# Copper Concentrates Benchmark Prices Estimation with an Assessment of the Trading Business

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# Chapter 1: Introduction

Commodity trading is one of the oldest forms of economic activity, albeit its functioning is yet scarcely understood nowadays. On the other hand, the large-scale expansion of fuels, minerals and grains trading in recent decades has been one of the vital elements which has facilitated globalization (Buchan and Errington, 2016).

The activity of sourcing the natural resources needed by the industry and society in general constitutes one of the pillars of the global economy, obtaining commodities from where there is abundance of and moving them to where they are most needed in time and form, equilibrating thus mismatches between offer and demand (Meersman et al., 2012). Commodity trading stretches from grain like wheat or corn, which comprises the base of human and animal feeding, to copper, iron or aluminium, metals that have laid the foundations the developing of information technologies, transportation and infrastructures, as well as oil and natural gas, key elements for the world's energy production.

The modern form of commodity trading dates back to the beginning of the 17th century, with the stablishing of the East India Company (EIC) in 1600, and the Vereeningde Oostindische Compagnie (VOC), or the Netherlands East India Company in 1602 (Robins, 2012). Both these two companies, which were in hands of private investors, possessed the concessions of their monarchies that granted them the exclusive right to trade all kind of commodities, mainly cotton, opium, spices or tea, between the middle and far east and their European metropolis.

Later on, the huge demand for metals that arose in the beginning of the 19th century stemming from the advances of the industrial revolution fostered the creation of the London Metal Exchange (LME) in 1877 (Geman and Smith, 2013), formalizing a trading which had been taking place informally in the outskirts of the Royal Exchange of London since decades earlier. At the LME, copper suppliers from Chile or tin suppliers from Malaisia would find their buyers, enabling them to set prices for future deliveries. Some of the first standards withing commodity trading were then first set, remaining in force still today, such as the 3-month future contract for copper. This originally was equivalent to the time that copper shipments from Chile took to arrive to England (The London Metal Exchange, 2019). In a similar fashion, the first future exchanges were stablished across the world, like the Chicago Board of Trade (CBOT), which in1864 created the first standardized futures contract for grain, being still nowadays a reference market for setting of agricultural commodities prices.

Additionally, the generalization of the use of oil in the 20th century, mainly since in 1911 gasoline became the main product that Standard Oil's refineries made, overtaking kerosene (Murty, 2020), to meet the unstoppable increase of its demand due to the growing use of cars, transformed the commodities trading business. The Seven Sisters appeared in the 40s. This seven large vertically-integrated petroleum conglomerate, were capable of controlling every stages of the oil production, from extraction to refining, going through transportation and retailing, hence becoming the first oil cartel (Buchan and Errington, 2016).

The 1973 Oil Crisis, caused by the embargo by the Arab countries to western nations due to the Yom Kippur war, terminated with this model of vertical integration. This allowed a new model of free competence between private companies that traded with commodities to appear. These new independent companies, like March Rich & Co. transform commodities in space through logistical systems, in time through warehousing and in form through processing by the making of blends

(Pirrong, 2013). Many companies involved in commodity trading ceased therefore to be present in all the parts of the process and focused solely in arbitrating in commodity markets.

Since 1981, with the financialization of futures markets after the creation of the first futures contract for oil by the New York Mercantile Exchange (NYMEX), like the ones that already existed for copper or corn, and large investment banks opening dedicated commodities trading desks (Blas and Farchy, 2021), commodity trading reshaped to look like today. Commodity trading is now decentralised, dominated by companies that reach both physical and financial markets, for either hedging their exposure in the physical world as well as for speculating. These companies have developed sophisticated financial structures for their operations, reaching in some cases to acquire a meaningful importance in the economy, even to be considered as systemic -too big to fail- (Jacques and Simondet, 2016).

### *1.1. Copper and its trading*

Copper is a key element for the current transformations in energy and technology. It is the best non-precious metal conductor, which makes it a key component of electrical wires, generators, motors, transformers and renewable energy production systems. Additionally, copper and copper brass are key components for construction materials and industrial machinery and equipment, thanks to its resistance to corrosion, its durability, as well as its machinability and ability to be cast.

Also, copper plays a major role in transportation uses, being critical in the electrification of fossil-fuel-based transportation means, being intensively used an in Electrical Vehicles (EV). Indeed, a conventional car contains 23 Kg of copper on average, whereas a Hybrid Electric Vehicle (HEV) has almost twice as much on average, while a Plug-in Hybrid Electric Vehicle (PHEV) contains about 60 Kg (International Copper Study Group, 2022).

In 2021, the use of refined copper in the world reached its historical peak with 25.3 million tons consumed globally. Having achieved an annualized consumption increase rate of 3.3% since 1990. Meanwhile, the total used of copper in the world, also accounted for recycling, approached 32.5 million tons (International Copper Study Group, 2022).

### 1.1.1. Copper production

The world's copper production is essentially achieved through alternative processes which depend on the chemical and physical characteristics of the copper ores extracted. According to the USGS' 2017 Mineral Commodity Summary on Copper (U.S. Geological Survey, 2017), global identified copper resources contained 2.1 billion tons of copper as of 2014, of which about 80% are mainly copper sulphides, whose copper content has to be extracted through pyrometallurgical processes (Schlesinger and Davenport, 2011).

The main source of copper, despite the growing importance of scrap recycling, is still primary copper, which is obtained by the extraction and processing of copper ores. According to the U.S. Geological Survey (2023) the world's primary copper production reached 21.2 million tons in 2021, of which 3.9 million were only obtained through the Solvent Extraction/Electro-Winning (SX/EW) process, whereas the rest were achieved by smelting and refining of concentrates of different grades (International Copper Study Group, 2022).

The majority presence in the earth's crust of sulphide deposits, having low copper grades between 0.5% - 2.0%, (Schlesinger and Davenport, 2011) makes concentrates the only technically and economically feasible mining product from most copper mines. Copper concentrates are thus the main products offered by copper mines, representing 80% of the origin of all cathodic copper obtained to be used with industrial applications (Norgate and Jahanshahi, 2010). Concentrates are produced from copper sulphide ores, occurring naturally in different kinds of deposits, being porphyry deposits the most relevant ones. According to the USGS' 2015 Assessment of Undiscovered Copper Resources of the World (Hammarstrom et al., 2019) porphyry deposits represented 1.8 billion tons of all identified copper resources, whereas undiscovered deposits of these kind alone are estimated to contain 3.1 billion tons of copper.

Ore grade differs depending on the deposit class, which for the case of porphyry deposits oscillates between 0.3% and 2.0% of copper, with the top 25 mines by Cu content having a mean grade of 0.49% (Mudd et al., 2013). These copper deposits may also contain some other valuable elements, such as gold or silver with variable gradings, as is the case of the Indonesian mine Grasberg with an average gold grade in the ore mined during 2018 of 1.58 g/t, as well as having reached an average copper grade at the mill of 0.98% (Freeport-McMoRan, 2018).

Other relevant, though fewer, common classes of deposits, such as sediment-hosted copper deposits, which account for 0.31 billion tons of global total identified resources and 0.42 billion tons of undiscovered world resources, are likely to contain significant amounts of additional by-products, for example cobalt or silver, as well as higher average copper grades (Hammarstrom et al., 2019), such as the Katanga mine in D.R.C. with an average Cu grade of 3.49% and an average cobalt grade of 0.46% in 2018 (Katanga Mining Limited, 2018), and KGHM Polska Miedź's Polish mines whose ores graded on average 1.49% of copper and 48.6 g/t of silver throughout 2018 (KGHM Polska Miedz, 2018).

### 1.1.2. Copper concentrates

Except for oxidized ores, which are mainly treated through a hydrometallurgical process, ores undergo an enrichment process known as froth flotation to achieve concentrates. Through flotation, grades are significantly increased while the tonnage of the concentrate outflow is much less than the tonnage of the ore inflow current. Most of the copper content in the ore, as well as the by-products, pass into the concentrate with minimum losses of valuable elements being removed to the tail outflow current **(Figure 1)**.

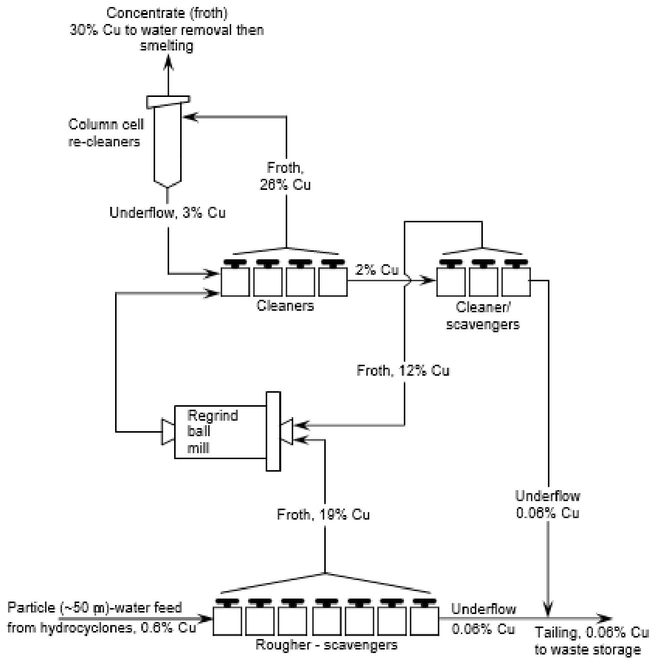


**Figure 1.** Cu ores flotation flowsheet.

Source: Schlesinger and Davenport (2011)

Copper sulphide ores must undergo froth-flotation to obtain concentrates containing around 30% of copper, as the average grades of ores mined globally range from 0.5% - 2% Cu which makes direct smelting unfeasible for economic and technical reasons (Glöser et al., 2013; International Copper Study Group, 2017).

Concentrates are latter further treated at smelting plants through pyrometallurgical processes (**Figure 2)** to achieve copper anodes, containing 99.5% of Cu. Copper is eventually refined electrochemically to achieve copper cathodes with 99.9% Cu (The London Metal Exchange, 2019).

In addition, gold, silver and other impurities in the concentrates are recovered from the anode slimes, produced at the electrorefining stage, using multiple techniques. The grade of these slimes is a direct reflection of the levels of these elements in the concentrate at the beginning of the smelting process (Schlesinger and Davenport, 2011).

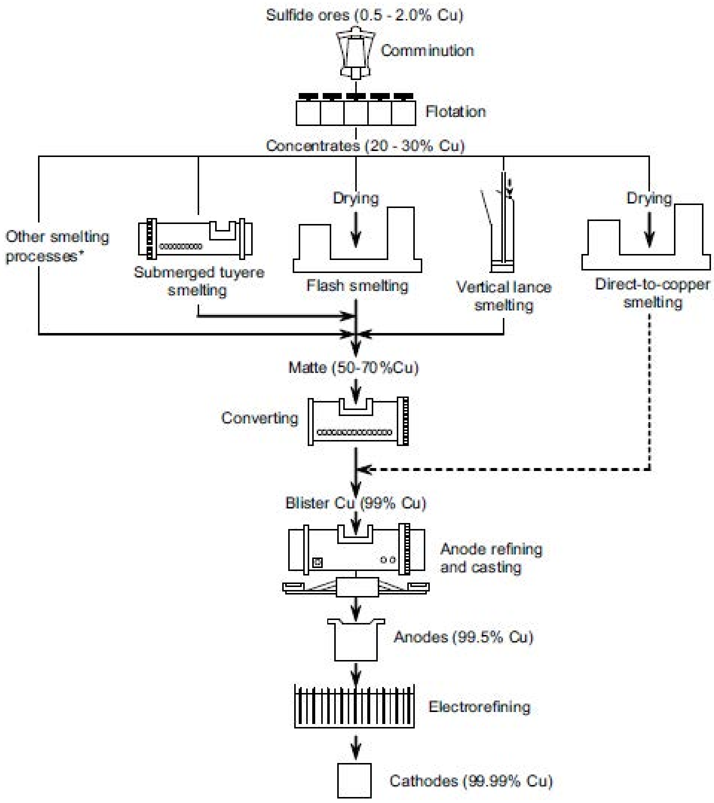


**Figure 2.** Industrial processing of copper sulphides ore to obtain copper cathodes.

Source: Schlesinger and Davenport (2011)

### 1.1.3. Copper trading

The relevance of copper trading is undeniable. In 2020 copper ores and concentrates trade globally reached 61.8 b $USD, being China the main importer and Chile the leading exporter followed by Peru (OEC, 2023). In 2016 exports of copper ores, concentrates, copper matte and cement copper increased by 1.5%, reaching 47.3 b $USD, while imports attained 43.9 b $USD (United Nations, 2016). In addition, the global mining capacity is expected to rise by 10% from the 23.5 million tonnes recorded in 2016 to 25.9 million tonnes in 2020, with smelter production having reached the record figure of 19.0 million tonnes in 2016 (International Copper Study Group, 2017).

Copper cathodes are regarded as the pure form of copper and are traded as the underlying asset of futures contracts at major physical future exchanges such as the London Metal Exchange (LME) or the New York Mercantile Exchange (COMEX). The price of copper on these exchanges is a reflection of the actual supply and demand of physical copper (Sánchez Lasheras et al., 2015), and is taken as the core reference for pricing all copper products.

Copper concentrates are usually sold directly by the mining companies that produce them to smelters and refiners that process them to obtain cathodic copper, frequently under the framework of long-term supply agreements. The absence of a reference market for concentrates generates that both producers and processors, as well as independent traders, agree on complex price fixing mechanisms, which are generally of exclusive application to each specific agreement. These mechanisms tend to be kept confidentially, as they are regarded as highly sensitive for the parties involved in concentrates trading. Nonetheless, they have broad repercussions in the financial aspects of such a relevant trade at a global scale.

Products that are less refined than copper cathodes, such as mattes or concentrates, are priced at a discount on the exchange price, while products requiring further processing than cathodes, such as copper wires or rods, are sold at the exchange reference price plus a premium. Discounts and premiums on the exchange reference prices are usually decomposed into a series of factors corresponding to the specifications of the products being traded. Discounts associated with concentrates widely vary in function of the content of the payable elements - copper, gold and silver - the excess of punishable elements, such as arsenic or bismuth, as well as the current availability of smelters demanding concentrates with those specifications, and the global supply and demand levels of copper, among other causes.

### *1.2. Objectives*

As previously mentioned, one of the main problems identified is the lack of a reference market for copper concentrates, unlike there is for cathodic copper or for other industrial or precious metals, as well as for other commodities. The absence of a specific reference market adds an additional layer of complexity to copper concentrate trading, as the market price for copper is not the only factor that affects the price of concentrates despite being the main one. Reference markets for commodities are of key importance. Apart from the ability of price discovery, market participants have also the possibility of using financial instruments to mitigate, transfer and, essentially, hedge, their risks related to commodities' high volatility.

Much previous research has primarily focused on possible modelling alternatives for reference commodity prices, such as Brennan and Schwartz (1985), Liu et al. (2017) and Dehghani and Bogdanovic (2018), who explore different alternatives to forecast copper prices at major commodity exchanges. Others, have modelled the exchanges' reference prices of some commodities to conduct applied studies, for instance Zhang et al. (2015), who employ the LBMA price of gold for the valuation of a mining project, and Guj and Chandra (2019), who model the LME copper price to apply to the real option valuation of a copper mine.

In addition, previous research have instead dodged the problem of copper concentrates pricing, focusing on cathodic copper price modelling. This has been applied to multiple projects that may be affected to a certain extent by copper price variations at some major metal futures exchange, such as the London Metal Exchange (LME). This simplification of reality, either referred to the mining business or the smelter side of the process, along with trading itself, falls short of what market participant come across on a daily basis when dealing with copper concentrates.

Therefore, we have set the main objective of this research to develop a model for copper concentrate benchmark prices, so we can set an useful reference for market participant to value their concentrates, as well as to be able to forecast potential price trends on copper concentrates. Through the analysis of available data, as well as the existent literature, this research aims to achieve a better understanding on the mechanisms for copper concentrates price setting, as well as on the agreements reached between traders, miners and smelters, along with the corporate finances of commodity traders.

On the other hand, there are also multiple examples in the literature on the finances of mining companies and on the valuation of mining projects, as well as the assessment of the economic aspects of mining investments considering the uncertainty involved. However, regarding to mineral commodities trading, specifically when treated as a standalone element, separated to the mining or smelting activities, little has been researched so far. Hence, aiming to improve our understanding of mineral commodities trading, we have set the second objective of this research the financial assessment of a copper concentrates trader through Real Options, as a case of analysis of mineral commodities trading. Treating the activity of trading in an isolated and independent way from the extractive and processing stages.

Additionally, this research pursues to provide practical, usable tools, that may be of application by market participants in their regular operations. Emphasizing on improving the current process of copper concentrate price setting, as well as developing price forecasting models, which may be especially beneficial for a better financial planning of any company that trades, process or produce copper concentrates.

### *1.3. Structure*

Following the introduction there is a chapter dedicated to the theoretical foundations in which we review the most recent advances on commodities pricing and trading, setting the theoretical framework for the development of our research. In this chapter, the methodology we have employed to develop the copper concentrates benchmark prices model is briefly explained, as well as the Real Options methodology and variables which we have resorted to for the valuation of the copper concentrates trader.

Additionally, the different concentrates' layouts that have been used to test the benchmark price model forecasting ability are also described. These concentrates have been chosen as they depict both regular trading operations, as well as most complex ones. Also, in this chapter we have set the parameters, such as the main error measures we have used, needed to develop our research and perform further testing of the models' forecasting capacity, as well as to carry out the Real Options analysis.

Chapter 3 is committed to find an accurate model that allows us to forecast copper TC/RC benchmark levels. The model should deliver reliable forecasts, while at the same time may be of practical application by industry participants and of easy integration in a latter, larger-scale copper concentrates benchmark prices model.

To find the best solution, we have compared five measures of error of different forecasts for copper TC/RC annual benchmark levels obtained using well-known models in literature: Geometric

Brownian Motion *(GBM)*, Mean-Reverting Orstein Uhlenbeck Process *(OUP)* and Linear Exponential Smoothing *(LES)*.

Three forecasting horizons were set, one, two and three years ahead, in order to compare the different models short- and longer-term forecasting capabilities. To carry out the different simulations, as well as the different models' calibration process, we have employed an historical dataset of copper TC/RC benchmark levels from 2004 to 2017.

The copper concentrates benchmark prices model is mathematically described in Chapter 4, where we have also performed a series of simulations to check its forecasting capacity. The model delivers short-term (up to one year ahead) price estimations for copper concentrates basing on independent forecast for metal prices (copper, gold and silver), as well as resorting to forecasts of main discounts (copper TC/RC and gold and silver RC), along with other relevant factors.

Both historical future contracts and spot price data have been used for copper, gold and silver from 2004 to 2018 to perform the Monte Carlo simulations to obtain concentrate prices forecasts. The results achieved in Chapter 3 on TC/RC benchmark levels have been integrated in this model to give full coherence to our research. The five concentrates described in Chapter 2 have been used to test the model's accuracy, while also employing the Mean Average Percentage Error (MAPE) as main indicator of precision.

Chapter 5, entitled '*Managing a High Uncertainty Scenario Through a Real Option Assessment: Evidence from a Copper Concentrate Trader'* is dedicated to the analysis of the copper concentrates trading business through the use of the Real Options methodology. Real Options allow to assess the uncertainties associated to the high volatility that concentrate prices experience, while also allowing to take advantage of potentially beneficial opportunities on the trading business development that may arise over time.

The trading company valuation is carried out by projecting ahead for a total of five years its market strategy, as well as its balance sheet and profits and losses statements. The concentrate benchmark price model developed in Chapter 4 is used here to obtain numerical results of the expected value of the trading company, which clearly depends of future concentrate price. Simulations have been carried out in a simplified version of the methodology proposed in Chapter 4, opting form an statistical approach for metal pricing, using the distributions of daily metal prices returns, instead of the Kalman-Filter technique which requires also futures contracts data and which increases the simulation complexity meaningfully.

To validate the proposed methodology, a single layout of concentrate has been assumed while expansion and abandonment criteria for the trader operations have been set as a function of potential profitability increases or declines as copper concentrate prices vary over time. The Real Options methodology, modelled with Monte Carlo simulations, have proven an useful and applicable tool for trading operations as well, like they had already been tested for mining operations in multiple previous research.

Finally, conclusions, limitations and future research are explained in Chapter 6. The results of our research indicate that the copper concentrate benchmark price model is sufficiently accurate for short-term price forecasts, delivering reliable estimations of copper concentrates prices for up to one year ahead. Additionally, the Linear Exponential Smoothing model is capable of providing a single forecast for TC/RC benchmark levels that may give market participants a proper indication of the potential market evolution based on preceding trends, while also being suitable to be included in the copper concentrates benchmark price model.

When applied to a real-life case, both the models and the Real Options methodology has applications of interest for the market participant to help them optimize their trading operations as well as to advert or prevent potentially loss-generating situation as a consequence of future market downturns.

On the other hand, the lack of publicly available data on commodity traders' financials, as well as on actual copper concentrates transactions, with precise details on prices and terms, presents the main limitation of our research, as well as the main future line of research of interest should further data be available.

# Chapter 2: Theoretical Foundations

## *2.1. Theoretical framework*

There is also an abundance of references in literature regarding flows of primary and secondary copper (Glöser et al., 2013; Jaunky, 2013; Mudd et al., 2013; Chen et al., 2016), which mainly focuses on analysing and quantifying where the copper used in human activities comes from, what it used for and what its transformations take place from an aggregated perspective. Many others focus their efforts in modelling and forecasting the price of copper and of copper future contracts using different methods (Kriechbaumer et al., 2014; Buncic and Moretto, 2015; Wets and Rios, 2015; Sverdrup, 2016; Liu et al., 2017; Dehghani and Bogdanovic, 2018; Guzmán and Silva, 2018).

It is also frequent to find examples in the literature where cathodic copper is taken as what mines produce, leaving out copper concentrates themselves due to their complexity to analyze them, or simply taking into consideration the copper content within the concentrates to simplify. Thus, works like Guj and Chandra (2019) use a forecast of cathodic copper prices at the London Metal Exchange (LME) and get rid of the complicated process of pricing concentrates through the assumption of a Net Smelter Value (NSV), as a percentage to apply to the value of copper in the concentrates sold by a certain mine.

Although these kinds of approaches might be valid, even desirable in certain context to specifically aimed towards a practical application, we must highlight that the reality, either regarding to mining operations, or just trading agreements, is notably different. Most mines produce concentrates containing form 20% to 40% of copper (Delbeke and Rodriguez, 2014), being its price linked to that of copper in international markets, but also to other important circumstances, as well as experiencing significant differences between mines or concentrates layouts, among other factors.

Thus, if precision is desired, one cannot assume homogenic prices for concentrates, whereas simplifications make no sense. The complex task of pricing copper concentrates implies considering both their content and the prices at reference metal markets of those metals that give value to the concentrates, such as copper, gold and silver. Also, it becomes necessary to reflect the remaining factors that, at least in an indirect way, have an impact on the price that concentrates are sold, such as the levels of concentrates supply/demand globally, or the processing capacity of the world's copper smelters at a given time.

The most common price setting method for copper concentrates prices using copper Treatment Charges/Refining Charges (TC/RC), along with other discounts and penalties, has scarcely been analysed in the literature, despite its importance for copper concentrate trading. Furthermore, TC/RC setting has not been analysed in depth, even though they represent the largest part of copper smelters' revenue. Being their levels a factor of major concern for not only smelters, but also copper mines and traders around the world.

To find an adequate forecasting model for copper TC/RC benchmark levels, in this research we have treated the historical available TC/RC benchmark data as a price timeseries, resorting to the available models for commodity price modelling of which there are abundance in the literature.
In this sense Xiong et al. (2015), compares various forecasting models after applying them to agricultural commodities, or Zhang et al. (2015) that employs a mean-reverting model for the price of gold, specifically applied to the valuation of a mining project, just like Brennan and Schwartz (1985) do for an unidentified commodity in their well-known work.

Finally, despite the great importance of trading activities within commodity markets, there is also a remarkable lack of systematic research in literature regarding this crucial aspect for commodities.

By the contrary, when the financial aspects of commodities are related to the productive or transforming stages, as well as with the extractive phases, literature becomes prolific (Saramak, 2011; Jovanović and Stanimirović, 2012; Jovanović et al., 2013; Boulamanti and Moya, 2016).

Additionally, the copper mining is also the research topic of many works, where not only the technical aspects are analysed, but also those closer to the financial optimization of copper mining operations, putting special attention to the uncertainties related to the variability of metals prices over time. Works like Dehglani et al. (2014) approach the problem from the Net Present Value (NPV) point of view of a mining project applying different techniques to forecast the price of copper along the proposed time horizon.

On the other hand, since Brennan & Schwartz (1985) first employed Real Options to value mining assets, this has become one of the main approaches, if not the main one, to perform the valuation of mining projects. Real Options share a similar view with financial options when valuing tangible assets under. They allow the conversion of uncertainty related to a mining project to management alternative based on probable situations.

As summarized by Savolainen (2016) in its extensive historical review, many recent works used Real Options to value metallic mining projects with different approaches when taking into consideration those factors considered to be generators of uncertainty, such as price fluctuations or exchange rates. Hence, works like Botín et al. (2012) pursues to find the optimal design for a copper mine while trying to maximize the NPV of the project using Real Options to account for the effect that horizontal dilution has on the deposit.

Another series of works have instead used Real Options to examine the complex process of mine design, considering the significant variation that the profitability of the project is prone to experience as the price of metals move. In Thompson and Bar (2014) Real Options has been used as a tool to determine the optimal cut-off grade to set the mine planning of an open-pit mine, while assuming the stochastic behaviour of the prices of any mineral.

Equally, Asad (2007) resorts to Real Options as a tool to develop a decision policy for the active design of a copper open-pit mine, looking to maximize its NPV, while simultaneously considering the dynamic evolution of prices as well as an increase of costs throughout all its lifetime.

