Programa de doctorado “Matemáticas”

Department of Applied Mathematics I

PhD DISSERTATION

KNAPSACK MODELS APPLIED TO THE SOLUTION OF COMPLEX PROBLEMS IN TRANSPORT PLANNING

Author
Ramón Piedra de la Cuadra

Supervisor
Francisco Alonso Ortega Riejos

March 13, 2023

Agradecimientos

Esta tesis es el final de una etapa de mi vida que comenzó con el trabajo fin de máster allá por el año 2017. Quisiera agradecer a todas las personas que han estado durante este tiempo a mi lado y a los que he ido conociendo y aprendiendo de ellos en estos años.

En primer lugar, y de manera especial, quiero agradecer a Paco todo el esfuerzo, trabajo, dedicación y cariño que le ha puesto a esta dirección de tesis. Gracias por el tiempo que le has dedicado incluso cuando no te sobraba, agradecerle también a Reyes por compartirte. Gracias por esas reuniones en el Rectorado, en el despacho o incluso por videollamada en pandemia y fuera de España. Gracias por tus consejos, de trabajo y de no trabajo, me han ayudado para estar aquí y afrontar lo que vendrá. Gracias por tu amistad.

También agradecer a todos los del grupo de investigación en localización por haberme acogido tan bien y por haber trabajado conmigo la mayoría de ellos (aún espero tener la oportunidad de colaborar con los restantes).

Quiero agradecer también a Giuseppe por haberme recibido en su tierra durante mi estancia. Agradecer también a todo el resto del equipo que me acogieron en Nápoles.

Gracias a todos con los que he tenido la suerte de compartir trabajo en estos años y por donde he podido pasar, gracias a José Miguel y todo su equipo, al personal de administración del IMUS, al departamento de Matemática Aplicada de la Universidad de Córdoba, al departamento de Matemática Aplicada II y I de la Universidad de Sevilla, gracias por enseñarme el funcionamiento de la universidad, a ser un investigador y profesor mejor.

A todos mis amigos de la facultad del grado y del máster por haberme dado la alegría de ir cada día a clase, por mis compañeros de doctorado por esos viajes a los congresos que a pesar de los nervios pudimos disfrutar, especialmente quiero recordar a Moisés, con quien tuve la suerte de ser compañero. A Carlos y a Abel, por apoyarnos mutuamente en los buenos y no tan buenos momentos, gracias por esas horas y horas hablando y por esos momentos tan bonitos que hemos pasado.

Agradecer a toda mi familia su apoyo en estos años, a mis padres, por todo el esfuerzo que les han dedicado a todos mis hermanos y a mí en particular para que podamos desarrollar nuestros estudios, especialmente a mi padre, por transmitirme la importancia del trabajo y la pasión por las Matemáticas.

A mis hijos, por alegrarme todos los días y enseñarme lo que de verdad importa.

A Raquel, gracias por tu paciencia, gracias por tu apoyo incluso cuando ni yo lo tenía claro. Gracias por acompañarme siempre, emocional y físicamente, incluso cuando tuve que salir fuera. Agradezco a Dios que haya entrelazado nuestras vidas.

Resumen

Esta tesis se ha elaborado durante el desarrollo de proyectos de investigación competitivos que tienen en común la promoción del uso de modelos matemáticos de optimización para la toma de decisiones en ámbitos de gran complejidad por la concurrencia de intereses diversos y, a menudo, contrapuestos (como los de los usuarios, las empresas de transporte y la administración), la existencia de múltiples condicionantes (debidos a una capacidad limitada, la obligada disponibilidad de ventanas temporales para poder realizar el servicio ofertado o la limitaciones de distancia para garantizar la cobertura del servicio) y, por último, la consideración de variables exógenas (como el comportamiento no determinista del usuario en su toma de decisión, la ocurrencia de episodios de congestión del tráfico en horas punta, la posibilidad de utilizar intermodalidad en los desplazamientos). Los problemas en la planificación del transporte dentro de estos ámbitos de gran complejidad requieren de formulaciones novedosas, si bien pueden estar basadas inicialmente (o al menos, inspiradas) en aquellas que en contextos similares han demostrado eficacia y eficiencia, tanto desde la perspectiva del uso de modelos exactos como la proporcionada por medio de los heurísticos (incluyendo metaheurísticas y matheurísticas).
Los problemas de localización y los de transporte comparten en sus formulaciones un origen común procedente del ámbito de la optimización matemática, caracterizado por el uso de variables de naturaleza diversa (continua, entera o binaria) para construir funciones objetivo compatibles, preferentemente, con un comportamiento lineal. La versatilidad que proporcionan las variables de decisión introducidas permiten representar mediante manipulación algebraica las restricciones características que están presentes en tales modelos de optimización. Además de las restricciones, es posible describir, también mediante adecuadas expresiones algebraicas de dichas variables, las posibles estrategias a seguir por parte de los agentes intervinientes (usuarios individuales, administración o empresas).
Un modelo de optimización paradigmático por su versatilidad en poderse adaptar a un gran número de contextos reales y por sus posibilidades de extensión (añadiendo nuevas líneas de restricción para la búsqueda de soluciones, incorporando niveles complementarios de optimalidad y/o modificando la linealidad de las expresiones algebraicas) es el denominado problema mochila (KP). En la

mayoría de las soluciones propuestas a los problemas analizados en esta tesis, los problemas mochila han constituido una herramienta útil para su formulación. Esta es la razón por la que en la memoria de tesis se hace referencia a dicho problema de optimización combinatoria.
En cuanto a los contextos reales analizados desde la perspectiva de la optimización matemática, hemos de admitir que el centro docente donde el doctorando se ha formado (Departamento de Matemática Aplicada de la Escuela Técnica Superior de Arquitectura de la Universidad de Sevilla) ha tenido una innegable influencia:

- El diseño de líneas de tránsito rápido se ha planteado desde una novedosa perspectiva consistente en adquirir un mayor nivel de cohesión territorial, minimizando la necesidad de la movilidad debida a desplazamientos obligados por la existencia de desequilibrios territoriales.

- La búsqueda de soluciones para la localización efectiva de contenedores de residuos y el despliegue eficiente de rutas de recogida de carácter selectivo se ha complementado con la consideración adicional de un comportamiento solidario por parte del usuario que, convenientemente motivado, podría estar dispuesto a depositar su demanda de recogida de residuos en un punto diferente al más cercano.

- La planificación de servicios de recogida de residuos no habituales mediante contenedores itinerantes de múltiples compartimentos (ecopuntos) se ha analizado en un capítulo separado, donde se han formulado modelos de optimización para diferentes estrategias de resolución que han sido comparadas en términos de eficiencia.

- El despliegue de electrolineras basado en la previa existencia de una red de gasolineras convencionales es otro de los problemas abordados en esta tesis y en el que se ha aportado como novedad la concurrencia de la perspectiva de la administración gubernamental, que exige que el número de puntos de recarga eléctrica proporcione una cobertura reforzada a los usuarios que emprendan un viaje a lo largo del territorio, y los intereses empresariales, seleccionando las localizaciones más prometedoras debido al flujo de viajes que pasa por ellas.

- Las restricciones de accesibilidad a los actuales centros urbanos en vehículos motorizados de uso privado han sido tratadas en otro de los capítulos de la tesis, aportándose un modelo de decisión para la elección óptima de una instalación park-and-ride que combina criterios de distancia a destino, tiempos de viaje sometidos a la fluidez del tráfico, posibilidad de uso de intermodalidad y disponibilidad de plazas libres de aparcamiento.

- Finalmente, la gestión del tiempo de espera de los usuarios en los nodos de la red de transporte puede generar interesantes estrategias para la optimización de tiempos de viaje, tanto en términos globales (reduciendo el número de paradas intermedias para favorecer el tiempo de recorrido de la mayoría de los pasajeros), como individuales (aguardando en un nodo que el arco que proporcione la salida del nodo pueda ser transitable en un tiempo sensiblemente menor).

Esta tesis doctoral comienza con un capítulo inicial donde se introducen conceptos claves para entender la temática que se expone. Dicho capítulo 0 incluye una breve descripción de los contenidos de los tres capítulos que la componen, y concluye enumerando las principales publicaciones derivadas de tales contenidos. Las personas que, junto con el doctorando y el director de la tesis, han contribuido a la difusión de los resultados de esta investigación son las siguientes:

- Soly Ventura Pérez (Universidad de Sevilla, España).
- Juan Antonio Mesa López-Colmenar (Universidad de Sevilla, España).
- Miguel Ángel Pozo Montaño (Universidad de Sevilla, España).
- Giuseppe Bruno (University of Naples Federico II, Italia).
- J. David Canca Ortiz (Universidad de Sevilla, España).
- Eva Barrena Algara (Universidad Pablo de Olavide, España).
- Guido Marseglia (University of Salento, Italia).

Abstract

This thesis has been developed during the competitive research projects aimed at promoting the use of mathematical optimization models for decision-making in complex contexts where there are often conflicting interests (such as those of users, transportation companies, and administration), multiple constraints (due to limited capacity, required availability of time windows to perform the offered service, or distance limitations to ensure service coverage), and exogenous variables (such as the user's non-deterministic behavior in decision-making, traffic congestion episodes during peak hours, and the possibility of using intermodality in travels). Planning transportation within such complex contexts requires innovative formulations, which may be initially based on those that have proven effective and efficient in similar contexts, using both exact and heuristic models (including metaheuristics and matheuristics).

Location and transportation problems share a common origin in mathematical optimization, characterized by the use of variables of different nature (continuous, integer, or binary) to construct compatible objective functions, preferably with linear behavior. The versatility provided by the introduced decision variables allows representing the characteristic constraints present in such optimization models through algebraic manipulation. In addition to constraints, possible strategies to be followed by the involved agents (individual users, administration, or companies) can also be described through suitable algebraic expressions of those variables.

A paradigmatic optimization model due to its versatility to adapt to a large number of real contexts and its possibilities of extension (by adding new lines of constraint for solution searching, incorporating complementary levels of optimality and/or modifying the linearity of algebraic expressions) is the so-called knapsack problem (KP). In most of the proposed solutions to the analyzed problems in this thesis, the knapsack problems have been a useful tool for their formulation. This is why the thesis references this combinatorial optimization problem.

Regarding the real contexts analyzed from the perspective of mathematical optimization, we must admit that the academic center where the Ph.D. student has been trained (Department of Applied Mathematics of the Technical School of Architecture at the University of Seville) has had an undeniable influence:

- The design of fast transit lines has been approached from a novel perspective of acquiring a higher level of territorial cohesion, minimizing the need for mobility due to forced displacements caused by territorial imbalances.

- The search for solutions for the effective location of waste containers and the efficient deployment of selective collection routes has been complemented by the additional consideration of solidarity behavior by the user who, suitably motivated, could be willing to deposit their waste collection demand at a point different from the nearest one.

- The planning of non-habitual waste collection services through itinerant containers with multiple compartments (ecopoints) has been analyzed in a separate chapter, where optimization models have been formulated for different resolution strategies that have been compared in terms of efficiency.

- The deployment of electric charging stations based on the prior existence of a network of conventional gas stations is another problem addressed in this thesis, in which the perspective of the government administration has been contributed as a novelty, which requires that the number of charging points provide reinforced coverage to users who undertake a journey throughout the territory, and the business interests, selecting the most promising locations due to the flow of travel that passes through them.

- The restrictions on accessibility to current urban centers in private motor vehicles have been dealt with in another chapter of the thesis, providing a decision model for the optimal choice of a park-and-ride facility that combines criteria of distance to destination, travel times subject to traffic flow, possibility of using intermodality, and availability of free parking spaces.

- Finally, the management of waiting time for users at transport network nodes can generate interesting strategies for optimizing travel times, both in global terms (reducing the number of intermediate stops to favor the travel time of most passengers), and individual terms (waiting at a node for the arc that provides the exit from the node to be passable in a significantly shorter time).

This doctoral thesis begins with an initial chapter where key concepts are introduced to understand the topic that is presented. This chapter 0 includes a brief description of the contents of the three chapters that compose it, and concludes by listing the main publications derived from such contents. The people who, together with the doctoral student and the thesis director, have contributed to the dissemination of the results of this research are the following:

- Soly Ventura Pérez (Universidad de Sevilla, España).

- Juan Antonio Mesa López-Colmenar (Universidad de Sevilla, España).

- Miguel Ángel Pozo Montaño (Universidad de Sevilla, España).
- Giuseppe Bruno (University of Naples Federico II, Italia).
- J. David Canca Ortiz (Universidad de Sevilla, España).
- Eva Barrena Algara (Universidad Pablo de Olavide, España).
- Guido Marseglia (University of Salento, Italia).

Contents

Chapter 0

Introduction

0.1 Knapsack Problem

Introducing the Knapsack Problem

The *Knapsack Problem* (KP) is inspired by the preparation of the necessary luggage that a mountaineer deposits in his knapsack to go on a trip. To prepare the knapsack, the hiker will have a multitude of objects that will have more or less utility throughout the trip but the storage capacity of the knapsack cannot be exceed. Hence, the problem will consist of maximizing the benefit/utility of the objects that the hiker carries in the knapsack without the weight of the objects exceeding the maximum weight that the knapsack supports.
Formally it would be defined like this: We have an instance of the knapsack problem for a set of objects with n items identify by j, each one of them with a benefit p_j and a weight w_j , being the capacity value of the knapsack is limited by c. The goal is to select a subset of this objects such that the total benefit of the selected objects is maximized and the total weight does not exceed c. Alternatively, KP can be formulated as a solution of the following Integer Linear Programming (ILP) model:

$$\text{(KP) max} \quad \sum_{j=1}^{n} p_j x_j \tag{0.1}$$

$$\text{subject to} \quad \sum_{j=1}^{n} w_j x_j \leq c, \tag{0.2}$$

$$x_j \in \{0, 1\}, \quad j = 1, \ldots, n. \tag{0.3}$$

Where $x_j \in \{0, 1\}$ are n binary variables that correspond to the selection in the j-th binary decision to select ($x_j = 1$) the object j to put it in the knapsack or leave it outside ($x_j = 0$).
Also, without loss of generality, let us assume that $w_j < c$ for any $j = 1, \ldots, n$, so that it can be ensured that each item under consideration is possible to fit

into the knapsack and that it is checked that $\sum_{j=1}^{n} w_j > c$, in order to avoid trivial solutions. We will denote as $x^* = \{x_1^*, \ldots, x_n^*\}$ the optimal solution vector and z^* as the optimal value for the objective function. The set X^* denotes the optimal solution set, i.e. the set of elements corresponding to the optimal solution vector.

The knapsack problem is one of the simplest optimization problems to understand and to model, however, it is highly complex to solve. In fact, it is part of Richard Karp's list of 21 NP-complete problems. Note that Karp (1972) motivated the study of NP-completeness and the inquiry into the famous question, whether P = NP.

The Knapsack Problem has been widely studied since Mathews (1896) showed that some constraints could be grouped into a single knapsack constraint. The fact that the problem has been so well studied is that many industrial problems can be formulated as knapsack problems: loading packages for shipment on vehicles, parallel computing of processors, stock-cutting, project selection with budget control, menu optimization from a nutritional perspective, etc. to mention a few examples. Furthermore, many combinatorial problems can be reduced to a KP model, and also the KP scheme frequently arises as a sub-problem in integer linear programming algorithms. However, as it is frequently the case with industrial applications, in practice several additional constraints, such as urgency and priority of requests, time windows for every request, packages with low weight but high volume, etc., have to be fulfilled. This leads to various extensions and variations of the basic model. Because this need of extending the basic knapsack model arose in many practical optimization problems, some of the more general variants of the KP have become standard problems of their own. Also, it should be noted that many more complex problem solving methods employ the knapsack problem (sometimes iteratively) as a subproblem. Therefore, a thorough study of the knapsack problem carries many advantages for a wide range of mathematical models (Kellerer et al., 2004).

This has meant that different solution techniques have been addressed during the last decades. Dantzig (1957) presented an efficient method to determine the solution for the continuous version of KP (CKP), and therefore, provide an upper bound for the discrete problem. Kolesar (1967) proposed the first branch-and-bound (B&B) algorithm for KP. During the 70's, the (B&B) methods continued being developed, thanks to which it was possible to solve problems with a large number of variables. The best known algorithm from this period is due to Horowitz (1978). Martello and Toth (1977) proposed the first upper bound that improved the value of continuous relaxation obtained for the CKP. In the 80's, research was oriented towards the search of solutions for large problems. Balas and Zemel (1980) presented a new approach to solve the KP considering, in many cases, only a small subset of the variables. From then on, variants of the problem began to be studied, such as its bounded and non-bounded versions, as well as the multiple-choice knapsack problem. Martello and Toth (1990) published an exhaustive review of the different theoretical re-

sults and solution methods existing up to that time.
New branching strategies for (B&B) approaches were developed by Morales and Martínez (2020); Yang et al. (2021). The sensitivity analysis to perturbations of item profits or weight was studied by Hifi et al. (2005, 2008); Belgacem and Hifi (2008); Pisinger and Saidi (2017). Improvements over existing *Fully Polynomial Time Appoximation Schemes* (FPTAS) were developed by Chan (2018) and Jin (2019).

