### Perpendicular-field EHD instabilities visualized in a tip-plane configuration

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**Abstract:** EHD interfacial instabilities can be induced by corona discharge from a tip electrode onto a liquid-air interface. The corona current charges the liquid surface and instabilities may appear depending on the applied voltage and the liquid properties. We present a technique to detect the onset of instability based on image processing. To establish the instability threshold we apply an edge-detection algorithm to images of the liquid surface acquired through a video camera.

### Introduction

Two types of EHD instabilities may be observed when the surface of a poorly conducting liquid is subject to corona discharge from a tip. If the liquid conductivity is very low, in such a way that the electric relaxation time,  $\epsilon/\sigma$ , is greater than the time the ions need to traverse the liquid thickness,  $d^2/(KV)$ , a space charge distribution appears. (Here  $\epsilon$  is the liquid permittivity,  $\sigma$  its conductivity, d its thickness, K the ion mobility in the liquid and V the voltage across the liquid layer.) The existence of this space charge gives place to the EHD volume instability induced by unipolar injection. The relevant parameter is in this case  $T = \epsilon V/(K\eta)$  ( $\eta$  the liquid viscosity), and above  $T \sim 100$  the liquid is set into motion with a pattern in the form of convective hexagonal cells. The size of these cells is of the same order as the liquid thickness. The instability due to unipolar injection is a classic problem in Electrohydrodynamics and has been the subject of many studies [1, 2, 3, 4].

In a tip-plane configuration the voltage drop across the liquid is determined by the corona current, and this is, in turn, determined by the voltage applied to the tip, being the dependence on the tip-plane distance rather weak [5]. Therefore, the typical pattern of EHD volume instability due to the space charge is observable applying a few kilovolts to a point over a grounded plate where we have disposed silicone oil, for example. The pattern appears independently of the tip-plane distance, provided the current is of the order of 1  $\mu$ A.

If the liquid conductivity is low, but high enough to prevent the existence of space charge (this occurs if  $\epsilon/\sigma < d^2/(KV)$ ), the conduction in the liquid is ohmic. In this case the EHD volume instability is not observable, but another type of instability is possible. This is referred to as rose-window instability and is due to the electric pressure exerted on the charged surface [6]. Being the liquid conductivity low, the electric charge on the surface is of the same polarity as the injecting electrode. Therefore the electric pressure pushes down the liquid and the instability arises as a consequence of the competition between electric and gravity forces. On the contrary, if the conductivity of the liquid is high, the polarity of the surface charge is of opposite sign to the applied field. In this case the electric pressure pulls up the liquid surface [7]. In both cases the relevant parameter is  $U = \epsilon_0 V_p^2 / \rho g h^3$ , where  $V_p$  is the potential applied to the tip and h the distance between the tip and the plane electrode [8].

Cells of size large compared with the liquid depth are characteristic of the rose-window instability, and this size decreases as the electric field on the liquid increases. Theoretically, it seems that the linear instability first appears for an infinite wavelength [8], and this may be the reason why the pattern is initially formed by large cells. Since rosewindow instability is associated to the effect of the electric pressure onto the liquid surface, an this is of order  $\epsilon_0 V_p^2/h^2$ , only if the tip-plane distance is low the instability is observable. For example, for a 1 mm depth layer of corn oil, the instability appears when the voltage applied to the tip is close to 4 kV and the distance is of the order of 3 cm. Although the rose-window instability seems to be caused by the action of surface electric forces it appears only if there is injection of charge into the surface, and the mechanism of instability is essentially different from that in classic studies [9].

The rose-window instability appears also in liquids where the EHD volume instability is present. Usually it appears at voltages above the volume instability threshold, when the surface has already been deformed. In these situations both patterns, that of small and that of large cells, are superimposed [10].

Although the rose-window instability can be detected by looking at the surface with unaided eye, a more reliable detection procedure is desirable. We have developed a detecting technique based on digital image processing. The same technique can be useful for detecting other fluid instabilities.



**Figure 1**: Point to plate configuration for the observation of EHD instabilities.

 Table 1: Physical properties of the liquids used in the experiments.

	Silicone oil	Castor oil
$\varepsilon_r$	2.73	4.69
$ ho(kg/m^3)$	960	958
$\nu(m^2/s)$	$50\cdot10^{-6}$	$600 \cdot 10^{-6}$
$K(m^2/Vs)$	$5\cdot10^{-10}$	$4 \cdot 10^{-11}$
$\sigma_c(\Omega^{-1}m^{-1})$	$6 \cdot 10^{-13}$	$7 \cdot 10^{-11}$

# Setup

The experimental setup is shown in figure 1 and described in detail elsewhere [6, 10]. A high-voltage source is connected to a point electrode. Below the point, there is a plane circular electrode which is grounded. A few milliliters of poor conducting liquid are disposed onto the electrode. The electrode is mounted on a frame made of plexiglass that prevents the liquid from flowing out of the electrode. The surface of the unperturbed liquid is a circle of diameter 4.5 cm. An electrometer connected between the plane electrode and the ground allows to monitor the electric current.

In order to have a good visualization, the plate is a piece of glass treated with  $TiO_2$  to make a transparent electrode. A video camera is placed below the electrode and connected to a computer, where the images are stored for further processing. A piece of paper glued around the point acts as light diffuser and provides a good background for the images.

Although several liquids have been tested (silicone oil, corn oil, castor oil, olive oil), the results that we present here correspond to castor oil and silicone oil, whose properties are enumerated in table 1.