It becomes evident that in high uncertainty environments, particularly in those related to extractive industries like mining, the valuation of investment projects is a complex challenge. In the first place, widely applied traditional methos like Discounted Cash Flows (DCF) have important deficiencies as they require a high level of concretion of future scenarios. This, due to the nature of certain types of investments, either requires making too many assumptions or simply unfeasible. As can be seen in Samis et al. (2005), Real Options offer clear advantages when the uncertainty of an investment is high, provided their ability to treat with flexibility natural resource assets, being capable of reflecting the uncertainty of future cash flows in an efficient way.

The advantages that the Real Options method show when valuing mining projects, and especially when tackling the variability of metal prices and deposit uncertainties are also explained in Abdel Sabour et al. (2008), which compares the Real Option method to traditional discounted cash flow methods by making a comparative ranking of twelve possible designs of an open-pit copper mine and a gold mine. The results of this analysis show that mine designs chosen following the Real Option methodology outperforms those obtained using the other methodology.

## *2.2. Copper concentrates prices*

Market participants lack a market-wide accepted reference for copper concentrates pricing. The only possible reference available is the exchanges' price for cathodic copper for both current and future expected prices, as well as certain information on part of the discounts agreed by the largest miners and smelters at the LME Week. This is usually provided by some accredited specialized consultant, such as Platts or Wood Mackenzie.

Copper concentrates supply agreements set specific mechanisms to price the copper concentrates to be traded according to their chemical specifications. Usually, most pricing systems establish the price of copper concentrates as the sum of the payable elements present in the concentrates minus the deductions and the penalties (Soderstrom, 2008; Seabridge Gold, 2010; Teck, 2012, 2015).

In this research we have followed the same logic to model copper concentrate prices, treating payable elements separately from discounts and punishable elements. Thus, determining the price of concentrates as the sum of the amount paid for all payable elements, which is the amount paid for the copper, gold and silver content in the concentrate, *M*, minus the sum of deductions applied by smelters and refiners, *D*, along with the sum of penalties, *P*, due to the presence of impurities and punishable elements that worsen the final properties of cathodic copper or make the concentrates harder to process.

$$C = M - D - P$$

### 2.2.1. Payable metal terms

Contract terms for payable metals may vary depending on the market conditions as well as on the layout of the concentrates. **Table 1** shows typical recovery factors and minimum deductions for copper and silver, which have also been used in this research.

**Table 1**
Recovery factors (r) and minimum deductions (MD) for copper and silver.

| | *r* | *MD* |
|---|---|---|
| Cu | 96.5% | 1.0% |
| Ag | 90.0% | 30 g/dmt |

Source: Own elaboration. Based on Teck (2015), Ghaffari et al. (2016) and OECD et al. (2017).

In addition, recovery factors and minimum deductions for gold are shown in **Table 2**. Unlike copper and silver, payable gold usually increases as its grade rises in concentrates.

**Table 2**
Recovery factors (*r*) and minimum deductions (*MD*) for gold.

| *g* (g/dmt) | *r* | *MD* (g/dmt) |
|---|---|---|
| < 1 | 00.00% | 1 |
| 1 – 3 | 90.00% | 1 |
| 3 – 5 | 93.00% | 1 |
| 5 – 7 | 95.00% | 1 |
| 7 – 10 | 96.50% | 1 |
| 10 – 20 | 97.00% | 1 |
| 20 – 30 | 97.50% | 1 |
| > 30 | 97.75% | 1 |

Source: Own elaboration. Based on Teck (2015), Ghaffari et al. (2016) and OECD et al. (2017).

**2.2.2. Discounts**

The main discounts involved in copper concentrates price determination are copper Treatment Charges and Refining Charges, most commonly known as TC/RC, along with gold and silver Refining Charges. These discounts comprise the bulk of deductions in most copper concentrate sale agreement.

TC/RC levels for copper concentrates continuously vary throughout the year, relying on private and individual agreements between miners, traders and smelters worldwide. Nonetheless, the TC/RC benchmark fixed during the *LME week* each October is used by market participants as the main reference to set actual TC/RC levels for each supply agreed upon for the following year. Hence, the year's benchmark TC/RC is taken here as a good indicator of a year's TC/RC average levels.

Analysed time series of benchmark TC/RC span from 2004 through to 2018, as shown in **Table 3,** as well as the source each value was obtained from. We have not intended to reflect the continuous variation of TC/RC for the course of any given year, though we have however considered benchmark prices alone as we intuitively assume that annual variations of actual TC/RC in contracts will eventually be reflected in the benchmark level that is set at the end of the year for the year to come.

Historical TC/RC benchmark levels in **Table 3** have been used as the in-sample dataset for one-step ahead forecasts in Díaz-Borrego et al. (2019), which seeks the most suitable forecasting model for copper TC/RC benchmark levels among frequently-used options for commodity prices.

Copper TC and RC normally maintain a 10:1 relation with different units, with TC being expressed in US dollars per metric tonne and RC in US cents per pound of payable copper content in concentrates. In fact, historical benchmark data shows that only in 2010 those values of TC and RC did not conform to this relation, though it did remain close to it (46.5/4.7).

**Table 3**
Copper TC/RC benchmark levels.

| | TC (USD/dmt) | RC (USc/lb) |
|---|---|---|
| 2004 | 45 | 4.5 |
| 2005 | 85 | 8.5 |
| 2006 | 95 | 9.5 |
| 2007 | 60 | 6.0 |
| 2008 | 45 | 4.5 |
| 2009 | 75 | 7.5 |
| 2010 | 46.5 | 4.7 |
| 2011 | 56 | 5.6 |
| 2012 | 63.5 | 6.35 |
| 2013 | 70 | 7.0 |
| 2014 | 92 | 9.2 |
| 2015 | 107 | 10.7 |
| 2016 | 97.35 | 9.735 |
| 2017 | 92.50 | 9.25 |
| 2018 | 82.25 | 8.225 |

Note: Data available free of charge. Source: Johansson (2007), Svante (2011), Teck (2012), Willbrandt and Faust (2013, 2014), Aurubis AG (2015), Drouven and Faust (2015), Shaw (2015), Aurubis AG (2016), EY (2017), Schachler (2017), Nakazato (2017).

On the other hand, gold and silver RC have been assumed to remain constant, since smelters tend to apply around 6.00 to 8.00 USD per troy ounce of payable gold content and 0.40 to 0.50 USD per troy ounce of payable silver content (Seabridge Gold, 2010; Teck, 2012, 2015; Ghaffari et al., 2016). Gold and silver RC may be seen in **Table 4**.

**Table 4**
Gold and silver refining charges values applied.

| | RC (USD/troz) |
|---|---|
| Au | 7.50 |
| Ag | 0.45 |

Source: Own elaboration. Based on Seabridge Gold (2010), Teck (2012), Teck (2015) and Ghaffari et al. (2016).

### 2.2.3. Penalty terms

Punishable elements are generally undesired in concentrates since some of them may hinder the concentrate smelting and refining process, such as Chlorine, which may condense as hydrochloric acid and cause corrosion problems. Others may seriously deteriorate the cathode's chemical, mechanical, thermal or electrical properties, such as Arsenic or Antimony which reduce copper conductivity, or could even represent an important hazard for human health and the environment if they are not properly managed or mitigated, such as Cadmium, that is a well-known carcinogen, or Lead, which is highly contaminating as well as toxic to humans (Organisation for Economic Co-operation and Development et al., 2017).

Normally, each country custom's authority sets the maximum allowance of the most toxic elements in imported concentrates See Announcement No. 106 of 2017 (AQSIQ, 2017). Nonetheless, smelters tend to have stricter requirements than authorities, depending on their ability to process these elements, considerably reducing the market for most so-called 'complex' concentrates, which sometimes are only accepted by traders for blending purposes with cleaner concentrates. Penalties applied to concentrates have also assumed to remain at constant levels for each element based on limits and unitary penalties outlined on reports by Ghaffari et al. (2016) and the OECD (2017). These penalties have hardly changed in recent years, as may be seen if current levels are compared with older reports, such as Ghaffari el al.'s (2016) and Seabridge Gold's (2010), which both assess the feasibility of the same mining project. Yet a certain downward pressure on the most environmentally harmful elements' limits is observable as well as a slight increase in some of the unitary penalties imposed. The parameters employed to determine penalties due to the excess of punishable elements may be seen in **Table 5**.

**Table 5**
Penalty parameters for punishable elements.

| | $F$ | $h$ | $\delta$ (USD) |
|---|---|---|---|
| Arsenic (As) | 0.10% | 0.10% | 3.00 |
| Antimony (Sb) | 0.10% | 0.10% | 3.50 |
| Bismuth (Bi) | 0.03% | 0.01% | 4.00 |
| Cadmium (Cd) | 10 ppm | 10 ppm | 3.00 |
| Chlorine (Cl) | 300 ppm | 100 ppm | 2.00 |
| Fluorine (F) | 300 ppm | 100 ppm | 1.50 |
| Lead (Pb) | 0.50% | 1.00% | 2.50 |
| Mercury (Hg) | 10 ppm | 10 ppm | 2.00 |
| Nickel (Ni) + Cobalt (Co) | 0.50% | 0.10% | 1.00 |
| Selenium (Se) | 0.05% | 0.01% | 4.00 |
| Tellurium (Te) | 0.03% | 0.01% | 4.50 |
| Zinc (Zn) | 2.00% | 1.00% | 2.50 |

Source: Own elaboration. Based on Seabridge (2010), Ghaffari (2016), OECD (2017).

### *2.3. Concentrates layout*

According to the statistical assessment made by Delbeke and Rodriguez (2014), 50% of world-wide copper concentrates grade less than 26.67% of copper, while 30% of the world's production grades more than 28.452% Cu. Also, according to the statistical distribution of punishable elements, 70% of world's copper concentrates can be regarded as clean.

On the other hand, less than 10% of all concentrates produced worldwide grade over 34% Cu, generally with significant levels of some impurities such as lead or arsenic, which qualifies them as complex concentrates. By the contrary, high values of precious metals are rare in concentrates, though may be frequent to some extent in those produced by some mines, such as CODELCO's Chuquicamata (CODELCO, 2018).

For the purpose of this research, we have selected five sample concentrates that may be representative of the full scope of possibilities according to the statistical distribution exposed by Delbeke and Rodriguez (2014). Hence being able to analyze both regular, good quality concentrates, as well as dirty concentrates, with too high levels of impurities, or precious concentrates with meaningful levels of either gold or silver. The layouts of the five concentrates considered in this study are shown in **Tables 6** and **7**.

**Table 6**
Concentrates analysed layout. Payable elements.

| | *Cu* | *Au* | *Ag* |
|---|---|---|---|
| C1 | 26.62% | 0.7 g/dmt | 18.4 g/dmt |
| C2 | 28.10% | 38.1 g/dmt | 51.6 g/dmt |
| C3 | 27.60% | 25.7 g/dmt | 187.0 g/dmt |
| C4 | 28.33% | 6.7 g/dmt | 81.4 g/dmt |
| C5 | 36.71% | 2.4 g/dmt | 48.3 g/dmt |

Source: Own elaboration. Based on Delbeke and Rodriguez (2014) for penalty levels.

**Table 7**
Concentrates analysed layout. Punishable elements.

| | *As* | *Sb* | *Bi* | *Cd* | *Cl* | *F* | *Pb* | *Hg* | *Ni* | *Co* | *Se* | *Te* | *Zn* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C1 | 0.002% | 0.002% | 0.003% | 50ppm | 49ppm | 210ppm | 0.04% | 0ppm | 0.004% | 0.009% | 0.002% | 0.001% | 0.22% |
| C2 | 0.110% | 0.010% | 0.004% | 40ppm | 105ppm | 128ppm | 0.14% | 6ppm | 0.002% | 0.005% | 0.004% | 0.001% | 0.62% |
| C3 | 0.272% | 0.042% | 0.033% | 28ppm | 332ppm | 258ppm | 1.47% | 1ppm | 0.010% | 0.024% | 0.035% | 0.021% | 1.87% |
| C4 | 0.62% | 0.21% | 0.051% | 73ppm | 44ppm | 105ppm | 3.05% | 4ppm | 0.000% | 0.003% | 0.021% | 0.005% | 3.12% |
| C5 | 0.48% | 0.32% | 0.066% | 41ppm | 512ppm | 405ppm | 2.91% | 0ppm | 0.000% | 0.008% | 0.012% | 0.040% | 4.75% |

Source: Own elaboration. Based on Delbeke and Rodriguez (2014) for penalty levels.

### *2.4. Copper concentrates price modelling*

The fundamental problem underlaying beneath the surface of our research is the lack of reference for copper concentrates prices which market participants can rely to. Thus, the cornerstone of our research is indeed the development of a copper concentrates price model which can be easily used as a benchmark tool, that can be implemented to value any copper concentrate, while also capable of providing reliable forecasts for prices.

Throughout this research we have employed different approaches to obtain numerical results out of the copper concentrates pricing model, depending on whether the forecasting accuracy of the model was to be tested or if, on the other hand, we were to perform a practical valuation of a trader's copper concentrates business. The simulating workload was thus adjusted in order to focus in each specific objective's need at each time.

Thus, at time t, the price of the ton of concentrate can be expressed as:

$$C_t = M_t - D_t - P_t$$

Though copper concentrates may contain multiple payable elements apart from copper, should there be some gold and silver are the most common. Metals in concentrates are paid at market price, though the full content is not usually paid, as smelters are normally not capable of recovering all the content. Hence, the price of payable metals in the concentrates is calculated as the payable content (*pp*) multiplied by its market price at time t ($S_t$).

$$M_t = Cu_t + Au_t + Ag_t$$

$$Cu_t = pp_{Cu} \times S_t^{Cu}$$

$$Au_t = pp_{Au} \times S_t^{Au}$$

$$Ag_t = pp_{Ag} \times S_t^{Ag}$$

Additionally, the payable copper content in concentrates is also dependent on the metal grade ($g$), multiplied by a recovery factor ($r$), subject to a minimum deduction (*MD*).

$$pp_{Cu} = \begin{cases} g_{Cu} - MD_{Cu}, & g_{Cu} - MD_{Cu} < g_{Cu} \times r_{Cu} \\ g_{Cu} \times r_{Cu}, & g_i - MD_i \geq g_{Cu} \times r_{Cu} \end{cases} \quad \forall\ g_{Cu} \geq MD_{Cu}$$

$$pp_{Cu} = 0\ \forall\ g_{Cu} < MD_{Cu}$$

On the other hand, precious metals in concentrates are paid only if their content is above the minimum deduction levels, otherwise they are priced at zero. The final payable content of precious metals is determined by multiplying the metal grade by a recovery factor.

$$pp_i = \begin{cases} 0\ , & g_i \leq MD_i \\ g_i \times r_i, & g_i > MD_i \end{cases}$$

$$i = Au, Ag$$

In addition, Dt is the sum of deductions applied, including copper Treatment Charges (TC) and copper Refining Charges ($R^{Cu}$), as well as gold Refining Charges ($R^{Au}$) and silver Refining Charges ($R^{Ag}$).

$$D_t = TC_t + R_t^{Cu} + R_t^{Au} + R_t^{Ag}$$

Discounts linked to copper concentrates pricing, of which copper TC/RC represent the largest share, are normally set in each concentrate supply contract agreement between the two parties of the contract. Copper TC are quoted in US Dollars per tonne of concentrate, usually expressed on a dry basis, while copper RC is quoted in US cents of dollars per pound of payable copper content.

In addition, gold and silver RC are applicable as long as there is gold or silver payable content in the concentrate, gold RC being quoted in USD per troy ounce of payable gold content and silver RC in USD per gram of payable silver content.

$$R_t^i = RC_t^i \times pp_i$$

Finally, $P_t$ is the overall penalty for an n number of punishable elements in the concentrate which negatively affect its quality as well as the chemical, mechanical and electrical properties of the copper products made with that concentrate. These penalties are calculated as a function of the excessive content of the punishable element beyond the penalty-free limit, below which the concentrate would be exempt of penalty for that element.

$$P_t = \sum_{i}^{n} p_i \ \forall \ t$$

$$p_i = \begin{cases} 0, & g_i < F_i \\ (g_i - F_i) \times \rho_i, & g_i \geq F_i \end{cases}$$

Where $g_i$ is the content of the punishable element i, Fi is the penalty-free limit content for that element and $\rho_i$ is the penalty per unit of excessive content of the punishable element i beyond the penalty-free limit. The unitary penalty is set as the nominal penalty $\delta_i$, divided by the nominal punishable increment of element i, $h_i$.

$$\rho_i = \frac{\delta_i}{h_i} \ \forall \ i$$

# Chapter 3: Looking for an accurate forecasting of Copper TC/RC Benchmark Levels

### *3.1. Introduction*

There is a lack of research about the price at which mines sell copper concentrate to smelters, while numerous investigations have instead focused on forecasting copper prices. Smelters however obtain the copper they sell from the concentrate that most mines produce by processing the ore which they have extracted. It therefore becomes necessary to thoroughly analyse the price at which smelters buy the concentrates from the mines, besides the price at which they sell the copper. In practice, this cost is set by applying discounts to the price of cathodic copper, the most relevant being those corresponding to the smelters' benefit margin (*Treatment Charges-TC* and *Refining Charges-RC*).

The valuation of copper concentrates is a recurrent task undertaken by miners or traders following processes in which market prices for copper and other valuable metals such as gold and silver are involved, as well as relevant discounts or coefficients that usually represent a major part of the revenue obtained for concentrate trading, smelting or refining. The main deductions are applied to the market value of the metal contained by concentrates such as *Copper Treatment Charges (TC)*, *Copper Refining Charges (RC)*, *the Price Participation Clause (PP)* and *Penalties for Punishable Elements* (Soderstrom, 2008).

These are fixed by the different parties involved in a copper concentrate long-term or spot supply contract, where TC/RC are fixed when the concentrates are sold to a copper smelter/refinery. The sum of TC/RC is often viewed as the main source of revenue for copper smelters along with copper premiums linked to the selling of copper cathodes. Furthermore, TC/RC deductions pose a concern for copper mines as well as a potential arbitrage opportunity for traders, whose strong financial capacity and in-depth knowledge of the market make them a major player (Crundwell, 2008, p. 491).

Due to their nature, TC/RC are discounts normally agreed upon taking a reference which is traditionally set on an annual basis at the negotiations conducted by the major market participants during *LME Week* every October and, more recently, during the *Asia Copper Week* each November, an event that is focused more on Chinese smelters. The TC/RC levels set at these events are usually taken as benchmarks for the negotiations of copper concentrate supply contracts throughout the following year.

Thus, as the year goes on, TC/RC average levels move up and down depending on supply and demand, as well as on concentrate availability and smelters' available capacity. Consultants, such as *Platts, Wood Mackenzie* and *Metal Bulletin,* regularly carry out their own market surveys to estimate the current TC/RC levels. Furthermore, *Metal Bulletin* has created the first TC/RC index for copper concentrates ("Metal Bulletin TC/RC Index," 2017).

### *3.2. The need for accurate forecasts of Copper TC/RC*

The information available for market participants may be regarded as sufficient to draw an accurate assumption of market sentiment about current TC/RC levels, but not enough to foresee potential market trends regarding these crucial discounts, far less as a reliable tool which may be ultimately applicable by market participants to their decision-making framework or their risk-management strategies. Hence, from an organisational standpoint, providing accurate forecasts of copper TC/RC benchmark levels, as well as an accurate mathematical model to render these forecasts, is a research topic that we have explored in depth. This is a question with undeniable economic implications for traders, miners and smelters alike, due to the role of TC/RC in the copper trading revenue stream.

As a first step, our research seeks to determine an appropriate forecasting technique for TC/RC benchmark levels for copper concentrates that meets the need of reliability and accuracy. To achieve these three different and frequently applied techniques have been preselected from among the options available in the literature. Then, their forecasting accuracy at different time horizons have been tested and compared. These techniques (Geometric Brownian Motion -GBM-; the Mean Reversion -MR- ;Linear Exponential Smoothing -LES-), have been chosen primarily because they are common in modelling commodities prices and their future expected behaviour, as well as in stock indices' predictive works, among other practical applications (Pinheiro and Senna, 2016; Savolainen, 2016; Zhang et al., 2017; Hloušková et al., 2018).

The selection of these models is further justified by the similarities shared by TC/RC with indices, interest rates or some economic variables that these models have already been applied to. Also in our view, the predictive ability of these models in commodity prices such as copper is a major asset to take them into consideration. The models have been simulated using historical data of TC/RC annual benchmark levels from 2004 to 2017 agreed upon during the LME Copper Week. The dataset employed has been split into two parts, with two thirds as the in-sample dataset and one third as the out-of-sample one.

With our study we aim to provide a useful and applicable tool to all parties involved in the copper trading business to forecast potential levels of critical discounts to be applied to the copper concentrates valuation process. To carry out our research, we have based ourselves on the following premises: 1) GBM would deliver good forecasts if copper TC/RC benchmark levels vary randomly over the years, 2) a mean-reverting model, such as the Ornstein-Uhlenbeck Process (OUP), would deliver the best forecasts if TC/RC levels were affected by market factors and consequently they move around a long-term trend, 3) a moving average model would give a better forecast than the other two models if there were a predominant factor related to precedent values affecting the futures ones of benchmark TC/RC.

In addition, we have also studied the possibility that a combination of the models could deliver the most accurate forecast as the time horizon considered is increased, since there might thus be a limited effect of past values, or of sudden shocks, on future levels of benchmark TC/RC. So, after some time, TC/RC levels could be 'normalised' towards a long-term average.

The remainder of this chapter is structured as follows: Section 3 revises the related work on commodity discounts forecasting and commodity prices forecasting techniques, as well as different forecasting methods; Section 4 presents the reasoning behind the choice of each of the models employed, as well as the historic datasets used to conduct the research and the methodology followed; Section 5 shows the results of the simulations of the different models, comparing the different error measures used to evaluate the best forecasting alternative for TC/RC benchmark levels amongst all those presented; Section 6 contains the conclusions of the chapter and discusses the results.

### *3.3. Related Work*

The absence of any specific method in the specialised literature in relation to copper TC/RC leads us to revisit previous literature in order to determine the most appropriate model to employ for our purpose, considering those that have already been used in commodity price forecasting as the logical starting point due to the application similarities that they share with ours.

Commodity prices and their forecasting have been a topic intensively analysed in much research. Hence, there are multiple examples in literature with an application to one or several commodities, such as Xiong et al. (2015), where the accuracy of different models was tested to forecast the interval of agricultural commodity future prices; Shao and Dai (2018), whose work employs the Autoregressive Integrated Moving Average (ARIMA) methodology to forecast food crop prices such as wheat, rice and corn; and Heaney (2002), who tests the capacity of commodities future prices to forecast their cash price were the cost of carry to be included in considerations, using the LME Lead contract as an example research.

**Table 8** summarizes similar research in the field of commodity price behaviours.

**Table 8**
Summary of previous literature research

| AUTHOR | RESEARCH |
|---|---|
| Shafiee & Topal (2010) | Validates a modified version of the long-term trend reverting jump and dip diffusion model for forecasting commodity prices and estimates the gold price for the next 10 years using historical monthly data. |
| Li *et al.* (2012) | Proposes an ARIMA-Markov Chain method to accurately forecast mineral commodity prices, testing the method using mineral molybdenum prices. |
| Issler *et al.*(2014) | Investigates several commodities' co-movements, such as Aluminium, Copper, Lead, Nickel, Tin and Zinc, at different time frequencies, and uses a bias-corrected average forecast method proposed by Issler and Lima (Issler and Lima, 2009) to give combined forecasts of these metal commodities employing RMSE as a measure of forecasting accuracy. |
| Hamid & Shabri (2017) | Models palm oil prices using the Autoregressive Distributed Lag (ARDL) model and compares its forecasting accuracy with the benchmark model ARIMA. It uses an ARDL bound-testing approach to co-integration in order to analyse the relationship between the price of palm oil and its determinant factors. |
| Duan *et al.* (2018) | Predicts China's crude oil consumption for 2015-2020 using the fractional-order FSIGM model. |
| Brennan & Schwartz (1985) | Employs the Geometric Brownian Motion (GBM) to analyse a mining project's expected returns assuming it produces a single commodity. |
| McDonald & Siegel (1986) | Uses GBM to model the random evolution of the present value of an undefined asset in an investment decision model. |
| Zhang *et al.* (2015) | Models gold prices using the Ornstein-Uhlenbeck Process (OUP) to account for a potentially existent long-term trend in a Real Option Valuation of a mining project. |
| Sharma (2016) | Forecasts gold prices in India with the Box Jenkins ARIMA method. |

From the summary above, it is seen that moving average methods have become increasingly popular in commodity price forecasting. The most broadly implemented moving average techniques are ARIMA and Exponential Smoothing. Both consider the entire time series data as well as do not assign the same weight to past values than those closer to the present as they are seen as affecting greater to future values.

The Exponential Smoothing models' predictive accuracy has been tested by Makridakis and Hibon (1979), concluding that there are small differences between them (Exponential Smoothing) and ARIMA models. Also, GBM and MR models have been intensively applied to commodity price forecasting. Nonetheless, MR models present a significant advantage over GBM models which allows them to consider the underlying price trend.

This advantage is of particular interest for commodities that, according to Dixit and Pindyck (1994) – pp. 74, regarding the price of oil: "*in the short run, it might fluctuate randomly up and down (in responses to wars or revolutions in oil producing countries, or in response to the strengthening or weakening of the OPEC cartel), in the longer run, it ought to be drawn back towards the marginal cost of producing oil. Thus, one might argue that the price of oil should be modelled as a mean-reverting process.*"

### *3.4. Methodology*

This section presents both the justification of the models chosen to forecast copper TC/RC benchmark levels, as well as the core reference dataset used as inputs for each model compared in the methodology. The dataset employed comprises publicly available annual benchmark levels for copper TC/RC from 2004 to 2017, which are taken as market-wide reference to agree each years' contracts specific TC/RC. The steps followed to carry out an effective comparison in terms of the forecasting ability of each model are also laid out herein.