Variants and Extensions of the Knapsack Problem

As we have commented in the previous section, KP is one of the most active research areas of combinatorial optimization. In the year 2022, doing a Google Scholar search for knapsack problem, there were more than 6000 publications, Knapsack Problem - Google Académico (n.d.). Due to the simplicity of the problem it has been modified and extended in many different ways. In this section we will explain the most basic ones. Kellerer et al. (2004) has published a book specifically dedicated to this area with many more variants of the KP. For further and recent variants of the Knapsack Problem, the reader may refer to the recent surveys Cacchiani et al. (2022a,b).
The first of the classic variants of the KP problem is the *Subset Sum Problem* (SSP) which consists of finding a subset of the values w_j whose sum is as close as possible without exceeding a given target value c.

$$\text{(SSP) max} \quad \sum_{j=1}^{n} w_j x_j \tag{0.4}$$

$$\text{subject to} \quad \sum_{j=1}^{n} w_j x_j \le c, \tag{0.5}$$

$$x_j \in \{0,1\}, \quad j = 1, \ldots, n. \tag{0.6}$$

SSP can be considered as a special case of the knapsack problem arising when the profit and the weight associated with each item are identical. It has a multitude of practical uses: designing better lower bounds for scheduling problems (see Guéret and Prins, 1999; Hoogeveen et al., 1994), solving combinatorial problems (see Pisinger, 1999), formulated as a decision problem has a particular interest in cryptography (see Kate and Goldberg, 2011).
Another consideration of the original problem arises if we do not consider all the different objects but instead have b_j copies of the same item and it is possible

to select only a subset of them. Formally,

$$
\begin{aligned}
\text{(BKP) max} \quad & \sum_{j=1}^{n} w_j x_j && (0.7)\\
\text{subject to} \quad & \sum_{j=1}^{n} w_j x_j \leq c, && (0.8)\\
& 0 \leq x_j \leq b_j, \quad x_j \quad \text{integer}, \quad j = 1, \ldots, n. && (0.9)
\end{aligned}
$$

The resulting problem is called the *Bounded Knapsack Problem* (BKP). Classical applications are given in the computation of the most profitable loading of a ship or the space shuttle (see Gilmore and Gomory, 1966; Barry et al., 2003) and for cutting problems (Gilmore and Gomory, 1965; Dyckhoff, 1990). A special variant of KP is the *Unbounded Knapsack Problem* (UKP), it is also known as *integer knapsack problem* where instead of a fixed number b_j a very large or an infinite amount of identical copies of each item is given. In this case, constraint 0.9 in BKP is replaced by

$$
x_j \geq 0, \quad x_j \quad \text{integer}, \quad j = 1, \ldots, n. \qquad (0.10)
$$

Hu et al. (2009) reviewed exact and approximation approaches to the UKP and Becker and Buriol (2019) reported the results of extensive computational experiments on several exact algorithms from the literature for this problem.
The *Multidimensional Knapsack Problem* (MdKP) has d different capacities c_i and weight w_{ij} for the i-th capacity. Formally,

$$
\begin{aligned}
\text{(MdKP) max} \quad & \sum_{j=1}^{n} p_j x_j && (0.11)\\
\text{subject to} \quad & \sum_{j=1}^{n} w_{ij} x_j \leq c_i, \quad i = 1, \ldots, d, && (0.12)\\
& x_j \in \{0, 1\}, \quad j = 1, \ldots, n. && (0.13)
\end{aligned}
$$

Surveys devoted to the MdKP have been published by Fréville (2004); Fréville and Hanafi (2005); Laabadi et al. (2018).
The *Multiple Knapsack Problem* (MKP) is the natural generalization of the KP. The goal of the MKP is to select m disjoint subsets of items so that the total profit of the selected items is a maximum, and each subset is assigned to a knapsack whose capacity is no less than the total weight of the items in the

subset. MKP can be formulated as follows:

$$
\begin{aligned}
\text{(MKP) max} \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} p_j x_{ij} && (0.14)\\
\text{subject to} \quad & \sum_{j=1}^{n} w_j x_{ij} \leq c_i, \quad i = 1,\dots,m, && (0.15)\\
& \sum_{i=1}^{m} x_{ij} \leq 1, \quad j = 1,\dots,n, && (0.16)\\
& x_{ij} \in \{0,1\}, \quad i = 1,\dots,m; j = 1,\dots,n.
\end{aligned}
$$

MKP has also many variants and applications, one of the most interesting for its use in real applications is the *Multiple Knapsack Assignment Problem* (MKAP) formulated by Kataoka and Yamada (2014) as an extension of the MKP in which the item are partitioned into disjoint sets and each knapsack may only have assigned items from one of the sets in the partition. Dimitrov et al. (2017) studied the variants that arise in emergency reallocation and Homsi et al. (2021); Simon et al. (2017) tackled a variant arisen in military and humanitarian situations which involves loading constraints.
Another variant appears if an item j has a corresponding profit p_{jj} and an additional profit p_{ij} which is redeemed only if item j is transported together with another item i that may reflect how well the given items fit together. This problem is expressed as the *Quadratic Knapsack Problem* (QKP) which is formally defined as follows:

$$
\begin{aligned}
\text{(QKP) max} \quad & \sum_{i=1}^{n}\sum_{j=1}^{n} p_{ij} x_i x_j && (0.17)\\
\text{subject to} \quad & \sum_{j=1}^{n} w_j x_j \leq c, && (0.18)\\
& x_j \in \{0,1\}, \quad j = 1,\dots,n.
\end{aligned}
$$

A thorough review of the QKP was provided by Pisinger (2007). Kellerer and Strusevich (2010) presented a FPTAS for a variant of the QKP, the *Symmetric Quadratic Knapsack Problem* (SQKP), which has several applications in machine scheduling. Schulze et al. (2020) introduced the *Rectangular Knapsack Problem* (RKP) as a special case of the QKP in which items have profit $p_j = 0$, weight $w_j = 1$, and the extra profit has the form $p_{ij} = a_i b_j$ with $(a_1,\dots,a_n)$ and $(b_1,\dots,b_n)$ positive integer vectors.
The *Bin Packing Problem* (BPP) can be considered as a variation of the KP if all the profits are 1, we will try to maximize the number of items which would not exceed the knapsack capacity, and if we have a number of containers (of the same size), and we wish to pack all n items in as few containers as possible, we get the bin packing problem, which is modelled by having indicator variables

$y_i = 1$ if container i is being used. This problem can be modelled as follows:

$$
\begin{aligned}
\text{(BPP) min} \quad & \sum_{i=1}^{n} y_i && (0.19)\\
\text{subject to} \quad & \sum_{j=1}^{n} w_j x_{ij} \leq c y_i, \quad i = 1, \ldots, n, && (0.20)\\
& \sum_{i=1}^{m} x_{ij} = 1, \quad j = 1, \ldots, n, && (0.21)\\
& x_{ij} \in \{0,1\}, \quad i = 1, \ldots, n; \quad j = 1, \ldots, n. \\
& y_i \in \{0,1\}, \quad i = 1, \ldots, n.
\end{aligned}
$$

A review of exact approaches for this problem has been presented by Delorme et al. (2016), while an exhaustive treatment of approximation algorithms ca be found in Coffman et al. (2013).

0.2 Measuring the urban sprawl

The analogy between city and information theory was developed in the 1960's, providing through the concept of urban entropy various formal approaches to generate simulation models that explain the interaction between the origins and destinations within the city, caused by the functional heterogeneity of the districts.

Shannon's entropy formula measures the diversity of information in a message (Shannon, 1948). Logically, the greater the diversity of data provided in a message, the greater the difficulty in predicting the exact meaning of the transmission. Margalef (1991) used a mathematical expression of the entropy to calculate the diversity of species in an ecosystem. He assimilated the different species to symbols of a message, so that the quantity of information of that message is equal to the diversity of species. Note that, if we assume that the different existing species are adapted to the environment, the measure of the diversity gives us also an idea of the degree of the organization of the ecosystem. Later, this methodology has been used by other authors to assess the organization degree or complexity of the urban context. Instead of species, the symbols are activities (trades, facilities, offices, services, etc.) and Shannon's entropy is conveniently used as an effective technique for monitoring and measuring the urban sprawl (Yeh and Li, 2001).

For this purpose, Shannon's formula measures the diversity of activities existing in the i-th zone and also the global entropy of the city for a given splitting into urban districts. Formally,

$$
H_i = -\sum_{k=1}^{m} p_{ik} \cdot log_2(p_{ik}); \quad H = \sum_{i=1}^{n} H_i \qquad (0.22)
$$

where

- H_i is the entropy of the i-th zone (or urban district).
- p_{ik} is the probability or proportion of the k-th variable (activity) in the i-th district.
- m is the total number of zone variables that have been analyzed.
- H is the whole entropy of the city.
- n is the total number of districts considered.

As is emphasized in Coward and Salingaros (2016), the city can be understood as a complex system where there is a continuous exchange of information in the form of flows of people, goods, situations, data, etc. (i.e. species, characteristic variables). Note that the organization of such a system is all the more complex the greater the number of species, and, therefore, diversity can be understood as an organization measure. In that sense, the greater the value of the entropy, the greater the degree of the system organization. In addition, discontinuities in the urban context of the territory represent obstacles to the flow of information. As a result of the increase of accessibility, the interconnection between two zones through a rapid transit system will lead to, in the medium and long terms, the territorial cohesion between both and the effective compensation of its characteristic variables.
As a real application of his thesis, Sierra (2009) elaborates a zonal division of the city of Glasgow based on a collection of data concerning to the uses of the buildings, number of floors and diversity of commercial establishments. Moreover, Sierra (2009) emphasizes the influence that some zones, provided with high levels of urban entropy, can induce over their adjacent ones. For instance, a district of high complexity transfers such characteristic to the adjacent cells, through connecting streets as influence corridors. The lowest the travel time between adjacent cells, the highest the degree of complexity is transferred.
Walker et al. (2011) analyzed the urban areas of La Serena-Coquimbo (Chile) according to the number of services covered at walking distances less than 10 minutes. Empirically, they show in that paper that the higher the entropy level of the zone, the lower the energy consumption and, from an urban perspective, propose the formation of small poly-centers of multi-services (more than three different services) that cover, with the help of a rapid transit system, an area of urban population of approximately 78 ha.

0.3 Shortest path problems

The shortest path problem is a classic problem in computer science and operations research that involves finding a path with the minimum distance or cost between two vertices in a graph. Vertices can represent physical locations, cities, network nodes, or any other abstract adimensional objects, while edges can represent its connections between these vertices weighted by means of distances or costs.

The shortest path problem is widely used in various real-world applications, including transportation, logistics, network design, and telecommunications. For instance, when planning a route in a road network, vehicle navigation systems usually provide the option of choosing among several different optimization criteria, or cost functions, such as a shortest route which can exhibit preference to highways or avoiding toll highways. In real-life scenarios, the quality of a planned route is influenced by the presence of traffic jams along the suggested itinerary. Information on traffic jams, road works, and incidences which can affect the road conditions can be received via smart-phone applications, so that the database, which describes the cost of crossing every arc in the road network, remains continuously updated in the GPS navigation system.

There are several algorithms for solving the shortest path problem, including Dijkstra's algorithm, Bellman-Ford algorithm, A* algorithm, and others. Dijkstra's algorithm, described by Dijkstra (1959), is a popular method for finding the shortest path in a graph with non-negative edge weights. The Bellman-Ford algorithm, first presented by Bellman (1958), is useful for finding the shortest path when some of the edge weights are negative. The A* algorithm, first proposed by Hart et al. (1968), is a combination of Dijkstra's algorithm and a heuristic function, that function takes a node as input and returns a value that corresponds to the cost to reach the target node from that node. In each iteration, the graph traversal is continued from the node with the lowest combined cost.

In addition to these algorithms, there are also several variations of the shortest path problem, for instance over a large dynamic network in real-time where the cost of an arc is the travel time of it, and their objective is to find paths having a minimum length with respect to time-dependent travel costs. Cooke and Halsey (1966) proposed a shortest path problem in networks where costs for traversing nodes were included. Dreyfus (1969) proposed a modification of the Dijkstra's static shortest path algorithm to calculate shortest paths between two nodes for a given departure time, by assuming that the travel times on the arcs are positive integers for every time period.

From a multiobjective perspective, Martins (1984) developed a multiple labeling version of Dijkstra's label setting algorithm to generate all Pareto shortest paths from the source node to every other node in the network. In Hamacher et al. (2006) the classical shortest path problem by considering two objective functions in a setting of time-dependent data has been generalized. In that paper, a complete survey of the relevant literature is provided and an algorithm for the time-dependent bicriteria shortest path problem with non-negative data is developed.

The multimodal shortest path problem consists of finding a path from an origin to a destination while the total associated cost is minimized by means of the use of several transportation modes, such as a personal car, taxi, subway, tram, bus, and walking. Modesti and Sciomachen (1998) introduced an approach based on the classical shortest path problem for finding multi-objective shortest paths in urban multimodal transportation networks, taking the overall time-cost and the users' inconveniences into consideration. In a multimodal network, a node represents a place where one has to select between continuing with the current mode or changing it. A change of mode or modal transfer is represented by an arc called transfer arc. An arc connecting two nodes by only one travel mode is called travel arc. In Lozano and Storchi (2001) label correcting techniques in an ad hoc multimodal shortest path algorithm to find the shortest viable path have been introduced. Those paths with illogical sequences in the use of transportation modes are eliminated in order to reduce the complexity of the connection graph. The same authors have extended their algorithm to calculate the viable hyperpath (Lozano and Storchi, 2002). Finally, the problem of finding the non-dominated viable shortest paths, considering the minimization of both the travel time and the number of modal transfers is addressed in Artigues et al. (2013).

0.4 Park-and-ride

Park and ride is a transportation management strategy that encourages drivers to park their cars in designated areas outside of a congested urban center and continue their journey using public transportation, such as buses, trains, or light rail. The idea behind park and ride is to reduce traffic congestion, improve air quality, and make public transportation more accessible and convenient for people who live in suburban or rural areas.
Park and ride facilities typically include ample parking spaces, sidewalks, bike racks, and other amenities to make it easy and safe for drivers to leave their cars and transfer to public transportation. These facilities can be located at transit hubs, shopping centers, or other convenient locations and are often strategically placed near major highways or transportation corridors to minimize travel time and increase the likelihood of attracting users.
Park and ride has been widely implemented in many cities around the world and has been shown to be an effective way of reducing the number of cars on the road and increasing the use of public transportation, see for instance Parkhurst and Richardson (2002); Zijlstra et al. (2015). It can also help to address the first-mile/last-mile problem in public transportation, which refers to the challenge of getting people from their homes to a transit stop and from a transit stop to their final destination.
From the attractiveness of a park facility perspective, there are several variables related to the setting of park-and-ride facilities, namely location, size, and price. In order to fix these variables the behavior decision of potential users has been extensively studied. In fact, several studies discuss the users' preferences complexity for choosing park-and-ride (see, e.g., Bonsall and Palmer, 2004). Most parking choice models have applied logit models with data from stated choice surveys, conducted on resident and non-resident drivers. For example, in Hess and Polak (2004) parking choice based on a stated preference data set in the center of several cities of the United Kingdom using mixed logit models

have been investigated. In Habib et al. (2012) the relationship between parking choice and an activity scheduling process based on data coming from a survey conducted in Montreal have been explored. Based on Decision Field Theory, in Qin et al. (2013), with the objective of assisting policymakers for planning park-and-ride facilities, decision behavior have been analyzed. In Ibeas et al. (2014) a mixed logit, that includes three parking scenarios and three variables: parking access time, parking fee and final time to destination, has been used to study drivers' behavior. On the other hand, factors affecting mode change behavior of commuters were explored by using a multinomial logit in Islam et al. (2015).

The evidence has shown that the attractiveness of a parking facility is related to different cost attributes that can be combined with relative importance weights to build an overall measure of disutility. In Thompson and Richardson (1998) three types of costs were distinguished: (1) access cost including in-vehicle travel time from the vehicle's current location to the car park as well as the time spent searching for an empty parking space, (2) waiting time cost that occurs when drivers have to queue at a car park before being able to enter, (3) usage cost associated with a car park. When assessing these costs, travel time cost on foot or public transit to the destination would also have to be included. Moreover, the attractiveness of a parking lot depends on its condition, such as the capacity and the chance of finding a vacant parking space (Hensher and King, 2001). Chen et al. (2014) propose a recommendation model to choose the best station for park-and-ride users. Availability of parking space, target time to the train stations, frequency of trains at stations, and service quality of stations were the criteria chosen to estimate a departure station of a commuter line for park-and-ride users. Besides, a fuzzy logic method is applied to estimate parking availability. The paper by Du and Wang (2014) considers three modes of transportation: only railway, only auto, and combined auto-railway with parking. The applied criteria are times for each mode, operating cost for only auto mode, crowding cost for the railway, parking fee, and railway fare. The study aims at determining the share of each mode by a continuous equilibrium model for the case of a continuum park-and-ride services along the corridor. The relationship between prices and capacity of the parking lot has been studied in García and Marín (2002), in which a bilevel programming model with the decisions on size and fees at the upper level and the reaction of the users at the lower level has been proposed. The problem is solved by simulated annealing. Wang et al. (2020) minimize the system travel cost by controlling the supply of parking-and-ride spots and parking permit numbers. The variables included in the generalized costs are travel times, the number of commuters in each of the three modes, parking fees and parking subsidies. Values of capacities, early and late arrival times, both for auto and transit modes, are taken into account in the coefficients. Specifically, the number of available places at each parking facility for a given time could be estimated from the current occupancy, take into consideration the observed evolution of this parameter in similar settings of the reference day, and the proximity to peak hours of traffic (Hössinger et al., 2014). In addition to these above mentioned research streams, several authors (see, for instance, Flinsenberg, 2004) have suggested improving the smart-phone applications for navigating in the city to the optimal park-and-ride facility by means of the incorporation of different aspects that have influence in the preferences of the drivers, such as parking costs, occupancy rates, walking time, the possibility of shuttle buses, etc.

0.5 Electric vehicle

Research on Electric Vehicle (EV) and its related infrastructure has gained momentum in the last decade. Ghosh (2022) classifies this researching activity in five main areas, a) Needs assessment and gaps, b) Energy & Finance, c) Site design standards, d) User experience and e) Locational models. Pagany et al. (2019) presents an extensive overview as well as an in-depth review of the literature dealing with the location of Charging Stations (CS) for EV.
Types of economic approaches used to locate charging stations can be classified in theoretical models and empirical applications. Theoretical works have concentrated on deriving mathematical models to assess locations for EV infrastructure with simulated data whereas empirical models focus on real spaces with spatial characters often with georeferenced data and provide the results obtained from different mathematical locational models. EV charging infrastructure site selection is a multiple criteria decision making problem as it is determined by several factors often contradictory. One part of existing literature presents strategies for locating charging stations; in particular, which type of CS and where should be placed those installations within a selected area or along a road network are investigated. While some proposals attempt to identify those locations by calculating the spatial distribution of charging demand, other studies aim to find the best CS location with an optimization approach using traffic demand along road networks Ghosh (2022).
The location of electric charging stations is a much-discussed topic in the literature. Lee (2022) the location models that have been employed in charging facilities for EVs and alternative energy-powered vehicles are reviewed: p-median problem, set covering problem, fixed charge location problem, and those based on demand of Origin-Destination trips. Moreover, contributions regarding the sizing problem of EV charging stations with different objective functions are also examined.
Kuby and Lim (2005) introduced the Flow Refueling-Location Model (FRLM) to maximize the impact of a given investment in refueling infrastructure for alt-fuel vehicles. Such model optimizes the location of a given number of refueling facilities within a network with the goal of enabling the maximum number of trips by vehicles with a limited range. The most basic input to the FRLM is a set of origin–destination (O–D) pairs and the flow volumes between them. For each O–D pair, one must calculate the shortest path between origin and destination and then determine all the combinations of facilities that can refuel a round trip along that path. These combinations of refueling stations, and the flow volume associated with each configuration, are evaluated by the FRLM's

mixed integer programming (MIP) formulation to determine optimal facility locations.
The FRLM is an un-capacitated model; it implicitly assumes that a single facility can refuel an infinite amount of flow. This may not be a realistic assumption. To address this concern, Kuby and Lim (2007) introduces the capacitated flow refueling location model (CFRLM) that limits the amount of flow that any facility can refuel. The objective consists of locating p stations on a network to maximize the refueling of origin–destination flows. Due to the limited driving range of vehicles, network vertices do not constitute a finite dominating set. Such authors propose to add candidate sites along arcs using minimax and maximin methods. The maximin criterion is motivated by the idea of not wasting candidate sites by putting any too close together, while the minimax objective aims to avoid long arcs with no candidate sites. Nevertheless, none of the methods reaches to generate a finite dominating set.
Upchurch et al. (2009) extended the FRLM by considering that only limited number of vehicles can be refueled by a station. To formulate this constraint, a variable was introduced to indicate the proportion of the traffic flow on each path being refueled by each station combination.
Kim and Kuby (2012) study simple-path deviations (excluding cycles) from the shortest paths. The deviations are calculated by a k-shortest path algorithm before the model is solved until a predefined user tolerance deviation is reached. Due to the preprocessing time in this deviation flow refueling location model is excessive when deviations are considered in extensive networks, Kim and Kuby (2013) propose a heuristic to solve realistic-sized problems.
Another formulation of the FRLM was proposed by Capar and Kuby (2012), which does not require the pre-generation of all feasible station combinations. For this purpose, authors introduced two binary variables on each node along every path indicating whether a station exists at that node and whether a driver at that node can reach another station further down the path without running out of fuel.
A generalized maximum covering model is proposed by Wen et al. (2014b) for the Flow Refueling-Location Problem (FRLP) without using extra variables as in Capar and Kuby (2012) and without pre-generating facility combinations as in the other maximum covering models. A set of sub-paths is defined for each path, in such a way that if each of these sub-paths contains a replenishment station, the entire path flow is captured.
Minimizing infrastructure cost has been dealt on Li and Huang (2014). Huang et al. (2015) extend the set cover model by allowing shortest path deviations, where the deviation paths are exogenously determined, and fuel level is still tracked on every node.
Literature contains another type of contributions for the FRLP which is based on the set covering problem. Wang and Lin (2009) presented a model to capture all the traffic flow with least station location cost. In their model, a variable is defined at each node on each path indicating the remaining amount of fuel when vehicles reach that node, such that a trip can be refueled if the remaining amount of fuel at each node along the path is non-negative.

