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Comment on "Breakup length of forced liquid jets" [Phys. Fluids 15, 2469 (2003)]

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Kalaaji *et al.*[1] presented experimental evidence of the influence of the surrounding air at rest on the destabilization of a liquid jet issuing from a nozzle for moderate jet velocities (first wind-induced regime). Comparisons of their data were done with respect to the classical Rayleigh[2] theory for capillary viscous jets without influence of the outer gas; the Weber[3] theory, which includes the Kelvin-Helmholtz mechanism of destabilization; and finally, the Sterling and Sleicher[4] semiempirical approach, which accounts for the nonzero viscosity of the outer gas. The most relevant experimental results were reported in Figs. 8(a)–8(c) of Ref. 1. There, the nondimensional rates of amplification of axisymmetric perturbations, β', for different liquids and wave numbers are plotted versus the jet velocity, V, and compared to the three mentioned theories. For the highest jet velocities, the experimental points seem to be closer to the Sterling and Sleicher predictions, but for low velocities, for which the three theoretical curves become indiscernible, the experimental points deviate systematically below them. This paradoxical fact is interpreted as a consequence of an experimental artifact, once the use of the more realistic spatial analysis[5] is shown to be nonessential. The experimental difficulty is revealed by means of a spectral decomposition of the temporal evolution of the diameter along the jet for two cases of very different velocities (Figs. 4 and 10 in Ref. 1) using the laser shadow technique.[6] For the highest velocity, the signal has a fundamental component with amplitude well above that of the harmonics, which does not happen for the lowest velocity. The exponential growth of perturbations occurs after an initial plateau dependent on the stimulation voltage. This fact could modify the rate of amplification obtained from the breakup-length method.[6]

Our Comment is motivated by the existence of a systematic error in the method of measurement of the jet velocity. However, we do not intend to question the main conclusions of the article because the error is not significant for high velocities. The consequences, on the contrary, may help to explain the surprising behavior found for low velocities.

The jet velocity was measured assuming that the wavelength of the perturbation λ is the distance between consecutive main drops. If f is the imposed stimulation frequency, the relation used is $V=\lambda f$ [Eq. (11) in their article]. The authors were aware of the approximate nature of this assumption, because the condition of validity $V/V_{\mathrm{cap}} \gg 1$ is stated just at the beginning of the paragraph [$V_{\mathrm{cap}} = \sqrt{\gamma/(\rho R)}$ is the capillary velocity, with γ and ρ the surface tension and liquid density, respectively, and R the jet radius]. However, the error is not always negligible and can be estimated, for instance, from the formula supplied by Dressler[7] for a monodisperse jet

$$V_d = V(1 - V_{\mathrm{cap}}^2/V^2) + V_s^2/(2V), \qquad (1)$$

where V_d is the drop velocity and V_s is the amplitude of the perturbation in the jet velocity at the stimulation zone. This formula comes from a careful balance of momentum and mass applied to the whole flow. Neglecting the existence of satellites, the error in the jet velocity is some 0.5%, from an estimation based on data from Fig. 38 in the work of Eggers.[8] If the last term is also negligible, we obtain an approximate relation between the velocity used by Kalaaji *et al.*, V_d, and the actual jet velocity V. For some of the experimental values reported in Fig. 8 of Ref. 1 the discrepancies reach 8%. The main consequence is a systematic error in the evaluation of the temporal rate of amplification, β, which is obtained from the spatial rate of amplification, b, and the jet velocity, according to $\beta = Vb$ [Eq. (4) in Ref. 1]. Other minor effects are errors in the nondimensional wave numbers, $k=2\pi R/\lambda$, and the aerodynamic Weber number, $\mathrm{We}_a = \rho_a V^2 R/\gamma$, used in the calculation of the theoretical curves. A replot of Figs. 8(a)–8(c) of Ref. 1 is presented in Figs. 1(a)–1(c), with all the described errors corrected. Stars are the original experimental points; squares are the same data when multiplied by the correction factor V/V_d evaluated from Eq. (1); the solid line is the Rayleigh's prediction; and, finally, the dashed line is the Sterling and Sleicher's prediction. We omit a third curve present in the original work (the Weber's prediction) as it deviates clearly from the experimental points. The curves have been calculated using the