### 3.4.1. Models in Methodology

**Geometric Brownian Motion (GBM)**

GBM has been used in much earlier research as a way of modelling prices that are believed not to follow any specific rule or pattern, hence seen as *random*. Black and Scholes (Black and Scholes, 1973) first used GBM to model stock prices and since then others have used it to model asset prices as well as commodities, these being perhaps the most common of all, in which prices are expected to increase over time, as does their variance (Zhang et al., 2017). Hence, following our first premise, concerning whether TC/RC might vary randomly, there should not exist a main driving factor that would determine TC/RC future benchmark levels and therefore GBM could to a certain extent be a feasible model for them.

GBM can be written as a generalisation of a Wiener, a continuous time-stochastic Markov process, with independent increments and whose changes over any infinite interval of time are normally distributed, with a variance that increases linearly with the time interval (Dixit and Pindyck, 1994).process:

$$dx = \alpha x dt + \sigma x dz$$

Where, according to Marathe and Ryan (Marathe and Ryan, 2005), the first term is known as the Expected Value, whereas the second is the Stochastic component, with α being the drift parameter and σ the volatility of the process. Also, *dz* is the Wiener process which induces the abovementioned stochastic behaviour in the model:

$$dz = \epsilon_t \sqrt{dt} \rightarrow \epsilon_t \sim N(0,1)$$

The GBM model can be expressed in discrete terms according to Equation 3:

$$\Delta x = x_t - x_{t-1} = \alpha x_{t-1}\Delta t + \sigma x_{t-1}\epsilon\sqrt{\Delta t}$$

In GBM percentage changes in $x$, $\Delta x/x$ are normally distributed, thus absolute changes in $x$, $\Delta x$ are *lognormally* distributed. Also, the expected value and variance for $x(t)$ are:

$$\mathbb{E}[x(t)] = x_0 e^{\alpha t}$$

$$var[x(t)] = x_0^2 e^{2\alpha t}(e^{\sigma^2 t} - 1)$$

**Orstein-Uhlenbeck Process (OUP)**

Although GBM or 'random walk' may be well suited to modelling immediate or short-term price paths for commodities, or for TC/RC in our case, it lacks the ability to include the underlying long-term price trend should we assume that there is one. Thus, in accordance with our second premise on benchmark TC/RC behaviour, levels would move in line with copper concentrate supply and demand as well as the smelters' and refineries' available capacity to transform concentrates into metal copper. Hence, a relationship between TC/RC levels and copper supply and demand is known to exist and, therefore, is linked to its market price, so to some extent they move together coherently. Therefore, in that case, as related works on commodity price behaviour such as Foo, Bloch and Salim (Foo et al., 2018) do, we have opted for the MR model, particularly the OUP model.

The OUP process was first defined by Uhlenbeck and Orstein (Uhlenbeck and Ornstein, 1930) as an alternative to the regular Brownian Motion to model the velocity of the diffusion movement of a particle that accounts for its losses due to friction with other particles. The OUP process can be regarded as a modification of Brownian Motion in continuous time where its properties have been changed[1] (Tresierra Tanaka and Carrasco Montero, 2016).

[1] Stationary, Gaussian, Markov and stochastic process.

These modifications cause the process to move towards a central position, with a stronger attraction the further it is from this position. As mentioned above, the OUP is usually employed to model commodity prices and is the simplest version of a mean-reverting process (Dixit and Pindyck, 1994):

$$dS = \lambda(\mu - S)dt + \sigma dW_t$$

Where *S* is the level of prices, $\mu$ the long-term average to which prices tend to revert and $\lambda$ the speed of reversion. Additionally, in a similar fashion to that of the GBM, $\sigma$ is the volatility of the process and $dW_t$ is a Wiener process with an identical definition. However, in contrast to GBM, time intervals in OUP are not independent since differences between current levels of prices, *S*, and long-term average prices, $\mu$, make the expected change in prices, *dS*, more likely either positive or negative.

The discrete version of the model can be expressed as follows:

$$S_t = \mu(1 - e^{-\lambda\Delta t}) + e^{-\lambda\Delta t}S_{t-1} + \sigma\sqrt{\frac{1-e^{-2\lambda\Delta t}}{2\lambda}}dW_t$$

Where the expected value for $x(t)$ and the variance for $(x(t) - \mu)$ are:

$$\mathbb{E}[x(t)] = \mu + (x_0 - \mu)e^{-\lambda t}$$

$$var[x(t) - \mu] = \frac{\sigma^2}{2\lambda}\left(1 - e^{-2\lambda t}\right)$$

It can be derived from previous equations that as time increases prices will tend to long-term average levels, $\mu$. In addition, with large time spans, if the speed of reversion, $\lambda$ becomes high, variance tends to 0.

On the other hand, if the speed of reversion is 0 then $var[x(t)] \to \sigma^2 t$ , making the process a simple Brownian Motion.

### Holt's Linear Exponential Smoothing (LES)

Both GBM and MR are Markov processes which means that future values depend exclusively on the current value, while the remaining previous time series data are not considered. On the other hand, moving average methods employ the average of a pre-established number of past values in different ways, evolving over time, so future values do not rely exclusively on the present, hence behaving as though they had only a limited memory of the past.

This trait of the moving average model is particularly interesting when past prices are believed to have a certain, though limited, effect on present values, which is another of the premise for this research. Existing alternatives of moving average techniques pursue considering this 'memory' with different approaches. As explained by Kalekar (2004), Exponential Smoothing is suitable only for the behaviours of a specific time series, thus Single Exponential Smoothing (SES) is reasonable for short-term forecasting with no specific trend in the observed data, whereas Double Exponential Smoothing or Linear Exponential Smoothing (LES) is appropriate when data shows a cyclical pattern or a trend. In addition, seasonality in observed data can be computed and forecasted through the usage of Exponential Smoothing by the Holt-Winters method, which adds an extra parameter to the model to handle this characteristic.

Linear Exponential Smoothing models are capable of considering both levels and trends at every instant, assigning higher weights in the overall calculation to values closer to the present than to older ones. LES models carry that out by constantly updating local estimations of levels and trends with the intervention of one or two smoothing constants which enable the models to dampen older value effects. Although it is possible to employ a single smoothing constant for both the level and the trend, known as Brown's LES, to use two, one for each, known as Holt's LES, is usually preferred since Brown's LES tends to render estimations of the trend 'unstable' as suggested by authors such as Nau (2014). Holt's LES model comes defined by the level, trend and forecast updating equations, each of these expressed as follows, respectively:

$$L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + T_{t-1})$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1}$$

$$\hat{Y}_{t+k} = L_t + kT_t$$

With $\alpha$ being the first smoothing constant for the levels and $\beta$ the second smoothing constant for the trend. Higher values for the smoothing constants imply that either levels or trends are changing rapidly over time, whereas lower values imply the contrary. Hence, the higher the constant, the more uncertain the future is believed to be.

### 3.4.2. Models Comparison Method

The works referred to in the literature usually resort to different measures of errors to conduct the testing of a model's forecasting capacity regardless of the specific nature of the forecasted value, be it cotton prices or macroeconomic parameters. Thus, the most widely errors used include *Mean Squared Error (MSE), Mean Absolute Deviation (MAD), Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE)* (Makridakis et al., 1982; Heaney, 2002; Kalekar, 2004; Khashei and Bijari, 2010, 2011; Shafiee and Topal, 2010; Gómez-Valle and Martínez-Rodríguez, 2013; Choudhury and Jones, 2014; Xiong et al., 2015). These error measures come defined as follows:

**Mean Square Error (MSE)**

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{Y}_i - Y_i)^2$$

Where $\hat{Y}_i$ are the forecasted values and $Y_i$ those observed.

**Mean Absolute Deviation (MAD)**

$$MAD = \frac{1}{n}\sum_{i=1}^{n}\left|\hat{Y}_i - \bar{Y}\right|$$

Where $\hat{Y}_i$ are the forecasted values and $\bar{Y}$ the average value of all the observations.

**Mean Absolute Percentage Error (MAPE)**

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{\left|\hat{Y}_i - Y_i\right|}{Y_i}$$

The above formula is expressed in parts-per-one and is the one used in the calculations conducted here. Hence, multiplying the result by 100 would deliver percentage outcomes.

**Root Mean Square Error (RMSE)**

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\hat{Y}_i - Y_i)^2}$$

In this research Geometric Brownian Motion (GBM), Ornstein-Uhlenbeck Process (OUP) and Linear Exponential Smoothing (LES) models have been used to forecast annual TC/RC benchmark levels, using all the four main measures of error mentioned above to test the predictive accuracy of these three models. The GBM and OUP models have been simulated and tested with different step sizes, while the LES model has been analysed solely using annual time steps.

The GBM and OUP models were treated separately to the LES model, thus GBM and OUP were simulated using Monte Carlo (MC) simulations, whereas LES forecasts were simple calculations. Monte Carlo simulations (MC) were carried out using Matlab software to render the pursued forecasts of GBM and OUP models, obtaining 1000 simulated paths for each step size. MC simulations using monthly steps for the 2013 – 2017 timespan were averaged every 12 steps to deliver year forecasts of TC/RC benchmark levels.

On the other hand, forecasts obtained by MC simulations taking annual steps for the same period were considered as year forecasts for TC/RC annual benchmark levels without the need for extra transformation. Besides, LES model forecasts were calculated at different time horizons to be able to compare the short-term and long-term predictive accuracy. LES forecasts were obtained for one-year-ahead, hence using known values of TC/RC from 2004 to 2016; for two-year-ahead, stretching the dataset from 2004 to 2015, and for five-year-ahead, thus using the same input dataset as for the GBM and OUP models, from 2004 to 2012.

Finally, for every forecast path obtained, we have calculated the average of the squares of the errors with respect to the observed values, MSE, the average distance of a forecast to the observed mean, MAD, the average deviation of a forecast from observed values, MAPE, and the square root of MSE, RMSE. The results of MSE, MAD, MAPE and RMSE calculated for each forecast path were averaged by the total number of simulations carried out for each case. Averaged values of error measures of all simulated paths, $\overline{\mathrm{MSE}}$, $\overline{\mathrm{MAD}}$, $\overline{\mathrm{MAPE}}$, $\overline{\mathrm{RMSE}}$, for both annual-step forecasts and monthly-step forecasts have been used for cross-comparison between models to test predictive ability at every step size possible.

Also, to test each model's short-term forecasting capacity against its long-term forecasting capacity, one-year-ahead forecast errors of the LES model were compared with the errors from the last year of the GBM and OUP simulated paths. Two-year-ahead forecast errors of LES models were compared with the average errors of the last two years of GBM and OUP, and five-year-ahead forecast errors of LES models were compared with the overall average of errors of the GBM and OUP forecasts.

### 3.4.3. Models Calibration

In a preliminary stage, the models were first calibrated to forecast values from 2013 to 2017 using available data from 2004 to 2012 for TC and RC separately, hence disregarding the well-known inherent 10:1 relation. The GBM and OUP models were calibrated for two different step sizes, -monthly steps and annual steps- in order to compare forecasting accuracy with each step size in each model. The available dataset is thus comprised of data from 2004 to 2017, which make the calibration data approximately 2/3 of the total.

**GBM Calibration**

Increments in the logarithm of variable *x* are distributed as follows:

$$\Delta(lnx) \sim N((\alpha - \frac{\sigma^2}{2})t, \sigma t)$$

Hence, if m is defined as the sample mean of the difference of the natural logarithm of the time series for TC/RC levels considered for the calibration and n as the number of increments of the series considered, with n=9:

$$m = \frac{1}{n}\sum_{t=1}^{n}(lnx_t - lnx_{t-1})$$

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(lnx_t - lnx_{t-1} - m)^2}$$

$$m = \alpha - \frac{\sigma^2}{2}$$

$$s = \sigma$$

**OUP Calibration**

The OUP process is an AR1 process (Dixit and Pindyck, 1994) whose resolution is well presented by Woolridge (2011) using OLS techniques, fitting the following equation:

$$y_t = a + by_{t-1} + \varepsilon_t$$

Hence, the estimators for the parameters of the OUP model are obtained by OLS for both TC and RC levels independently:

$$\hat{\lambda} = -\frac{lnb}{\Delta t}$$

$$\hat{\mu} = \frac{a}{1-b}$$

$$\hat{\sigma} = \varepsilon_t \sqrt{\frac{2\ln{(1+b)}}{(1+b)^2 - 1}}$$

**LES Calibration**

A linear regression is conducted on the input dataset available to find the starting parameters for the LES model, the initial Level, $L_0$, and the initial value of the Trend, $T_0$, irrespective of TC values and RC values. Here, as recommended by Gardner (2006), the use of OLS is highly advisable due to the erratic behaviour shown by trends in the historic data, so the obtaining of negative values of $S_0$ is prevented. Linear regression fulfils the following equations:

$$Y_t = at + b$$

$$L_0 = b$$

$$T_0 = a$$

By fixing the two smoothing constants, the values for the forecasts, $\hat{Y}_{t+k}$, can be calculated at each step using the model equations. There are multiple references in the literature on what is the optimum range for each smoothing constant; Gardner (2006) speaks of setting moderate values for both parameter less than 0.3 to obtain the best results. Examples pointing out the same may be found in Brown (1963), Coutie (1964), Harrison (1967) and Montgomery and Johnson (1976). Also, for many applications, Makridakis and Hibon (1979) and Chatfield (1978) found that parameter values should fall within the range of 0.3-1. On the other hand, McClain and Thomas (1973) provided a condition of stability for the non-seasonal *LES* model given by:

$$\begin{cases} 0 < \alpha < 2 \\ 0 < \beta < \dfrac{4 - 2\alpha}{\alpha} \end{cases}$$

Also, the largest possible value of α that allows the avoidance of areas of oscillation is proposed by McClain and Thomas (1973) and McClain (1974):

$$\alpha < \frac{4\beta}{(1 + \beta)^2}$$

However, according to Gardner, there is no tangible proof that this value improves accuracy in any form. Nonetheless, we have opted to follow what Nau (2014) refers to as 'the usual way', namely, minimising the Mean Squared Error (MSE) of the one-step-ahead forecast of TC/RC for each input data series previously mentioned. To do so, Matlab's fminsearch function has been used with function and variable tolerance levels of 1x10-4 as well as a set maximum number of function iterations and function evaluations of 1x106 to limit computing resources. In **Table 9** the actual number of necessary iterations to obtain optimum values for smoothing constants is shown. As can be seen, the criteria are well beyond the final results, which ensured that an optimum solution was reached with assumable computing usage (the simulation required less than one minute) and with a high degree of certainty.

**Table 9**
Iterations on LES smoothing constants optimization

| | Iteration | Function Evaluations |
|---|---|---|
| TC 1-Step | 36 | 40 |
| TC 2-Steps | 249945 | 454211 |
| TC 5-Steps | 40 | 77 |
| RC 1-Step | 32 | 62 |
| RC 2-Steps | 254388 | 462226 |
| RC 5-Steps | 34 | 66 |

### *3.5. Analysis of Results*

Monte Carlo simulations of GBM and OUP models render 1000 different possible paths for TC and RC, respectively, at each time step size considered. Accuracy errors for both annual time steps and averaged monthly time steps, for both GBM and OUP forecasts, were first compared to determine the most accurate time step size for each model. In addition, the LES model outcome for both TC and RC at different timeframes was also calculated and measures of errors for all the three alternative models proposed at an optimum time step were finally compared.

#### 3.5.1. GBM Forecasts

The first 20 of 1000 Monte Carlo paths for the GBM model with a monthly step size using Matlab software may be seen in **Figure 3** for both the TC and RC levels compared to their averaged paths for every 12 values obtained. The tendency for GBM forecasts to steeply increase over time is easily observable in the non-averaged monthly-step paths shown.

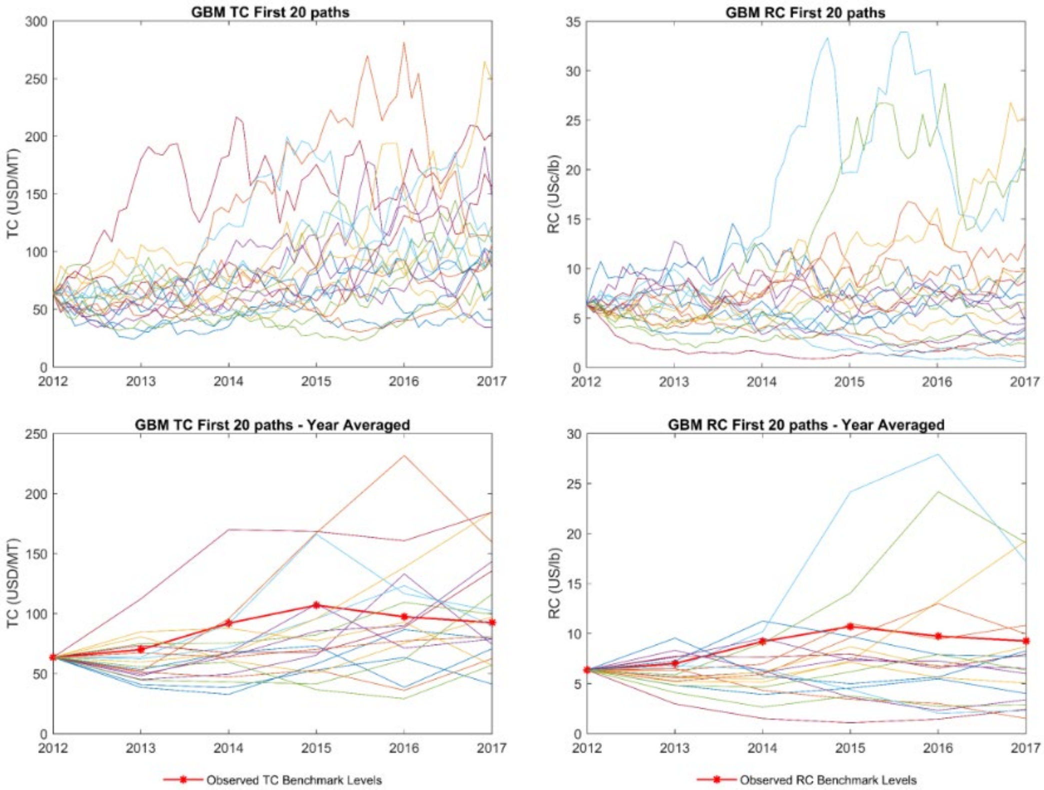


**Figure 3.** The upper illustrations show the first 20 Monte Carlo paths for either TC or RC levels using monthly steps. The illustrations below show the annual averages of monthly step forecasts for TC and RC levels.

The average values of error of all 1000 MC paths obtained through simulation for averaged monthly-step and annual-step forecasts are shown in **Table 10** for both TC and RC discounts over the period from 2013 to 2017, which may lead to preliminary conclusions in terms of an accuracy comparison between averaged monthly steps and annual steps.

**Table 10**
Average of main measures of error for GBM after 1000 MC simulations from 2013 to 2017.

| | $\overline{MSE}$ | $\overline{MAD}$ | $\overline{MAPE}$ | $\overline{RMSE}$ |
|---|---|---|---|---|
| TC Averaged Monthly Steps | $5.60\text{x}10^3$ | 46.98 | 0.50 | 56.38 |
| TC Annual Steps | $5.20\text{x}10^3$ | 49.02 | 0.52 | 58.57 |
| RC Averaged Monthly Steps | 58.74 | 4.55 | 0.48 | 5.46 |
| RC Annual Steps | 50.85 | 4.91 | 0.52 | 5.84 |

However, a more exhaustive analysis is shown in **Table 11**, where the number of times the values of each error measure is higher for monthly steps is expressed over the total number of MC simulations carried out. The results indicate that better values of error are reached the majority of times for averaged monthly-step simulations rather than for straight annual ones.

**Table 11**
Number of paths for which measures of error are higher for monthly steps than for annual steps in GBM.

| | $MSE$ | $MAD$ | $MAPE$ | $RMSE$ |
|---|---|---|---|---|
| TC | 457/1000 | 455/1000 | 453/1000 | 453/1000 |
| RC | 439/1000 | 430/1000 | 429/1000 | 429/1000 |

In contrast, short term accuracy was also evaluated by analysing the error measures of one-year-ahead forecasts (2013) in **Table 12**.

**Table 12**
Average for main measures of error for GBM after 1000 MC simulations for 2013.

| | $\overline{MSE}$ | $\overline{MAD}$ | $\overline{MAPE}$ | $\overline{RMSE}$ |
|---|---|---|---|---|
| TC Averaged Monthly Steps | 336.58 | 14.55 | 0.21 | 14.55 |
| TC Annual Steps | 693.81 | 20.81 | 0.30 | 20.81 |
| RC Averaged Monthly Steps | 2.97 | 1.38 | 0.20 | 1.38 |
| RC Annual Steps | 6.57 | 2.05 | 0.29 | 2.05 |

Two-year-ahead forecasts (2013–2014) are shown in **Table 13**. The results indicate, as one may expect, that accuracy decreases as the forecasted horizon is widened, with the accuracy of averaged monthly-step forecasts remaining higher than annual ones as found above for the five-year forecasting horizon.

**Table 13**
Average for main measures of error for GBM after 1000 MC simulations for 2013-2014.

| | $\overline{MSE}$ | $\overline{MAD}$ | $\overline{MAPE}$ | $\overline{RMSE}$ |
|---|---|---|---|---|
| TC Averaged Monthly Steps | 994.92 | 21.87 | 0.31 | 24.35 |
| TC Annual Steps | 1.33x103 | 28.33 | 0.40 | 31.04 |
| RC Averaged Monthly Steps | 10.35 | 2.40 | 0.34 | 2.71 |
| RC Annual Steps | 13.13 | 2.84 | 0.34 | 3.11 |

### 3.5.2. OUP Forecasts

Long-term levels for TC and RC, μ, the speed of reversion, λ, and the volatility of the process, σ, are the parameters determined at the model calibration stage, which define the behaviour of the OUP, shown in **Table 14**. The calibration was done prior to the Monte Carlo simulation for both TC and RC, with each step size using available historical data from 2004 to 2012. The OUP model was fitted with the corresponding parameters for each case upon MC simulation.

**Table 14**
OUP parameters obtained in the calibration process.

| | μ | λ | σ |
|---|---|---|---|
| TC Monthly | 63.45 | $4.792 \times 10^5$ | 2.534 |
| TC Annual | 63.45 | 2.974 | 1.308 |
| RC Monthly | 6.35 | $4.763 \times 10^5$ | 2.519 |
| RC Annual | 6.35 | 2.972 | 1.305 |

**Figure 4** shows the Mean Reversion MC estimations of the TC/RC benchmark values from 2013 to 2017 using monthly steps. The monthly forecasts were rendered from January 2012 through to December 2016 and averaged every twelve values to deliver a benchmark forecast for each year. The averaged results can be comparable to actual data as well as to the annual Monte Carlo simulations following Mean Reversion. The lower-side figures show these yearly-averaged monthly-step simulation outcomes which clearly move around a dash-dotted red line, indicating the long-term run levels for TC/RC to which they tend to revert.

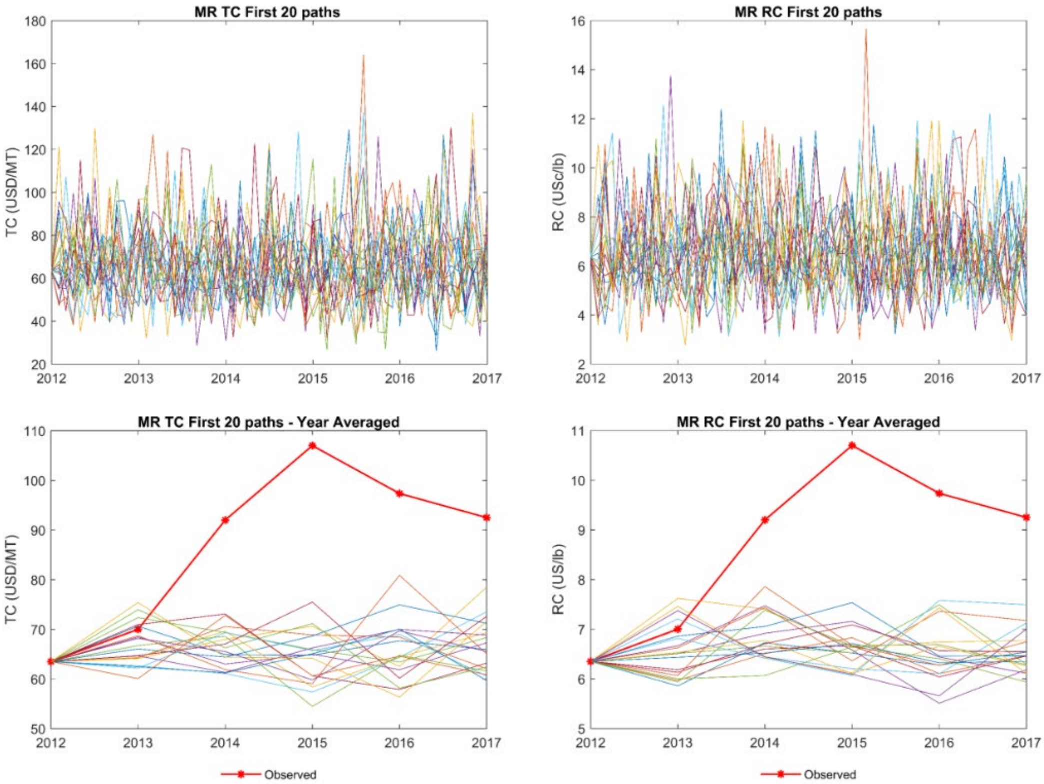


**Figure 4.** The first 20 Monte Carlo paths following the OUP model using monthly steps are shown on the upper illustrations for either TC and RC. The annual averaged simulated paths every 12 values are shown below.