Wang and Wang (2010) consider that each node is associated with a population and if a station is located in the node, the corresponding population is said to be covered. For this context, the authors extended the FRLP model to multiple objectives: to minimize the cost of locating the stations and also to maximize the coverage of the population.

MirHassani and Ebrazi (2013) proposed a new formulation whose objective function changed to a maximum-covering in terms of a maximum-covering and more constraints were added to the optimization model.

Special work has been done from the point of view of the company, maximizing the flow captured or minimizing the cost of the infrastructures. Optimization procedures employ both exact methods Asamer et al. (2016); Li et al. (2016); Wang et al. (2016) as well as heuristic techniques Chen et al. (2015a); Hidalgo et al. (2016); Salmon (2016); Sebastiani et al. (2016). They aim to find the optimal location for CS by minimizing total cost, reducing trip length, or waiting time.

Furthermore, some studies deal with the interactions between electricity and transport networks. In Riemann et al. (2015) an optimal location problem of wireless charging facilities with a given number of wireless charging facilities has been deployed combining the flow capturing location and the stochastic user equilibrium model.

Zhang et al. (2016) study optimal planning of EV fast-charging stations considering the interactions between the transportation and electrical networks. Authors propose the capacitated flow refueling location model (CFRLM) to explicitly capture EV charging demands on the transportation network under driving range constraints. Then, a mixed-integer linear programming model was formulated for plug-in electric vehicle fast-charging station planning considering both transportation and electrical constraints based on CFRLM.

The locations and sizes of fast-charging stations in a transportation network should satisfy EV driving demands, while simultaneously ensuring the security operation constraints of power systems, e.g., distribution line current limits and nodal voltage limits. In addition, authors in this field consider that an appropriate fast-charging station planning method should minimize the investment costs of both charging stations and corresponding power grid upgrades.

In Li et al. (2016) a multi-period multipath refueling location model has been developed to expand EV charging network to dynamically satisfy OD trips and determined the cost-effective station rollout scheme on both spatial and temporal dimensions.

In Xiang et al. (2016) a solution has been proposed to integrate EV and optimally determine the siting and sizing of charging stations (CSs), considering the interactions between power and transportation industries.

In Motoaki (2019) only a location problem with the objective of providing a geographical coverage of the demand is considered. In this case, the objective is to have a maximum number of EVs with access to a potentially available station, and the charging station locations are uncorrelated to the charging station sizes.

Sun et al. (2020) used a mixed-method approach, with location of fast charging

stations for vehicle interception and a node-based approach to place slow charging stations in places where a long charging time is acceptable.
Bilevel modeling has also been used in the literature to study different objectives in the location of charging stations. Jing et al. (2017) developed a bi-level model to maximize coverage of EV flows by deploying a given number of charging stations on a network with mixed conventional vehicles and EVs.
Zheng et al. (2017) used the bi-level structure to optimally locate charging stations to minimize travel time and energy consumption while considering traffic equilibrium. Guo et al. (2018) developed a bi-level integer programming model to locate charging stations in the manner of minimizing the construction cost and deviation cost while maximizing the number of served EV by the charging service.
He et al. (2018) proposed a bi-level programming model with the consideration of EV's driving range, for finding the optimal locations of charging stations: the upper level is to optimize the position of charging stations to maximize the path flows that use the charging stations, while the user equilibrium of route choice with the EV's driving range constraint is formulated in the lower level.
Makhlouf et al. (2019) developed a bilevel problem where the upper-level problem is a max-cover type station location and sizing problem, and the lower-level problem represents the preference of EV user behavior in terms of making the minimum number of stops to reach their destination.
Huang and Kockelman (2020) considered congested travel flows and congested stations under elastic demand to maximize profits of electric vehicle charging station owner by means of a genetic algorithm.
Tran et al. (2021) developed a bi-level program to determine the optimal location of public fast-charging stations while simultaneously considering heterogeneous vehicle classes, the installation cost of charging stations, link congestion and route choice behaviors of travelers with multiple recharging locations.

0.6 Vehicle Route and Bin Packing Problems in waste collection management

Waste collection and transportation problems are one of the most difficult operational issues in the development of an integrated waste management system (Nuortio et al., 2006; Franca et al., 2019). Eiselt and Marianov (2015) provide a survey of 64 studies on the landfill siting problem that include applications across the world, for which the main aspects of interest of the contributions are summarised in a complete table (see Ref. Eiselt and Marianov, 2015), by selecting country, technique, criteria, objectives, and type of facility to be located.
Furthermore, the intrinsic nature of Municipal Solid Waste (MSW) collection relates to the development of effective vehicle route models that optimise the total travelling distance of vehicles, environmental emissions, and investment costs (Apaydin and Gonullu, 2011). Vehicle Routing (VR) is a scheduled process that enables vehicles to load waste at collection sites and to dump it at a

landfill with the result being oriented by a single objective or multiple objectives (Tung and Pinnoi, 2000).
In real-life scenarios, the waste collection system is distributed across a set of zones. Each zone has an associated starting and ending node. These nodes are used for vehicle routes and landfill points where the rubbish collected in the visited containers can be delivered. A planning horizon must also be considered in order to schedule a sequence of services within its bounds. A succession of routes (one per day, belonging to the same or to different distribution zones and performed by the same vehicle along the planning horizon) is called a circulation.
Yeomans (2007) underlined the importance of using a mathematical approach in the sustainable waste collection and transportation in urban areas. Mohsenizadeh et al. (2020) developed a mathematical model in order to reduce pollutant emissions from vehicles during the collection of waste, while considering the Ankara case.
Teixeira et al. (2004) analysed the Vehicle Routing Problem (VRP) while separating the collection of paper, glass, plastic, and metal waste materials. Their model is based on three stages: the definition of a zone for every truck; the identification of the type of waste to be collected every day; and the choice of the points to visit and of the sequence order.
Bing et al. (2014) investigated the plastic waste collection problem based on eco-efficiency terms of the proper balance between environmental impacts, social issues, and costs. They modelled the urban plastic waste collection based on a VRP. Different cases were considered by analysing key factors, such as the type of truck, collection frequency, and collection method. These authors solved the VRP by means of a tabu search algorithm capable of solving real cases. Their results showed that, based on the proposed algorithm, the eco-efficiency performance of the current collection paths could be improved by 7%.
Beliën et al. (2014) presented a review of solid waste management problems where VRPs are classified into several categories. Han and Ponce Cueto (2015) provided a detailed analysis of the VRP for waste collection.
The most efficient methods of VR and Bin Packing (BP) problems are based on heuristic and metaheuristic solution models, Willemse and Joubert (2016); Corberán and Prins (2010); Paquay et al. (2018). Buhrkal et al. (2012) applied an Adaptive Large Neighbourhood Search (ALNS) metaheuristic to solve the waste collection with time windows for various real cases. Their objective aims to minimise the distance driven which was also linked to a reduction in fuel consumption and pollutant emissions. Akhtar et al. (2017) developed a modified Backtracking Search Algorithm (BSA) in Capacitated Vehicle Routing Problem (CVRP) models based on the "intelligent bin" idea to optimise path design in the waste management system. Their results for four days presented a 36.80% reduction in distance for 91.40% of the total waste collection, with an increase of the mean waste collection efficiency of 36.78% and a reduction in fuel consumption of 50%, in fuel cost of 47.77%, and in CO2 production of 44.68%, respectively.
Kim et al. (2006) considered multiple disposal paths and drivers' lunch breaks

using an extension of Solomon (1987). Angelelli and Speranza (2002) presented a tabu search algorithm to optimise the operating costs of various waste collection processes for two case studies.

Das and Bhattacharyya (2015) minimised the length of every waste collection and transportation route. They proposed a heuristic methodology based on the Travelling Salesman Problem (TSP) and obtained a reduction of approximately 30% of the total length of the waste collection route. Laporte et al. (2010) validated a heuristic algorithm for a Capacitated Arc Routing Problem (CARP) based on stochastic demand. In Perea et al. (2016) a Mixed Integer Linear Programming (MILP) is developed so that decision about which collection sites should be collected on each service day can be done by guaranteeing that no container overflows, the amount of waste collected per service day is within a given maximum deviation, and the collection sites visited on each service day are close to each other, over a weekly planning horizon. The MILP proposed cannot solve instances of realistic sizes and a clustering method, based on heuristics for the TSP is proposed for this purpose.

More recently, Tirkolaee et al. (2018) investigated the periodic CARP by considering the work of the drivers and their crews in order to analyse the demand change. Their model is based on an objective function for the minimisation of the total transport route and total costs in terms of the number of needed vehicles. A simulated annealing algorithm is employed to improve the solution data.

By considering capacity, time, and distance restrictions, Willemse and Joubert (2016) extended the VRP to deal with the problem of Waste Management (WM). They proposed four heuristic models to compute feasible solutions for the two issues of WM and VRP to minimise the total cost and/or the fleet dimension. They analysed CARP under time restrictions with intermediate facilities, and carefully modelled the collection of waste, based on the development of a constructive heuristic model.

A numerical approach based on MILP is proposed in Chu et al. (2005) and considers the VRP for every period to reduce the number of trucks, and in turn the total costs during a defined period. Mes et al. (2014) presented a heuristic methodology based on the optimisation of various tunable parameters for every day of the week. Thanks to their model, they obtained a cost reduction of approximately 40% for a specific case study in the Netherlands. Son and Louati (2016) developed a mathematical model that considered multiple transfer stations for urban solid waste management. They validated their model by applying it to a case study and obtained a reduction of path length and working hours of the trucks. Ghiani et al. (2012) investigated the issue of locating waste collection points in urban areas. They developed a model to identify: the optimal sites for the location of the waste collection bins; the required number of bins; and the features of the bins sited at the various collection stations. The results obtained from their numerical model based on heuristic procedures demonstrated a reduction of approximately 62% for the waste collection points and a decrease of approximately 73% of the number of bins allocated.

Recently, various researchers have underlined the importance of optimising not only the logistic aspects of VRP, but also the bin packing methodologies to improve the performance of waste collection, Jank et al. (2015). Rodrigues et al. (2016) based the kerbside collection of WM on the classification of the bin components, truck components, and collection methods. They analysed the collection frequency by considering one bin for the single packaging waste flow (yellow colour for lightweight packaging; green for glass; blue for paper and cardboard). Martinho et al. (2017) studied the importance of identifying the proper recycling methods and analysed two different waste collection methods for the recycling in 3 districts of Portugal. In their research, they highlighted the importance of carrying out an itemised characterisation of the end waste to identify the overall quantity of waste packaging material that can be recycled. They defined certain key "performance recycling indicators", such as waste characterisation, recycling rate, and separate waste collection rate, and also "logistic performance indicators", such as the distance travelled by car to collect waste, the number of workers involved in the collection, and the effective collection time.

An eco-point container is actually a multi-compartment (block) vehicle. The multi-product vehicle routing problem has been extensively researched in recent years. A very important special case is the multi- compartment vehicle routing problem (MCVRP) in which each compartment can only load a single type of product. Applications of the MCVRP include the collection of hazardous materials (Paredes-Belmar et al., 2017a), inventories-routing (Coelho and Laporte, 2013), maritime logistics (Bertazzi et al., 1997), groceries distribution (Martins et al., 2019), and milk collection (Paredes-Belmar et al., 2016, 2017b), among others. A very recent review (Ostermeier et al., 2021) classifies the MCVRP according to the kind of application, and regarding waste collection states: "across all publications, single waste types are always collected completely by one vehicle if a customer is visited, and therefore customer demands are not split", in section 2.4 this reflextion is assumed and a mathematical model is developed by considering the posibility of several vehicles can visite a same customer while selectively collecting waste.

0.7 Contributions

This PhD dissertation is divided into three main chapters. Chapter 0 offers a basic background for the contents that are presented in the remaining chapters. The main contributions of this PhD dissertation are the following:

Chapter 1. In this chapter, we have addressed three different problems related to the issue of urban expansion and increasing traffic congestion. The first problem involved locating a rapid transit line using new criteria that maximizes the global entropy of the metropolitan system, assuming that the interconnection of two zones produces homogeneity of their urban characteristics in the medium and long term. This reduces territorial disparities that occur in sprawled cities. Additionally, we solved the problem for a real case in the city of Seville with 47 zones using a greedy algorithm.

In the next point, we modeled the problem of route design for a user whose destination is in a zone with private traffic restrictions, such that there are parking zones around that zone with connections to various modes of public transportation or walking. We modeled this problem as an integer programming problem that minimizes the total travel time, parking cost, and the possibility of not finding a parking space (depending on occupancy). In addition to considering all these attributes, our contribution also consisted of considering different user profiles depending on the available information on the parking zones. We also solved the problem for the city of Seville to empirically see the sensitivity of the model.

In the last point of this first chapter, we addressed the problem of transforming conventional gas stations into electric charging stations. We approached the problem with a dual public-private perspective so that we maximize the benefit with a maximum budget and ensure that all nodes have an electric charging station nearby. In the case that a station is not functional or cannot be considered for any reason, there is another station nearby as an alternative. To solve this dual vision, we modeled the problem as a bilevel problem, reformulated it as a single-level problem, and performed a computational experiment.

Chapter 2. In this chapter, we have addressed three different problems related to urban waste collection. The Multiple Knapsack Model provides an appropriate theoretical framework for analyzing the territorial deployment of fixed containers for selective collection of urban solid waste, as well as for planning routes and stops for the so-called eco-points, i.e., waste containers with less frequent generation by society in the urban area and with potential to be pollutants to the environment.

In the first and second points, we tackled the same problem of deploying containers for selective urban waste collection. In the first point, we solved the model, due to its complexity, with a greedy algorithm and carried out a computational experiment in a real area with random data. In the second point, we now consider that the user may have a cooperative behavior and may go to throw the trash in a container that may not necessarily be the closest. We modified the greedy algorithm to take this new version into account and also carried out a computational experiment like the previous one.

In the last point of this second chapter, we dealt with the problem of eco-points, specifically, the problem of considering some mobile eco-points and some waste-generating points, and determining the optimal route that minimizes costs. Due to the complexity of the model, we proposed a heuristic and carried out a computational experiment with the Sioux Falls network.

Chapter 3. In this chapter, we have addressed two theoretical problems related to efficient strategies aimed at minimizing wait times for users/operators in transportation networks.

In the first point, we developed a methodology for implementing a service redistribution along a railway transit line, which must be carried out by the operator by choosing new train schedules within a series of feasible space-time windows previously established by the railway infrastructure manager. The objective is to minimize the loss of users (minimizing wait time), who may perceive a deterioration in the quality of service they have been receiving until now.

In the second point, we have proposed a new algorithm to solve the shortest path problem in a network but considering time-dependent arcs. We have proposed to modify the original graph by exploiting those nodes that belong to the time-dependent arcs, and with this new graph, we perform the Dijkstra algorithm.

0.8 Related Publications

The material presented in this thesis is based on the following papers, some of them published and others submitted (or about to be submitted) for publication in journals in the area of Operation Research:
Chapter 1. *Providing cohesion to the territory through the location of collective services.*

- Ortega, F.A., **Piedra-de-la-Cuadra, R.** & Ventura, S. (2018). Applying An Entropic Analysis To Locate Rapid Transit Lines In Sprawled Cities. International Journal of Sustainable Development and Planning 13 (4), 626- 637.

- Mesa, J. A., Ortega, F. A., Pozo, M. A., & **Piedra-de-la-Cuadra, R.** (2021). Assessing the effectiveness of park-and-ride facilities on multimodal networks in smart cities. Journal of the Operational Research Society, 1-11.

- **Piedra-de-la-Cuadra, R.**, Bruno, G. & Ortega, F. A. (2022). Bilevel optimization for the location of charging stations in corridors. Book of Abstracts of the XXVII EURO Working Group on Locational Analysis. p. 97-98. Submitted at Computers & Operations Research.

Chapter 2. *Location and routing of containers (fixed and mobile) for the selective collection of urban solid waste.*

- Barrena, E., Canca, D., Ortega, F.A. & **Piedra-de-la-Cuadra, R.** (2019). Optimizing container location for selective collection of urban solid waste. WIT Transactions on Ecology and the Environment, 231, 1-9.

- Barrena, E., Canca, D., Ortega, F.A. & **Piedra-De-La-Cuadra, R.** (2020). Solidarity behaviour for optimizing the waste selective collection. International Journal of Sustainable Development and Planning, Vol. 15, No. 2, pp. 133-140.

- Marseglia, G., Mesa, J. A., Ortega, F. A., & **Piedra-de-la-Cuadra, R.** (2022). A heuristic for the deployment of collecting routes for urban recycle stations (eco-points). Socio-Economic Planning Sciences, 82, Part A, 101222.

Chapter 3. *Efficient strategies based on waiting time for operators/users in transport networks.*

- Ortega, F. A., Mesa, J. A., **Piedra-de-la-Cuadra, R.**, & Pozo, M. A. (2019). A matheuristic for optimizing skip-stop operation strategies in rail transit lines. International Journal of Transport Development and Integration, 3(4), 306-316.

- Ortega, F. A., Marseglia, G., Mesa, J. A. & **Piedra-de-la-Cuadra, R.** (2022). Algorithm for planning faster routes in urban networks with time-dependent arcs and the possibility of introducing waiting periods at nodes. WIT Transactions on The Built Environment, 212, 25-36.

The diffusion given to the developed research is evidenced through the following selected presentations that have been presented in scientific congresses of the specialty.

- Juan A. Mesa, Francisco A. Ortega & Ramón Piedra-de-la-Cuadra. *Determing alignments to design rapid transit lines in urban setting.* **SEIO Transportation Working Group Meeting.** 2017.

- Ramón Piedra-de-la-Cuadra. *Modelling the navigation to nearest park-and-ride facility.* **Workshop "Mathematical Models of Optimization for Transportation Planning".** 2017.

