability that the event occurs.

The following assumptions considering those parameters p_1, p_2, p_3 are normally distributed / $\Omega \sim N(\mu,\sigma)$ are represented in the Fig. 6, showing the calculation of expected outcomes lead to a Gaussian function Eq. 12.

$$\Omega(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{12}$$

Table 3. Normal distributions fitted to the outcomes obtained after the Monte Carlo simulation has been carried out

Description	Parameter	Normal distribution
Wiring length	p_1	N (2.46, 1.89)
Number of components	p_2	N (3.66, 1.36)
Protective sheath	p_3	N (0.61, 0.74)

The purpose of the normal distributions is to obtain a probability function to help to understand the variability of the system simulated and to give an evidence of its reliability. In the future, they can also be useful to make predictions of the parameters.

These considerations in terms of modelling validation are necessary to be taken into account. The auto failure-predictor prioritizes to determinate accurate predictions from a given dataset parameters for model validation [8].

The Fig. 6 shows these parameters are normally distributed, the x-axis corresponds to the value of this parameter and the y-axis the density function of probability showing how likely is to observe that value. Each colour represents the different Gaussian distributions for each of those parameters selected for the Monte Carlo simulation.
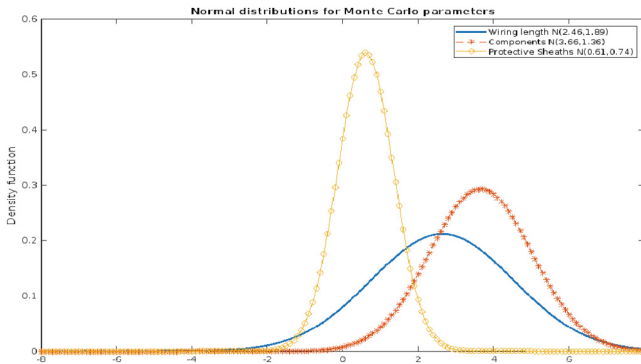
Fig. 6. Normal or Gaussians distribution for the dataset parameters p_1, p_2, p_3

The Fig. 7 depicts the dataset within the time and frequency domain. The series of points in left figure are referred to the parameters of the model and show the signal changes within time. The blue line represents the first parameter p_1, which is the wiring length. The red line represents the second parameter p_2, which is the number of electrical components and the third parameter p_3, represents by the orange line is the protection sheaths. The 'y axis' refers to the amplitude of the signal what is the scale of each point of the dataset. The 'x axis' refers to each point of the dataset on a single harness out of 157. The information obtained from wiring length and number of components show that the harness with more wiring length will also present more number of electrical components, so these parameters are positively correlated. It is also observed they have similar amplitude between peaks. In harness with longer wiring length it is shown that the blue line is above the red line showing that in this group of harness the amplitude of this parameter is greater than the number of components. This type of harness are the most critical harness since the present high risk matrix. From the orange line, the protective sheath is the unpredictable parameter, in some points the dataset has 'zero values' what it means there are harness which are not in presence of this parameter. This parameter is also in phase with the other two parameters, what it means, if the parameter value is greater than 0, the increasing of the wiring length and number of electrical components will also reflect an increasing of the protective sheath parameter. The use of this representation helps to predict future values based on the previous history data.
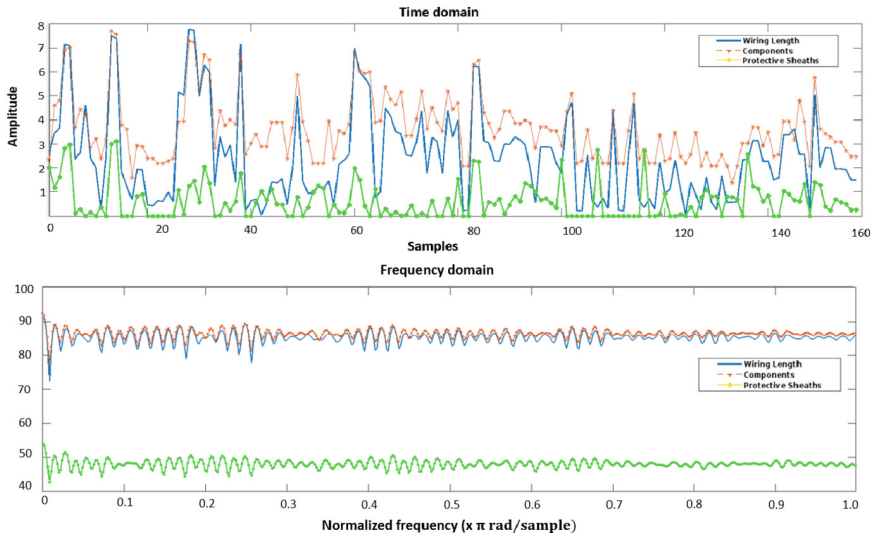