The accuracy of monthly steps against annual steps for the TC/RC benchmark levels forecast was also tested by determining the number of simulations for which average error measures became higher. **Table 15** shows the number of times monthly simulations have been less accurate than annual simulations for five-year-ahead OUP forecasting by comparing the four measures of errors proposed. The results indicate that only 25-32% of the simulations drew a higher average error, which clearly results in a better predictive accuracy for monthly-step forecasting of TC/RC annual benchmark levels.

**Table 15**
Number of paths for which measures of error are higher for monthly steps than for annual steps in MR-OUP 5-Steps MC simulation.

| | $MSE$ | $MAD$ | $MAPE$ | $RMSE$ |
|---|---|---|---|---|
| TC | 311/1000 | 283/1000 | 266/1000 | 266/1000 |
| RC | 316/1000 | 281/1000 | 250/1000 | 250/1000 |

The averaged measures of errors obtained after the MC simulations of the OUP model for both averaged-monthly-steps and annual steps giving TC/RC benchmark forecasts from 2013 to 2017 are shown in **Table 16**.

**Table 16**
Average for main measures of error for MR-OUP after 1000 MC simulations 2013-2017.

| | $\overline{MSE}$ | $\overline{MAD}$ | $\overline{MAPE}$ | $\overline{RMSE}$ |
|---|---|---|---|---|
| TC Averaged Monthly Steps | 842.54 | 26.14 | 0.27 | 28.95 |
| TC Annual Steps | $1.14\text{x}10^3$ | 29.31 | 0.31 | 33.25 |
| RC Averaged Monthly Steps | 8.40 | 2.61 | 0.27 | 2.89 |
| RC Annual Steps | 11.48 | 2.94 | 0.31 | 3.33 |

The error levels of the MC simulations shown above point towards a higher prediction accuracy of averaged-monthly-step forecasts of the OUP Model, yielding an averaged MAPE value that is 12.9% lower for TC and RC 5-step-ahead forecasts. In regard to MAPE values, for monthly steps, only 26.6% of the simulations rise above annual MC simulations for TC, and 25% for RC 5-step-ahead forecasts, which further underpins the greater accuracy of this OUP set-up for TC/RC level forecasts. A significant lower probability of higher error levels for TC/RC forecasts with monthly MC OUP simulations is reached for the other measures provided. In addition, short-term and long-term prediction accuracy was tested by comparing errors of forecasts for one-year-ahead in **Table 17,** two-year-ahead in **Table 18**, as well as for five-year-ahead error measures above in **Table 16.**

**Table 17**
Average for main measures of error for MR-OUP after 1000 MC simulations 2013.

| | $\overline{MSE}$ | $\overline{MAD}$ | $\overline{MAPE}$ | $\overline{RMSE}$ |
|---|---|---|---|---|
| TC Averaged Monthly Steps | 39.14 | 5.22 | 0.07 | 5.22 |
| TC Annual Steps | 366.43 | 15.55 | 0.22 | 15.55 |
| RC Averaged Monthly Steps | 0.39 | 0.51 | 0.07 | 0.51 |
| RC Annual Steps | 3.63 | 1.54 | 0.22 | 1.54 |

**Table 18**
Average for main measures of error for MR-OUP after 1000 MC simulations 2013-2014.

| | $\overline{MSE}$ | $\overline{MAD}$ | $\overline{MAPE}$ | $\overline{RMSE}$ |
|---|---|---|---|---|
| TC Averaged Monthly Steps | 371.65 | 15.67 | 0.18 | 19.00 |
| TC Annual Steps | 682.44 | 21.85 | 0.26 | 23.24 |
| RC Averaged Monthly Steps | 3.76 | 1.57 | 0.18 | 1.91 |
| RC Annual Steps | 6.89 | 2.19 | 0.26 | 2.43 |

With a closer forecasting horizon error, measures show an improvement of forecasting accuracy when average monthly steps are used rather than annual ones. For instance, the MAPE values for 2013 forecast for TC is 68% lower for averaged monthly steps than for annual steps, also MAPE for 2013 – 2014 were 30% lower for both TC and RC forecasts. Similarly, better values of error are achieved for the other measures for averaged monthly short-term forecasts than in other scenarios. In addition, as expected, accuracy is increased for closer forecasting horizons where the level of errors shown above become lower as the deviation of forecasts is trimmed with short-term predictions.

### 3.5.3. LES Forecasts

In contrast to GBM and OUP, the LES model lacks any stochastic component, so non-deterministic methods such as the Monte Carlo are not required to obtain a forecast. Nonetheless, the LES model relies on two smoothing constants which must be properly set in order to deliver accurate predictions, hence the values of the smoothing constants were first optimised. The optimisation was carried out for one-year-ahead forecasts, two-year-ahead forecasts, and five-year-ahead forecasts by minimising the values of MSE for both TC and RC. The different values used for smoothing constants, as well as the initial values for level and trend obtained by the linear regression of the available dataset from 2004 through to 2012 are shown in **Table 19**.

**Table 19**
LES Model Optimised Parameters.

| | $L_0$ | $T_0$ | α | β |
|---|---|---|---|---|
| TC 1 year | 71.3611 | -1.5833 | -0.2372 | 0.1598 |
| RC 1 year | 7.1333 | -0.1567 | -0.2368 | 0.1591 |
| TC 2 years | 71.3611 | -1.5833 | $-1.477\times10^{-4}$ | 777.4226 |
| RC 2 years | 7.1333 | -0.1567 | $-1.448\times10^{-4}$ | 789.8336 |
| TC 5 years | 71.3611 | -1.5833 | -0.2813 | 0.1880 |
| RC 5 years | 7.1333 | -0.1567 | -0.2808 | 0.1880 |

Compared one-year-ahead, two-year-ahead and five-year-ahead LES forecasts for TC and RC are shown in **Figure 5,** clearly indicating a stronger accuracy for shorter term forecasts as the *observed* and *forecasted* plotted lines overlap.

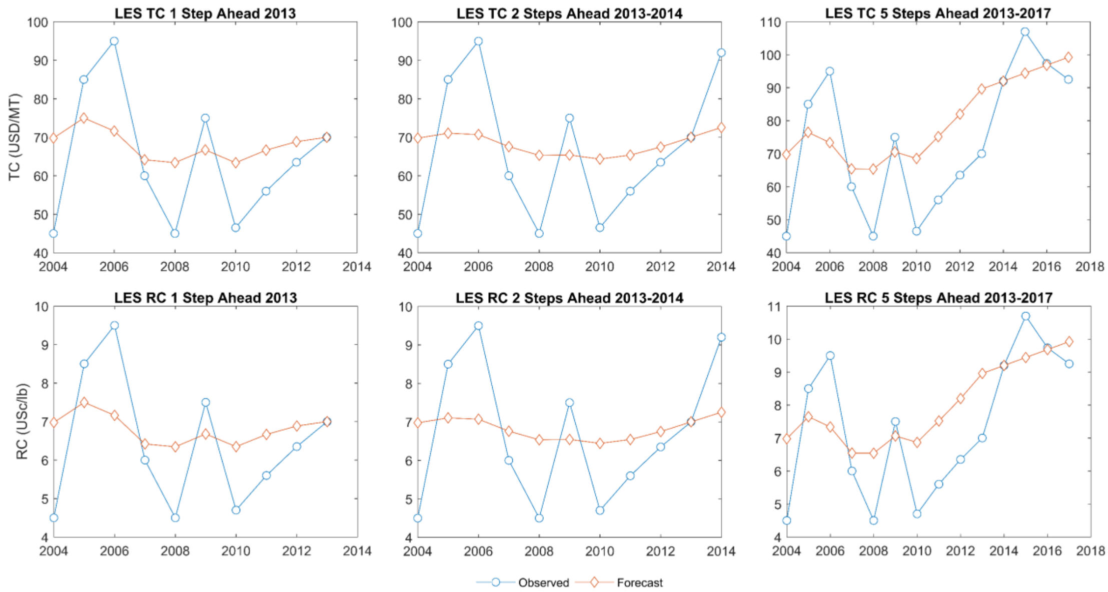


**Figure 5.** One-step-ahead forecasts (2013), two-step-ahead forecasts (2013-2014) and five-step-ahead forecasts (2013-2017) for TC and RC using the Linear Exponential Smoothing model (LES).

Minimum values for error measures achieved through the LES model parameter optimisation are shown in **Table 20**. The values obtained confirm the strong accuracy for shorter-term forecasts of TC/RC seen in the figures.

**Table 20**
LES error measures for different steps-ahead forecasts.

| | $\overline{MSE}$ | $\overline{MAD}$ | $\overline{MAPE}$ | $\overline{RMSE}$ |
|---|---|---|---|---|
| TC 1 year | $1.0746 \times 10^{-5}$ | 0.0033 | $4.6831 \times 10^{-5}$ | 0.0033 |
| RC 1 year | $5.1462 \times 10^{-8}$ | $2.2685 \times 10^{-4}$ | $3.2408 \times 10^{-5}$ | $2.2685 \times 10^{-4}$ |
| TC 2 years | 189.247 | 9.7275 | 0.1057 | 13.7567 |
| RC 2 years | 1.8977 | 0.9741 | 0.1059 | 1.3776 |
| TC 5 years | 177.5531 | 7.8881 | 0.0951 | 10.8422 |
| RC 5 years | 1.1759 | 0.7889 | 0.0952 | 1.0844 |

### *3.6. Discussion and Model Comparison*

The forecasted values of TC/RC benchmark levels could eventually be applied to broader valuation models for copper concentrates and their trading activities, as well as to the copper smelters' revenue stream, thus the importance of delivering as accurate a prediction as possible in relation to these discounts to make any future application possible. Each of the models presented may be a feasible method on its own with eventual later adaptations to forecasting future values of benchmark TC/RC. Nonetheless, the accuracy of these models as they have been used in this research requires, firstly, a comparison to determine whether any of them could be a good standalone technique and, secondly, to test whether a combination of two or more of them would deliver more precise results.

When comparing the different error measures obtained for all the three models, it is clearly established that results for a randomly-chosen simulation of GBM or OUP would be more likely to be more precise had a monthly-step been used to deliver annual forecasts instead of an annual-step size. In contrast, average error measures for the entire population of simulations with each step size employed showing that monthly step simulations for GBM and OUP models are always more accurate than straight annual step forecasts when a shorter time horizon, one or two-year-ahead, is taken into consideration. However, GBM presents a higher level of forecasting accuracy when the average for error measures of all simulations is analysed, employing annual steps for long-term horizons, whereas OUP averaged-monthly-step forecasts remain more accurate when predicting long-term horizons. **Table 21** shows the error improvement for the averaged-monthly-step forecasts of each model. Negative values indicate that better levels of error averages have been found in straight annual forecasts than for monthly-step simulations.

**Table 21**
Error Average Improvement for averaged monthly steps before annual steps.

| | $\overline{MSE}$ | $\overline{MAD}$ | $\overline{MAPE}$ | $\overline{RMSE}$ |
|---|---|---|---|---|
| GBM TC/RC 1 Year | 53.26%/56.53% | 32.72%/35.87% | 33.33%/35.48% | 32.72%/35.87% |
| GBM TC/RC 2 Years | 26.95%/26.35% | 25.75%/15.97% | 26.19%/2.86% | 24.34%/13.69% |
| GBM TC/RC 5 Years | -11.73%/-11.04% | 8.73%/7.09% | 9.26%/7.69% | 7.47%/5.65% |
| OUP TC/RC 1 Year | 88.08%/87.08% | 63.71%/62.50% | 62.91%/63.19% | 63.68%/62.24% |
| OUP TC/RC 2 Years | 46.28%/44.21% | 27.71%/26.17% | 29.99%/30.75% | 22.16%/20.75% |
| OUP TC/RC 5 Years | 26.02%/25.53% | 10.93%/9.97% | 13.12%/12.18% | 13.06%/12.69% |

Considering the best results for each model, and comparing their corresponding error measures, we can opt for the best technique to employ among the three proposed in this paper. Hence, GBM delivers the most accurate one-year forecast when averaging the next twelve-month predictions for TC/RC values, as does the MR-OUP model. **Table 22** shows best error measures for one-year-ahead forecasts for GBM, OUP and LES models.

**Table 22**
TC/RC best error measures for 2013 forecasts after 1000 MC simulations.

| | $\overline{MSE}$ | $\overline{MAD}$ | $\overline{MAPE}$ | $\overline{RMSE}$ |
|---|---|---|---|---|
| GBM TC/RC 1 Year* | 331.11/3.36 | 14.25/1.43 | 0.20/0.20 | 14.25/1.43 |
| OUP TC/RC 1 Year* | 41.05/0.42 | 5.36/0.54 | 0.0766/0.0777 | 5.36/0.54 |
| LES TC/RC 1 Year | $1.07 \times 10^{-5}$/ $5.14 \times 10^{-8}$ | 0.003/ $2.26 \times 10^{-4}$ | $4.68 \times 10^{-5}$/ $3.24 \times 10^{-5}$ | 0.003/ $2.26 \times 10^{-4}$ |

*Averaged monthly steps.

Unarguably, the LES model generates minimal error measures for one-year-ahead forecasts, significantly less than the other models employed. A similar situation is found for two-year-ahead forecasts where minimum error measures are also delivered by the LES model, shown in **Table 23**.

**Table 23**
TC/RC best error measures for 2013 – 2014 forecasts after 1000 MC simulations.

| | $\overline{MSE}$ | $\overline{MAD}$ | $\overline{MAPE}$ | $\overline{RMSE}$ |
|---|---|---|---|---|
| GBM TC/RC 2 Steps* | $1.03\text{x}10^3$/10.06 | 22.00/2.42 | 0.31/0.34 | 24.47/2.71 |
| OUP TC/RC 2 Steps* | 373.31/3.76 | 15.73/1.58 | 0.1802/0.1813 | 19.00/1.91 |
| LES TC/RC 2 Steps | 189.24/1.89 | 9.72/0.97 | 0.1057/0.1059 | 13.75/1.37 |

*Averaged monthly steps.

Finally, accuracy measures for five-year-ahead forecasts of the GBM model might result in somewhat contradictory terms for MSE reaching better values for annual steps than for averaged monthly steps, while the other figures do better on averaged monthly steps. Addressing the definition of MSE, this includes the variance of the estimator as well as its bias, being equal to its variance in the case of unbiased estimators. Therefore, MSE measures the quality of the estimator but also magnifies estimator deviations from actual values since both positive and negative values are squared and averaged.

In contrast, RMSE is calculated as the square root of MSE and, following the previous analogy, stands for the standard deviation of the estimator if MSE were considered to be the variance. Though RMSE overreacts when high values of MSE are reached, it is less prone to this than MSE since it is calculated as its squared root, thus not accounting for large errors as disproportionately as MSE does.

Furthermore, as we have compared an average of 1000 measures of errors corresponding to each MC simulation performed, the values obtained for average RMSE stay below the square root of average MSE, which indicates that some of these disproportionate error measures are, to some extent, distorting the latter. Hence, RMSE average values point towards a higher accuracy for GBM five-year forecasts with averaged monthly steps, which is further endorsed by the average values of MAD and MAPE, thus being the one used for comparison with the other two models as shown in **Table 24**.

**Table 24**
TC/RC Best error measures for 2013 – 2017 forecasts after 1000 MC simulations.

| | $\overline{MSE}$ | $\overline{MAD}$ | $\overline{MAPE}$ | $\overline{RMSE}$ |
|---|---|---|---|---|
| GBM TC/RC 5 Steps* | $6.00 \times 10^3$/61.65 | 46.51/4.59 | 0.49/0.48 | 55.71/5.51 |
| OUP TC/RC 5 Steps* | 843.34/8.43 | 26.16/2.62 | 0.2715/0.2719 | 28.96/2.89 |
| LES TC/RC 5 Steps | 177.55/1.17 | 7.88/0.78 | 0.0951/0.0952 | 10.84/1.08 |

*Averaged monthly steps.

The final comparison clearly shows how the LES model outperforms the other two at all average measures provided, followed by the OUP model in accuracy, although the latter more than doubles the average MAPE value for LES.

The results of simulations indicate that measures of errors tend to either differ slightly or not at all for either forecast of any timeframe. A coherent value with the 10:1 relation can then be given with close to the same level of accuracy by multiplying RC forecasts or dividing TC ones by 10.

# Chapter 4: Estimating Copper Concentrates benchmark prices under dynamic market conditions

### *4.1. Introduction*

Copper concentrates are the primary product sold by copper mines to traders or smelters. Their price is set in private agreements following unofficial market practices which, along with the fact of the wide variety of concentrates layouts traded worldwide, make problematic for any market participant to have a clear price reference for a specific concentrate. Despite the large number of transactions of copper concentrates taking place, there is no market-wide accepted concentrates price reference from an official, publicly accessible and reliable source that can be taken as a benchmark. By the contrary, miners, smelters and traders alike are forced to price concentrates following indirect references and, sometimes, complex and poorly transparent calculation procedures.

In this research a copper concentrates benchmark price model was developed, which provides suitable short-term price estimations based on metals and discounts forecasts, as well as on copper, gold and silver spot and future prices data from the LME and COMEX. The model, which redeems considerably low forecast error values for the short-term concentrate prices, constitutes a useful and applicable tool for miners, traders and smelters to set a benchmark price level for their copper concentrate transactions, also helping them optimize their operations, as well as estimate their immediate liquidity needs or their actual necessity to hedge the price risks associated to their concentrate trading.

In addition, five different concentrates layouts have been analysed to test the model's behaviour with the most common specification and blends of copper concentrates demanded by the market, as well as to portray the model's forecasting capacity and its ability to convey information on the future relevance of the different components of pricing in the most frequent timeframe in which copper concentrate trading takes place.

To address this our concentrates benchmark price model provides suitable short-term price estimations as a function of the concentrate's chemical specifications and the market conditions at each moment. Our model puts together separate forecasts for the LME copper and COMEX gold and silver spot prices, along with the main deductions and penalties applied by smelters, to achieve copper concentrates price forecasts based on their layout.

The model resorts to historical data on spot and futures prices at different maturities for copper, gold and silver as well as to historical values on copper TC/RC benchmark levels to render copper concentrate price forecasts. These concentrate benchmark price model constitute a powerful source of information for market participants, helping them to determine the price optimality of the copper concentrates being traded as well as their future conditions. It also serves as an applicable tool for miners, smelters and traders alike, allowing them to improve their production planning, to increase the benefits from their trading operations, as well as helping them optimize their risk management and their short-term liquidity needs.

### *4.2. Copper concentrates benchmark price model*

Copper concentrates supply agreements set specific mechanisms to price the copper concentrates to be traded according to their chemical specifications, as already explained in Chapter 2. Usually, most pricing systems establish the price of copper concentrates as the sum of the payable elements present in the concentrates minus the deductions and the penalties (Soderstrom, 2008; Seabridge Gold, 2010; Teck, 2012, 2015). The model presented in this research is based on the same logic, treating payable elements separately from discounts and punishable elements, though differing from usual market practices in its forecasting capabilities for concentrates prices as metals and copper TC/RC are separately modelled instead of employing the available market data, and their values are projected ahead. Thus, at time t, the price of the ton of concentrate can be expressed as:

$$C_t = M_t - D_t - P_t$$

Though copper concentrates may contain multiple payable elements apart from copper, should there be some gold and silver are the most common. Thus, $M_t$ may be defined as the sum of the amount paid for all payable elements, which is the amount paid for the copper, gold and silver content in the concentrate. Metals in concentrates are paid at market price, though the full content is not usually paid, as smelters are normally not capable of recovering all the content. Hence, the price of payable metals in the concentrates is calculated as the payable content (*pp*) multiplied by its market price at time t ($S_t$).

$$M_t = Cu_t + Au_t + Ag_t$$

$$Cu_t = pp_{Cu} \times S_t^{Cu}$$

$$Au_t = pp_{Au} \times S_t^{Au}$$

$$Ag_t = pp_{Ag} \times S_t^{Ag}$$

Payable metal prices, including the market price for copper, gold and silver, are incorporated separately with the inclusion of the Schwartz-Smith's (2000) two-factor model, which decomposes the log-market price of metals as the sum of two stochastic factors: $\chi_t$, that comprises the short-term deviation of prices to reflect rapid and temporary changes in market conditions, and $\xi_t$, representing the long-term equilibrium price, thus including the long-term market dynamics in prices, such as supply and demand changes.

$$\ln\left(S_t^i\right) = \chi_t^i + \xi_t^i$$

$$i = Cu, Au, Ag$$

Short-term price fluctuations are modelled to revert towards zero, following and Orstein-Uhlenbeck process, whereas the long-term equilibrium price level is assumed to follow a Brownian motion with drift.

$$d\chi_t^i = -\kappa^i \chi_t^i dt + \sigma_\chi^i dz_\chi^i$$

$$d\xi_t^i = \mu^i dt + \sigma_\xi^i dz_\xi^i$$

$$dz_\chi^i dz_\xi^i = \rho_{\chi\xi}^i dt$$

$$i = Cu, Au, Ag$$

Where $\kappa$ is known as the short-term mean-reversion rate, $\sigma_\chi$ and $\sigma_\xi$ are the short-term and the equilibrium volatilities, respectively, $\mu_\xi$ is the equilibrium drift rate, while $dz_\chi$ and $dz_\xi$ are two correlated increments of standard Brownian motions with correlation parameter $\rho_{\chi\xi}$. According to Schwartz-Smith (2000) the log of future spot prices is normally distributed, whereas the spot price itself is log-normally distributed, being:

$$S_t^i = \exp\left(\chi_t^i + \xi_t^i\right)$$

$$\mathrm{E}\left[S_t^i\right] = \exp\left(\mathrm{E}\left[\ln\left(S_t^i\right)\right] + \frac{1}{2}\mathrm{Var}\left[\ln\left(S_t^i\right)\right]\right)$$

$$\ln\left(\mathrm{E}\left[S_t^i\right]\right) = \mathrm{E}\left[\ln\left(S_t^i\right)\right] + \frac{1}{2}\mathrm{Var}\left[\ln\left(S_t^i\right)\right] =$$

$$= e^{-\kappa^i t}\chi_0^i + \xi_0^i + \mu_\xi^i t + \frac{1}{2}\left(\left(1 - e^{-2\kappa^i t}\right)\frac{\left(\sigma_\chi^i\right)^2}{2\kappa^i} + \left(\sigma_\xi^i\right)^2 t + 2\left(1 - e^{-\kappa^i t}\right)\frac{\rho_{\chi\xi}^i \sigma_\chi^i \sigma_\xi^i}{\kappa^i}\right)$$

$$i = Cu, Au, Ag$$

Hence, the payable amount for metals in concentrates can be rewritten using the notation of the two-factor model as follows:

$$Cu_t = pp_{Cu} e^{\left(\chi_t^{Cu} + \xi_t^{Cu}\right)}$$

$$Au_t = pp_{Au} e^{\left(\chi_t^{Au} + \xi_t^{Au}\right)}$$

$$Ag_t = pp_{Ag} e^{\left(\chi_t^{Ag} + \xi_t^{Ag}\right)}$$

$$M_t = Cu_t + Au_t + Ag_t = pp_{Cu} e^{\left(\chi_t^{Cu} + \xi_t^{Cu}\right)} + pp_{Au} e^{\left(\chi_t^{Au} + \xi_t^{Au}\right)} + pp_{Ag} e^{\left(\chi_t^{Ag} + \xi_t^{Ag}\right)}$$

Additionally, the payable copper content in concentrates is also dependent on the metal grade ($g$), multiplied by a recovery factor ($r$), subject to a minimum deduction (*MD*).

$$pp_{Cu} = \begin{cases} g_{Cu} - MD_{Cu}, & g_{Cu} - MD_{Cu} < g_{Cu} \times r_{Cu} \\ g_{Cu} \times r_{Cu}, & g_i - MD_i \geq g_{Cu} \times r_{Cu} \end{cases} \quad \forall\, g_{Cu} \geq MD_{Cu}$$

$$pp_{Cu} = 0 \ \forall \ g_{Cu} < MD_{Cu}$$

On the other hand, precious metals in concentrates are paid only if their content is above the minimum deduction levels, otherwise they are priced at zero. The final payable content of precious metals is determined by multiplying the metal grade by a recovery factor.

$$pp_i = \begin{cases} 0\ , & g_i \leq MD_i \\ g_i \times r_i, & g_i > MD_i \end{cases}$$

$$i = Au, Ag$$

In addition, Dt is the sum of deductions applied, including copper Treatment Charges (TC) and copper Refining Charges ($R^{Cu}$), as well as gold Refining Charges ($R^{Au}$) and silver Refining Charges ($R^{Ag}$).

$$D_t = TC_t + R_t^{Cu} + R_t^{Au} + R_t^{Ag}$$

Copper TC are quoted in US Dollars per tonne of concentrate, usually expressed on a dry basis, while copper RC is quoted in US cents of dollars per pound of payable copper content. To apply copper TC/RC discounts to concentrate pricing, we have assumed that they remain constant at benchmark levels throughout their year of validity. TC/RC benchmark levels have been forecasted using the Holt's Linear Exponential Smoothing (LES) method, which delivers the best forecasts of those analyzed in Chapter 3, treating TC and RC separately instead of observing the usual 10:1 relation. Holt's LES model may be expressed by the combination of level, trend and forecast updating equations, written as follows:

$$L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + T_{t-1})$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1}$$

$$\hat{Y}_{t+k} = L_t + kT_t$$

In addition, gold and silver RC are applicable as long as there is gold or silver payable content in the concentrate, gold RC being quoted in USD per troy ounce of payable gold content and silver RC in USD per gram of payable silver content.

$$R_t^i = RC_t^i \times pp_i$$

Finally, $P_t$ is the overall penalty for an n number of punishable elements in the concentrate which negatively affect its quality as well as the chemical, mechanical and electrical properties of the copper products made with that concentrate. These penalties are calculated as a function of the excessive content of the punishable element beyond the penalty-free limit, below which the concentrate would be exempt of penalty for that element.

$$P_t = \sum_i^n p_i \;\; \forall \, t$$

$$p_i = \begin{cases} 0, & g_i < F_i \\ (g_i - F_i) \times \rho_i, & g_i \geq F_i \end{cases}$$

Where $g_i$ is the content of the punishable element i, $F_i$ is the penalty-free limit content for that element and $\rho_i$ is the penalty per unit of excessive content of the punishable element i beyond the penalty-free limit. The unitary penalty is set as the nominal penalty $\delta_i$, divided by the nominal punishable increment of element i, $h_i$.

$$\rho_i = \frac{\delta_i}{h_i} \ \forall \ i$$

### *4.3. Copper concentrates benchmark price forecasting*

Copper concentrates daily prices have been modelled using Matlab software employing historical daily official spot and futures data from the two main commodity markets (LME - copper; COMEX - gold and silver) obtained from Refinitiv EIKON as well as historical copper TC/RC benchmark levels from 2004 to 2018 to calibrate the model. Daily forecasts for each concentrate layout have been obtained using Monte Carlo simulations performing 1000 trials for each concentrate to up to one year ahead (2019).