- Ramón Piedra-de-la-Cuadra, Giuseppe Bruno & Francisco A. Ortega. *Bilevel optimization for the location of charging stations in corridors* **XXVII EURO Working Group on Locational Analysis**. 2022.

- Eva Barrena, David Canca, Francisco A. Ortega & Ramón Piedra-de-la-Cuadra. *Optimizing container location for selective collection of urban solid waste.***Waste Management 2018.** 2018.

- Eva Barrena, David Canca, Francisco A. Ortega & Ramón Piedra-de-la-Cuadra. *Solidarity behavior for optimizing the waste selective collection.* **International Workshop on Locational Analysis and Related Problems.** 2019.

- Ramón Piedra-de-la-Cuadra. *Optimizing the development of mobile eco-points for a selective collection of municipal waste.* **ISOLDE XV**. 2021.

- Ramón Piedra-de-la-Cuadra, Juan A. Mesa, Francisco A. Ortega & Miguel Ángel Pozo. *Optimal fleet allocation for skip-stop strategies in rail transit lines.* **30th European Conference on Operational Research.** 2019.

- Francisco A. Ortega, Guido Marseglia, Juan A. Mesa & Ramón Piedra-de-la-Cuadra. *Algorithm for planning faster routes in urban networks with time-dependent arcs and the possibility of introducing waiting periods at nodes.* **Urban and Maritime Transport 2022**. 2022.

All this work has been financially supported by the following research projects:

- *Optimización Matemática en Problemas de Localización y Planificación de Redes*. PID2019-106205GB-I00.

- *Modelos matemáticos de optimización en redes multicapa. Aplicación en sistemas multimodales de transporte en áreas metropolitanas*. US-1381656.

- *Ayudas para la formación de profesorado universitario (FPU)*. Ministerio de Universidades.

Chapter 1

Providing cohesion to the territory through the location of collective services

1.1 Introduction

In general, urban sprawl refers to certain forms of city spatial expansion toward suburbs and peripheral areas with, low density, single-use, extensive road and highway networks, car dependent, open up vast space of territory, scattered and ribbon development in an mono-centric urban structure (Zhang, 2001), see for example Figure 1.1.

From a socio-economic perspective, urban sprawl leads to excessive infras-

Figure 1.1: Example of a sprawled city. Suburbs of Scottsdale (Arizona).

tructure and public service costs, the decline of public space, reducing social

cohesion, loss of a sense of community, loss of cultural values, increase of the income inequality and polarization, traffic congestion, longer travel distances and limited access, especially for non-driver people (Mosammam et al., 2017).
The increase in transport demand caused by the urban model of sprawled city generates a massive use of the means of locomotion that saturate the roads destined to mobility and produce periodic episodes of pollution (Bhatta, 2010).
To cope with urban sprawl, road traffic pollution and growing traffic congestion, several cities throughout the world have turned to metro, rapid rail transit (RRT) and light rail transit (LRT) systems with connections, on many occasions with large parking areas located on the outskirts (park-and-ride) and the promotion of the use of electric vehicles.
Building rapid transit systems requires long term commitments and sizeable capital investments. What constitutes a good infrastructure configuration is by no means obvious, due to planners, engineers, users, environmentalists, and other interest groups do not, as a general rule, agree on a common set of objectives and constraints (Gendreau et al., 1995). The core problem consists of designing a transit network capable of transporting a large number of people efficiently and effectively. This problem is highly complex. Standard network design techniques cannot easily be applied to such contexts because the problem is typically of very large scale and involves non-linearities, as well as a multiplicity of criteria. It is more common for planners to generate a set of basic scenarios that are later analyzed, modified and refined. Some authors have proposed various indices to assess the topological configurations of rapid transit systems (see, e.g. Refs. Musso and Vuchic, 1988; Laporte et al., 1994, 1997) but such indices are more easily used as a way of measuring the quality of proposed or existing networks than as a means of generating good solutions.
Maximizing the covered population by means the optimal location of stations is a common objective (Dufourd et al., 1996) or combining that objective with data on origin-destination demand (as proposed in Bruno et al., 2002). For a survey on rapid transit network design, see Laporte et al. (2011).
In the sprawl city model, districts only serve for a residential function and they are almost empty of other contents. The complexity of the urban setting is based on the diversity of functions that are inside located (residence, education, health, commerce, industry, etc.). Numerous researchers ensure that the shortage of urban complexity, that the districts of a city exhibit, causes an increase in traffic density (Salat and Bourdic, 2012). Assuming this hypothesis, promoting the functional diversity of the districts should consistently imply a reduction in the forced mobility of their inhabitants, while promoting territorial cohesion.
As can be seen, the design of rapid transit lines in urban contexts (metro, tram, BRT), by using as main objective the maximization of the functional diversity of the districts that are crossed by the infrastructure, is a problem that has not been enough treated in the literature from a purely scientific perspective.
In section 1.2, maximizing the global entropy of the district system is proposed as a design criterion for rapid transit lines.
Another key transport issues facing many countries in the world is the increase in congestion in urban areas and their accesses, due to the heavy dependence

on the use of private cars. Since park-and-ride facilities allow citizens to use their cars for a part of the trip, while completing the rest by means of public transportation, such facilities have been identified as a tool that can help to reduce traffic congestion (Mesa and Ortega, 2001). During recent decades increased attention has been paid to park-and-ride facilities. Reports commissioned by urban or transportation agencies of cities, metropolitan areas or governments, and papers published in specialized journals have informed about inquires and research related to several aspects of park-and-ride facilities. In the report Gleave (2017) typical objectives of park-and-ride facilities are listed. Searching-for-parking traffic comprises a significant amount of the total traffic volume, with average values reported equivalent to 30–50% of the total traffic volume at certain hours and districts (Arnott and Williams, 2017; Shoup, 2006; Chaniotakis and Pel, 2015).

Holguín-Veras et al. (2012) compare the generalized cost of the only drive trip with that of using park-and-ride. The generalized cost for the only drive mode is a combination of the in-vehicle cost, the value of time, and out-of-pocket expenses, whereas that of the combined mode consists of in-vehicle and transit costs, the values of time, and out-of-pocket expenses both for the auto and transit branches.

Hence, reducing the parking-search time could lead to significant improvements in terms of travel time, production, traffic flows, fuel consumption, pollution, and noise emission. This is one of the reasons why searching for parking process has been investigated in recent years (see, e.g., Benenson et al., 2008; Kaplan and Bekhor, 2011).The study of drivers' behavior regarding uncertainties about search times for finding vacant spot has been carried out in Chaniotakis and Pel (2015),where stated preference experiments are applied to several discrete choice models. One of the findings of this research is that most of the drivers searching for a parking lot who make a trip for shopping purposes start the searching process when they approach or reach their destinations. This fact supports the need for reducing the cruising traffic by introducing reservation systems and guiding tools (see, e.g., Arnott and Williams, 2017; Karaliopoulos et al., 2017).

During the past two decades, traffic authorities of some cities and several companies have implemented devices to inform and guide drivers towards parking facilities with variables message signals in real-time. These signalling systems show if park facilities are available and/or the number of parking places available (see Figure 1.2).

Intelligent transport systems (ITS) includes parking applications that inform, guide and reserve available parking facilities(Thompson and Bonsall, 1997). For example, Thompson and Richardson (1998) introduced a simulation parking search model that accepts or rejects the vacant parking places based on a disutility function. Current commercial navigation systems manage those data in order to plan routes between start and destination nodes reasonably fast, but they are notable to optimally deal with complex scenarios, such as when the route must necessarily be multimodal.

Figure 1.2: Signaling system of park facilities in the city of Seville.

For instance, if traffic restrictions in the neighbourhood of the destination point prevent access by private vehicle, it is essential in the configuration of optimal routes to consider a nearby car park to temporarily leave the car and walk (park and walk) or change to another authorized means of transportation (park and ride). The incorporation of daily occupancy patterns of each of the candidate carparks, as well as their main characteristics (rates, number of available free spaces, the possibility of commuting to another means of transportation inside the parking facility), can favour the determination of the best real routes for this type of problem. However, in current navigation systems, this possibility of improvement in the optimal determination of complex routes such as those described above has not yet been incorporated. Smart Parking PCT Cartuja is a comprehensive decision-making support system developed by the company Telefónica in the context of an agreement with the City Council of Seville. Its purpose is to provide users of the PCT Cartuja with information about the degree of occupancy of each of the parking blocks, and to advise them, using Big Data techniques, on the best option for their specific mobility needs (Proyecto de aparcamiento inteligente en el Parque Científico y Tecnológico Cartuja, 2019). Finally, another key transport issues facing many countries in the world is the emission of greenhouse gases causing the climate change. According to the Climate Plan decided at the 26th session of the Conference of the Parties (COP26) to the United Nations Framework Convention on Climate Change (UNFCCC), many governments have taken multiple strategic policy initiatives in the energy and transport sector to steer their respective nations to the path of reducing total projected carbon emissions by one billion tons from now to 2030. The European Union (EU) is the world's third-largest emitter of greenhouse gases behind China and the USA, followed by India and Russia. The European Commission has set out the objective of leading the world in the transition to a carbon neutral economy and established a goal of net-zero economy-wide emis-

sions by 2050.
Transport was responsible for close to a quarter of CO2 emissions in the EU in 2019, of which 71.7% came from road transport, according to the annual European Environment Agency report. To reduce CO2 emissions and achieve the climate neutrality of the European Green Deal, greenhouse gas emissions from transport would have to be reduced by 90% by 2050, compared to 1990 levels. However, current projections put the decline in transport emissions by 2050 at just 22%, well below current targets. There are two ways to reduce CO2 emissions from cars: make vehicles more efficient or change the fuel used. In 2019, most of the road transport in Europe used diesel (66.70%), followed by gasoline (24.55%).
Electric cars (EC) have a series of very important advantages over conventional internal combustion engine models linked to fuel savings, tax exemption in some countries, subsidies, maintenance costs, and zero emission of polluting gases to the atmosphere. For these reasons, electric cars are gaining ground and have accounted for 11% of all new passenger vehicles registered in 2020. Sales of electric vehicles (battery electric vehicles and plug-in hybrid electric vehicles) have soared since 2017 and have tripled in 2020, when the current CO2 targets began to apply. Electric vans represent 2.3% of the market share of new vans registered in 2020 (The future of the eu automotive sector — think tank — european parliament. (n.d.)).
The recent market shift towards electric vehicles (EVs) in Europe has been impressive. In 2020, despite the contraction of overall car sales in Europe, EV registrations more than doubled to 1.4 million and reached 10% of the market, while this number stood at 6% in China and 2% in the US (Executive summary – Global EV Outlook 2022 – Analysis - IEA).
The availability of infrastructure to serve the electric car market varies considerably between EU Member States. For example, in the Netherlands there are more than 32,000 charging points and there are more than 119,000 registered electric vehicles; while in Greece, only 40 charging points are available for just over 300 EVs Niestadt and Bjørnåvold. In addition, consumers' perception of electric vehicles is that they cannot cover the desired intercity distance without recharging Berkeley et al. (2017). Norway has one of the largest EV fleets in the world. Studies from the Norwegian EV market show the growing EV penetration requires large scale public charging infrastructure in addition to home based charging system. A well spread public charging network has also shown increased propensity to encourage complete shift to EV in the household Lorentzen et al. (2017).
In the context of Spain's Recovery, Transformation and Resilience Plan, a goal has been set for 2023 of at least 100,000 charging points and 250,000 electric vehicles, as well as the development of the value chain, new business models and new dynamics that favor the progressive electrification of mobility, the reduction of emissions and the fulfillment of energy and climate objectives. The financing planned in Spain for this purpose will reach 140,000 million euros in transfers and credits over the next six years.
In the case of electric mobility, it is planned to deploy fast or ultra-fast recharg-

ing infrastructure along corridors that make it possible to structure the entire territory, both on interurban roads of special national and regional relevance, as well as on those corridors connected with neighboring countries.
The industrial companies that manage gas stations are currently in a general process of adaptation with the aim of installing charging points for electric cars in their service stations. These energy companies develop strategic plans to adapt their infrastructures with the aim that electric vehicle drivers can have refueling points distributed throughout the territory for recharging energy, where these ones being not too far from each other to make it possible to circulate with supply guarantees throughout any corridor contained in the territory. Figure 1.3 shows an informative sign indicating the proximity of an EV refueling station along a road in the Algarve (Portugal).

Figure 1.3: Example adapted gas station

Once the specific contexts in which these complex problems arise whose solutions significantly determine territorial cohesion have been introduced, we describe the methodologies used to address them.
Section 1.2 addresses the problem of locating a rapid transit line with the objective of maximize the functional diversity of the districts traversed by the alignment. Section 1.3 develops an Intelligent Transport Systems to reduce the searching-for-parking traffic and time while taking into account drivers' behaviour, provide a methodology to evaluate the effectiveness of routes between two points of a smart city, starting them with a private vehicle but necessarily ending them with another means of transportation with the compulsory use of a park-and-ride facility. Users differ from each other, firstly according to those who will reserve a spot in advance and those who will look for a spot when getting there. Secondly, the information that users could have about the parking occupancy may differ. In this sense, users may be able to estimate the number of vacant places that parking facility will offer at a future time or they may also check from a device the free places at a parking facility at a present time. According to the type of user (and their available information), three criteria will have an influence on the decision towards choosing a park-and-ride facility: (1) the total travel time from origin to destination, (2) the parking fee and public transportation fare in case of using after parking and, (3) the attractiveness of

the parking facility as a factor that will depend on the risk of not having an available spot at the parking facility at the arrival time. Section 1.4 develops, from a double public-private vision, a procedure to optimally select, among a group of candidate sites where gas stations are already located, enough charging points in such a way as to guarantee that any electric vehicle can make its journey without a problem of energy autonomy and that each selected charging station has another at least that serves as coverage in case of failure (reinforced service). For this purpose, we propose a bilevel model that minimizes the number of refueling points necessary to guarantee a reinforced service coverage for all users who transit from their origin to destination inside a territorial zone and, as a second level, maximize the volume of served demand subject to budgetary restrictions. With the first of these objectives, we are meeting the usual requirement of the administration, which consists of guaranteeing the viability of the solutions, and the second of the objectives is a criterion typically used by the private sector initiative of profit maximization.

1.2 Applying an entropic analysis to locate rapid transit lines in sprawled cities

1.2.1 Assessing the location of a transit line

To cope with urban sprawl and increasing traffic congestion, many cities around the world have turned to metro, Rapid Rail Transit (RRT) and Light Rail Transit (LRT) systems with connections, often to large areas. parking located outside (park-and-ride). Building rapid transit systems requires long-term commitments and considerable capital investment. Maximizing the coverage of travel demand is a common objective when transit lines are located in the territory because it is possible, through said infrastructures, to improve the mobility of the population. However, some authors argue that certain investments can lead to increase rather than reduce regional disparities.

In the early stages of the development of a network, the transport policy is usually geared towards efficiency, but as the offer of infrastructures increases, the optimal strategy for the implementation of transport infrastructures must address the acquisition of a balance between criteria of efficiency, cohesion and environment.

Since rapid transit systems are planned in the long term, other complementary criteria could be applied to determine the layout of public transport infrastructures. Among them, maximizing the global entropy of the metropolitan system, assuming that the interconnection of two zones produces homogeneity of its urban characteristics in the medium and long term.

The most widely-used decision criteria in transportation network design are related to the maximisation of the population covered by the lines Mesa and Ortega (2001). For instance, the methodology applied to determine the most effective metro lines in terms of trip coverage in the metropolitan area of Seville, illustrated in Figure 1.4, was based on maximizing the capture of origin-destination traffic.

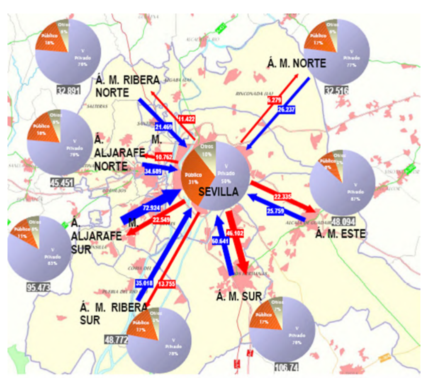

Figure 1.4: Generated and attracted trips to/from zones in the metropolitan area of Seville.

Once the metropolitan area had been divided into macro zones, the generated/attracted trips from/to each zone to/from its adjacent ones were counted. The width of each arrow in Figure 1.4 is proportional to the number of trips. Line 1 of the Seville Metro, the only one currently just built, was the most efficient for the criterion of covering travel demand and was consequently designed to connect zones A. Aljarafe Sur with A.M. Sur.
Since rapid transit systems are designed for the long term, other criteria can be applied to determine alignments which serve as the basis for the construction of public transit infrastructures. Among them, to maximize the global entropy of the metropolitan system, assuming, as was previously pointed out, that the inter-connection of two zones produces homogeneity of their urban characteristics in the medium and long term.
The following properties are necessary to support the performance of the design algorithm proposed to construct a rapid transit line whose objective is the maximization of the global entropy of the urban districts system.
Property 1.
Let be the function

$$h(x) = -xlog_2(x), x \in (0, 1].$$

For any pair of values $p, q \in (0, 1]$, the following inequality holds:

$$h(p) + h(q) \leq 2h\left(\frac{p+q}{2}\right).$$

Property 2.
Let be the entropic function

$$H(\vec{x}) = -x_k log_2(x_k),$$

for any vector $\vec{x} = (x_1, \dots, x_k)$,$x_k \in (0, 1], \forall k = 1, \dots, m$. For any pair of vectors $\vec{p} = (p_1, \dots, p_m), \vec{q} = (q_1, \dots, q_m)$;$p_k, q_k \in (0, 1]$ algebraic expression

$$2H\left(\frac{\vec{p}+\vec{q}}{2}\right) - H(\vec{p}) - H(\vec{q})$$

reaches its minimum value when both vectors $\vec{p}, \vec{q}$, coincide; i.e. for $p_k = q_k, \forall k = 1, \dots, m$.