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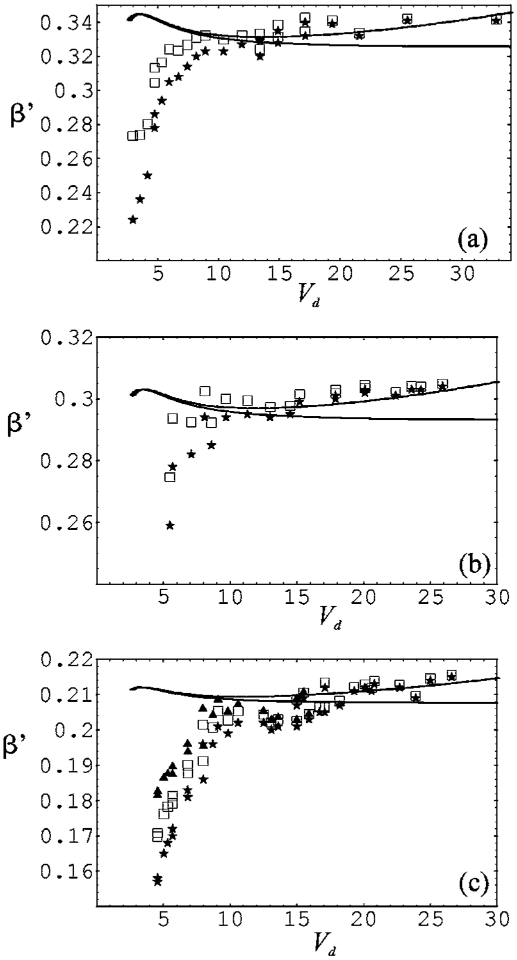

FIG. 1. Nondimensional rate of amplification of perturbations vs the velocity of the resulting drops for the three cases reported in Fig. 8 of Ref. 1. Stars are the original data from Ref. 10 and squares are these data once the correction in the jet velocity is applied. The curves give the predictions from the Rayleigh (solid lines) and Sterling-Sleicher (dashed lines) theories, respectively. Triangles in (c), appearing for $V_d < 16$ m/s, illustrate the possible effect of changes in the actual values of the radii (see the text).

spatial dispersion relation and mainly for this reason they exhibit a different behavior for low velocities than in the original work. It is evident the better correlation, although not in quantitative agreement, between the replotted experimental points and the theoretical curves in the low velocity range.

Another issue to be discussed is the measurement of the jet radius. Kalaaji *et al.* presented in their Fig. 5 a comparison between some measured radii for different Reynolds numbers with respect to Gavis and Modan's[9] findings. However, these authors obtained their data in experimental conditions for which the Weber number is high enough to disregard the effect of capillary forces (We > 160), whereas the Weber number in the present case may be as small as We = 10 for the smallest jet velocities (fluid 0). In any case, the Reynolds number of the data reported in Fig. 5 of Ref. 1 ranges from 20 to 200, whereas for the data corresponding to fluid 2 (the one with the greatest deviations after the velocity correction) we have $10 < \mathrm{Re} < 60$. Correspondence with the authors has clarified this point and revealed that no measurement of the radius has been made below $V = 10$ m/s.[10] Consequently, the changes in jet radius for the lowest velocities could reduce further the remaining disagreement between theory and experiments. Indeed, as the growth rates are made nondimensional with the capillary time $t_{\mathrm{cap}} = (\rho R^3/\gamma)^{1/2}$, an increase in the radius for low velocities (apparent in Fig. 5 of Ref. 1) would raise the experimental points in Fig 8 of Ref. 1 as $R^{3/2}$. For illustration purposes, we have represented in Fig 1(c), by means of triangles, the tentative locations of all the experimental points affected by this last correction, assuming applicable the dependence of the radius with the Reynolds number found by Gavis and Modan (Fig. 5 of Ref. 1).

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