This horizon has been chosen as the price of concentrates beyond a year may be of scarce interest for market participants as most copper concentrate transaction occur in a shorter timeframe, whilst this still allows to understand the model's behaviour while key pricing components, such as metals prices, evolve over time.

Payable metals prices have been modelled separately to each other, calibrating the model within the in-sample timeframe of the datasets (2004-2018) using both spot and future data for each metal. Metals prices forecasts are put together, along with the projected values of discounts and penalties, to build the concentrate forecasts.

### 4.3.1. Parameters estimation

The whole set of parameters for the Schwartz-Smith two-factor model for metal prices $(\kappa^i, \sigma^i_\chi, \sigma^i_\xi, \mu^i, \rho^i_{\chi\xi})$ are estimated following the process proposed by Barlow et al. (2004), which running the Kalman filter (Kalman, 1960) recursively, like the original approach by Schwartz and Smith (2000), maximizing the log-likelihood function calculated with the parameters obtained from the previous Kalman filter estimation over the model's state-space form. The Schwartz-Smith two-factor model used to obtain the market prices of payable metals in the concentrates is expressed in its state-space form by the *transition equation* and the *measurement equation*, being the first:

$$x_t = c + G_{t-1} + \omega_t, \qquad t = 1, \dots, n_T$$

where,

$$x_t \equiv [\chi_t, \xi_t], \qquad a\ 2 \times 1\ vector\ of\ state\ variables;$$

$$c \equiv \left[0, \mu_\xi \Delta t\right], \qquad a\ 2 \times 1\ vector;$$

$$G \equiv \begin{bmatrix} e^{-\kappa\Delta t} & 0 \\ 0 & 1 \end{bmatrix}, \qquad a\ 2 \times 2\ matrix;$$

With $\omega_t$ being a $2 \times 1$ vector of serially uncorrelated, normally distributed disturbances with $\mathrm{E}[\omega_t] = 0$ and $\mathrm{Var}[\omega_t] = W \equiv Cov[(\chi_{\Delta t}, \xi_{\Delta t})]$, $\Delta t$ the length of the time period and $n_T$ the number of time periods in the dataset.

Being the measurement equation:

$$y_t = d_t + F'_t x_t + v_t, \qquad t = 1, \dots, n_T$$

where $y_t \equiv [\ln F_{T1}, \dots, \ln F_{Tn}]$ is a $1 \times n$ vector of log future prices with time maturities $T_1, T_2, \dots, T_n$; $d_t \equiv [A(T_1), \dots, A(T_2)]$ is a $n \times 1$ vector, $F_t \equiv [e^{-\kappa T_1} 1, \dots, e^{-\kappa T_n} 1]$ is a $n \times 2$ matrix and $v_t$ is a $n \times 1$ vector of serially uncorrelated, normally distributed disturbances with $\mathrm{E}[v_t] = 0$ and $Cov[v_t] = V$.

The parameters estimation procedure has been run independently for each payable metal using spot and futures data from 2004 to 2018. The different maturities of contracts considered for the parameters estimation for each metal are shown in **Table 25**. The maximum number possible of maturities of contracts have been considered to calibrate the models in order to increase the metal's price forecasts reliability and longer-term accuracy. Furthermore, according to Schwartz-Smith (2000), the model's state variable may ultimately be estimated within a range of around ±8% if only spot data are used, whereas if multiple future contracts with different maturities are used the uncertainty will be reduced.

**Table 25**
Contracts considered for parameters estimation.

| | Exchange | Contracts |
|---|---|---|
| Cu | LME | Cash, 3M, 15M, 27M, 63M |
| Au | COMEX | Cash, 2M, 3M, 6M, 9M, 12M, 15M, 18M, 20M |
| Ag | COMEX | Cash, 2M, 3M, 6M, 9M, 12M, 15M, 18M, 20M |

### *4.4. Metals and Copper TC/RC Forecasts*

In-sample daily metals prices from 2004 to 2018 along with their estimation using Schwartz and Smith's (2000) model with parameters adjusted through the Kalman filter maximum likelihood recursive method proposed by Barlow (2004) are shown in **Figure 6**.


**Figure 6.** Observed and Schwartz and Smith's two-factor model estimation for 2004-2018 daily metals prices.

The optimum parameters estimations obtained using historical daily data of spot and futures prices for each metal are shown in **Table 26**, along with the different error estimations for the measurement equation with the future contracts employed with each metal at every maturity. Values of the log likelihood function have been maximized to achieve the parameters that render the best forecast for each commodity, thus enhancing the model fitting to historical data and forecasting capacity of metal prices and of concentrates prices consequently.

**Table 26**
Parameters values for Schwartz and Smith's model for Copper, Gold and Silver obtained through the Kalman Filter and maximum likelihood using futures and spot prices data from 2004 to 2018.

| | Copper | Gold | Silver |
|---|---|---|---|
| $\kappa$ | 0.2427 | 0.0440 | 0.0455 |
| $\sigma_\chi$ | 0.2418 | 0.4369 | 0.4139 |
| $\mu$ | 0.0958 | 0.0606 | -0.2718 |
| $\sigma_\xi$ | 0.3121 | 0.4660 | 0.4230 |
| $\rho_{\chi\xi}$ | -0.2889 | -0.8852 | -0.5549 |
| $logL$ | $5.5948 \times 10^{4}$ | $1.3311 \times 10^{5}$ | $1.2112 \times 10^{5}$ |
| $s_1$ | $8.8710 \times 10^{-5}$ | $1.155 \times 10^{-5}$ | $5.038 \times 10^{-6}$ |
| $s_2$ | $1.3519 \times 10^{-4}$ | $2.401 \times 10^{-7}$ | $1.017 \times 10^{-5}$ |
| $s_3$ | $1.000 \times 10^{-7}$ | $2.757 \times 10^{-6}$ | $1.114 \times 10^{-5}$ |
| $s_4$ | 0.0040 | $1.865 \times 10^{-6}$ | $8.453 \times 10^{-6}$ |
| $s_5$ | | $1.047 \times 10^{-6}$ | $2.549 \times 10^{-6}$ |
| $s_6$ | | $2.044 \times 10^{-4}$ | $6.944 \times 10^{-5}$ |
| $s_7$ | | 0.002 | $5.695 \times 10^{-4}$ |
| $s_8$ | | 0.0055 | 0.0236 |

On the other hand, the LES model's forecasts for 2019 are 84.1247 USD/dmt for TC and 8.4125 USc/lb for RC. As it may be seen, the usually observed 10:1 historical relation in TC/RC levels slightly differs in the values obtained, as TC and RC have been modeled individually. The LES model has been calibrated following the procedure explained in Chapter 3, minimizing the Mean Absolute Percentage Error (MAPE) for the one-year-ahead forecast, achieving a value of 4.12%.

*4.5. Copper concentrates benchmark price forecasts and errors*

The five different copper concentrates, whose layouts have been defined in Chapter 2, have been used to test the model's fitting errors and forecasting accuracy. As previously explained, these five concentrates have been used to be able to check the model's behaviour in common conditions, pricing regular clean concentrates, as well as complex concentrates, having high levels of punishable elements, or concentrates with meaningful levels of gold or silver.


**Figure 7.** Estimations of concentrates 1 and 2 prices compared to objective price for the in-sample period (2004 - 2018).

Model price estimations for the in-sample period (2004 – 2018) for concentrates 1 and 2 compared to their objective price calculated with spot data are shown on Figure 6. In addition, fitting errors for the five concentrates considered are shown on **Table 27**. Error values have been measured with respect to their market price calculated using historical spot metals prices during the same historical period.

**Table 27**
Fitting errors for concentrates price estimation (2004 – 2018).

| | C1 | C2 | C3 | C4 | C5 |
|---|---|---|---|---|---|
| MD | -4.0332 | -0.9911 | -1.8450 | -3.6765 | -5.3456 |
| MAD | 10.2647 | 10.1856 | 10.1189 | 10.7307 | 14.1090 |
| MSE | 279.4339 | 236.0696 | 243.5428 | 299.6983 | 525.5189 |
| RMSE | 16.7163 | 15.3646 | 15.6059 | 17.3118 | 22.9242 |
| MPE | -0.0018 | $-4.0298 \times 10^{-5}$ | $-4.0160 \times 10^{-4}$ | -0.0014 | -0.0017 |
| MAPE | 0.0071 | 0.0041 | 0.0046 | 0.0064 | 0.0069 |

For the 2019 out-of-sample forecasts of concentrates prices we have performed 1000 simulations for each of the five concentrates during the period considered. The first forecast obtained for every concentrate is shown on **Figure 8** compared to the objective concentrates prices for the same year, as well as to the averages of the 1000 simulations for the one-year ahead daily prices forecasts for each concentrate considered, which represent the expected values for the concentrates' prices by the model over time.

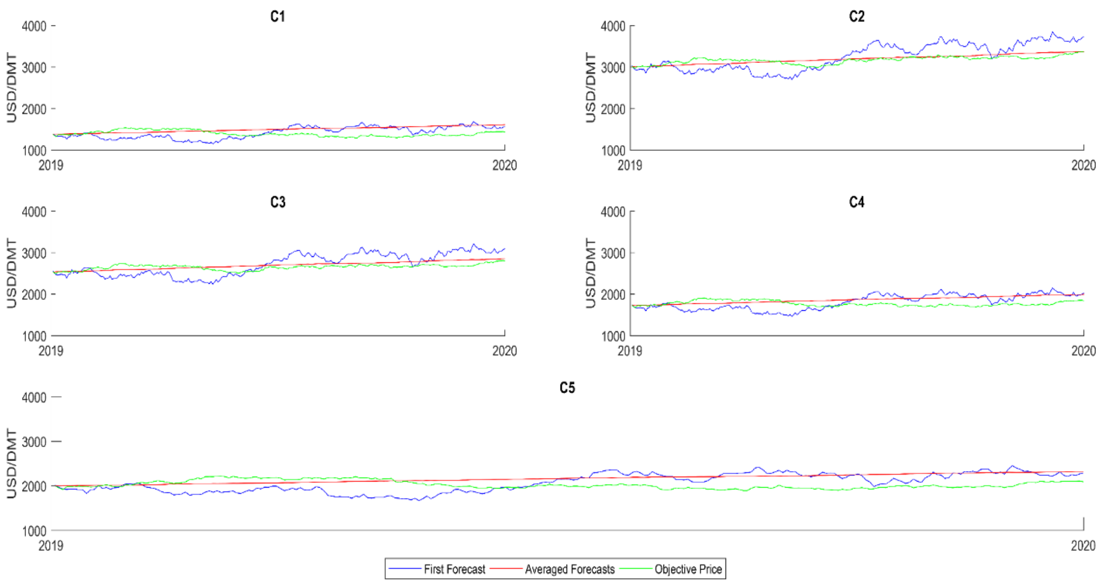


**Figure 8.** 2019 Model's forecasts for concentrates, expected prices and objective spot prices.

Short-term and longer-term average forecasting error measures achieved for each concentrate may also be seen on **Table 28**. Forecasting error measures have also been calculated by comparing forecasted values with concentrates prices calculated for the year 2019 using SPOT prices data of payable metals.

**Table 28**
Errors for concentrates price forecasting.

| | | C1 | C2 | C3 | C4 | C5 | Average |
|---|---|---|---|---|---|---|---|
| 1 day | MD | -1.452 | 4.320 | 2.816 | -0.365 | -1.554 | 0.753 |
| | MAD | 21.448 | 36.414 | 32.476 | 25.591 | 30.707 | 29.327 |
| | MSE | 699.021 | 2006.750 | 1598.975 | 996.447 | 1433.446 | 1346.928 |
| | RMSE | 21.448 | 36.414 | 32.476 | 25.591 | 30.707 | 29.327 |
| | MPE | -0.001 | 0.001 | 0.001 | 0.000 | -0.001 | 0.000 |
| | MAPE | 0.016 | 0.012 | 0.013 | 0.015 | 0.015 | 0.014 |
| 5 days | MD | -26.908 | -12.971 | -16.123 | -25.393 | -35.894 | -23.458 |
| | MAD | 45.171 | 62.898 | 57.549 | 50.514 | 63.286 | 55.883 |
| | MSE | 3403.150 | 6694.986 | 5595.886 | 4278.982 | 6694.282 | 5333.457 |
| | RMSE | 45.171 | 62.898 | 57.549 | 50.514 | 63.286 | 55.883 |
| | MPE | -0.020 | -0.004 | -0.006 | -0.015 | -0.018 | -0.013 |
| | MAPE | 0.033 | 0.021 | 0.023 | 0.030 | 0.032 | 0.028 |
| 1 month | MD | -13.209 | 3.010 | -0.883 | -10.518 | -16.861 | -7.692 |
| | MAD | 73.552 | 120.444 | 107.985 | 86.825 | 104.925 | 98.746 |
| | MSE | 9419.109 | 25761.431 | 20670.874 | 13223.143 | 19216.173 | 17658.146 |
| | RMSE | 73.552 | 120.444 | 107.985 | 86.825 | 104.925 | 98.746 |
| | MPE | -0.010 | 0.001 | 0.000 | -0.006 | -0.009 | -0.005 |
| | MAPE | 0.053 | 0.040 | 0.042 | 0.050 | 0.052 | 0.048 |
| 3 months | MD | 41.757 | 64.676 | 58.815 | 48.841 | 59.422 | 54.702 |
| | MAD | 134.769 | 220.363 | 197.857 | 159.356 | 192.387 | 180.946 |
| | MSE | 31407.277 | 84012.894 | 67726.073 | 43933.814 | 64014.626 | 58218.937 |
| | RMSE | 134.769 | 220.363 | 197.857 | 159.356 | 192.387 | 180.946 |
| | MPE | 0.027 | 0.020 | 0.022 | 0.026 | 0.027 | 0.025 |
| | MAPE | 0.092 | 0.070 | 0.075 | 0.087 | 0.090 | 0.083 |
| 6 months | MD | -2.881 | 4.668 | 3.158 | -1.298 | -3.251 | 0.079 |
| | MAD | 183.731 | 301.582 | 270.094 | 217.130 | 262.186 | 246.945 |
| | MSE | 60179.870 | 159490.231 | 128294.522 | 83681.170 | 122385.032 | 110806.165 |
| | RMSE | 183.731 | 301.582 | 270.094 | 217.130 | 262.186 | 246.945 |
| | MPE | -0.004 | 0.001 | 0.001 | -0.002 | -0.004 | -0.002 |
| | MAPE | 0.128 | 0.097 | 0.103 | 0.121 | 0.126 | 0.115 |
| 1 year | MD | -100.988 | -23.680 | -41.838 | -90.147 | -132.711 | -77.873 |
| | MAD | 267.058 | 424.151 | 378.967 | 309.632 | 378.378 | 351.637 |
| | MSE | 137882.891 | 322525.588 | 260476.025 | 181401.640 | 274961.672 | 235449.563 |
| | RMSE | 267.058 | 424.151 | 378.967 | 309.632 | 378.378 | 351.637 |
| | MPE | -0.076 | -0.008 | -0.016 | -0.052 | -0.068 | -0.044 |
| | MAPE | 0.193 | 0.133 | 0.142 | 0.175 | 0.188 | 0.166 |

MAPE values for concentrates price forecasts barely exceed 1.5% for the one-day ahead horizon, and 5% for the one-month ahead. On the most frequent horizons for copper concentrates trading, the one-month ahead and the three-month ahead, average MAPE values of the model varies from 4.8% for the first and 8.3% for the latter, thus giving reasonable estimations over the final price that market participants would pay for concentrates, as well as a clearer picture of the level of commodity-price-related risk their taking, their short-term liquidity needs or their hedging necessities, among other concerns they face. Additionally, non-existing references in literature on copper concentrate pricing, making this the first research to address the lack of an appropriate model for this, impedes comparing the values of errors achieved with any pre-existing benchmark.

The influence of high gold or silver grade is clearly a driving factor as may be understood by the prime paid for C2 and C3 in comparison to the rest of the concentrates. We also find that the presence of high levels of gold and silver in the concentrate act as a stabilizing factor for concentrates prices as precious metals tend to stay less volatile than copper. This has a direct impact in the model's accuracy when forecasting the future prices of these kind of concentrates. Thus, MAPE values for C2 and C3 for the three-month horizon stay between 7.0-7.5%, whereas it goes above 8.5% for C4 and 9.0% for C1 and C5. This effect is amplified as the forecasting horizon is extended, reaching differences in MAPE values of 2.4 percentage points between C3 and C5 and 6.0 percentage points between C1 and C2 for the one-year horizon.

Taking the full scope of concentrates layout considered and using MAPE as the benchmark to compare the model's forecasting ability for a particular concentrate, the group of those presenting higher levels of precious metals are forecasted more accurately by the model (C2 and C3). On the other hand, the price of those with lower levels of precious metals are tougher to forecast by the model (C4, C5 and C1). By the contrary, RMSE values for the different concentrates indicate a larger dispersion of residuals for C2 and C3, as well as the opposite for the others, at every horizon studied, indicating that the influence of high levels of gold and silver, whilst stabilizing the average forecast, may be tending to introduce a higher variance in individual predictions.

In addition, high values of copper may even compensate the amounts of penalties due to punishable elements in the concentrate and make them commercially feasible as is the case of C5, grading over 8 percentage points more of copper than C4. Nevertheless, the difference in value between highly complex concentrates due to high levels of payable elements, such as C4, and clean concentrates with an average content of payable elements, such as C1, might not be sufficient to compensate the complexity and additional costs incurred in treating the most complex ones at specialized smelters versus the cleanest. In this case, this difference in value is an average of 24.4% more for C4 than C1 for the one-year ahead forecast.

#### 4.2.1. Effects of discounts and punishable elements over the price of concentrates

The value of the discounts as well as the penalties due to punishable elements and the total amount of deductions applied expressed in USD per dry metric tonne of concentrate for the year 2019 (one-year ahead forecast) may be seen in **Table 28** for each of the concentrates considered. According to our results, the impact of punishable elements on the overall deduction, and thus on the final price paid for concentrates is considerable, as they just represent 8.35% of the total amount subtracted for C1, which is regarded as a clean concentrate, while it is a 30.43% for C5, with a net increment of 54.02 USD/dmt, over four times as much as for the previous case.

**Table 28**
Discounts and Penalties in USD/dmt of concentrate for the 2019 forecast.

| | C1 | C2 | C3 | C4 | C5 |
|---|---|---|---|---|---|
| Cu TC | 84.12 | 84.12 | 84.12 | 84.12 | 84.12 |
| Cu RC | 47.52 | 50.26 | 49.33 | 50.69 | 65.70 |
| Au RC | 0.00 | 8.98 | 6.04 | 1.53 | 0.52 |
| Ag RC | 0.00 | 0.67 | 2.43 | 1.06 | 0.63 |
| Penalties | 12.00 | 9.30 | 14.83 | 55.93 | 66.02 |
| Total | 143.64 | 153.34 | 156.76 | 193.33 | 216.99 |

Notwithstanding, the copper content in C5 is noticeably higher than for C1 which presumably to some extent offsets the increase in penalties and in copper RC. On the other hand, regarding C4, the penalties have increased by 366% compared to the first concentrate, now accounting for 28.93% of the overall deductions, while the copper content has barely increased from 26.62% to 28.33%. This could even make C4 less valuable than C1 if it were not for the important difference in gold and silver content which helps boost its price.

Finally, the discounts' relative weight with respect to payable metals content fluctuates over time for a specific concentrate layout as metal prices and discounts themselves also do. The forecasted relative weight of the overall discounts on the payable metal content of each of the concentrates is represented by the boxplots in **Figure 9**.

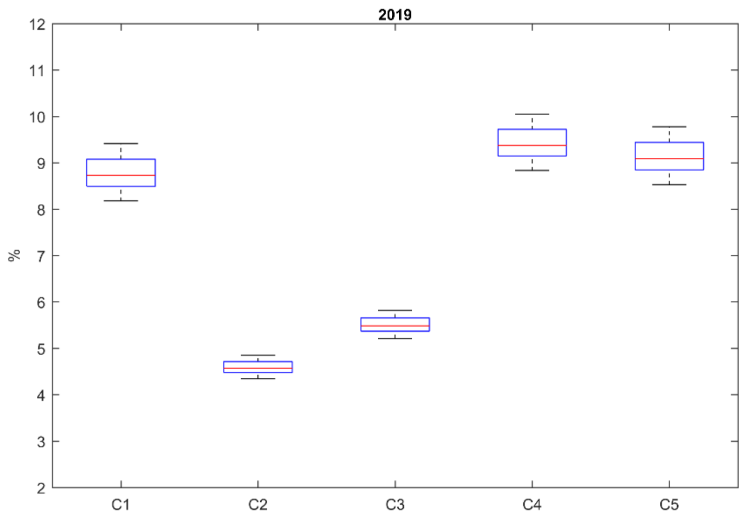


**Figure 9.** Discounts' relative weight in payable metal content in concentrate.

The mean value of the discounts' relative weights with respect to the payable metals content for each concentrate is shown by the middle line of the boxplots, while the lower and upper limits represent the 25th and 75th percentiles, respectively. Also, the whiskers stretch to the furthermost values in the forecasted timeseries that have not been considered as outliers.

# Chapter 5: Managing a High Uncertainty Scenario through a Real Option Assessment: Evidence from a Copper Concentrate Trader

### *5.1. Introduction*

Most commodity traders participate to some extent in the extraction or processing stages of the commodities supply chain. However, independent traders, also known as trading houses, neither produce nor process the commodities that they trade with, and yet they play a major role in commodity markets. Independent traders buy and sell a commodity seeking to take advantage of potentially profitable arbitrage opportunities in time and space, connecting primary producers to processors. Their activity normally involves bespoke agreements to ensure supply, while mastering logistics and simultaneously merging financial markets with physical transactions to mitigate risks and ensure margins.

As a subset of commodity traders, metal traders focus on the most universally used industrial and precious metals, not limiting themselves to the refined form of the metals, but also engaging in the trading of unrefined forms, semi-processed or raw ores. For the case of copper, copper concentrate is the main form of unrefined copper and the primary product sold by most copper miners, being massively traded by independent trading houses.

Traders acquire copper concentrate from miners, taking the other side of the trade from smelters and refiners, which process concentrates to obtain copper cathodes. Their role is predominant in the global copper market. An example is Glencore, which in 2020 traded 3.4 million tons of metal copper and concentrates, almost as much as the mining production of Peru and China combined, the second and third leading copper producers in the world respectively. Meanwhile, its main competitor, Trafigura, traded 11.1 million tons of copper, zinc, lead and nickel concentrates as well as alumina and cobalt hydroxide just in 2020 alone (Trafigura Beheer B.V., 2021).

Consequently, in this chapter we analyse the copper concentrate trading business resorting to the Real Options methodology to assess the high volatility and uncertainty which dominates in this area of commodity markets. The results obtained indicate that this method can be applied by copper concentrate traders to separate the price uncertainty from their business operative and financial planning for the future. This provides them with a reliable tool to lower the level of uncertainty affecting their long-term goals, as well as keeping the risk that they are taking under control. It also gives them higher managerial flexibility.

### *5.2. Related Work*

Traditionally, commodity trading houses have developed their activities in utter discretion, having little incentive to disclose their most sensitive information, if any at all. Hence, details related to the conditions of the agreements that traders reach with either primary producers or their clients, the pricing mechanisms for concentrates, as well as for most of the commodities they trade with, or the structure of some usual financial tools that they employ such as the pre-payment and pre-finance agreements, are widely unknown except by those directly involved in commodity trading.

This lack of understanding of the behaviour of commodity traders has not been straightforwardly addressed in the academic literature either. This has mainly focused on studying commodities pricing methods, such as Brennan and Schwartz (1985), Schwartz and Smith (2000), Liu et al. (2017), and Dehghani and Bogdanovic (2018), who analyse different alternatives to forecast copper prices as well as assessing the value of commodity-related projects. Jia and Kang (2022) formulate two claims about spot and future price predictions by investigating six industrial metals traded in the London Metal Exchange (LME).

Furthermore, a significant number of works have instead centred on evaluating the feasibility of commodity-related investments, such as Sarkis and Tamarkin's (2005), which applies Real Options analysis to a particular petroleum project that would generate greenhouse gases emissions permits. Many works have employed Real Options as their main tool to assess a certain project's feasibility and its long-term value. Real Options are quite suitable to determine the value of projects which rely on commodities, either energy, mineral, agricultural or soft commodities, and which require the estimation of financial statements. This method also helps to draw more accurate financial plans as it allows considering a stretch of different possible scenarios.