The problem of determining an alignment so that global entropy is maximised can be formulated as follows: *Given an urban system composed of n districts, where a set of centroids (candidate stations)* $S = \{s_1, \dots, s_j, \dots s_n\}, |S| = n$, *represent the different urban areas with their specific characteristics (an unique centroid per zone) collected in the matrix* $P = p_{jk}, j = 1, \dots, n; k = 1, \dots, m$ *determine a subset L of S and an alignment* $Align(L)$ *on points of L such that global entropy is maximized without exceeding a constraint of maximum length (LMAX) for* $Align(L)$. Formally,

$$MAX_{L\subset S}H(L;S) = |L|\sum_{k=1}^{m} r_k log_2(r_k) - \sum_{j\in S\setminus L}\sum_{k=1}^{m} p_{jk}log_2(p_{jk}) \tag{1.1}$$

s.t.

$$r_k = \frac{1}{|L|}\sum_{j\in J} p_{jk}; k = 1, \ldots, m, \tag{1.2}$$

$$Length[Align(L)] \leq LMAX. \tag{1.3}$$

Hence, our measure of effectiveness, derived from having selected a set of nodes to build line L, is obtained by additively cumulating the partial entropies of each zone, taking into account that the characteristics of all the zones selected to form the alignment have been recalculated in order to force a common coincidence with their mean values. The objective function to be maximized, formulated by means of expression (1.1), describes the overall entropy of the urban district system, once a subset of them has been selected for the construction of the transit alignment. The constraint set (1.2) states that the characteristics of the selected districts must match their mean values. Constraint (1.3) ensures that the alignment length does not exceed the budget.
On the other hand, the common constraint (see, e.g., Bruno et al., 2002) consisting of imposing that distance between two consecutive stations must lie within an interval $[L_{min}, L_{max}]$, typically [0.5 km, 2 km], can be easily included in our model, by considering that each district has only one associated centroid and the district size is enough large so that distance between neighbouring centroids lies within the typical variation range.
This problem of combinatorial nature can be assumed as a variant of the Multiple Knapsack Problem (MKP) (Pisinger and Toth, 1998). The 0–1 Knapsack Problem is the problem of choosing a subset of items such that the corresponding profit sum is maximized, without the weight sum to exceed a prefixed capacity. The Multiple Knapsack Problem is a generalisation of the standard 0–1 Knapsack Problem, where instead of considering only one knapsack, one tries to fill several knapsacks of different capacities. The MKP problem is strongly NP-complete and the need for algorithms that give a good heuristic solution is justified by the computational complexity of this problem. A model similar to the one previously proposed has been investigated in Laporte et al. (2002), whereby heuristics were designed in a computationally feasible way and consistent with the approach. Tests carried out on randomly generated data have shown that a simple greedy extension heuristic yields the best results if the inter-station spacing is sufficiently large. Otherwise, an insertion heuristic followed by a post-optimization phase is the tool required to obtain the most efficient design.
Taking these precedents into account, we propose heuristic GINA (Greedy Insertion of Nodes along an Alignment) for solving our optimization model for determining the most effective metro line in terms of urban entropy.

Heuristic GINA

1. **Read** locations of centroids $S = \{s_1, \ldots, s_j, \ldots s_n\}$ and their characteristics $\{p_{j1}, \ldots p_{jm}\}, \forall j = 1, \ldots, n$.

2. **Find** the pair of centroids (u, v) which generates the most effective link. (In the case of a tie, consider the shortest edge).

3. Let $Align(L) = \{u, v\}$.

4. **While** length of alignment $Align(L)$ is less than boundary $LMAX$ **do**:

   4.1 **Find** the node w^*, not yet included in $Align(L)$, where the highest effectiveness is reached. (In the case of a tie, consider the option which generates a minor increase in length).

   4.2 **Determine** the position $pos(w^*)$ where node w^* should be inserted along the node sequence $Align(L)$ to produce the smallest increase in its length.

   4.3 **Insert** node w^* at position $pos(w^*)$ along the sequence $Align(L)$. **Rename** the new resulting alignment, once added node w^*, as $Align(L)$.

1.2.2 Computational experience

A computational experience has been carried out on a district system (composed of 47 zones associated to the metropolitan area of Seville) in order to test the performance of heuristic GINA. The experiment consists of designing a rapid transit line connecting nine city zones by means an alignment passing through them, using two different criteria: maximizing the population covered (assuming that the inhabitants are covered by the transport service if they live in an area whose centroid is included in the transit corridor) and maximize urban entropy (the areas connected through the corridor share the same level for the attributes considered in the analysis of urban diversity). First objective was already applied in Bruno et al. (2002) to locate one alignment on data from the city of Milan. Second objective, unpublished at this time, is focused on increasing the diversity of the interconnected districts.
Based on data corresponding to the Statistical Yearbook 2015 of Seville (available at `http://www.sevilla.org/ayuntamiento/competencias-areas/area-de-h/anuario_2015`), values of seven characteristics (population, medical services, job positions, intermodality, study centres, commercial centres and tourist attractions) have been estimated and normalized for the 47 zones under consideration. Such attributes were already identified in Gendreau et al. (1995) as the main mobility generators. Hence, seven different intensity maps can be obtained when each attribute is separately considered. Between them, as instances, in Figure 1.5 normalized population densities are shown on the district map; on the other hand, Figure 1.6 shows the distribution of densities of tourist attractions.

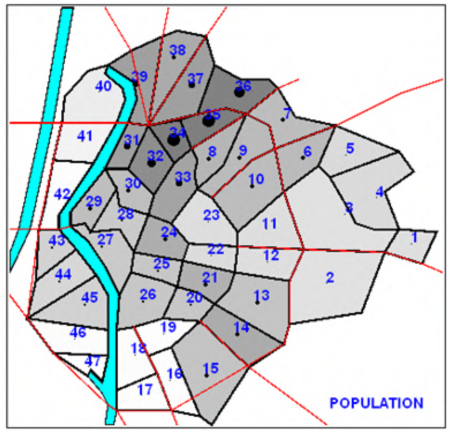

Figure 1.5: District system (47 zones) associated to Seville. Map of normalized densities of population (Source: Laporte et al., 2002)

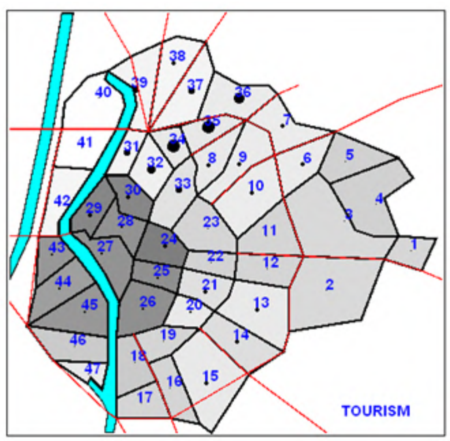

Figure 1.6: District system (47 zones) associated to Seville. Map of normalized densities of tourist attractions.

Applying the GINA algorithm on this Seville map, the alignment is generated in an iterative way. In the first step, the zone with the highest entropy level (La Calzada, the 24th zone) is connected to the area with the lowest level (Isla Mágica, the 41th zone). Next, different nodes are incorporated in sequence to finally generate the alignment of 9 nodes that produces a greater entropy level for the global system: 46, 18, 24, 30, 29, 41, 40, 34, 35 (the length of that line, shown in Figure 1.7, is 13889 meters). Initially, the entropy had the value 35.8011; at the end of the process, the value of the entropy amounts to 36.8560 (2.94% higher).

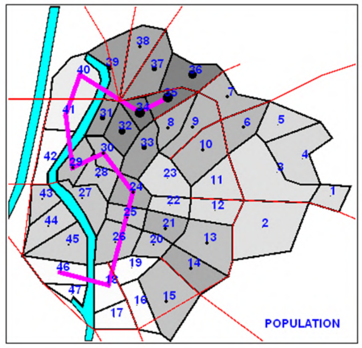

Figure 1.7: 9-station line obtained for the criterion of maximizing the global urban entropy.

On the other hand, by using the maximization of the covered population, as a criterion of node incorporation in the alignment, we obtain an optimal corridor composed of nodes 38, 39, 31, 32, 33, 34, 35, 37, 36 (the line length shown in Figure 1.8 is 8960 meters). This alignment has an entropy value of 36.0856, 2.13% lower than previously achieved.

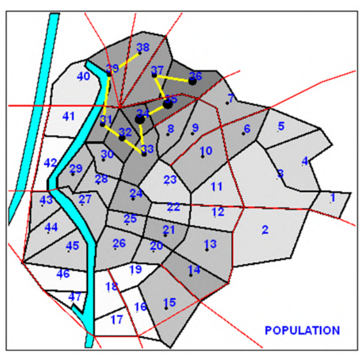

Figure 1.8: 9-station line obtained for the criterion of maximizing the population covered

First alignment, obtained by applying a criterion of maximizing the global urban entropy, presents two desired advantages: from an urban planning perspective, the improvement of the territorial cohesion within the district system of the city, and, from the point of view of sustainability, the reduction of the forced mobility of the inhabitants caused by the lack of opportunities of the districts where they live.

1.2.3 Conclusions

A methodology for the design of a rapid transit alignment, that increases the urban entropy of a district system, has been proposed in this article. A greedy algorithm has been formulated to solve the proposed mathematical programming model, identified as a version of the well-known problem of Multiple Knapsack Problem. To evaluate the performance of the proposed methodology, a computational experience has been carried out on an urban system composed of 47 districts with data from the metropolitan area of Seville (Spain). The evaluation of the generated transit line shows that the methodology meets the objective of efficiently designing an alignment that improves the functional diversity of the areas where it crosses.

A possible continuation of this research would be the determination with entropy maximization criteria of a new rapid transit line, conditioning its design to the previous existence of another line.

1.3 Assessing the effectiveness of park-and-ride facilities on multimodal networks in smart cities

1.3.1 Problem description and formulation

In this section, we provide first a detailed description of the problem by introducing the notation for all the elements involved. Second, we develop a mathematical programming model that integrates the different aspects that are considered in the decision of the user.

Problem description

We consider a user that aims to travel from an origin to a destination splitting his trip into two parts. The first part will be carried out by private vehicle mode (car or motorcycle) ending in a park-and-ride facility to be determined. The second part of the trip, up to the destination, will be carried out by means of public mode (including bus, tram, train, or subway) and/or walking mode. Once the park-and-ride facility has been chosen, the shortest time paths from origin to park-and-ride and from park-and-ride to the destination, can be optimally determined.

As previously mentioned, users differ from each other, firstly according to the fact of looking for (or not) a reservation at a parking facility. Secondly, the information that users could have may differ. In this sense, users may be able to estimate the number of free places that parking facilities will offer at a future time according to the knowledge/experience that the user may have parking in a given area. Users may also check from a device the free places at a parking facility at a present time, which could be useful for estimating the availability of parking spaces at the arrival time at the parking facility. If such information could not be provided by the device, maybe it could show if the parking facility is full or not at a present moment. In any case, we assume that users always know the size of each parking facility.

According to the type of user (and their available information), three criteria will have influence on the decision towards choosing a park-and-ride facility: (1) the total travel time from origin to destination, (2) the parking fee and public transit fare, and (3) the "attractiveness" of the parking facility. We understand attractiveness as a factor (for users without a reservation in a parking facility) that will depend on the risk of not having an available spot at the parking facility at the arrival time. Other characteristics of the parking facility such as safety, parking space cleanliness, etc., could be taken into account. However, in order to keep the model as simple as possible, we have not included these features.

Let us consider an initial directed graph $G = (N, A)$, where N is the node set and A is the arc set. We decompose $N = \{o\} \cup V \cup P \cup \{d\}$, where:

- o: node associated with the origin of the path, that is, the geographical position of the private vehicle mode (car or motorcycle) in the city.
- d: node associated with the destination site in the city center, whose access is forbidden to cars.
- $P = P^+ \cup P^-$ where P^+ and P^- are respectively the entry nodes and the exit nodes of the car parks.
- V is the set of intermediate nodes where the different transportation modes operate.

In order to construct the arc set A we take into account the following issues:

1. There are no arcs entering at node o and all arcs leaving this node correspond to the private vehicle mode.
2. There are no arcs leaving node d and all arcs entering in this node correspond to walking mode, individual mode (electric scooters, bicycle, etc.), or other public mode.
3. All arcs entering at a parking site $k \in P^+$ correspond to the private vehicle mode. From each node $k \in P^+$ only a single arc connects with the exit node $k' \in P^-$ of the parking site. Therefore, note that if $k \in P^+$ and $k' \in P^-$ are connected by an arc (k, k') then both k and k' belong to the same park site. Additionally, all arcs leaving node $k' \in P^-$ correspond to the walking mode or other public mode.
4. All arcs entering and departing from a node $i \in V$ correspond to the same transportation mode. If there were a geographical node in which different transportation modes could be feasible for arriving at/departing from that node, it would be virtually replicated, once for each feasible transport mode.

Note that according to the graph construction, each feasible path connecting o and d must traverse a parking facility where the private vehicle mode changes to the walking mode or other public mode. Additionally, several transportation modes could be included after leaving the parking facility just by means of including the corresponding transfer nodes and arcs between the different modes. In this section, we assume an operational time horizon that is conveniently discretized in a set of time slots T. Let $l_{ij} \in \mathbb{R}^+$ be the length associated with arc $(i, j) \in A$ and v_{ij}^t the average transit speed along the arc (i, j) entering at node i at time $t \in T$. For the case of $(k, k') \in A : k \in P^+, k' \in P^-$, $l_{kk'}$ represents the transit time within parking facility k.
We define next some parameters related to parking facilities. Let f_k^t be the fee at parking facility k at time t and w_{kj}^t be the waiting time at parking facility exit node $k \in P^-$ during the transfer to node j at time t. In case of requiring a later means of transportation, we assume a fare g_j has to be paid by the user. As

previously mentioned, users differ from each other according to the information they have available (or not available). In this sense, let $q_k^{1,t}$ be the estimation of free places that parking facility k will offer at a future time t, this parameter is related to the knowledge/experience the user may have parking at k. The user may also check a device that could provide the free places q_k^2 at parking facility k at a present time that could be useful for estimating the availability of parking spots at the arrival time at the parking facility. If this information was not provided by the device, maybe it could be shown if the parking facility is full or not. In this sense, let q_k^3 be the size of parking facility k if the device indicates free spots at present; 0 otherwise. In any case, we assume that users always know the size of each parking facility $k \in P^+$, namely q_k^4 as well as the size Q of the biggest parking facility. Given $\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4 \in [0, 1]$ weighting parameters holding $\sum_{i=1}^{4} \gamma_i = 1$, we can define the attractiveness of a parking facility at time k as:

$$\gamma_0 \frac{\gamma_1 q_k^{1,t} + \gamma_2 q_k^2 + \gamma_3 q_k^3 + \gamma_4 q_k^4}{Q}$$

which its value is within $[0, 1]$. Note that there are some relations among parameters $\gamma_i, i \in \{0, 1, 2, 3, 4\}$ according to the user profile. First, users who look for a reservation (weighted as $\gamma_0 = 0$) are not affected by information of parking availabilities ($\gamma_1 = \gamma_2 = \gamma_3 = 0$). Second, available information of free places at parking facility k at a present time (weighted with a $\gamma_2 \neq 0$) includes the information of availability of the parking at a present time (that is, $\gamma_2 \neq 0$ implies $\gamma_3 = 0$ and conversely, $\gamma_3 \neq 0$, implies $\gamma_2 = 0$).

In addition, each user might impose a minimum level of attractiveness $\Lambda \in [0, 1]$ accepted for the chosen parking. The reader may observe that parameters $\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4$ and Λ require to be properly weighted/calibrated according to the particular user's features and preferences. This might require an empiric procedure that is out of the scope of this research.

Mathematical programming model

For each $t \in T$ and each arc (i, j) let $x_{ij}^t \in \{0, 1\}$ be the binary variable that takes value 1, if the arc (i, j) is traversed at time t, and 0 otherwise. We present next an integer programming formulation to determine a "minimum generalized cost" path from node o to node d in our setting. We understand that the path cost is determined by the minimization of three weighted criteria; namely, (1) the travel time, (2) the parking fee, and (3) the non-attractiveness of the parking facility. For this aim, let α, β, γ_0 be the weighting parameters for each of these criteria respectively.

$$
\begin{aligned}
\min \quad & \alpha \sum_{t\in T}\left(\sum_{(i,j)\in A}\left\lceil \frac{l_{ij}}{v_{ij}^t}\right\rceil\ x_{ij}^t + \sum_{(k,j)\in A: k\in P^-} w_{kj}^t x_{kj}^t\right) + \\
& +\beta \sum_{t\in T}\left(\sum_{(k,k')\in A: k\in P^+, k'\in P^-} f_k^t x_{kk'}^t + \sum_{(k',j)\in A: k'\in P^-} g_j x_{k'j}^t\right) + \\
& +\gamma_0\left(\sum_{t\in T}\sum_{(i,k)\in A: k\in P^+} x_{ik}^t\left(1-\frac{\gamma_1 q_k^{1,t}+\gamma_2 q_k^2+\gamma_3 q_k^3+\gamma_4 q_k^4}{Q}\right)\right) && (1.4)\\
\text{s.t.} \quad & \sum_{t\in T}\sum_{(o,j)\in A} x_{oj}^t = 1, && (1.5)\\
& \sum_{t\in T}\sum_{(i,d)\in A} x_{id}^t = 1, && (1.6)\\
& x_{ij}^t \le \sum_{(j,j')\in A} x_{jj'}^{t+\left\lceil \frac{l_{ij}}{v_{ij}^t}\right\rceil}, \quad (i,j)\in A: j\in V\cup P, t\in T, && (1.7)\\
& x_{ij}^t \in \{0,1\}, \quad (i,j)\in A, t\in T. && (1.8)
\end{aligned}
$$

The objective function (1.4) minimizes a weighted sum of three criteria/components, namely travel time, parking cost, and parking non-attractiveness. Constraint (1.5) guarantees that the path will begin at the origin (by using the car as a means of transportation). Constraint (1.6) guarantees that the path will end at the destination (by using a means of alternative transportation to the car). Constraints (1.7) ensure flow conservation for node subsets V and P. When arc (i,j) is traversed at time t, another arc departing from j needs to be traversed at time $t+\left\lceil \frac{l_{ij}}{v_{ij}^t}\right\rceil$. Note that, since time is discretized, we need to round up the time required to traverse edge $(i,j)\in A$. In addition, the generalized cost minimization ensures that no more than one arc departs from a node.

We recall that according to the graph construction there is no need to impose on the model that no arcs enter in node o, no arcs leave from node d and one parking facility has to be visited.

Next, we propose two additional constraints to bound the travel time and the parking facility attractiveness:

$$
\sum_{t\in T}\sum_{(i,d)\in A}\left(t+\left\lceil \frac{l_{ij}}{v_{ij}^t}\right\rceil\right)x_{id}^t \le \max\{t\in T\}, \tag{1.9}
$$

$$
\sum_{t\in T}\sum_{(i,k)\in A: k\in P^+} x_{ik}^t\frac{\gamma_1 q_k^{1,t}+\gamma_2 q_k^2+\gamma_3 q_k^3+\gamma_4 q_k^4}{Q} \ge \Lambda. \tag{1.10}
$$

Constraint (1.9) implies the arrival time to destination is upper bounded whereas constraint (1.10) ensures a minimal attractiveness for the chosen parking facility. Problem (1.4)-(1.10) can be efficiently solved by using a modified Dijkstra's algorithm. More precisely, note that problem (1.4)-(1.10) is composed of a shortest path problem, namely (1.4)-(1.8), plus two additional constraints (1.9)-(1.10). Therefore, the guidelines of a modified Dijkstra's algorithm for solving (1.4)-(1.10) consists first in start solving (1.4)-(1.8) with a standard Dijkstra's shortest-path algorithm. We recall that on each iteration, a new edge has to be added to the last node of the shortest (costless) path constructed so far. Therefore, such shortest path cost until the node is known. Additionally we could also keep track of the time and attractiveness associated to such path, that can be discarded if its travel time is above $\max\{t \in T\}$ of if its a attractiveness is below Λ. In this way, if the arc costs induce no negative cycles on G (as it is assumed in the problem description), the problem (1.4)-(1.10) can be efficiently solved in polynomial time.