Fig. 7. Time/frequency domain representation for the dataset parameters p_1, p_2, p_3

From the representation in the frequency domain, the right Fig. 7 shows how much the amplitude of the signal is present in each frequency interval. The times series analysis and the observed values are represented in a data spectrum. The information from this representation includes in detail the phase difference between the signals. The wiring length and number of components are in phase. There is a minimal difference in phase

at low frequency (f = 0.05 Hz). At this frequency the wiring length decreases, however the number of electrical components increases. In this specific situation it is expected that the number of electrical components should increase as well. This situation occurs in avionics harness where the number of components is in percentage very closed to the wiring length percentage. There are many electronic devices to be connected in a very small area.

At f = 0.35 Hz is another particular case where the signals are not in phase and there is a difference between the phases on both signals. This situation illustrates only one electrical harness presents more components than wiring length.

The protective sheaths parameter is presented in a constant rate and the information obtained shows is an unpredicted parameter, concluding that this parameter should not be considered as a relevant parameter for validation of the model.

5 Discussion

For the purpose to validate the system, the Monte Carlo simulation is a proper method to analyse how the performance is affected by the parameters. The input parameters are stochastically assigned for each dataset. The random behaviour of the components aim to define sensitivities indexes to quantify the outcomes and inform about the importance of variation in the features of the system.

From the predictive results, it is shown that potentially a new harness within this aircraft is expecting to have greater than 2 m wiring length, 45 components and 1.49 m of protective sheath. This event occurs with a probability higher than 50%. The expected values in summary are shown in the following Table 3.

Table 4. Expected values of the input parameters and probability of occurrence after Monte Carlo simulation

Description	Expected value
Wiring length	2.01 m
Number of components	45 units
Protective sheath	1.49 m

The impact of these components and their metrics contribute to assess the risk of the system to fail. Failures in aerospace are not only costly and time-consuming but they can involve catastrophic consequences [9]. The outcomes are in accordance with the expected values and within the range for this aircraft. The correct behaviour of the outcomes is relevant to consider the system as a reliable and a safe system. This represents the best model prediction (Table 4).

From the risk matrix, this type of harness is defined as a medium-low risk what it means there is low probability to create an error during the creation of manufacturing process. This situation shows very reliable results since mostly harness in this medium light aircraft are medium size presenting not a high risk in terms of error creation. This

information provided in advanced generate a lot of benefits, such as better performance on the end-to-end process shortening the potential error creation between stages. Additionally, the whole process system gets more reliable providing a full visibility of the entire process.

The aging effect on these parameters is also important to take into consideration. The wiring length aging can influence to the wiring properties and make them shorter. The number of components can be also reduced due to material wear decreasing components efficiency. The protection sheaths aging can increase the risk of short circuit after damaging the wires.

Additionally, the accuracy of the model is ensured by calculation of the minimum square error. The results show a value of 0.13 what implies that the model is predicting well the outcomes for the given input parameters. The expected values obtained from the cumulative function of density in Table 3 are also good metrics to ensure accuracy of the modelling parameters.

6 Conclusions

Safety is a main priority and necessary to be established between all main actors involved in the end-to-end process. The positive communication enhances not only safety but also commitment with high requirements in order to anticipate before the undesirable situations occur. Starting from this research developed within aerospace the aim is to define the sensitivity respect three main inputs parameters which are the most relevant for electrical harness in order to define the model as reliable to be used as a reliable model. The importance of these parameters defines effectiveness for the risk matrix calculation and sensitivity analysis. In this paper we presented evidences that the model used for prediction of errors creation in a military aircraft case study is a reliable auto-failure detector. The model has used the following relevant parameters in the electrical harness definition such as wiring length, number of electrical components and length of protective sheaths. For the sensitivity analysis the sample size used n = 1000 for these factors were modelled in a Monte Carlo simulation. After analysis, results show these parameters are highly correlated, showing that the wiring length and number of components are proportional and reliable parameters to be used to define the risk matrix of the model. Additionally, we observed the avionics harness are critical presenting both parameters in very similar proportion. The protection sheath shows an unpredictable behaviour and to be used as an estimator of the risk matrix needs to be consider together with another parameter, either wiring length or number or components. The dataset has been validated using real data manufactured by one of the main aircrafts making this system as reliable model and safe. This research also establishes future trends applicability to keep safety at the highest level, not only in aerospace but also in other disciplines. This is the first milestone in order to be implemented in the industry. The results show that the model can be implemented as an auto-failure predictor and can be used to make realistic estimation of these parameters.

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