# Image processing

The human-eye-based determination of the voltage at which the instability begins has some shortcomings. First, the measure is based on a human decision on a unique measure (the minimum voltage for which the instability is apparent). This is somewhat subjective, i.e., dependent on the experimenter aptitudes or training. Second, it cannot be implemented automatically. An alternative, machinebased method is desirable for automatic implementation and validation of the first method.

We have implemented an image-processing method that allows to determine accurately the minimum voltage for which the instability appears. We acquire a set of images of the liquid surface for different values of the applied voltage, from well below the instability threshold up to a voltage at which the instability is clearly developed. For each image, a numerical algorithm computes a number related to the total length of optical edges, these being characterized by having a local maximum of light intensity gradient. This number is expected to be a constant while the applied voltage is under its critical value; above this voltage, new edges, associated to cells, develop suddenly. Since the typical size of cells decreases as the voltage increases, the total length of edges in the image should increase monotonically. Therefore, the intersection of a linear or parabolic fit of the computed length of edges for high enough voltages with its value for zero voltage is expected to be a good measure of the critical voltage.

We have adopted the number of detected edgepixels as a measure of the total edge length. We apply firstly an edge-detection filter called SUSAN (from Smallest Univalued Segment Assimilating Nucleus, see [11] for details), whose response to each pixel of the original image is intense only in the vicinity of an edge, in the the same way that gradient maps. However, since the method is integral, it behaves very well in the presence of noise, unlike the more traditional derivative-based methods. These derivative-based methods could also be used, but previous noise filtering of images would be mandatory. Comparatively, the SUSAN edge filter is faster and more reliable. The number of non-black pixels of this response cannot be used to our purpose because the width of the edges is larger for lower voltages, giving more edge pixels than expected. It is necessary to thin the edges, i.e., select the pixels which locally have maximum gradient. To this end, we select only the pixels that have maximum SUSAN intensity in the direction perpendicular to



Figure 2: Image of the liquid surface as seen from below by the camera. The voltage applied to the point is 8 kV (negative polarity), the electric current through the liquid is  $1.74\mu$  A. The liquid is castor oil.

the edge, using as neighborhood a  $3 \times 3$  mask. A simple linking method of pixels, based on vicinity and approximate direction of edges, allows discarding spurious edge points, as isolated pixels, usually originated by noise.

#### Results

Figure 2 shows a picture of the rose-window instability for a voltage at which the instability is well developed ( $V_p = 8 \text{ kV}$ , negative polarity). The liquid is castor oil and we use 2 ml, corresponding to a depth of d = 1.3 mm. The tip-plane distance is 3.5 cm. The diameter of the liquid surface is 4.5 cm.

In tables 2 and 3 we present the variation with the voltage applied to the tip of the average size of the cells, in rose-window instability. Table 2 corresponds to silicone oil, where a space charge density is expected and the conduction in the liquid is not ohmic. Table 3 corresponds to castor oil, where no space charge is present and the conduction is ohmic. In both types of liquid the size of the cells has the same behavior: it decreases if the electric field increases and increases if the liquid layer thickness increases. The typical size of the cells is large compared with the liquid depth and tends to be greater for the ohmic liquid than for the non-ohmic one.

The dependence of the cells size on the electric field intensity is responsible for the variation of this size from the center, where the field is more intense, to the periphery. This variation is very visible in figure 2. Although the cells tend to be hexagonal, the variation of their size induces the presence of many defects, more of them pentagons.



Figure 3: Bitmap of the edge points corresponding to the previous picture.

**Table 2**: Average size D of the rose-window cells in silicone oil as a function of the applied voltage  $V_p$ , for several liquid layer thicknesses. p = 2.5 cm.

	$V_p(kV)$	6.5	7.0	8.0	9.0	10.0
d = 0.6 mm	D(cm)	0.34	0.34	0.30	0.25	0.20
$d=1.3\ mm$	D(cm)	0.61	0.52	0.41	0.27	0.24

Figure 3 shows the bitmap with detected edge points corresponding to this image. The edges corresponding to the frame features are also present when the instability is not, and contribute to the reference number of edge pixels. Looking carefully at this picture it is noted that some of the central cells look to have a double boundary. This is a consequence of the shadow that the support of the point projects into the liquid surface. This effect warns us against the consequence of a bad background or a defective illumination. Unfortunately, we cannot remove the point from our set-up, and its shadow cannot be avoided.

Figure 4 is a plot of the number of pixels involved in edges as a function of the voltage applied to the tip. The results correspond to 1 ml volume of silicone oil (layer depth 0.6 mm) and the tip-plane distance is 2.5 cm. The two lines fitting the data intersect at a point that determine the critical voltage, 5.8 kV. The corresponding value of the parameter U is  $5.5 \times 10^{-3}$ , which is somewhat greater than the value expected in a purely plane geometry [8].

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**Table 3:** Average size D of the rose-window cells in castor oil as a function of the applied voltage  $V_p$ , for several liquid layer thicknesses. p = 3.5 cm.

	$V_p(kV)$	6.0	7.0	8.0	9.0	10.0
d = 0.6 mm	D(cm)	1.12	0.74	0.40	0.22	0.21
d = 1.3 mm	D(cm)	1.15	0.54	0.57	0.44	0.36

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Figure 4: Number of pixels (in thousands) involved in edges as a function of the applied voltage. The reference line corresponds to the pixels present in the liquid container edges. The critical voltage is unambiguously determined by the intersection of the two lines drawn. The liquid is silicone oil. Negative polarity.

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