1.3.2 Computational experiment

Next, we report on the results of a computational experiment that we have carried out in order to empirically show the model sensitivity by means of a parametric analysis of solutions. That is, we show different solution values of the problem (1)-(7) for an instance assuming different choices of the parking selected, two types of users (two different sets of $\alpha, \beta, \gamma_i,\ i \in \{0, 1, 2, 3, 4\}$ values) and other parameters fixed according to an urban setting of Seville (Spain).

Seville is the capital of the Andalusia region (Spain), provided with a large and well-preserved historical center which is approximately 2,200 years old. In fact, the Historic Center of Seville is one of the largest in Europe, along with those of Venice and Genoa. It has an approximately circular configuration, with an area of 3.94 km^2. During 2019, the city of Seville received the visit of 3.12 million tourists attracted by an old town that contains three world heritage sites and also many convents, churches, palaces, museums, and gardens. Motorized traffic in this sector of Seville is limited and/or forbidden in many streets that are predominantly narrow and one-way.

For this reason, there is a network of parking lots around the city center that provides, via signaling panels, information to the drivers about their current occupancy levels. We assume that this basic information to the users is complemented by the input data described along Subsection 1.3.1.

In Figure 1.9 we have supposed a driver with a starting point (labeled with O), and a destination (labeled with DES) located at the city center that is not accessible by private car. Six existing car parks have been selected around the city center with entrances labeled by p_k^+ and exits p_k^- for $k \in \{1, ..., 6\}$. Note that for the sake of visibility, exit parking nodes have been slightly separated from their real locations.

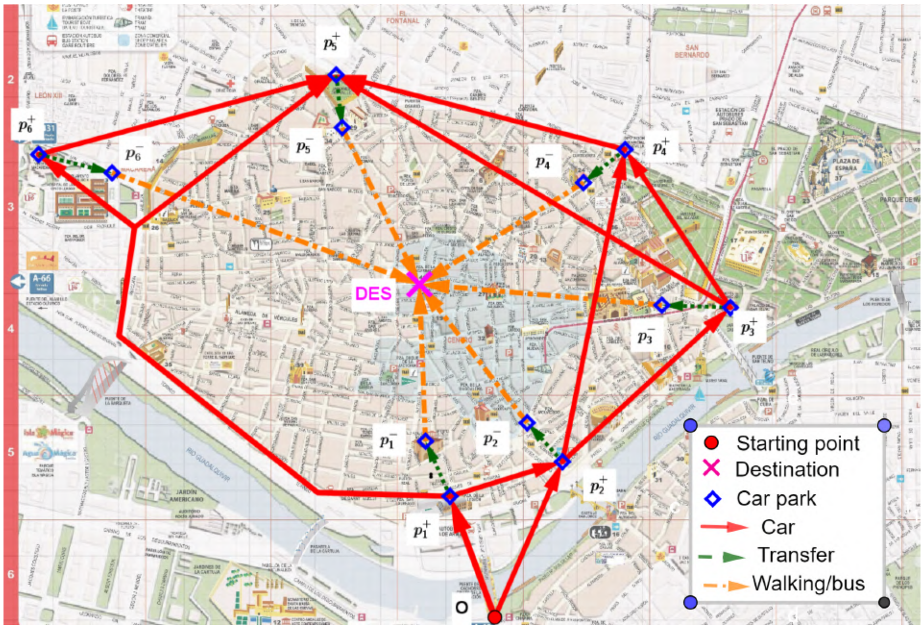

Figure 1.9: Graph associated to the selection of best strategy

We deploy three different arc types according to the transportation mode in use: car (continuous red), walking (dotted green) and walking or bus (both in dashed orange).

Park (k)	Time to Park	Time in Park	Waiting time (w^t_{kj})	Time to DES	Total time
1	8	2	0	12	22
2	7	1.5	0	13	21.5
3	11	1	2	6	**20**
4	14	2	5	13	34
5	15	1	2	9	27
6	10	1	3.5	15	29.5

Table 1.1: Travel times (min) involved in the shortest path from origin to destination.

Table 1.1 shows traveling times in minutes from the origin point (La Pañoleta, 41910-Seville, Spain) to the destination point (Plaza del Salvador, 41004-Seville, Spain) passing through each one of the 6 available parking lots. Column "Time to Park" shows the driving time from the origin to each parking entrance through a time-dependent shortest-path. Column "Time in Park" shows the time spent at the parking facility. Column "Waiting Time" shows the waiting time at the exit of the parking facility in case a bus transfer is required. Finally, column "Time to DES" shows the total travel time from the exit parking node to DES through a time-dependent shortest-path that might include bus and/or walking mode. Note that the best parking facility (marked in bold) in terms of the total time is parking lot 3.

Park (k)	Parking fee (f_k^t)	Fare rate (g_j)	Total price
1	1.17	0	1.17
2	1.01	0	1.01
3	0.3	0.74	1.04
4	0.5	0.69	1.19
5	0.35	0.74	1.09
6	0.16	0.69	**0.85**

Table 1.2: Park fees and fare rates involved in the trip.

Table 1.2 includes parking fees and fare rates if a later means of transportation is required. Note that the best option (cheapest) in this case (parking 6) changes with respect to the best option (parking lot 3) in the previous case (shortest).

Park (k)	Estimation of free places ($q_k^{1,t}$)	Free places (q_k^2)	Full or not (q_k^3)	Parking size (q_k^4)	Q
1	15	0	0	25	100
2	0	8	30	30	100
3	20	1	50	50	100
4	6	5	25	25	100
5	20	50	70	70	100
6	66	35	100	100	100

Table 1.3: Parking parameters involved in the non-attractiveness criterion.

Table 1.3 shows the different parking parameters involved in the non-attractiveness criterion of the objective function. In order to obtain a non-attractiveness value, these parameters require to be combined with weights $\gamma_i,\ i \in \{0, 1, 2, 3, 4\}$ according to the individual specific preferences and characteristics of the user into consideration.

Park (k)	α	Total time	β	Total price	γ_0	γ_1	γ_2	γ_3	γ_4	Non-attractiveness	Total cost
1	0.2	4.4	0.8	0.93	0	0	0	0	1	0	5.33
2	0.2	4.1	0.8	0.81	0	0	0	0	1	0	4.91
3	0.2	4	0.8	0.83	0	0	0	0	1	0	**4.83**
4	0.2	6.8	0.8	0.95	0	0	0	0	1	0	7.75
5	0.2	5.4	0.8	0.87	0	0	0	0	1	0	6.27
6	0.2	5.9	0.8	0.68	0	0	0	0	1	0	6.58

Table 1.4: Weighting parameters and generalized cost for a user looking for a reservation.

Tables 1.4 and 1.5 provide the objective function values (generalized total cost) when the three criteria are combined according to the objective function (1.4). In both cases weighting values $\alpha, \beta, \gamma_i,\ i \in \{0, 1, 2, 3, 4\}$ have been chosen for modeling to different user's profiles. In particular, Table 1.4 shows the weights of a user that chose a parking lot with a reservation procedure. For that reason, the objective value is not affected by the information of parking capacities.

Park (k)	α	Total time	β	Total price	γ_0	γ_1	γ_2	γ_3	γ_4	Non-attractiveness	Total cost	%
1	0.04	0.88	0.17	0.2	0.79	0	0.67	0	0.33	0.72	1.80	17.3
2	0.04	0.82	0.17	0.17	0.79	0	0.67	0	0.33	0.67	1.66	25.6
3	0.04	0.8	0.17	0.18	0.79	0	0.67	0	0.33	0.65	1.63	32.2
4	0.04	1.36	0.17	0.2	0.79	0	0.67	0	0.33	0.7	2.26	3.8
5	0.04	1.08	0.17	0.19	0.79	0	0.67	0	0.33	0.34	**1.61**	8.6
6	0.04	1.18	0.17	0.14	0.79	0	0.67	0	0.33	0.34	1.67	12.5

Table 1.5: Weighting parameters and generalized cost for a user with information about free parking lots.

Table 1.5 shows the weights of a user with knowledge of the number of free spots at the initial time as well as the size of the parking. Besides, this example assumes that this user is not able to estimate the number of free spots at his/her arrival time at the parking.

The last column of Table 1.5 shows the % in which each of the car parks turned out to be optimal, for an experiment of 1000 tests in which the vector of non-attractiveness parameters was random and uniformly distributed with values between 0.1 and 0.9 (the remainder parameters in Table 1.5 were kept constant). The results of this experiment show that optimality is achievable in any car park offered, although there preferably is a greater probability that the optimum will be reached in options 3, 2 or 1, which are geographically better aligned with the objective. The farthest car parks 6, 5 and 4 must compensate for their unfavorable locations with low values of non-attractiveness to be able to achieve an optimum generalized total cost.

If we take into account traffic congestion effects in the Seville metropolitan area at rush hour (see Figure 1.10) and its impact on the access time to parking lots 1 to 6, the arc cost must be recalculated in a part of the graph, following the approach of Pallottino and Scutella (2003).

In the example being analyzed, the access times would increase by 3 minutes to reach the car parks 1 and 2, as well as 1 minute to access the car parks 5 and 6. These specific modifications in the travel times of certain arcs do not change the final decisions in the scenarios analyzed, although they could do so if the levels of congestion were more pronounced.

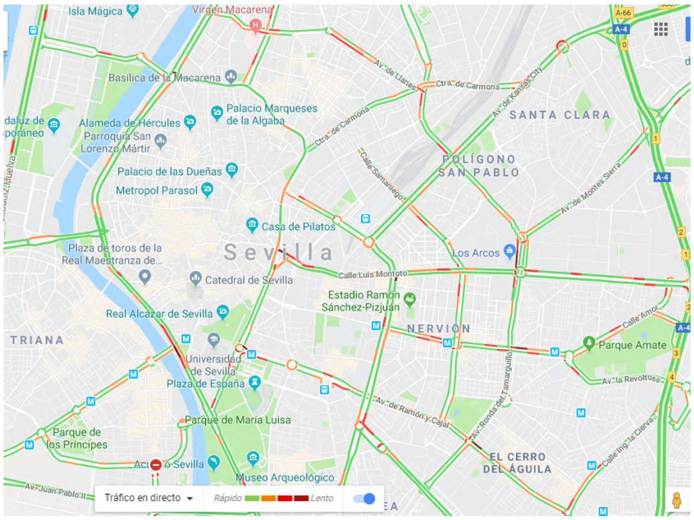

Figure 1.10: Map of traffic intensities in the city of Seville taken from Google Maps 04/27/2018 at 10 am

From this experiment, we conclude summarizing the following remarks. Firstly, the application to Seville shows an example of how the proposed methodology can be applied to a real context. Secondly, as previously mentioned, the model can be solved by using a modified Dijkstra shortest-path algorithm so the computational effort for solving this type of problem is not a big issue. For this reason, we have focused our experiment in showing the model sensitivity to the input parameters. For example, we show that two different user's profiles give rise to different shortest paths and generalized cost values. This demonstrates the relevance of the model that suits the diversity of several user profiles, as well as the information available for the different scenarios.

1.3.3 Conclusions

Intelligent car parking information systems are nowadays evolving to provide, through smart-phone applications, the best choice of park facility for a given destination, taking distances to candidate destinations and their occupancy levels into consideration.
This research has been aimed at improving the information that these types of applications could provide for those users who necessarily need to use a park and ride facility before reaching their destination located in a restricted traffic zone in the city. For that purpose, different aspects that have influence in the drivers' preferences (such as rates, walking time to destination, the estimated number of vacant places, the possibility of transferring to other means of transport, etc.) have been integrated into an optimization model. The proposed approach has consisted of an integer programming model where the generalized cost to

be minimized combines three criteria (total travel time, parking fee, and non-attractiveness for the parking facility under consideration), while a constraint set guarantees the feasibility of solutions into the ad hoc build graph. The optimization model can be solved by using an adapted shortest path algorithm. Therefore, the computation time required to obtain a solution is reasonably low. The main contribution of the chapter is twofold. First, the objective in the model integrates several cost attributes: determining shortest routes through a multimodal network with time-dependent transit times, minimizing parking costs and an attractiveness criterion related to the risk of not having an available space at the chosen parking facility. Second, the coefficient values allow us to collect, in the same optimization model, the diversity of several user profiles, as well as the information available for the different scenarios.

Results of a computational experiment located in the urban setting of Seville (Spain) have been reported in order to empirically show the model sensitivity to the input parameters.

Future research can expand the model to new scenarios where the three variables of the model can interact between them. For example, parking fees may vary based on the current level of parking occupancy, or alternatively, a discount on the price of the ticket for the subsequent bus trip (or at the rate of the shared bicycle rented at the station) could be offered to all users who had previously chosen to park their vehicle in that facility. Other new scenarios for future research could be motivated by the creation of low emission zones that represent areas prohibited for the circulation of certain types of vehicles declared polluting.

1.4 Bilevel optimization for the deployment of refueling stations for electric vehicles on road networks

1.4.1 Model development

Input data

In order to determine optimal location of charging stations in corridors a connected graph $G = (V, A)$ is assumed composed of a node set V representing gas stations or cities and an arc set A representing road sections between points, such that the existence of a shortest path in terms of distance (or travel time) between each pair of points of V is always guaranteed inside G.
The following notation is used in our formulation:
Origin-destination demand matrix $(d_{ij}), i, j \in V$.
(Γ_{ij}): shortest path matrix between pair of nodes $i, j \in V$, where
$\Gamma_{ij} = \{i, v_1, v_2, \ldots, v_k, j\}$ with $v_1, v_2, \ldots, v_k$ intermediate nodes.
$T = (t_{ij})$: distance matrix between pair of nodes through the shortest path.
Term q_k denotes the capacity of each node $k \in V$ to install charging stations, and p_k the unit price depending on site k.
Moreover, the following variables are required in the model:
x_k: Integer variable that indicates the number of charging facilities installed at point $k \in V$.
y_l: Binary variable that takes value 1 if we select point $l \in V$ to open at least one charging facility.
Note that the total installation cost at point $k \in V$ depends on the number of facilities x_k $(0 \leq x_k \leq q_k)$ that we open. It will be $x_k \cdot p_k$. Thus, the total cost in the whole network will be

$$\sum_{k \in V} x_k \cdot p_k$$

Preprocess

For each point k, the shortest paths containing point k as an intermediate node are identified. This collection is labeled as γ_k.

$$\gamma_k = \{\Gamma_{ij}; i, j \in V | k \in \Gamma_{ij}\}$$

Once set $\gamma \equiv (\gamma_k)$ is determined, the following weights are obtained

$$\omega_k = \sum_{\Gamma_{ij} \in \gamma_k} d_{ij}$$

that will serve to quantify the attractiveness of locating a service at point k.

Conditional Covering and Knapsack Models

Suppose we want each favorable facility decision ($x_k \geq 1$) to be complemented with the installation of another facility at a location $l \in V$ such that the travel time between points k and l (i.e., t_{kl}) does not exceed a quantity R which has previously been set by the experts.

Definition 1 *Let $G = (V, A)$ be a graph. It is called R-dense if $\forall p \in V, \exists q \in V(q \neq p)$ such that $dist_G(p, g) \leq R$.*

The Conditional Covering Problem (CCP) consists of minimizing the total number of facilities that must be established in order to cover all nodes and no facility can cover the site on which it is located, and must therefore be covered by another established facility. A site is said to be covered if its distance to the nearest facility is less than or equal to the covering radius R.

We can observe that if G is R-dense it guarantees us the possible existence of paths with conditional covering.
The CCP was first introduced Moon and Chaudhry (1984), where an integer programming model for this problem was proposed and linear programming relaxation methods were applied to them. In Chaudhry et al. (1987), the authors consider several greedy heuristics for solving CCP and provide computational results for the same. In Moon and Papayanopoulos (1995), authors discuss a slight variation of CCP on tree graphs. In this problem, each demand point has a specific radius such that a facility has to be located within that radii. In Lunday et al. (2005) an $O(n^2)$ algorithm for the CCP on paths with a covering radius is uniform for all the vertices and arbitrary positive costs are assigned to vertices has been presented. They also improve the result with an $O(n)$ time algorithm, when the covering radius is uniform and cost is unity for all vertices of the path. In Horne and Cole (2005), the $O(n^2)$ algorithm, obtained in Lunday et al. (2005), is extended to the case when vertices are assigned to an arbitrary covering radius. In Benkoczi et al. (2012), new upper bounds have been proposed for the conditional covering problem on paths, cycles, extended stars, and trees.
To incorporate the concept of conditional coverage to our model, we can define a parameter matrix $B = (b_{kl})$; $b_{kl} \in \{0, 1\}$, such that

$$b_{kl} = \begin{cases} 1 & \text{if} \quad t_{kl} \leq R \quad \text{and } k \neq l \\ 0 & \text{if} \quad t_{kl} > R \quad \text{or } k = l \end{cases}$$

From this matrix B we can extract the vector $B_k = \{l \in V(l \neq k) | b_{kl} = 1\}$ for the conditional covering between pair of selected nodes.
To ensure a reinforced coverage of services, the following conditional covering

model is proposed

$$
\begin{aligned}
\min \quad & \sum_{l \in V} y_l && && (1.11)\\
\text{s.t.} \quad & \sum_{l \in V, l \neq k} b_{kl} y_l \geq 1 && \forall k \in V, && (1.12)\\
& y_O = 1, && && (1.13)\\
& y_D = 1, && && (1.14)\\
& y_l \in \{0,1\} && \forall l \in V. && (1.15)
\end{aligned}
$$

The inclusion of restrictions guarantees the coverage of services at the points of origin (O) and destination (D) and along any itinerary established inside the graph G.

Thus, the CCP can solve the problem of locating power stations from the administration point of view, since it results in a network in which all nodes are covered. They are also covered in a reinforced way to avoid cases of collapse of certain points that have high demand or cases of breakdowns or technical problems.

To solve the problem from the point of view of the energy companies, the classic knapsack problem is used, which maximizes profit taking into account a maximum budget P.

$$
\begin{aligned}
\max \quad & \sum_{k \in V} \omega_k \cdot x_k && && (1.16)\\
\text{s.t.} \quad & \sum_{k \in V} x_k p_k \leq P, && && (1.17)\\
& x_k \in \mathcal{N}, 0 \leq x_k \leq q_k. && \forall k \in V && (1.18)
\end{aligned}
$$

The optimization model formulated corresponds to a bounded knapsack problem (see, Kellerer et al., 2004).

Bilevel optimization model and solution algorithm

According to Piedra-de-la Cuadra et al. (2022), the global problem can be modeled as a network optimal decision problem involving two nested objectives.

In the objective function of the leading problem, the aim is to minimize the number of refueling points necessary to guarantee a reinforced service coverage for all users. This objective is linked to the political decision maker interested in the viability of solutions.

The objective function of the follower problem is to maximize the volume of demand subject to budget constraints; a criterion typically used by the private sector initiative.