Many researchers have employed Real Options specifically focused on mining projects, as summarised by Savolainen (Savolainen, 2016), such as Inthavongsa et al. (Inthavongsa et al., 2016), who employ a real-option approach to analyse a hypothetical gold mine project. Similar research by Zhang et al. (2015) evaluated a gold mine with price uncertainty as well as analysing its optimal long-term production strategy. This has been extensively examined in the literature (Ajak and Topal, 2015; Aminrostamkolaee et al., 2017; Guj and Chandra, 2019; Hazra et al., 2019; Siña and Guzmán, 2019).

On the other hand, another branch of research has focused on valuing companies that extract, process and trade with commodities without considering the trading aspect of their business as a standalone element, or even as a key aspect for their profitability or for their company value (Kaiser, 2013; 2015). In this sense, Misund et al. (2015) valued a series of US and non-US oil and gas companies considering the primary factors affecting production. Sabet and Heaney (2017) examined the relation between oil and gas firms share prices, and the return, volatility and drilling activity of crude oil and natural gas, neither having taken into consideration the trading side. Moreover, Amram and Kulatilaka (1998) focused on using Real Options to manage the uncertainty of investments. Barton and Lawryshyn (2011) fit managerial cash flows estimates to a continuous cash flow process with changing growth and volatility parameters. Damodaran (2012) had in large part laid the basis for most of the currently-used investment valuation tools.

This chapter focuses on the copper concentrate trading business. Here the degree of uncertainty is notably higher than for other commodities because of the absence of a price reference for copper concentrates, which prevents market participants from discovering price outright, opposite to what happens with copper cathodes for instance. It also very seriously hinders their ability to hedge their exposure to the commodity. In contrast to all the previous research, this research analyses copper concentrate trading as a single and independent element of commodity markets. This is done through its highest exponent, represented by independent commodity traders, tackling the remarkable void in the literature on commodity traders' operations, financials, and inner mechanics, particularly regarding copper concentrates.

The copper concentrate trading business model is reviewed in our research through the valuation of a standardised copper concentrate trading house, which neither produces nor processes any copper concentrate, using classical discounted cash flows valuation techniques and Real Options. The main factors affecting the activity of copper concentrate trading are considered stochastically, resorting to publicly available historical data on spot prices for copper, gold and silver from COMEX and the London Metal Exchange (LME), and forecasted five years ahead to assess the trader's business future performance.

The methodology laid out in this chapter constitutes a first approach to commodity traders' dynamics, albeit a full-scale analysis would involve taking into account long-term hedging practices with futures and options. Nonetheless, this procedure enables assessing the impact that strategical decisions have on the future financial performance of any commodity trader, which is also subdued by commodities price fluctuations, among other factors, if an appropriate hedge is not implemented. Additionally, even though we have focused on copper concentrate trading, the process developed in this paper may well be applicable to commodity traders regardless of the commodity or commodities that they are trading with. This is because its main utility is to provide continuous managerial flexibility under very volatile contexts, such as those which unfold in commodity markets.

### *5.3. Materials and Method*

To assess the valuation of the copper concentrate trading company a five-year timeframe has been set from the years 2021 to 2025. 2020 being the last full fiscal year completed by the trader. The reference economical and financial data employed to develop the projections have been based on a commodity trader's 2020 annual income statement and balance sheet. Given the complexity of copper concentrate physical trading, the starting information has been limited to exclusively portraying the copper concentrate trades carried out by the company, not the remaining possible associated deals that it might have conducted. On the other hand, exchange rate fluctuations in the long run have not been accounted for to model the behaviour and profitability of the trader's activity, thus assuming a perfectly currency-hedge trader, with null foreign currency exposure.

#### 5.3.1. Concentrate Price

The trader's activity has been modelled employing a standard concentrate whose chemical layout has been detailed in **Table 29**. The concentrate's specifications were set in accordance with the statistical classification described in Delbeke and Rodriguez (2014), fitting the concentrate used within the 60th percentile of this classification, containing low levels of gold and silver.

**Table 29**
Copper Concentrate Specifications. (Source: Own elaboration based on Delbeke and Rodriguez (2014).

| | % | Ppm | g/DMT |
|---|---|---|---|
| Cu | 27.25% | | |
| Au | | | 1.50 |
| Ag | | | 30 |
| As | 0.002% | 20 | |
| Sb | 0.002% | 20 | |
| Bi | 0.003% | 30 | |
| Cd | 0.005% | 50 | |
| Cl | 0.005% | 49 | |
| F | 0.021% | 210 | |
| Pb | 0.040% | 400 | |
| Hg | 0.000% | 0 | |
| Ni | 0.004% | 40 | |
| Co | 0.009% | 90 | |
| Se | 0.002% | 20 | |
| Te | 0.001% | 10 | |
| Zn | 0.220% | 2200 | |
| Humidity | 8.00% | | |

The expected annual prices for the copper concentrate in each period, *t*, comprising the years from 2021 to 2025, have been obtained by averaging out 500 individual daily price paths for the copper concentrate over this timespan. The selling price for the concentrate on each day *d*, expressed in United States Dollars per dry metric ton (USD/DMT), is determined by the price model developed in Chapter 4.

To obtain the concentrate daily price paths over the forecasting horizon we have employed a slightly simplified, less computing-demanding procedure than the one exposed in Chapter 4. Hence 500 individual daily price paths for Copper, Gold and Silver have been first obtained, following the process shown in **Figure 10**. The Anderson – Darling test has been used to find out the distribution best fitting the historical data on daily spot prices returns from 2010 to 2020 of each metal. The

Anderson – Darling test has also been applied to obtain the distribution best fitting the historical data on annual TC benchmark levels from 2004 to 2021.

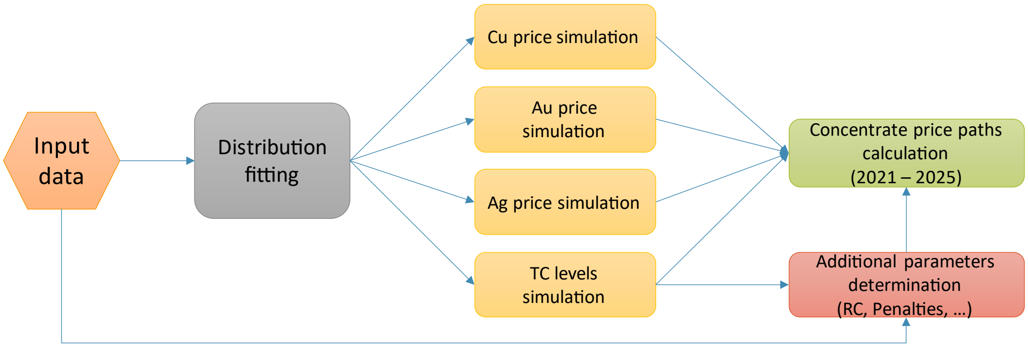


**Figure 10.** Concentrate price paths calculation process. (Source: Own elaboration).

Daily returns for Copper, Gold and Silver, $s_d^k$, for *the* N working days in each year from 2021 to 2025, thus $s_d^k = \{s_1^k, \dots, s_N^k\}$, as well as annual values for copper Treatment Charges benchmark levels, $TC_t$, from 2022 to 2025, have been forecasted by performing 500 independent Monte Carlo simulations for each metal and for TC using the corresponding best-fitting statistical distribution shown in Table 2. Historical annual benchmark values from 2010 to 2021 used in Chapter 3 have been used to estimate expected annual TC benchmark levels. Additionally, forecasts and distribution fittings have been obtained using Oracle's Crystal Ball add-in for Microsoft Excel.

**Table 30**
Statistical Distribution of Historical daily returns of metal prices. (Source: Own elaboration).

| | Distribution |
|---|---|
| Cu | Logistic (0; 0.01) |
| Au | t – Student (0; 0.01) |
| Ag | t – Student (0; 0.01) |
| TC | Weibull (66.34; 3.39) |

To find an adequate number of independent simulations to achieve reliable forecasts for both metal prices daily returns and TC benchmark levels, while also avoiding an unnecessarily high number of simulations, we have pursued the stabilisation of the simulated timeseries' standard deviation rate of decrease as the number of simulations is incremented as seen in **Figure 11.**

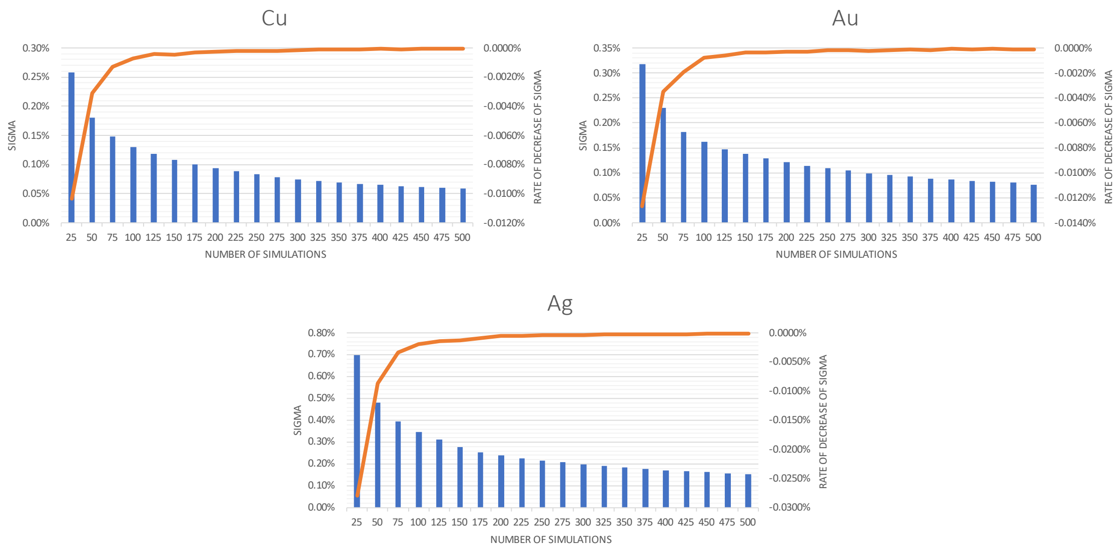


**Figure 11.** Rate of decrease of sigma for different number of simulations for each metal price forecasts.

Metals prices each day $d$, $S_d^k$, have been projected alternatively to the method presented in Chapter 4, in which a basket of future contracts was the necessary input for the model to deliver a valid forecast for the metal prices. To deliver a more purposeful approach that stays in line with the main objective of valuating the activity of the trader, rather than giving extremely precise and reliable price forecasts, a purely statistical method has been implemented.

This method, which sacrifices some precision in favour of simplicity, sets the closing price for each metal on December 31, 2020, as the initial reference, $S_0^k$, for the forecasted timeseries of metal prices. The 500 simulated independent timeseries of daily returns of each metal have been used to determine the expected accumulated annual returns over the 2021 – 2025 timespan. The expected accumulated annual returns to date, $a_d^k$, for each metal have been employed to draw individual expected price paths for copper, gold, and silver.

$$s_d^k = \{s_1^k, \dots, s_N^k\}$$

$$a_d^k = \prod_d (1 + s_d^k)$$

$$S_d^k = S_0^k \times (1 + a_d^k)$$

$$k = Cu, Au, Ag;\ t = 1, \dots, 5$$

$$\forall\, d \leq N \times t$$

Additionally, annual expected benchmark values for copper Treatment Charges (TC), the main deductions along with Refining Charges (RC) that smelters apply to determine copper concentrates prices, have only been projected from 2022 to 2025, as the benchmark level for 2021 is already known by the end of 2020. On the other hand, copper RC levels have not been projected independently as a nominal 10:1 relation throughout the historical data employed has been observed, following the explanation in Chapter 3.

Hence, the same relation for copper TC/RC has been used to project RC values based on projections of TC over the 2022 – 2025 period. On the other hand, the remaining parameters for copper concentrate pricing, shown in **Table 31**, are set at constant levels over the full timeframe applying the same unitary values previously used and calculated for the concentrate's specific layout.

**Table 31**
Constant parameters for Concentrate Pricing (2021 – 2025). (Source: Own elaboration).

| | Value |
|---|---|
| Gold RC | 10.13 USD/DMT |
| Silver RC | 12.15 USD/DMT |
| Penalties | 12.00 USD/DMT |

Lastly, individual forecasted price paths for each metal in the concentrate have been merged with the projections of copper TC/RC benchmark levels as well as with the remaining discounts to deliver 500 daily price paths for the copper concentrate over the 2021 – 2025 period. Daily price paths for the concentrate have not been used to model the trader's financial performance over the projected period, instead the concentrate's expected annual average price for each year have been employed. To obtain the expected annual average price for the concentrate, the daily price paths have been transformed into annual average price paths. The 500 annual average price paths have been averaged out to give a single expected annual price for the concentrate. Price paths have been converted into EUR/DMT, as that is the company reference currency, employing a fixed USD/EUR rate of 1.10.

### 5.3.2. Copper concentrate trading companies valuation model

The value of the trader within the established five-year horizon is based upon the approach of fundamental analysis that matches risk and return (Gajek and Kuciński, 2017; Li and Mohanram, 2019). This can be expressed as the sum of the value of the company's trading business, $V_0$, developed in accordance with the initial planning, along with the added value that the company may reach due to variations from this initial business plan, $V'$. Thus, the company's initial valuation, $V_0$, which is also deemed to be the most likely, is expressed through the discount of Expected Equity Free Cash Flows (EFCF) along with the business' Terminal Value, $TV_n$, at the end of the timeframe considered, $n = 5$.

$$V = V_0 + V'$$

$$V_0 = \sum_{t=1}^{n} \frac{EFCF_t}{(1+K)^t} + \frac{TV_n}{(1+K)^n} \qquad n = 5$$

On the other hand, the potential added value alternatives, $V'$, are analysed through Real Options, expressed as the sum of the expected company value if each option is exercised multiplied by its respective exercise probability as schematised in **Figure 12**. $u_1 v$ and $u_2 v$ are the company's values if either the first grow option or the second is exercised, whereas the $d_1 v$ is the company's value should the abandon option be exercised.

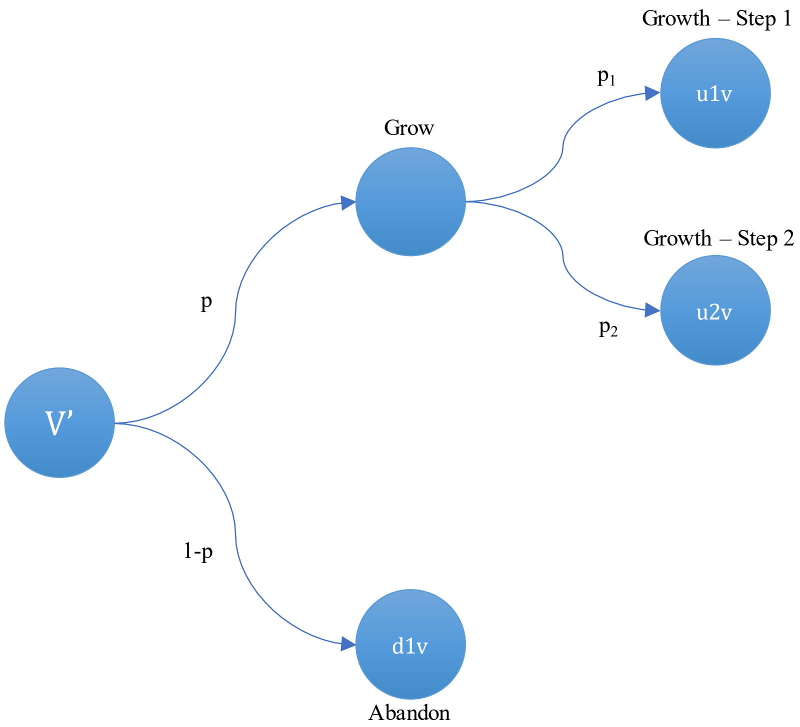


**Figure 12.** Real Options valuation scheme. (Source: Own elaboration).

$$V' = p[p_1 u_1 v + p_2 u_2 v] + (1 - p)\ [d_1 v]$$

### 5.3.3. Equity free cash flows

Equity free cash flows (Damodaradan, 2006; Yaari et al., 2016; Aharon et al., 2019; Smith and Pennathur, 2019) in each projected period, $EFCF_t$, are obtained through the application of the Market Strategy, the Financial Strategy, the Capital Strategy, the Financial Policy and the Working Capital Strategy.

$$EFCF_t = SF_t + d_t - I_t + F_t$$

Where both the self-financing at the end of each period, $SF_t$, and the dividends perceived by shareholders during that period, $d_t$, are fixed by the Financial Strategy, in which the distribution of returns or Earnings Before Interest Taxes Depreciation and Amortisation (EBITDA) is materialised. Seeking the preserving of the trader's financial capabilities, as well as minimising the necessity to resort to external capital, the Financial Strategy has set dividends payments to zero, $d_t = 0$, throughout the company's entire lifespan. On the other hand, the trader's self-financing at each period t, $SF_t$, has been calculated as the generated EBITDA, minus the financial expenses, $f_t$, and the amount of taxes paid by the company, $TX_t$.

$$SF_t = EBITDA_t - f_t - TX_t$$

On the other hand, EBITDA (Rozenbaum, 2019), is obtained through the Market Strategy. This sets up the trader's operative over the projected timeframe 2021 – 2025 by projecting the number of standard containers the trader is expected to sell in each period, expressed in Twenty-foot Equivalent Units (TEU), as well as the annual increase rate of sales of containers. The sold mass of containers in each period, expressed in Dry Metric Tons (DMT), $V_t$, is a function of the nominal loading capacity of each container, 27.6 MT/TEU, as well as the humidity of the concentrate.

$$EBITDA_t = (V_t \times C_t) - CM_t - CE_t - OE_t$$

The EBITDA in each period is hence calculated as the net value of sales, this being the mass of dry tons of copper concentrates sold multiplied by their forecasted price in €/DMT, $V_t \times C_t$, minus the costs of acquiring the concentrates, $CM_t$, the costs of employees, $CE_t$, and the operative expenses, $OE_t$. These costs and expenses that the trader faces in its business operation have been set in accordance with the observed data from the trading company taken as a reference, the cost of the concentrates being indirectly determined by fixing a gross margin over the sales value.

Next, by the application of the Capital Strategy, particularly through the Investment Policy, the company's investment needs, or Capital Expenditures (CAPEX), for each period t have been outlined, these being defined as the increases of the non-current assets, $\Delta NCA$, minus their decrements, $\nabla \mathrm{NCA}_t$, plus the change in the net working capital, $\Delta\nabla NWC_t$.

$$CAPEX_t = \Delta NCA_t - \nabla \mathrm{NCA}_t \pm \Delta\nabla NWC_t$$

Additionally, as part of the Investment Policy the Working Capital Strategy has been designed. This, given the nature of physical trading, and in a simplified form, establishes that payments to suppliers are executed only once the copper concentrates purchased are served at the trader's designated port for loading. On the other hand, the collection of payments from customers is estimated to be made once the cargo has been delivered at the destination port. The trader's business model has been standardised, restricting its activity to fetch concentrates from different origins in South America and delivering to smelters in China.

Finally, through the Financial Policy, the indebting capacity of the company has been projected, this being described as the increases of the non-current liabilities, $\Delta NCL_t$, minus their decrements, $\nabla NCL_t$, plus capital increases, $CS_t$.

$$F_t = \Delta NCL_t - \nabla NCL_t + CS_t$$

### 5.3.4. Terminal value

The Terminal Value depicts the expected income the trader should obtain beyond the year set as a limit for the planification horizon, year n, this being 2025. It can be established assuming as a constant the equity free cash flow from that moment on, $EFCF_n$, discounted at a factor K.

$$TV_n = \frac{EFCF_n}{K}$$

### 5.3.5. Discount factor

The discount factor for future cash flows, K, has been determined employing the formulation in the Capital Asset Pricing Model, based on Markowitz (1952) and Markowitz (1959), and later developed in subsequent works by Sharpe (1964) and Makwasha et al. (2019). This is a function of the perceived risk-free rate, $R_f$, plus the market risk premium, denoted as the difference between the demanded return by investors, $R_m$, with the risk-free rate, multiplied by a factor that correlates the risk of the company with the market, $\beta$.

$$K = R_f + (R_m - R_f) \times \beta$$

The $\beta$ employed has been determined qualitatively, given the inconvenience of calculated betas for non-publicly traded companies, which cannot be compared to one that is and concerning which there are enough historical data. For the specific case of the copper concentrates trader, there is no availability of data from a similar company that could be employed for the calculation of a generic beta. It is also unfeasible to fall back on data from large publicly traded corporations as these have a vertically integrated structure, the extractive, processing, trading, and supplying steps of the commodities supply chain being developed altogether.

Consequently, the MASCOFLAPEC method for qualitative betas described in Fernandez and Carabias (2007) has been used, adjusting the weighing to the ones shown in **Table 32**. The answers to two separate surveys undertook independently by executive staff from the reference trader, where each question could be answered grading from 1 to 5 according to the interviewed person's perceived level of risk from low to remarkably high, have been averaged out. The overall result of each questionnaire has also been normalised by employing a factor of 0.5 in both surveys, thus ensuring that the values of the qualitative betas fall within a reasonable range from 0.5 to 2.5.

**Table 32**
MASCOFLAPEC weighing. (Source: Own elaboration).

| | | Weight |
|---|---|---|
| M | Management | 10.0% |
| A | Assets; Business; Industry/Product | 25.0% |
| S | Strategy | 3.0% |
| C | Country risk | 15.0% |
| O | Operative leverage | 10.0% |
| F | Financial leverage | 15.0% |
| L | Liquidity of investment | 5.0% |
| A | Access to sources of funds | 5.0% |
| P | Partners | 2.0% |
| E | Exposure to other risks | 5.0% |
| C | Cash flow stability | 5.0% |

### 5.3.5. Real options

The different long-run managerial alternatives have been considered as the business faces uncertainties that might either play in its favour or against it. Should market conditions become favourable enough, the trader could opt for expanding its supply capacity at differentiated steps whether conditions are more or less advantageous. On the other hand, in the event of a critical market downturn the trader would choose to terminate its operations rather than continuing with the base case. The expected value of the trading company as a function of the possible modifications of the original business plan has been assessed using the Real Options method (Savolainen, 2016; Aminrostamkolaee et al., 2017; Berk and Podhraski, 2018; Yeh and Lien, 2020).

To value the different alternatives to be considered, mean annual concentrate prices at each year between 2021 – 2025 have been calculated based on previous simulations. As shown in **Figure 13**, each of the 500 simulated timeseries of daily concentrate prices have been averaged annually so each of the 1310 components is funnelled to just 4 elements. Mean annual prices are hence taken to populate a discrete distribution that allows for real option modelling under the concentrate price uncertainty.

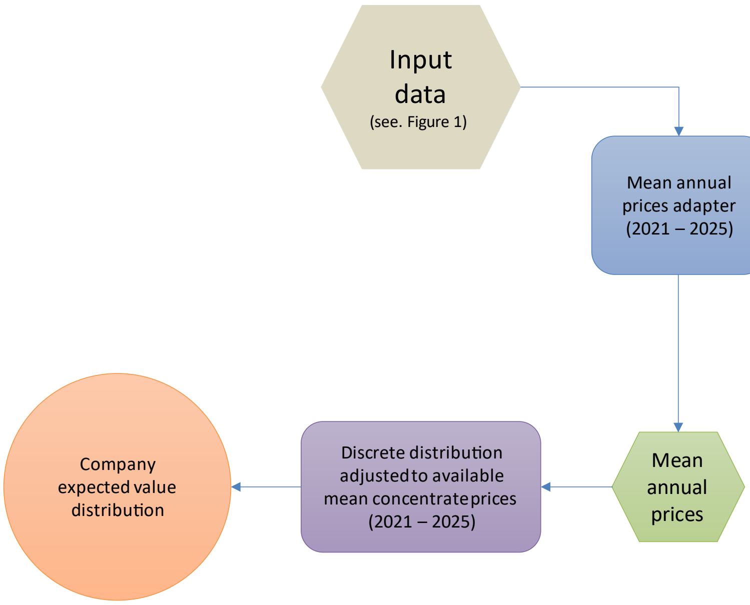


**Figure 13.** Global average concentrate prices and Company Expected Value calculation workflow. (Source: Own elaboration).

To assess the different scenarios the trader's management would face throughout the company's lifetime, three different kinds of real options have been considered: Abandonment, Continuity and Growth. The abandonment option has been first considered to model the company's terminal value if adverse market conditions occur. The option is set to be exercised if the company's expected value falls below the opportunity cost, which has been assumed to be the company's value estimated through the DCF method previously developed.

On the other hand, a growth option with two differentiated incremental steps has been modelled in which supply capacity would be first increased by 25 percentage points, or ultimately by 50 percentage points, rounded to give complete containers. These increases in the trader's supply capacity are meant to cope with sudden increases in copper concentrate prices. The increases in prices are considered as demand-driven, hence underpinning the expansion of the trader's supplying capacity as the whole concentrates' availability is assumed to be sold.

Both the two increase steps have been chosen over narrower alternatives, where smaller increments were considered and more steps were possible, since the differences in the company value become meaningful as steps are wider. In addition, provided that the defined growth options are exercised according to copper concentrate prices, smaller increments pose lower exercise thresholds. These would usually carry an inappropriately high exercise probability linked to the volatility of copper concentrates prices.

As any expansion in the trader's supply capacity is assumed to get immediately sold out, the trader would therefore to a certain extent be willing to pay a higher price for the concentrates acquired from producers, whilst it is also keen on marketing them to its customer at a less optimal price aiming to maximise sales. Hence, for each supply capacity increasement step, a selling penalty is borne by the trader, whilst the gross margin on the concentrates' acquisition costs also shrinks, indicating the traders wish to purchase the required concentrate tonnage. The two increase steps for the trader's supply capacity are displayed in **Table 33** as a percentage of the capacity fixed at the baseline scenario, where the corresponding selling price penalisation and gross margin decrease amount is also shown.