The full formulation is as follows:

$$
\begin{aligned}
\min \quad & \sum_{l \in V} y_l && && (1.19)\\
\text{s.t.} \quad & \sum_{l \in V, l \neq k} b_{kl} y_l \geq 1, && \forall k \in V && (1.20)\\
& \max \sum_{k \in V} \omega_k \cdot x_k && && (1.21)\\
& \text{s.t.} \quad \sum_{k \in V} x_k p_k \leq P, && && (1.22)\\
& \qquad y_k \leq x_k, && \forall k \in V && (1.23)\\
& \qquad \frac{x_k}{q_k} \leq y_k; \quad \frac{x_k}{q_k} \leq \sum_{l \in B_k} y_l, && \forall k \in V && (1.24)\\
& x_k \in \mathcal{N}, 0 \leq x_k \leq q_k, && \forall k \in V && (1.25)\\
& y_l \in \{0, 1\} && \forall l \in V && (1.26)
\end{aligned}
$$

Where (1.19) is the leading objective of the bilevel model. Constraints (1.20) guarantee the reinforced coverage of all refueling points. (1.21) is the follower objective of the bilevel model, adapting the number of charging points to potential demand. Constraints (1.22) are budget constraints. Constraints (1.23) establish that the places selected by the leader objective have recharge points. Constraints (1.24) ensure that all places where a recharging point is planned to be installed ($0 < \frac{x_k}{q_k} \leq 1$) must be covered in a reinforced way by another different place. Constraints (1.25)-(1.26) indicate the nature of the variables used in the model.
The Knapsack Problem is of combinatorial nature and, in computational complexity theory, is classified as NP-hard problem (Garey, 1979). Bilevel optimization problems are known to be intrinsically hard to solve. Even the models with both linear leader and follower's problems, which are generally the simplest to solve, are shown to be strongly NP-hard (Labbé and Marcotte, 2021). Typically, solution methods used for these studies are metaheuristics, such as a genetic algorithm or large-scale neighbourhood search, or a single-level reformulation of the bilevel problems is proposed to be able to solve the models using commercial solvers.
Taking these precedents into account, we propose this heuristic for solving the optimization problem in order to determine the locations and the number of charging points inside the network.

Heuristic

1. **Let** $S1$ be the solution of the Conditional Covering Problem in V (model 1.11, 1.12,1.15).
2. **Let** $S2$ be the solution of the Bounded Knapsack Problem in V forcing that the locations that are solutions of $S1$ are also solutions in $S2$ ($S1$ is a subset of $S2$, obviously).
3. **Let** $G = \{\}$
4. **While** $K = S2 \setminus S1$ not empty **do**:

   4.1 **Let** $k1 = \text{argmax}\{w_i/p_i | i \in K\}$.

   4.2 **Let** S1 the the solution of the Conditional Covering Problem in V, forcing that $k1$ is part of the solution ($y_{k1} = 1$).

   4.3 **Let** $S2$ the solution of the Bounded Knapsack Problem in V forcing that the locations that are solutions of $S1$ are also solutions in $S2$.

5. **End**

1.4.2 Our case study

In order to illustrate the developed methodology, we are inspired by a real case with existing gas stations in the southern region of Spain (Buscador de Estaciones de servicio Repsol (n.d.)), let suppose an area with 27 traditional gas stations and the distances between connections are equal to 1 shown in Figure 1.11. Let suppose that the cost of each charging point are equal to 1 ($p_k = 1$), the total budget is equal to 20 ($P = 20$), the capacity of each node is equal to 5 ($q_k = 5$), the attractiveness of each node is equal to the number in red, the solution graph has to be 1-dense ($R = 1$) and one charging station is covered by another if they have a distance of 1.

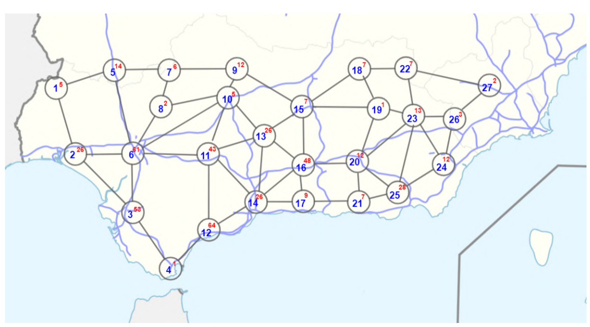

Figure 1.11: Example of a corridor with 27 traditional gas stations

Step 1. Solve the CCP with $V = \{1, 2, \ldots, 27\}$. The solution is

$$S1 = \{2, 3, 7, 9, 14, 17, 22, 23\}$$

Step 2. Solve the KP forcing that S1 are also solutions. The solution is

$$S2 = \{2, 3, 6, 7, 9, 12, 14, 17, 22, 23\}$$

with

$$x_2 = x_7 = x_9 = x_{14} = x_{17} = x_{22} = x_{23} = 1, \quad x_3 = 3, \quad x_6 = x_{12} = 5$$

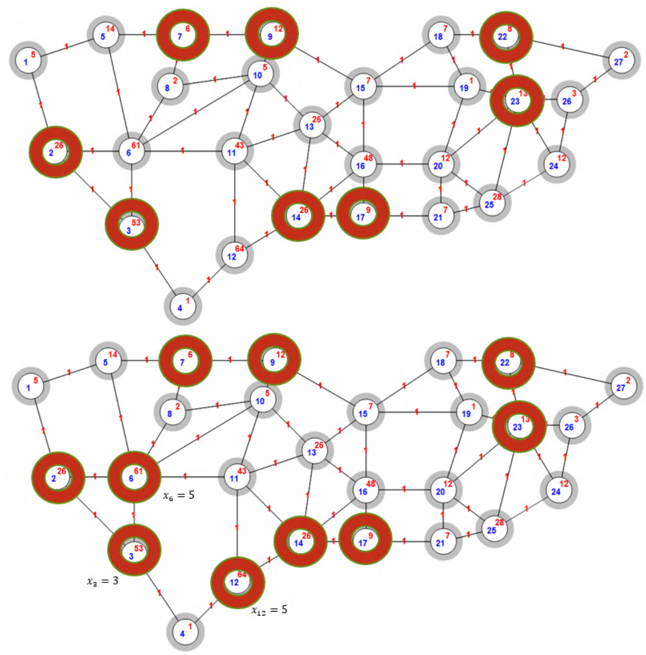

Figure 1.12: Step 1 and Step 2.

Step 3. $K = \{6, 12\}$ is not empty, k_1 is 12. Solve the CCP including k_1 as solution.

$$S1 = \{5, 6, 12, 13, 14, 15, 20, 22, 23\}$$

Step 4. Solve the KP forcing that S1 are also solutions.

$$S2 = \{3, 5, 6, 12, 13, 14, 15, 20, 22, 23\}$$

Step 5. $K = \{3\}$ is not empty. $k_1 = k_1 \cup \{3\}$. Solve the CCP including k_1 as solution.

$$S1 = \{2, 3, 7, 9, 12, 14, 20, 22, 23\}$$

Step 6. $S2 = \{2, 3, 6, 7, 9, 12, 14, 20, 22, 23\}$

Step 7. $K = \{6\}$ is not empty. $k_1 = k_1 \cup \{6\}$. Solve the CCP including k_1 as solution.

$$S1 = \{3, 5, 6, 12, 14, 15, 18, 23, 25, 26\}$$

Step 8. $S2 = \{3, 5, 6, 12, 14, 15, 18, 23, 25, 26\}$, with

$$x_5 = x_{14} = x_{15} = x_{18} = x_{23} = x_{25} = x_{26} = 1, \quad x_3 = 3, \quad x_6 = x_{12} = 5$$

Step 9. K is empty. Stop. The Solution is $S2$.

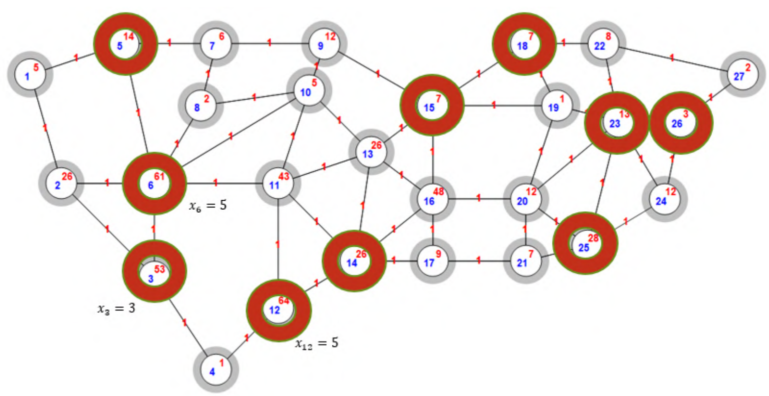

Figure 1.13: Step 9

The solution obtained through the algorithm is to open 10 electric gas stations that give us an attractiveness of 882. On the other hand, if we solve this same example with the exact model, the solution obtained is $S = \{2, 3, 7, 9, 14, 17, 22, 23\}$ with $x_7 = x_9 = x_{17} = x_{22} = x_{23} = 1, \quad x_2 = x_3 = x_{14} = 5$. In other words, the solution is to open 8 electric gas stations that give us an attractiveness of 573. The bilevel problem has been separated into two classic linear problems but both are still NP-hard. Consequently, we will use different heuristics from the literature to solve them (see, Lotfi and Moon, 1997; Deineko and Woeginger, 2011; Pisinger, 2000).

1.4.3 Others perspectives

Single level reformulation with primal-dual problems

In order to solve problem (1.19)-(1.26), we reformulate it as a single-level optimization problem by exploiting the primal-dual optimality conditions of linear programming. First we define the dual problem of the follower problem knowing that the upper level binary variable y_k is an input parameter whose value is fixed.

$$\min \quad P \cdot \lambda + \sum_{k \in V} q_k \beta_k + \sum_{k \in V} \left(\sum_{l \in B_k} y_l \right) \cdot \alpha_k + \sum_{k \in V} y_k \cdot \delta_k + \sum_{k \in V} y_k \cdot \gamma_k \tag{1.27}$$

$$\text{s.t.} \quad p_k \lambda + \beta_k + \frac{\alpha_k}{q_k} + \gamma_k + \frac{\delta_k}{q_k} \geq \omega_k, \qquad \forall k \in V \tag{1.28}$$

$$\lambda, \beta_k, \alpha_k, \delta_k \geq 0, \qquad \forall k \in V \tag{1.29}$$

$$\gamma_k \leq 0. \qquad \forall k \in V \tag{1.30}$$

According to the weak and strong duality theorems, if x_k is a feasible solution of primal problem, $\lambda, \beta_k, \alpha_k, \delta_k, \gamma_k$ is a feasible solution of dual problem and the optimal values of the primal and dual problem coincide, then x_k (resp. $\lambda, \beta_k, \alpha_k, \delta_k, \gamma_k$) is an optimal solution of primal (resp. dual). Problem (1.19)-(1.26) can be reformulated as the following single level problem.

$$\min \quad \sum_{l \in V} y_l \tag{1.31}$$

$$\text{s.t.} \quad \sum_{l \in V, l \neq k} b_{kl} y_l \geq 1, \qquad \forall k \in V \tag{1.32}$$

$$\sum_{k \in V} x_k p_k \leq P, \tag{1.33}$$

$$y_k \leq x_k, \qquad \forall k \in V \tag{1.34}$$

$$\frac{x_k}{q_k} \leq y_k; \quad \frac{x_k}{q_k} \leq \sum_{l \in B_k} y_l, \qquad \forall k \in V \tag{1.35}$$

$$p_k \lambda + \beta_k + \frac{\alpha_k}{q_k} + \gamma_k + \frac{\delta_k}{q_k} \geq \omega_k, \qquad \forall k \in V \tag{1.36}$$

$$P\lambda + \sum_{k \in V} q_k \beta_k + \sum_{k \in V} \left(\sum_{l \in B_k} y_l \right) \alpha_k + \sum_{k \in V} y_k \delta_k + + \sum_{k \in V} y_k \gamma_k = \sum_{k \in V} \omega_k x_k, \tag{1.37}$$

$$x_k \in \mathcal{N}, 0 \leq x_k \leq q_k, \qquad \forall k \in V \tag{1.38}$$

$$y_l \in \{0, 1\}, \qquad \forall l \in V \tag{1.39}$$

$$\lambda, \beta_k, \alpha_k \geq 0, \qquad \forall k \in V \tag{1.40}$$

$$\gamma_k \leq 0. \qquad \forall k \in V \tag{1.41}$$

Constraint (1.37) is the strong duality condition stating that the primal and dual objectives of the lower level problem must be equal, the blocks of constraints (1.31)-(1.32) represent the upper level problem constraints, (1.33)-(1.35) is the lower level primal problem constraints and (1.36) represent the lower level dual problem constraints.

Single level reformulation with exchange in the hierarchy criteria

In view of the results of our case study in the exact model, although the solution minimizes the number of electric stations that we open, it does not give good attractiveness results. Therefore, we carry out an exchange at the hierarchy levels:

$$
\begin{aligned}
\max \quad & \sum_{k\in V} \omega_k \cdot x_k && && (1.42)\\
\text{s.t.} \quad & \sum_{k\in V} x_k p_k \le P, && && (1.43)\\
& \frac{x_k}{q_k} \le y_k; \quad \frac{x_k}{q_k} \le \sum_{l\in B_k} y_l, && \forall k \in V && (1.44)\\
& \min \sum_{l\in V} y_l && && (1.45)\\
& \text{s.t.} \quad \sum_{l\in V, l\neq k} b_{kl} y_l \ge 1, && \forall k \in V && (1.46)\\
& \qquad y_k \le x_k, && \forall k \in V && (1.47)\\
& x_k \in \mathcal{N}, 0 \le x_k \le q_k, && \forall k \in V && (1.48)\\
& y_l \in \{0,1\} && \forall l \in V && (1.49)
\end{aligned}
$$

Using the same argument and results used in 1.4.3, we reformulate our problem, obtaining the following single-level model:

$$
\begin{aligned}
\max \quad & \sum_{k\in V} \omega_k \cdot x_k && && (1.50)\\
\text{s.t.} \quad & \sum_{k\in V} x_k p_k \le P, && && (1.51)\\
& \frac{x_k}{q_k} \le y_k; \quad \frac{x_k}{q_k} \le \sum_{l\in B_k} y_l, && \forall k \in V && (1.52)\\
& \sum_{l\in V, l\neq k} b_{kl} y_l \ge 1, && \forall k \in V && (1.53)\\
& y_k \le x_k, && \forall k \in V && (1.54)\\
& \sum_{l\in B_k} \alpha_k + \beta_k \le 1, && \forall k \in V && (1.55)\\
& \sum_{k\in V} \alpha_k + \sum_{k\in V} \beta_k x_k = \sum_{k\in V} y_k, && && (1.56)\\
& x_k \in \mathcal{N}, 0 \le x_k \le q_k, && \forall k \in V && (1.57)\\
& y_l \in \{0,1\}, && \forall l \in V && (1.58)\\
& \alpha_k, \beta_k \in \{0,1\}, && \forall k \in V && (1.59)
\end{aligned}
$$

Computational results

A computational experience, consisting of performing 25 experiments on the network in Figure 1.11 has been carried out in order to compare the solutions provided in both reformulations and the heuristics solutions. Computational experience was solved by means of the GuRoBi 9.1.2 in Python solver on a laptop with 16 GB of RAM and an Intel processor i7-1165G7 (with a 64-bit Windows 10 professional operating system and limiting the execution time to 2 h). Instance #1 is our case study. Instances between #2 and #8 have a single parameter randomly changed from the original example. Instances between #9 and #18 all the parameters have been taken randomly with values of attractiveness and budget less than 100, capacity less than 10, unit price less than 1/4 of the budget and coverage radii less than 3. Instances between #19 and #25 all the parameters have been taken randomly with values of attractiveness and budget less than 1000 and greater than 100, capacity less than 20 and all other parameters taken randomly in the same way.
Table 1.6 shows the obtained results. The columns of *NH, AH* show the number of nodes and the attractivity obtained with the heuristic solution respectively. The columns of *NE1, AE1* and *NE2, AE2* show the number of nodes and attractivity obtained with the exact method explained in section 1.4.3 and 1.4.3 respectively. The computation times (in seconds) of the heuristic, model 1 and model 2 are included in the columns *CPU H, CPU E1, CPU E2* respectively. In all the experiments carried out, the minimum number of nodes is given by the first exact model and the maximum attractivity is given by the second exact model.
The heuristic gives us a good solution between both methodologies, in all the experiments, the difference of nodes with the minimum numbers is less or equal than 4 and with the maximum attractivity has 8.49% of relative error. The heuristic gives solutions that are closer to the optimal value of the second model because once a solution with few nodes covering the network is achieved, the heuristic increases the number of nodes to increase attractiveness. On instance #12, #14, #19 and #23 the heuristic does not give us a solution of the problem because the initial CCP solution exceeds the budget.

1.4.4 Conclusions

In this section, a methodology has been developed to optimally select, among a group of candidate sites already equipped with refueling facilities, a series of recharging points in order to guarantee that an electric vehicle can autonomously transit within a territory and, in the event of a breakdown in the selected service station, be able to count on an alternative charging station that is within a reasonable distance radius. This concept of reinforced coverage has been formulated following the conditional covering model, which minimizes the number of installations required (Criterion 1). Complementarily, the maximization of the demand that could be satisfied subject to budgetary restrictions has been a second objective, which has been formulated as an instance of the knapsack

#	NH	AH	CPU H	NE1	AE1	CPU E1	NE2	AE2	CPU E2
1	10	862	**0.00**	**8**	573	0.19	10	**945**	0.03
2	9	1481	**0.00**	**8**	1457	0.13	10	**1627**	0.03
3	11	1600	**0.00**	**8**	747	0.19	11	**1605**	0.03
4	9	507	**0.00**	**8**	391	0.09	9	**542**	0.03
5	10	849	**0.00**	**8**	573	0.25	10	**871**	0.02
6	9	507	**0.00**	**8**	345	0.07	10	**648**	0.03
7	6	1052	**0.00**	**4**	635	0.48	6	**1060**	0.09
8	5	**1095**	**0.00**	**4**	495	0.14	5	**1095**	0.03
9	13	2405	**0.01**	**10**	1446	8.2	13	**2460**	0.08
10	**4**	558	**0.02**	**4**	412	0.78	5	**673**	0.05
11	8	**1692**	**0.01**	**4**	746	0.58	8	**1692**	0.05
12	**-**	-	**-**	**9**	**297**	0.07	**9**	**297**	**0.04**
13	8	1509	**0.01**	**5**	180	2.99	8	**1734**	0.06
14	-	-	-	**10**	434	0.09	11	**446**	**0.04**
15	5	1248	**0.01**	**4**	821	1,01	6	**1580**	0.03
16	6	1224	**0.00**	**4**	454	0.35	8	**1287**	0.01
17	5	911	**0.00**	**4**	344	0.07	6	**1033**	0.0
18	7	1223	**0.00**	**5**	293	0.62	7	**1223**	0.01
19	**-**	-	**-**	**9**	7074	0.38	**9**	**10338**	**0.01**
20	6	48977	**0.00**	**4**	38743	1.01	6	**53362**	0.02
21	6	**15561**	**0.00**	**5**	9479	1.64	6	**15561**	0.02
22	6	33497	**0.00**	**4**	15471	1.31	6	**40682**	0.02
23	-	-	-	**9**	7062	0.41	10	**14441**	**0.02**
24	7	38071	**0.00**	**4**	28073	0,37	7	**39529**	0.02
25	12	21797	**0.00**	**10**	7798	0,05	12	**23710**	0.01

Table 1.6: Results obtained from the computational experience.

problem (Criterion 2). This second criterion has been hierarchically combined with the previous one, giving rise to a bilevel model.
Since the computational complexities of both proposed models are NP-difficult, a heuristic has been designed to solve the two phases of the bilevel model separately, following an iterative scheme (Methodology 1). This first solution procedure has been compared with two other methodologies based on programming a single optimization level, by integrating the primal and dual versions of the involved constraints. The first of those methodologies has been directly obtained without modifying the hierarchy of the criteria initially considered (ie, minimizing the number of installations required as the main objective - Methodology 2), while the second methodology pursues the maximization of the user population covered as main objective (Methodology 3).
As future research, a possible continuation would be to take into account that due to the proximity between some gas stations they cannot be selected at the same time due to the electrical voltage or to carry out a study with different companies that have different locations and new restrictions.