**Table 33**
Trader's supply capacity increasement steps (Selling price penalty/Gross margin decrease). (Source: Own elaboration).

| | | 25% | 50% |
|---|---|---|---|
| Step 1 | Selling price penalty | -250 USD/DMT | |
| | Gross margin decrease | -125 bp | |
| Step 2 | Selling price penalty | | -500 USD/DMT |
| | Gross margin decrease | | -250 bp |

The exercise thresholds for the abandonment option have been determined by running 2000 independent Monte Carlo simulations for the price of the concentrate for each year, while the remaining prices for the following and previous years have been left at the global annual average prices. The minimum price levels are marked by the discount factor employed. To achieve price thresholds, the company valuation at each price level when the Monte Carlo simulation is carried out is compared to the company baseline valuation defined by the DCF method.

To determine the exercise threshold for each step of the growth option, 280 trials have been run with Crystal Ball's Opt Quest at incremental steps of 10 USD/DMT each for the threshold value, performing 2000 Monte Carlo simulations per trial. The minimisation of the mean of the company's expected value using the discounted EFCF method is sought. This is restricted to being higher than the base case set by the DCF valuation. In addition, each step's threshold and expected company mean value has been limited down to the levels reached in the previous steps, while the first step's threshold has been limited to the 2025 global annual average price.

Finally, the exercise probabilities, along with the company's expected value in either case have been estimated carrying out 2000 Monte Carlo simulations. The inverse cumulative distribution functions for each year between 2021 – 2025 for copper concentrate prices, as shown in **Figure 14**, have been employed to model Real Options. Each year's discrete distribution of concentrate prices has been taken directly from the results of the preceding simulations, thus disposing of 500 possible concentrate prices for each year within the valuation horizon.

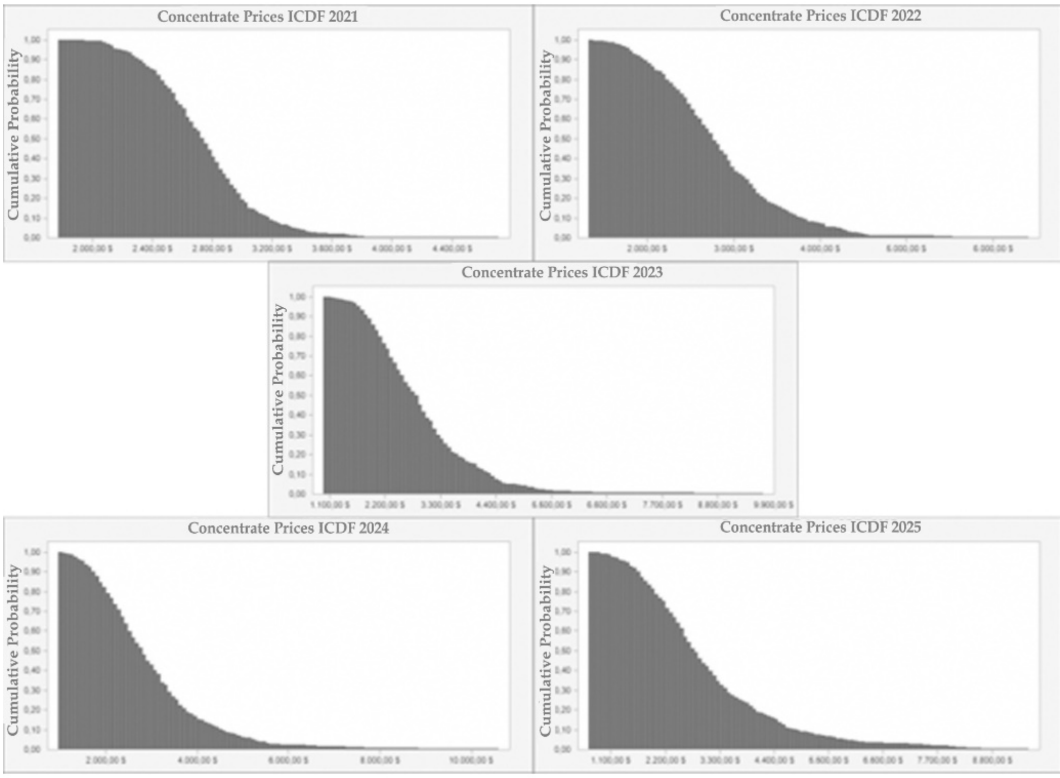


**Figure 14.** ICDF for annual Copper Concentrates prices used for ROV. (Source: Own elaboration).

### *5.4. Results and Discussion*

#### 5.4.1. Concentrate price forecasts

Five sample paths for copper concentrate price, determined as explained in subchapter 5.3.1., are shown in **Figure 15** out of the 500 paths simulated. Respectively, the annual average prices for each of the five sample paths are shown in **Figure 16**. Additionally, each year's expected annual average price for each year used to perform the trader's valuation through the DCF method are shown in **Figure 17**, being the average of the 500 annual average prices. The average daily price path is also shown in **Figure 17**.

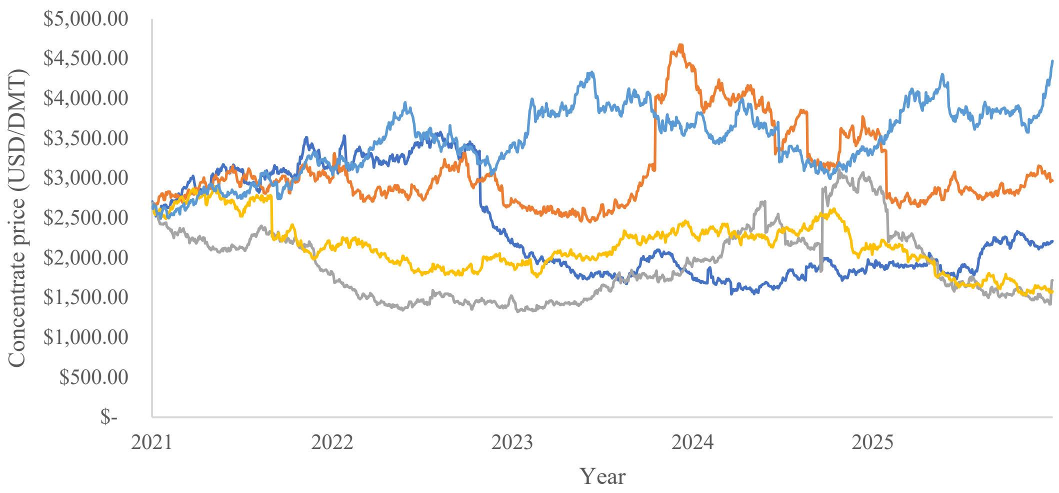


**Figure 15.** Five sample price paths for copper concentrate 2021 – 2025. (Source: Own elaboration).

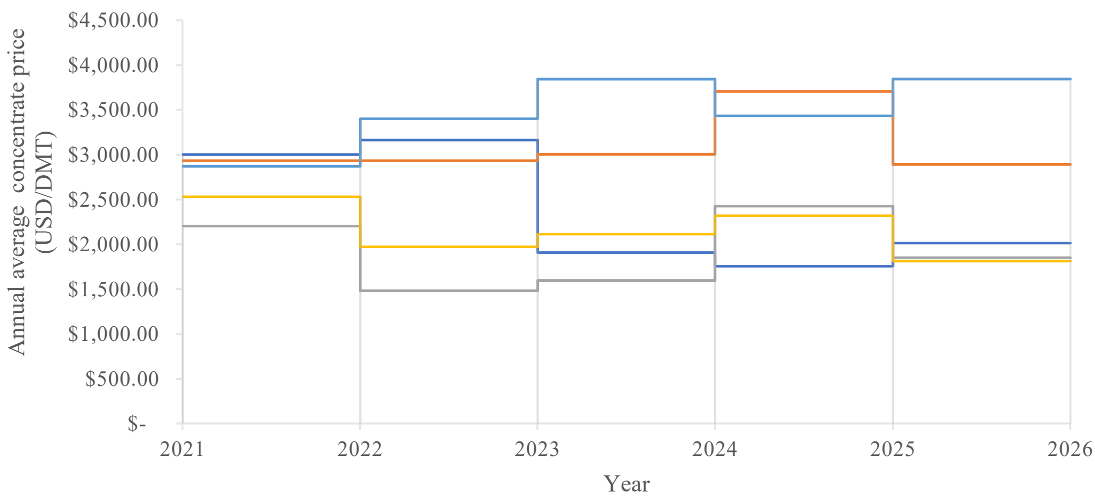


**Figure 16.** Annual average price paths for copper concentrates 2021 – 2025. (Source: Own elaboration).

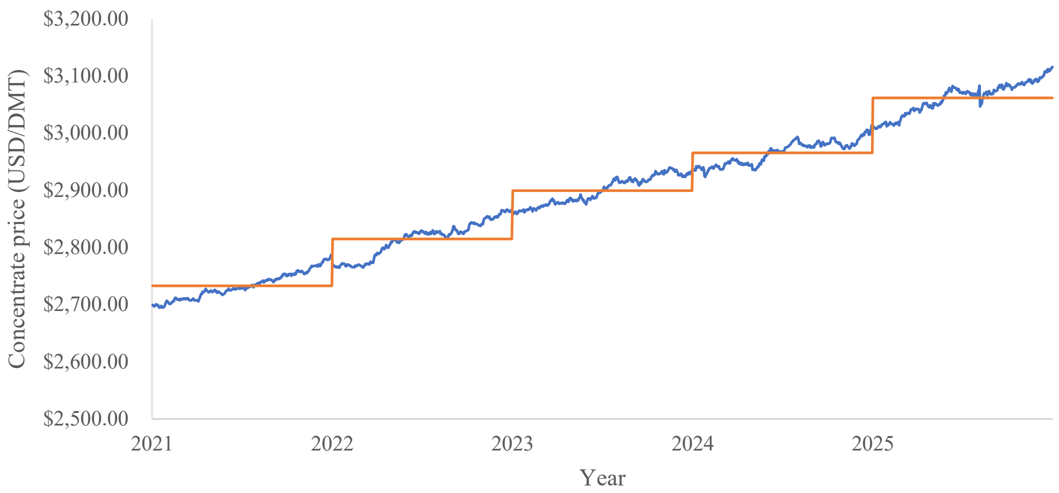


**Figure 17.** Average price path for 500 forecasts and expected annual average price for copper concentrate 2021 – 2025. (Source: Own elaboration).

The expected annual average prices for the copper concentrate used to model the trader's activity are also shown in **Table 34**.

**Table 34**
Expected Annual Average Price for Copper Concentrate 2021 – 2025. (Source: Own elaboration).

| | 2021 | 2022 | 2023 | 2024 | 2025 |
|---|---|---|---|---|---|
| Concentrate Price (USD/DMT) | 2,732.89 | 2,814.79 | 2,899.35 | 2,965.68 | 3,061.59 |
| Concentrate Price (EUR/DMT) | 2,484.45 | 2,558.90 | 2,635.77 | 2,696.07 | 2,783.27 |

### 5.4.2. Equity free cash flows

The trader's expected Equity Free Cash Flows (EFCF) have been estimated following a self-financing approach, where dividends have not been paid out to shareholders, leaving any possible external source for financing the company's activities untapped. The initial levels of self-financing achieved, shown in Table 7 alongside the forecasted levels for Equity Free Cash Flows, may only be feasible thanks to the relational capital that the project's promotors count on. This is provided from an initial and stable market share, knowledge of the commodity processes, as well as of their prices and trading mechanics.

**Table 35**
Expected Annual Average Price for Copper Concentrate 2021 – 2025. (Source: Own elaboration).

| | 2021 | 2022 | 2023 | 2024 | 2025 |
|---|---|---|---|---|---|
| Self-financing | 125,619.27 | 85,091.33 | 116,812.90 | 159,045.64 | 215,089.70 |
| Dividends | - | - | - | - | - |
| CAPEX | 118,176.64 | 71,679.89 | 59,914.91 | 76,030.16 | 99,049.76 |
| Financing | - | - | - | - | - |
| EFCF | 7,442.63 | 13,411.44 | 56,897.99 | 83,015.47 | 116,039.94 |

Expected EFCF levels achieved by the trader each year between 2021 to 2025 are subject to the annual forecasted price for the concentrate, along with the different strategies and policies preliminarily set by the company. The projected tonnage sold for each year has been assumed to increase at a constant level of around 25%, rounded to full containers. The value of sales for each year, on the other hand, varies at a different pace as it also depends on the forecasted fluctuation of concentrate prices. The price of concentrate has been set per dry metric ton of concentrate (DMT); that is, excluding the humidity. Nonetheless, humidity is to be accounted for as transportation costs are measured on the global tonnage carried, expressed in metric tons (MT).

The trader's anticipated EFCF levels are expected to increase on a yearly basis, although the rate of increase presents some deceleration beyond the year 2022 as this shows the effect of the trader's normalising its creation of stockpiles of concentrates in comparison with the first two years of operation. This is also the primary factor driving EBITDA up in 2021, as net concentrates stockpiles variations in the year levels up to 116,746.69 €, with EBITDA being 166,742.36 €. On the other hand, the net stockpiles variations in 2022 are less than a third of the increase shown in 2021, whilst EBITDA, excluding stockpile variations, increased by 58.03%, dropping 32.41% if concentrate stockpile variations are added in.

Additionally, EBITDA, and thus EFCF levels, are driven by the steady increase in sales during the whole timespan considered. On the other hand, the cost of the concentrates sold has been projected as a fraction of their sale price, as traders usually apply higher discounts to miners to determine the price paid for the concentrate that they acquire. The gross margin between the acquisition price and the sale price for the given concentrate has been observed to be at around 8.75%[2]. In addition, the efficiency and scalability of commodity traders help them to optimise structural costs, thus Personnel Costs have been assumed to remain constant at observed levels despite the projected increase in the trader's sales volume. This is because the additional workload caused by the increase in sales is assumed to remain sufficiently manageable by the trader's current staff.

Also, Operative Expenses increase at a constant pace of 1.35% annually. Hence, higher business revenues with marginal increments of main expenses, and maybe slight fluctuations in margins, are to be expected throughout the trader's lifetime. This is clearly shown in **Figure 18** and **Figure 19**, where the net increases (decreases) of revenues, EBITDA and self-financing are shown along with their relative yearly variations.

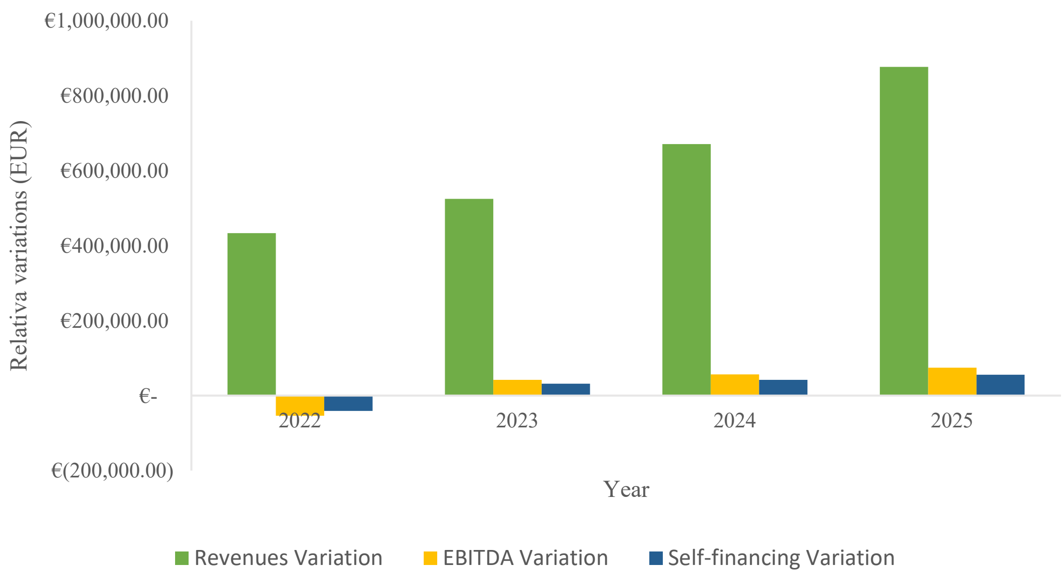


**Figure 18.** Revenue, EBITDA, Self-financing variations. (Source: Own elaboration).

2. As observed from non-publicly available and confidential economical and financial data employed as reference.

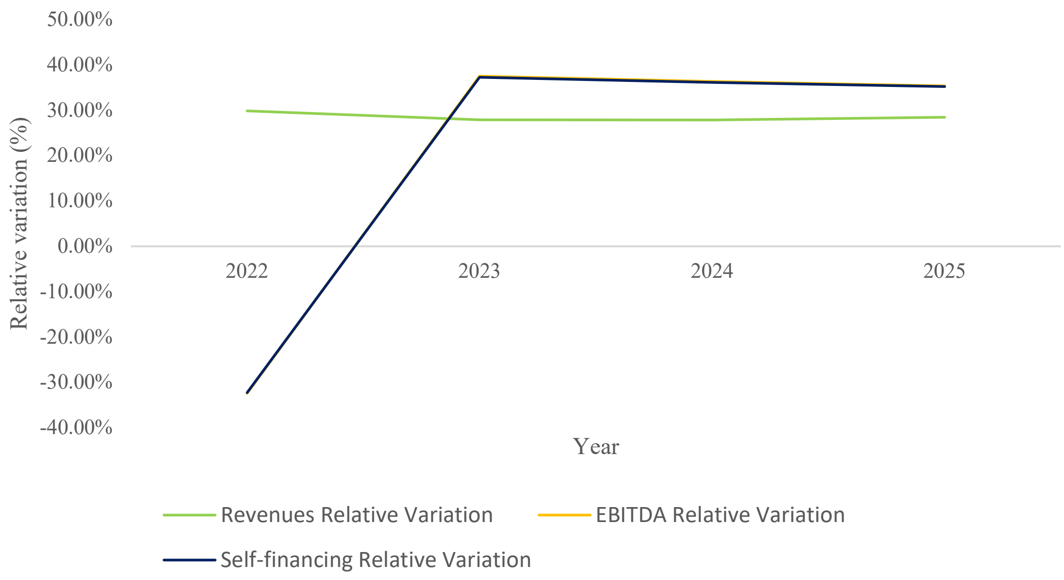


**Figure 19.** Revenue, EBITDA, Self-financing relative variations. (Source: Own elaboration).

As can clearly be seen, revenues experience a somewhat steadily increase while EBITDA and self-financing behave likewise, reaching a certain stabilisation of their respective rates of increase around 2024 even though operative expenses, excluding the costs of the concentrates sold, maintain their previously described control rise. Further detail on the trader's EBITDA construction as well as its planned Market Strategy may be found in **Table 36** and **37**.

Financial expenses have also been assumed to remain constant throughout the entire projected timespan, as most financial resources needed by the company are self-supplied. Just a small short-term recurring loan is available, with a maximum available amount of 40,000.00 EUR, of which 10,000.00 EUR are on average disposed of yearly. This loan has a fixed interest rate of 4.50% with an annual fee of 2.00%, being fully reimbursed at the end of every period. The trader lacks any other outstanding debt, either short- or long-term.

**Table 36**
Market Strategy 2021 – 2025 (EUR, unless otherwise stated).

| | 2021 | 2022 | 2023 | 2024 | 2025 |
|---|---|---|---|---|---|
| Sales (TEU) | 23 | 29 | 36 | 45 | 56 |
| Sales (DMT) | 584.02 | 736.37 | 914.11 | 1142.64 | 1421.95 |
| Operating Revenue | 1,450,955.90 | 1,884,292.08 | 2,409,391.48 | 3,080,640.54 | 3,957,667.29 |
| Cost of Concentrate | 1,334,212.32 | 1,732,682.37 | 2,215,532.39 | 2,832,772.91 | 3,639,234.29 |
| Cost of Employees | 47,160.00 | 47,160.00 | 47,160.00 | 47,160.00 | 47,160.00 |
| Other Operative Expenses | 19,587.90 | 25,437.94 | 32,526.78 | 41,588.65 | 53,428.51 |

**Table 37**
Expected Income Statements 2021 – 2025 (EUR).

| | 2021 | 2022 | 2023 | 2024 | 2025 |
|---|---|---|---|---|---|
| Revenues | 1,450,955.90 | 1,884,292.08 | 2,409,391.48 | 3,080,640.54 | 3,957,667.29 |
| Cost of Concentrates Sold | -1,334,212.32 | -1,732,682.37 | -2,215,532.39 | -2,832,772.91 | -3,639,234.29 |
| Cost of Employees | -47,160.00 | -47,160.00 | -47,160.00 | -47,160.00 | -47,160.00 |
| Other Operative Expenses | -19,587.90 | -25,437.94 | -32,526.78 | -41,588.65 | -53,428.51 |
| Stockpile Variations | 116,746.69 | 33,693.34 | 40,828.24 | 52,191.86 | 68,191.77 |
| EBITDA | 166,742.36 | 112,705.10 | 155,000.54 | 211,310.85 | 286,036.26 |
| Amortisations | -6,000.00 | -6,000.00 | -6,000.00 | -6,000.00 | -6,000.00 |
| Provisions | - | - | - | - | - |
| Financial Expenses | -1,250.00 | -1,250.00 | -1,250.00 | -1,250.00 | -1,250.00 |
| EBT | 159,492.36 | 105,455.10 | 147,750.54 | 204,060.85 | 278,786.26 |
| Taxes | -39,873.09 | -26,363.78 | -36,937.63 | -51,015.21 | -69,696.57 |
| Net Result | 119,619.27 | 79,091.33 | 110,812.90 | 153,045.64 | 209,089.70 |

On the other hand, projected annual CAPEX values have been calculated as the variations of the asset's book value, along with the net yearly requirements for working capital. Here current assets and liabilities are accounted for, including accounts payable, stockpile variations and tax payments. The trader's free cash flows are greatly decreased by heightened levels of CAPEX, mainly inflated due to working capital necessities. Unfortunately, this is a major disadvantage of almost any physical commodity trading operation, as payments to commodities suppliers, miners in the case of copper concentrates, need to be made earlier, whereas collecting accounts pending for the sale of the same concentrates to customers is deferred more or less time depending on the conditions agreed for shipping the commodities, usually Cost Freight (CFR) or Cost Insurance Freight (CIF) at the destination port. Thus, prior to collecting payments from customers, the concentrates must be shipped to the customers, usually at large distances, and several tests and proceedings need to be conducted. To reflect this effect, we have assumed the average accounts payable days to be 5, while accounts receivable days are 30 and days inventory outstanding are 25.

### 5.4.3. Discounted Cash Flows Valuation

Projected free cash flows have been discounted at a rate of 10.69% yearly. The employed discount factor, K, has been calculated using the parameters shown in **Table 38**, the risk-free rate being the 10-year yield of US treasuries, while the demanded return by investors is the mean daily annual returns of the MSCI World Index from 2010 to 2020. Also, the value of $\beta$, which has been determined following the MASCOFLAPEC method previously explained under the chapter of *Discount Factor*, has been found to be 1.470.

**Table 38**
Values for the calculation of the Discount Factor, K. (Source: Own elaboration).

| | Reference | Value |
|---|---|---|
| $R_f$ | US T-BILL 10Y | 0.50% |
| $R_m$ | MSCI World Index | 7.43% |
| $\beta$ | MASCOFLAPEC | 1.470 |

Additionally, the trader's expected cumulative discounted EFCF throughout the whole timespan considered from 2021 to 2025 amounts to 184,776.60 €, according to **Table 39**, this being the most likely valuation horizon. Also, the EFCF obtained by the trader in the last year of operations, 2025, is projected to be 116,039.90 €, which gives a TV for the company of 1,803,983.94 €. Equity free cash flows are estimated to remain constant from the year 2025 on, as further assumptions beyond this point on concentrates prices, market conditions or the company operation itself are hardly quantifiable.

**Table 39**
Discounted Equity Free Cash Flows 2021 – 2025 (EUR). (Source: Own elaboration).

| | 2021 | 2022 | 2023 | 2024 | 2025 |
|---|---|---|---|---|---|
| EFCF | 7,442.63 | 13,411.44 | 56,897.99 | 83,015.47 | 116,039.94 |
| Discount Factor | 1.1069 | 1.2252 | 1.3561 | 1.5010 | 1.6614 |
| Discounted EFCF | 6,724.02 | 10,946.65 | 41,957.14 | 55,305.84 | 69,842.93 |
| Most likely Valuation Horizon | | | 184,776.60 | | |
| Terminal Value | | | 1,803,983.94 | | |
| Company Value | | | 1,988,760.54 | | |

Finally, the overall Company Value, calculated as the sum of the TV and the discounted cumulative EFCF obtained from 2021 to 2025, or the most likely cumulative value, is 1,988,760.54 €. This, if compared to the TV of the trader's equity shown on the expected Balance Sheets for the year 2025, 811,658.83 €, is 145% higher, giving a positive outlook for the trader's business and therefore for its growth perspectives.

### 5.4.4. Real options valuation

The optimised exercise concentrates price thresholds for the growth option's capacity increase steps can be seen in **Table 40**, while each year's optimum exercise price thresholds for the abandonment option are shown in **Table 41**. Also, as no scale-down or reversal option has been considered, the increases in the trader's supplying capacity or the closure of the company are thus made irreversibly once a certain price threshold has been exceeded. Additionally, so long as the market price for the concentrate stays within the limits marked by the abandonment option price thresholds and the price threshold of the first step of the growth option, the trader's operative will remain unaltered.

**Table 40**
Concentrate exercise price for growth option increase steps (USD/DMT). (Source: Own elaboration).

| | Growth 25% | Growth 50% |
|---|---|---|
| Exercise Price | 3,810 | 5,780 |

**Table 41**
Concentrate exercise price for abandonment option in each year (USD/DMT). (Source: Own elaboration).

| | 2021 | 2022 | 2023 | 2024 | 2025 |
|---|---|---|---|---|---|
| Exercise Price | 2,732 | 2,811 | 2,895 | 2,965 | 3,057 |

### 5.4.5. Abandonment option

The trader would consider halting its activity in the event of the price of copper concentrates dropping below the threshold levels for each year. In this case, the sales of the company in the remaining time until the end of 2025 after the triggering event occurred are expected to be zero. Also, the abandonment option has been supposed to be exercised at the end of the corresponding year, thus both the exercise year's incomes and costs are fully realised. Nonetheless, subsequent annual cash flows from the exercising option are assumed to be zero, as no remaining stock still possessed by the company would be sold and neither would other assets be liquidated afterwards.

**Table 42**
Discounted Equity Free Cash Flows 2021 – 2025 (EUR). (Source: Own elaboration).

| | 2021 | 2022 | 2023 | 2024 | 2025 |
|---|---|---|---|---|---|
| Exercise Probability | 52.60% | 53.95% | 57.00% | 56.05% | 59.55% |
| Company Mean Expected Value (EUR) | 6,724.02 | 17,670.68 | 59,627.82 | 114,933.67 | 184,776.60 |
| Option Value (EUR) | 3,536.83 | 9,533.33 | 33,987.86 | 64,420.32 | 110,034.47 |

In **Table 42** the exercise probabilities of the abandonment option in each year are shown along with the mean expected value of the company and the option value itself if the option were exercised at the end of that year. The company mean expected value when the option is to be exercised is calculated as the sum of the discounted equity free cash flows up to that moment, excluding the business' terminal value, as it will not resume operations later. Thus, the intrinsic option value can be calculated as the sum of the realised option value at each execution opportunity, hence being 221,512.81 €.

### 5.4.6. Growth Option

On the contrary, should the price of the concentrate exceed the exercise threshold levels, the trader would automatically expand its capacity in accordance, albeit assuming certain rises in the price it pays to acquire the concentrate from suppliers. The option exercise probability is 54.55%, existing 44.10% of just expanding capacity by 25% as the second threshold would not be surpassed. Nevertheless, there is a 10.45% probability of the trader increasing its capacity by 50%, as the price of concentrate would soar pass the mark of 5,780 USD/DMT at some point between 2021 and 2025.

**Table 43**
Growth Option Probabilities and Value. (Source: Own elaboration).

| | Growth 25% | Growth 50% |
|---|---|---|
| Exercise Probability | 44.10% | 10.45% |
| Mean Expected Value (EUR) | 1,800,234.94 | 530,913.92 |
| Option Value (EUR) | 793,903.61 | 55,480.50 |

In **Table 43** the results achieved for the two steps of the growth option can be seen. An expectancy of 793,903.61 € is estimated if the first step of the growth option is triggered, while an additional expectancy of 55,480.50 € would be added if the second step is too. Consequently, the total value of the growth option is assumed to be 849,384.11 €.

The time effect has not been taken into consideration for the growth option to make the valuation more straightforward. However, it is worth mentioning that the results present a quite meaningful level of dispersion certainly due to this factor, as it would not have the same impact on the company value to exercise the option in an earlier stage than later. This is because the trader would enjoy a higher selling tonnage and, presumably, higher incomes. But it is also true that the contrary might well happen, mainly when the triggering event takes place early in the company's life. As concentrate prices have much more time to freely fluctuate and hence fall back below the growth threshold, the business model that is developed when the growth step is taken, which contemplates the worsening of the buying and selling prices of the copper concentrate, can easily become inefficient, as it is notably more price sensitive.

### 5.4.7. Company Value

The company value resides not only in the management's projections which allow setting a baseline valuation scenario, but also in the potential behind the developing opportunities the company could come across along the way. These opportunities need to be assessed to attain a fair valuation of the company, hence accounting for the initial plan as well as for the evolution potential. As for the copper concentrate trading company, the DCF method brings forward the most likely value of the company, this being kept within the original volume of tons sold per year and its forecasted average annual price from 2021 to 2025. Moreover, the potential within the business model is portrayed by the Real Options Valuation method. This allows considering price uncertainty, which is indeed the main source of risk and of returns the trader is exposed to.

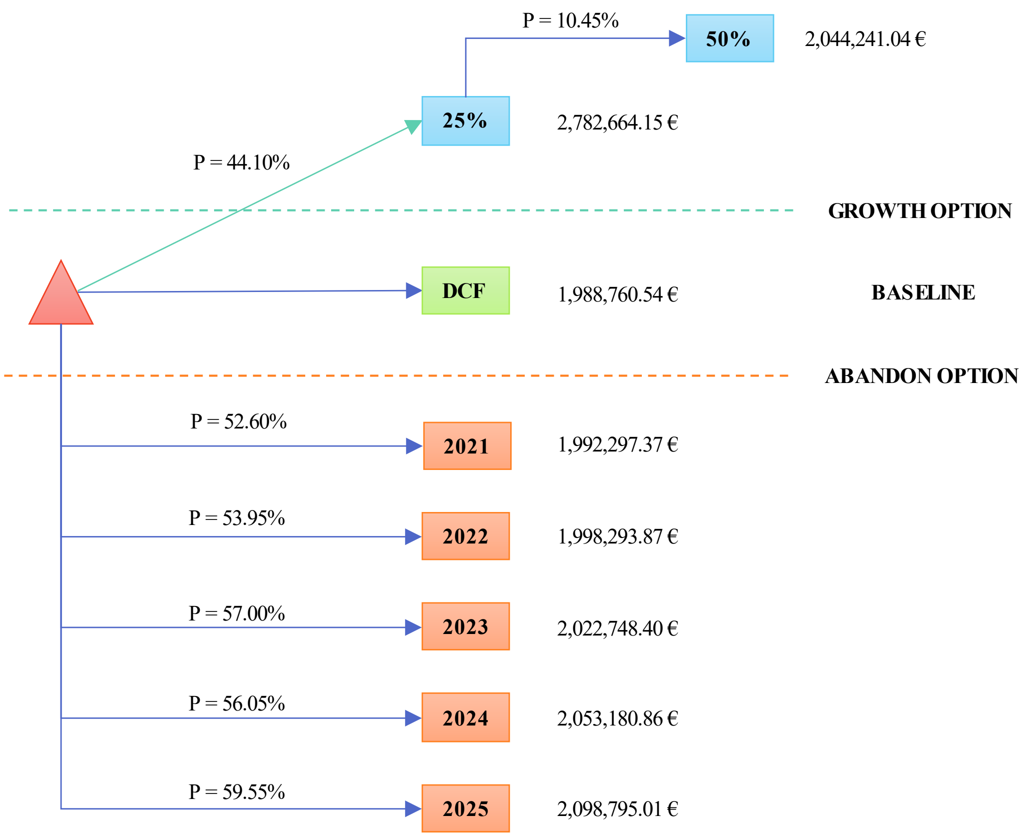


**Figure 20.** Real Options valuation alternative for Copper Concentrate Trader. (Source: Own elaboration).

As shown in **Figure 20** the company would develop its business model reaching a total value slightly under 2.0 M€ if everything goes as initially planned. This implies that concentrate prices, and therefore metal prices and annual benchmark TC/RC levels, stay within the range delimited by the exercise thresholds of both the growth and the abandonment options. Also, so long as reference prices remain within limits, the minimum demanded return to the trader's business would be met, making it undesirable for investors to move away to look for higher-yielding investment opportunities elsewhere. On the other hand, if the opportunity cost is reached because the company fails to deliver the returns required, the abandonment option shall be executed by investors, who would then seek for investments that yielded at least what concentrate trading was meant to. In such a case, as time passes, investors would have accumulated more cash flows from their investment in the trader, thus increasing the value of the abandonment option over time.

When it comes to capacity growth, the company valuation is increased as the growth option is exercised in either of its steps. However, the potential improvement for the company valuation if the trader's supply capacity is increased by 50% instead of 25% is not as much for the first step as for the second. This, nonetheless, can still be interpreted as a strategical decision by the company's management that, while optimising the moment to take the increase decision, perceives that expanding the supply capacity would eventually lead to a higher market share which, in the end, could be traduced as a competitive advantage as well as higher survival odds for the company in the long run.

# Chapter 6: Conclusions

Throughout this research we have developed a copper concentrates benchmark price model that provides market participants involved in copper concentrates production, processing or trading of a neutral, unbiased price reference. The methodology proposed resorts to copper, gold and silver spot and futures data, as well as well-known forecasting techniques, to circumvent the problem of lacking a market-wide reference for copper concentrates.

Despite not having access to most confidential contract terms of copper concentrate supply agreements, the findings of this research suggest that some of the developments may well be of application by market participants. For instance, the ability to forecast with a sufficient degree of accuracy the TC/RC benchmark levels can have a great financial importance for either copper miners and smelters, as both can manage to set in advance more advantageous TC/RC for their contracts if considering the information delivered by the model.

Furthermore, the practical usability and potential advantages of using the benchmark price model for copper concentrates by a copper concentrates trader has also been analysed using Real Options, which additionally shows how having good forecasts of copper concentrates prices can substantially improve the trader's financial performance, while also allows to take to action when prices are expected to fall.

## *6.1. Implications of developing forecasts for Copper TC/RC*

Copper TC/RC are a keystone for pricing copper concentrates which are the actual feedstock for copper smelters. The potential evolution of TC/RC is a question of both economic and technical significance for miners, as their value decreases the potential final selling price of concentrates.

Additionally, copper miners' revenues are more narrowly related to the market price of copper, as well as to other technical factors such as ore dilution or the grade of the concentrates produced.

Smelters, on the contrary, are hugely affected by the discount which they succeed in getting when purchasing the concentrates, since that makes up the largest part of their gross revenue, besides other secondary sources. In addition, eventual differences between TC/RC may give commodity traders ludicrous arbitrage opportunities. Also, differences between short and long-term TC/RC agreements offer arbitrage opportunities for traders, hence comprising a part of their revenue in the copper concentrate trading business, as well copper price fluctuations and the capacity to make economically optimum copper blends.

As far as we are aware, no rigorous research has been carried out on the behaviour of these discounts. Based on historical data on TC/RC agreed upon in the LME Copper Week from 2004 to 2017, three potentially suitable forecasting models for TC/RC annual benchmark values have been compared through four measures of forecasting accuracy at different horizons. The models chosen in this research were chosen, firstly, due to their broad implementation and proven capacity in commodity prices forecasting that they all share and, secondly, because of the core differences in terms of price behaviour with each other.

Our research contributes by delivering a formal tool for smelters or miners to make accurate forecasts of TC/RC benchmark levels. The level of errors attained indicates the LES model may be a valid model to forecast these crucial discounts for the copper market. In addition, our work further contributes to helping market participants to project the price of concentrates with an acceptable degree of uncertainty, as now they may include a fundamental element for their estimation. This would enable them to optimise the way they produce or process these copper concentrates. Also, the precise knowledge of these discounts' expected behaviour contributes to let miners, traders and smelters alike take the maximum advantage from the copper concentrate trading agreements that they are part of. As an additional contribution, this work may well be applied to gold or silver RC, which are relevant deduction concentrates when these have a significant amount of gold or silver.

### *6.2. Accuracy of TC/RC benchmark levels forecasts*

Focusing on the MAPE values achieved, those obtained by the LES model when TC and RC are treated independently have been significantly lower than for the rest of the models. Indeed, one-year-ahead MAPE measures for TC values for the GBM model (20%) almost triple those of the OUP model (7.66%), in contrast with the significantly lower values from the LES model (0.0046%). This gap tends to be narrowed when TC/RC values are forecasted at longer horizons, when most measures of error become more even. The GBM and OUP models have proven to deliver better accuracy performance when the TC/RC values are projected monthly and then averaged to obtain annual benchmark forecasts. Even so, the LES model remains the most accurate of all with MAPE values of 10% at two-year-ahead forecasts, with 18% and 31% for TC for OUP and GBM, respectively.

Despite TC and RC being two independent discounts applied to copper concentrates, they are both set jointly with an often 10:1 relation as our data reveals. This relation also transcends to simulation results and error measures, hence showing a negligible discrepancy between the independent forecasting of TC and RC, or the joint forecasting of both values, keeping the 10:1 relation. This is, for instance, the case of the five-year-ahead OUP MAPE values (0.2715/0.2719) which were obtained without observing the 10:1 relation in the data. A similar level of discrepancy was obtained at any horizon with any model, which indicates that both values could be forecasted with the same accuracy using the selected model with any of them and then applying the 10:1 relation.

Our findings thus suggest that both at short and long-term horizons TC/RC annual benchmark levels tend to exhibit a pattern which is best fit by an LES model. This indicates that these critical discounts for the copper trading business do maintain a certain dependency on past values. This would also suggest the existence of cyclical patterns in copper TC/RC, possibly driven by many of the same market factors that move the price of copper.

### *6.3. The importance of having a benchmark on copper concentrates prices*

We have developed a copper concentrates benchmark price model which aims to address the absence of market-wide accepted references that can orientate market participants to set a fair price for the copper concentrates they trade with. The relevance of a reliable reference to set the price of copper concentrates is key as its trade involve most copper mines, smelters and commodity traders worldwide, which are currently forced to employed indirect references and incomplete information to find out the just price of these.

The methodology followed helps to deliver estimations on copper concentrates pricing, enabling comparing physical trades agreements to a comprehensible benchmark based on both official, publicly available metal spot and future prices and main deductions forecasts. The model has been formulated in a way that makes it suitable for pricing any concentrate available in the market as there is not a typical concentrate layout, though we have standardized our simulations by carrying them out with five sample concentrates, being all representative of the complete spectrum of layouts from a statistical standpoint.

In addition, our model offers a deep insight into the different components of the price of concentrates by giving further understanding of the future role that each of the elements involved in pricing the concentrates will have as the market conditions evolve. Thus, this tool brings new possibilities for optimizing mining production, using the expected value of the concentrate and the expected role of penalties to target the ores that would maximize profits according to the expectations provided by the model. Also, as the model provides greater foresight on the behavior of penalties regarding the final price of concentrates, blending activities, mainly carried out by miners and traders, may be optimized to obtain the maximum return possible for more complex concentrates that require some level of blending before being sold.

### *6.4. Applying the models and methodologies to anticipate future business performance*

A copper concentrate trading company has been valued, computing the uncertainties related to copper concentrate price fluctuations over time through the modelling of copper concentrate prices. Alternative valuations for the trading company have been determined employing Real Options, where the growth or abandon conditions for the trader's activity have been linked to the copper concentrate price in each year.

Our research focuses on the trading of copper concentrates as a standalone element, being separated from the extraction or the transformation stages, which fills a gap in literature. The outcome suggests that the methodology laid out in this paper may well be employed by commodity traders, regardless of the commodities they trade with, to anticipate potentially profitable opportunities or avoid unnecessary risks.

As far as we know, this research manages for the first time to separate the trading element from the extraction and processing factors in commodity markets, tackling the existent void in the literature by shedding light on trading houses' financials. At the same time it provides further insight into physical commodity trading and, more specifically, on how copper concentrate traders run their businesses. The results of our research indicate that the methodology employed can be applied for estimating the future performance of copper concentrate traders through the valuation of their company.

The outcome achieved suggests that the methodology explained in this research may well be employed primarily, but not exclusively, by copper concentrate traders to anticipate potentially profitable opportunities that may arise in the foreseeable future. It may also add flexibility to their strategical decisions or help to mitigate upcoming threats for their businesses.

To better understand the physical commodities trading business and the uncertainties associated with it, a copper concentrate trading company has been valued, using copper concentrate trading as a sample of the richer and more complex physical commodities trading business model. The uncertainties under which copper concentrate trading is carried out have been reckoned to properly address the valuation problem with precision. An acceptably reliable outcome is also obtained, such as prices and discount fluctuations, resorting to real market data to reflect the conditions a copper concentrate trader is submitted to, as well as estimating its most likely evolution.

### *6.5. Limitations and future lines*

As a limitation of this research, mainly with what regards to TC/RC forecasts, we should point out the timespan of the data considered, compared to those of other forecasting works, on commodity prices for example, which use broader timespans. For our case, we have considered the maximum available, and sufficiently reliable data on TC/RC benchmark levels, starting back in 2004, as there is no reliable data beyond this year.

In addition, we have used four measures of error which are among the most frequently used to compare the accuracy of different models. However, other measures could have been used at an individual level to test each model's accuracy. Once TC/RC annual benchmark levels are able to be forecasted with a certain level of accuracy, future research should go further into this research through the exploration of the potential impact that other market factors may have on these discounts.

Furthermore, a similar problem appears when analysing trading companies' activities through Real Options, since publicly available data from companies engaged in concentrates trading, processing or production is fairly limited as well. These companies tend to keep utter discretion in relation to the economic and technical details of their transactions. Nonetheless, the reliability of our methodology is very dependable on the accuracy of the individual forecasting techniques employed for each of the components that build up the price of concentrates. In this sense, one of the limitations of our research is the fact that our copper concentrate benchmark price model's forecasting accuracy decreases for further-out forecasting horizons, mainly beyond six months or one year, although it doesn't have a serious impact in the practical usefulness of the model, as copper concentrate transactions usually occur in shorter time periods.

The accuracy of the copper concentrate benchmark price model is also a function of the concentrate layout, even though it is related to the accuracy of Schwartz and Smith's two-factor model and that of the Linear Exponential Smoothing model for TC/RC forecasting, having different levels of accuracy for concentrates with only one payable element than for those with more than one. Future research should strive to contrast the model's outcomes with data from market participants if they became available to evaluate potential ways of improvement if needed.

Our research has centred on copper concentrate trading to understand the usefulness of our methodology in a context of high uncertainty with low available information and where price volatility tends to be on the upper side. Nonetheless, the main limitation lies on making a well-balanced approach to the trader's business model without adding excessive complexity to the analysis. Thus, as a first approach, long-term supply contracts normally priced below broader market conditions, in combination with hedging practices, mostly to secure contango margins (higher prices in further-out future contracts), have not been included in this research. Future research should also consider these common practices of commodity traders to help differentiate more from a common retailer and to define more precisely the specificities of commodity trading.

In addition, further extensions of this research should be adapted to not only contemplate abandonment as the only alternative if market conditions become adverse, hence introducing a scale-down or diversification options. These different Real Options approaches may help give a clearer picture of the strategical decisions implemented by commodity traders. They would more probably rather diversify their commodity portfolio or simply reduce their trading operations to a minimum level if market conditions are not favourable, instead of shutting down their operations completely. This is provided there is a low level of their fixed costs along with the importance of keeping their relational capital.

On the other hand, the access to useful and reliable data on copper concentrate traders is notably constrained, as most major traders are still in private hands and therefore are not required to disclose much of the information regarding their operations or assets ownership. Deeper access to data from larger and more diversified commodity traders would thus provide more accurate results. This is one of the most relevant lines of development for this research. Additionally, the inclusion of several frequent activities in copper concentrates trading which are key to maximise returns, such as blending, would certainly provide a richer perspective. Nonetheless, as the market becomes more volatile and unstable, also given the situation stemming from the COVID-19 pandemic, the usefulness of the real option technique has become more interesting, as it can also be adapted to different sectors and business models. This enables companies across the board to anticipate key managerial decisions without having to take them in a rush and under pressure due to a critical market downturn.

Finally, Environmental, Social and Governance criteria (ESG) have not been taken into account in this research, as we have focused in purely technical and financial challenges within copper concentrate markets. However, future research should consider including ESG impact on copper concentrate market, as copper concentrate prices might ultimately get affected by restrictions imposed onto the importation of copper concentrate with high levels of certain punishable, highly contaminating elements. The effects of ESG policies over the copper concentrates trading business would also be another topic of interest for potential future research, analysing how observing the elemental practices of ESG would impact the traders' normal workflow and financials.

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# Appendix: Published Papers

- Díaz, F.J.; Miras, M.M.; Escobar, B. (2019): "Looking for accurate forecasting of Copper TC/RC Benchmark Levels". *Complexity*, 2019 –ID8523748. JCR, Impact factor 2019: 2.462 (Q2 Mathematics, Interdisciplinary Applications -28/106). https://doi.org/10.1155/2019/8523748

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*Research Article*

## Looking for Accurate Forecasting of Copper TC/RC Benchmark Levels

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Forecasting copper prices has been the objective of numerous investigations. However, there is a lack of research about the price at which mines sell copper concentrate to smelters. The market reality is more complex since smelters obtain the copper that they sell from the concentrate that mines produce by processing the ore which they have extracted. It therefore becomes necessary to thoroughly analyse the price at which smelters buy the concentrates from the mines, besides the price at which they sell the copper. In practice, this cost is set by applying discounts to the price of cathodic copper, the most relevant being those corresponding to the smelters' benefit margin (*Treatment Charges-TC* and *Refining Charges-RC*). These discounts are agreed upon annually in the markets and their correct forecasting will enable making more adequate models to estimate the price of copper concentrates, which would help smelters to duly forecast their benefit margin. Hence, the aim of this paper is to provide an effective tool to forecast copper TC/RC annual benchmark levels. With the annual benchmark data from 2004 to 2017 agreed upon during the LME Copper Week, a three-model comparison is made by contrasting different measures of error. The results obtained indicate that the LES (*Linear Exponential Smoothing*) model is the one that has the best predictive capacity to explain the evolution of TC/RC in both the long and the short term. This suggests a certain dependency on the previous levels of TC/RC, as well as the potential existence of cyclical patterns in them. This model thus allows us to make a more precise estimation of copper TC/RC levels, which makes it useful for smelters and mining companies.

### 1. Introduction

*1.1. Background.* The relevance of copper trading is undeniable. In 2016 exports of copper ores, concentrates, copper matte, and cement copper increased by 1.5%, reaching 47.3 b $USD, while imports attained 43.9 b $USD [1]. In addition, the global mining capacity is expected to rise by 10% from the 23.5 million tonnes recorded in 2016 to 25.9 million tonnes in 2020, with smelter production having reached the record figure of 19.0 million tonnes in 2016 [2].

The world's copper production is essentially achieved through alternative processes which depend on the chemical and physical characteristics of the copper ores extracted. According to the USGS' 2017 Mineral Commodity Summary on Copper [3], global identified copper resources contained 2.1 billion tonnes of copper as of 2014, of which about 80% are mainly copper sulphides, whose copper content has to be extracted through pyrometallurgical processes [4]. In 2010, the average grades of ores being mined ranged from 0.5% to 2% Cu, which makes direct smelting unfeasible for economic and technical reasons. So, copper sulphide ores undergo a process known as froth-flotation to obtain concentrates containing ≈ 30% Cu, which makes concentrates the main products offered by copper mines [2, 5]. Concentrates are later smelted and, in most cases, electrochemically refined to produce high-purity copper cathodes (Figure 1). Copper cathodes are generally regarded as pure copper, with a minimum copper content of 99.9935% Cu [6]. Cathodes are normally produced by integrated smelters that purchase concentrates at a discounted price of the copper market price and sell cathodic copper at the market price, adding a premium when possible.

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# Estimating copper concentrates benchmark prices under dynamic market conditions

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ABSTRACT

Copper concentrates are the primary product sold by copper mines to traders or smelters. Their price is set in private agreements following unofficial market practices which, along with the fact of the wide variety of concentrates layouts traded worldwide, make problematic for any market participant to have a clear price reference for a specific concentrate. This paper presents a copper concentrates benchmark price model which provides suitable short-term price estimations based on metals and discounts forecasts, as well as on copper, gold and silver spot and future prices data from the LME and COMEX. The model, which redeems considerably low forecast error values for the short-term concentrate prices, constitutes a useful and applicable tool for miners, traders and smelters to set a benchmark price level for their copper concentrate transactions, also helping them optimize their operations, as well as estimate their immediate liquidity needs or their actual necessity to hedge the price risks associated to their concentrate trading. In addition, different concentrates layouts have been analyzed to test the model's behavior with the most common specification and blends of copper concentrates demanded by the market, as well as to portray the model's forecasting capacity and its ability to convey information on the future relevance of the different components of pricing in the most frequent timeframe in which copper concentrate trading takes place.

## 1. Introduction

The copper concentrates market lacks tangible references, public, official or regulated markets which may help market participants to establish the reference official price of the concentrates being traded, while these references do exist for the main commodities present in the concentrates, allowing participants to have only a partial notion of the price of their concentrates.

Copper concentrates are traded globally between mines, traders and smelters, constituting the current main source of refined copper, accounting, as of 2017, for 67% of the world's production, while SX-EW represents 16% and secondary copper processing reaches 17% (International Copper Study Group, 2018). Concentrates are produced from copper sulphide ores, occurring naturally in different kinds of deposits, being porphyry deposits the most relevant ones. According to the USGS' 2015 Assessment of Undiscovered Copper Resources of the World (Hammarstrom et al., 2019) porphyry deposits represented 1.8 billion tons of 2.1 billion tons of identified copper resources, whereas undiscovered deposits of these kind alone are estimated to contain 3.1 billion tons of copper.

Ore grade differs depending on the deposit class, which for the case of porphyry deposits oscillates between 0.3% and 2.0% of copper, with the top 25 mines by Cu content having a mean grade of 0.49% (Mudd et al., 2013). These copper deposits may also contain some other valuable elements, such as gold or silver with variable gradings, as is the case of the Indonesian mine Grasberg with an average gold grade in the ore mined during 2018 of 1.58 g/t, as well as having reached an average copper grade at the mill of 0.98% (Freeport-McMoRan, 2018). Other relevant, though fewer, common classes of deposits, such as sediment-hosted copper deposits, which account for 0.31 billion tons of global total identified resources and 0.42 billion tons of undiscovered world resources, are likely to contain significant amounts of additional by-products, for example cobalt or silver, as well as higher average copper grades (Hammarstrom et al., 2019), such as the Katanga mine in D.R.C. with an average Cu grade of 3.49% and an average cobalt grade of 0.46% in 2018 (Katanga Mining Limited, 2018), and KGHM Polska Miedź's Polish mines whose ores graded on average 1.49% of copper and 48.6 g/t of silver throughout 2018 (KGHM Polska Miedz, 2018).

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