1.5 Chapter conclusions

In this chapter we have presented three different key transport issues facing many countries in the world. Urban sprawl is a phenomenon that leads to an extensive use of motorized transport modes with negative environmental impacts such as congestion, time wasted in traffic jams, air and noise pollution and additional costs incurred by using non-renewable energy. Increasing the existing infrastructures is a decision, which often generates the installation of new urban settlements, whose degree of isolation is mitigated with a new increase in the demand for transport. This vicious circle can be broken by reducing the need of transport imposed by the urban model, which is only possible by bringing citizens closer to those services they demand. In the model of sprawled city, housing predominates as land use in the residential areas, where other complementary uses (such as commercial, cultural, institutional and industrial ones) are excluded in the urban development. When the urban districts do not present enough complexity, an increase in traffic density between different zones into the city arises. Such forced mobility could be reduced if the functional diversity of the districts were greater, or if there was an urban rapid transit system connecting the areas that generate the greatest imbalances. To measure the complexity of the urban districts system, the Information Theory developed in the 1960s proposes the use of urban entropy. Section 1.2 addresses the problem of locating a rapid transit line (metro, tram, BRT) with the objective of maximize the functional diversity of the districts traversed by the alignment. In order to illustrate the proposed model a computational experience is carried out by using data from the metropolitan area of Seville (Spain).
In section 1.3 we have presented an optimization procedure to choose a parking facility according to different criteria: total travel time including transfers, parking fee and a factor depending on the risk of not having an available spot in

the parking facility at the arrival time. An integer programming formulation has been proposed to determine an optimal strategy of minimum cost considering the available information, different scenarios, and each user profile. To evaluate the performance, a computational experience has been carried out on Seville (Spain), where a historical city center restricts the traffic of private vehicles and encourages the use of parking facilities. we have worked on the park-and-ride problem. We have designed an ITS to reduce the parking search time taking into account the behavior of the drivers, it has been taken into account: the price of the trip, the time of the trip, the number of free spaces in the selected car park. We have provided a methodology to evaluate the efficiency of routes between two points starting with a private vehicle but necessarily ending with another means of transport with the mandatory use of a park-and-ride facility. In section 1.4 we have developed a procedure to optimally select, among a group of candidate sites where gas stations were already located, a sufficient number of charging points in order to guarantee that an electric vehicle can make its journey without a problem of energy autonomy and that each selected charging station has another one that serves as an alternative support in case of failure (reinforced coverage service). For this purpose, we have proposed a bilevel model that, in a former level, minimizes the number of refueling points necessary to guarantee a reinforced service coverage for all users who transit from their origin to destination and, in a second level, maximizes the volume of demand that can be satisfied subject to budgetary restrictions. With the first of the objectives we are addressing the typical demand of the administration, which consists of guaranteeing the viability of the solutions, and the second of the objectives is a criterion typically used by the private sector initiative, compatible with the profit maximization.

Chapter 2

Location and routing of containers (fixed and mobile) for the selective collection of urban solid waste

2.1 Introduction

The Multiple Knapsack Model can establish an adequate theoretical framework to analyze the territorial deployment of fixed containers for the selective collection of urban solid waste and also, for the planning of routes and stops of the so-called eco-points, that is, waste containers with less frequency of generation by society in urban areas and that have the potential to pollute the environment.

Nowadays, due to greater attention to the quality of life and the interest in sustainable energy resource utilisation, the majority of institutions are moving towards efficient solutions to combat the urban waste collection issue. The Sustainable Development Goals defined by the United Nations in the 2030 Agenda have brought the imperative to make cities more sustainable to the fore of development discussions, thereby improving the quality of life for citizens (Nations, 2015). European Nations have recently defined several policies in favour of the circular economy (da Silva, 2018; van Ewijk and Stegemann, 2020). Therefore, due to the inherent social implications, the optimisation of urban waste collection assumes a fundamental role in each city today (Alçada-Almeida et al., 2009).

Municipal Solid Waste (MSW) includes used paper, discarded cans and bottles, food scraps, yard trimmings, and other items. Proportionally, household waste accounts for usually up to 75 per cent of all the municipal solid wastes. MSW management includes several functional phases such as waste generation, storage, collection, transportation, processing, recycling and disposal in a suitable landfill (Khan and Samadder, 2014). Collecting solid waste involves storage at the generation and pick-up points, pick up by the crew, trucks driving around the neighbourhood, and truck transport to a transfer station or disposal point. These tasks are difficult, complex and costly. Therefore, the objective of an efficient service should be the minimization of solid waste collection costs, together with the provision of an adequate and regular service to all of the target area of Technology Assessment (1989). Providing an efficient collection service to a city often requires a combination of techniques and equipment, to accommodate the different challenges of the various neighbourhoods within the city (Coffey and Coad, 2010).

Waste collection and transportation phases are closely related, since the deployment of containers along the city determines both the vehicle fleet size required for picking up the collected waste into the containers and the design of efficient routes needed for that purpose. Typically, collection costs represent 80–90% and 50–80% of municipal solid waste management budget in low income and middle income countries, respectively (Aremu, 2013). Therefore, waste collection and transportation problems are considered as one of the most difficult operational problems when developing an integrated waste management system (Nuortio et al., 2006). Eiselt and Marianov (2015) provide a compilation of 64 papers that include applications throughout the world, where the main aspects of interest of the contributions have been summarized in a Table, and classified according to country, technique, criteria, objectives and type or facility to be located.

Management of solid-waste collection services is intrinsically linked to the development of effective vehicle routing (VR) models that optimize the total traveling distance of vehicles, the environmental emission and the investment costs (Apaydin and Gonullu, 2011). An optimal VR is a scheduled process that allows vehicles to load waste at gather sites and dump it at a landfill by satisfying multiple objectives (Tung and Pinnoi, 2000). Through a Route optimization for Waste management (WM), both the residential routing problem and the commercial routing problem settings can be solved. Beliën et al. (2014) present a review of the available literature on solid waste management problems, with a particular focus on vehicle routing problems that are classified into different categories.

In real scenarios, the waste collection system is distributed in a set of zones. The purpose of the zoning phase is to determine collection districts. The districts must be defined such that the total solid waste loads within each one does not exceed the capacity of the vehicles used to perform the waste collection. The problem of districting is not widely addressed in the literature, or in many cases it is assumed to be solved a priori, neglecting the influence it could have on the subsequent routing phase. Male and Liebman (1978) proposed a districting heuristic based on the construction of an auxiliary graph, in which nodes

represent trips and edges represent feasible trips aggregations. Eisenstein and Iyer (1997) devised flexible schedules for garbage trucks in the city of Chicago. Hanafi et al. (1999) studied a weekly zoning schedule problem with the aim of determining a fixed number of sectors which must be balanced with respect to the daily total waste collection time. They proposed an optimization model which can be applied to small-size instances. For large-size scenarios these authors develop a local search heuristic that is based on the definition of a zoning matrix. The proposed methods are tested on three real-world instances and 28 randomly generated instances. Labelle et al. (2002) presented several models and heuristics for partitioning a city into sectors, with respect to snow disposal operations, and for assigning the sectors to disposal sites. The problem results quite similar to the problem encountered in garbage collection operations. Sahoo et al. (2005) present a discussion on how to divide the area from which waste is collected into districts, with the aim of subdividing the problem, making it more manageable. Authors proposed both, a mathematical model and a two phase insertion algorithm, in which a feasible solution is first generated and later, improved; see also the works of Kim et al. (2004); Solomon (1987); Taillard et al. (1997), which are used to address the two phase method. Each zone has a set of starting and ending nodes associated in order to determine the tours for vehicles responsible for carrying the garbage collected in the visited containers. A planning horizon must also be considered in order to schedule a sequence of services within the useful life of each vehicle. A succession of routes (one per day, belonging to the same or to different distribution zones and performed by the same vehicle along the planning horizon) is called a circulation. Plans for determining the vehicle circulation in transportation networks are described by, for instance, Barrena et al. (2016); Canca and Barrena (2018).
Community containers are the locations in the street where the waste can be transferred to the collection agency at a short distance from the dwellings where garbage is generated. Sites, where community storage facilities should be located, must depend on the customer behaviour. If a community is willing to co-operate in their proper use by carrying their waste to the containers, rather than dropping it in the street or on open plots nearer to their homes or businesses. In these cases, the task of collection will be transferred to the street sweeping service which is more expensive than collecting from containers.
Collection vehicles visit community containers at frequent intervals, usually once daily or every second day, to remove accumulated waste. A planning horizon must also be considered in order to schedule a sequence of services within the useful life of each vehicle. The set of collecting routes, belonging to the same or to different distribution zones in the city and performed by the same vehicle along the planning horizon, is called a vehicle circulation. Plans for efficiently determining vehicle circulations in transportation networks are described by Barrena et al. (2016).
Summarizing, the problem of managing selective collection of waste containers can be performed in three sequential phases devoted to: first, the location of containers along the streets; then, the determination of the minimum fleet size required to perform all collecting services; and finally, the design of optimal

routes, in terms of total and balanced number of kilometres travelled by the trucks. The decisions to be taken in these three phases can be advised through the use of optimization models. Obviously, the result of the first phase (location of the containers) highly influences the procedure since this will determine the decisions to be taken for the subsequent phases (route of collection vehicles and service programming).

The choice of permanent or temporary collection points in the various areas of each city is of extreme importance. Indeed, every waste container presents a specific capacity, cost, and environmental impact. In this respect, the actual challenge for most of the administrations involves the suitable definition of key parameters, such as the type, the number, and the position of the containers for every area in order to dispose of all types of waste produced in a defined period. A further important aspect for consideration in the analysis of this issue is that of the size of the city examined. In fact, in highly populated cities, waste collection is managed by different municipalities or organisations responsible for specific established zones. Typically, the containers should be distributed so that the distance between any two containers is not excessive. In the cities which have historic core areas it may not be possible to locate containers at the most convenient distances, because community containers can only be located along the main streets and in places where there is enough space for the container itself and for operating the collection vehicle.

In the scenario regarding municipalities, it will also be problematical to predict future modifications in waste collection and recycling. The composition of municipal solid urban waste is influenced by the standard of living of the population, the economic activity of their inhabitants, and the climate of the region (Bandara et al., 2007). Certain products will eventually become more commonly used in relation to these factors and, subsequently, various waste modalities will be generated in varying proportions.

According to the report entitled Statistics on Waste Collection and Treatment for the Year 2018 of the National Institute of Statistics (INE) in Spain, the main materials produced in Spain are paper and cardboard (24.1%), organic matter (22.9%), glass (18.9%), plastic and mixed packaging (16.8%), and others (representing 17.3%). This latter group requires special attention, since certain items can be considered as hazardous waste. Waste can be characterised as hazardous if it possesses any one of the following four features: ignitability, corrosiveness, reactivity, and toxicity. Hazardous waste, which is usually the waste by-product of our industrial processes, presents immediate or long-term risks to humans, animals, plants, or the environment.

In Spain, many municipalities have combined the need to collect this type of potentially hazardous waste with the promotion of environmental policies and use containers with an aesthetically attractive design, which help spread the commitment to the selective collection of urban solid waste.

The so-called eco-points are large waste containers with separate non-homogeneous sections for the collection of various kinds of items, including mobiles, batteries, chargers, syringes and needles, used low-energy lamps, radiographs and photographic material, books for the exchange between citizens, toner and cartridges of ink, aluminium and plastic coffee capsules, and CDs and DVDs A real example of an eco-point located in the city of Seville (Spain) is shown in Figure 2.1.

The decision regarding the best configuration for an eco-point container in-

Figure 2.1: An eco-point situated in city of Seville.

cludes the design of the distribution of volume of their sections. The eco-point lay-out is linked to the Bin Packing (BP) problem. In agreement with Garey (1979), the BP issue is a combinatorial problem that belongs to the class of NP-hard problems. Various real applications of this kind of issue are presented in Garey and Johnson (1981); Falkenauer (1996). In addition to this connection with the BP problem, MSW collection is intrinsically connected to the VR model (Carrese et al., 2019; Marseglia et al., 2019) in terms of optimising different criteria, such as the total distance travelled by vehicles, the emission of environmental pollutants, and the investment costs (Tung and Pinnoi, 2000).
Our interest in this issue is focused on a type of mobile container for the collection of solid waste, composed of several sections for the separate storage of different items, which can either all be of the same size or can have heterogeneous volumes, depending on the demand of the place where they are temporarily located. Eco-points deployed in the region of Cantabria (Spain), such as that shown in Figure 2.2, represent real instances of the type of containers of interest. The identification of the allocation of the multi-block container is one of the two decisions to be adopted. It should be borne in mind that mobile eco-points

Figure 2.2: An example of an eco-point deployed in Cantabria.

follow an established route and visit all the neighbourhoods of the city in an itinerant way. The container is placed at identified stops on public roads for a temporary period (for example, on a Monday, whereby it is moved to a new point in the city on the following Monday). The calendar is previously made known to all residents in the neighbourhood.

Section 2.2 focus on the first phase of the problem of managing selective collection of waste containers: the location of collecting facilities (waste containers), where facility-customer distances must be considered in the collecting design system, as well as other considerations such as the size of container groups, their capacities in accordance with the closest population and the installation cost of those containers in specific sites along the streets. Section 2.3 also develops the location of collecting facilities but taking into account the influence of customer solidarity behaviour on this location. For this purpose, we consider parameters such as container customer distances, the size of container groups, their capacities in accordance with the closest population, and the installation cost of those containers in specific sites along the streets. Section 2.4 addresses the double determination of BP configurations and of container routes. One of the ultimate aims involves the cost minimisation of the resources employed. In order to solve the problem of waste collection and vehicle routes in an optimal way, an adaptive algorithm of overflow deviated to the immediate neighbourhood is developed. This algorithm strives to solve the proposed mathematical programming model, whose computational complexity justifies the use of heuristics to address large real-life scenarios. The evaluation of the performance of the developed methodology has been carried out through a computational experience in a toy network using two strategies to design the layouts of the mobile multi-block containers that visit the demand nodes.

2.2 Optimizing container location for selective collection of urban solid waste

2.2.1 Model formulation

We assume the following mathematical context, associated to the characteristics of the problem, that consists of a connected graph $G = (V, A)$, composed of a node set V (portals) and an arc set A (directed edges representing street sections). The arcs of set A connect the nodes belonging to set V, so that the existence of a shortest path in terms of distance or travel time between each pair of points of V is guaranteed. Let us suppose that set V is composed of nodes where urban waste is generated (set I) and by points where it is possible to locate the containers to deposit them (set J). We also assume that the inclusion sequence $J \subset I \subset V$ is maintained. Note that any node i of V located at the entrance gate of a building could be identified as a generating point of waste; in that case, node i would belong to set I. Alternatively, node i could simply be a feasible site along the street, where the waste container could temporarily be located (in that case, node ri would also belong to set J). The following notation is used in our waste containers formulation:

- I: set of demand nodes ($i \in I$). There is a population p_i associated to each demand point $i \in I$.
- J: set of possible location nodes to locate waste containers ($j \in J$). There is an upper bound (cap_j,$j \in J$) in terms of capacity associated to each candidate point $j \in J$.
- K: set of main types of solid waste generated in the urban area (for instance, cardboard, plastic, organic waste, scrap metal, etc.) ($k \in K$).

Additionally, we assume a compensation cost $\beta_j^k > 0$ associated with the economic value that the municipal cleaning company would be willing to pay to maintain a container of modality k in node j during the planning horizon. This means that the cleaning company should have previously negotiated a payment reduction with the inhabitants nearby node j, due to the inconveniences generated by permanently establishing the container of type k close to their dwellings. So, if node j is located in the public domain far from any building, the compensation cost could be considered 0; on the other hand, if the location of the container in the proximity of a house is technically unfeasible, this cost could be associated to an infinite value.
The parameters involved in our optimization model are the following:

- Each node i has a known weight w_i^k (which can be identified with the amount of waste in kg or dm^3 generated in node i of modality k, i.e., organic material, glass, packaging or paper units) associated.
- The shortest distances between nodes of set V, along network G, have previously been determined and recorded in the matrix $D = (d_{ij}), d_{ij} \geq 0$.

Inhabitants associated to node i would experience a displacement cost (discomfort) C_{ij}^k when having to take their type k waste to the container located at point j. This discomfort implicitly requires a restriction on travel distances. In practice, this restriction is modelled by the assignation of a feasible coverage radius from point i. A point i can be considered covered by another point j if the distance between them does not exceed a radius of displacement R^k.

Let us assume that each customer is willing to use any container, as long as a maximum walking distance from their residence to the assigned container is not exceeded. That container might not be the closest, but this must lie within a predefined radius. In our model, a portion of inhabitants associated with node i could take their garbage to the container j and another portion of the population of the same node would be willing to take out their garbage to another unfilled container j that is not excessively distant. This solidary behavior of the clients would allow an efficient deployment of the containers in the area under analysis, reducing their total number and grouping them in the points of lowest cost.
Let $q_{ij}^k \in 0, 1$ be a binary expression that takes value 1 if the demand point i can be covered by site j by means of a container of modality k (note that $q_{ij}^k = 1$ implies that $d_{ij} \leq R^k$, and value 0, otherwise). Additionally, let $N_j > 0$ be an integer parameter that indicates the maximum number of containers that could be installed at location j. We assume that all containers are provided with the same capacity Q.
Moreover, the following variables are required in the model.

Variables

y_j^k Binary variable that takes value 1, if container location j is activated to collect type k garbage, and value 0, otherwise.

x_{ij}^k Number of type k containers to be installed at location j.

n_j^k Percentage of type k garbage that the client corresponding to node i will deliver at location j.

The nature of the variables used in the model yields varied formulations to face different objectives. In our case, the following integer programming formulation determines the minimum number of container groups to be installed in the area under consideration. Note that the lower the number of garbage deposit points, the more efficient the collection procedure will be.

Objective and constraints

$$
\begin{aligned}
z_1 \equiv \min \quad & \sum_{j\in J}\sum_{k\in K} \beta_j^k n_j^k && && (2.1)\\
\text{s.t.} \quad & \sum_{j\in J} q_{ij}^k y_j^k \geq 1, && \forall i \in I, k \in K, && (2.2)\\
& \sum_{j\in J} q_{ij}^k x_{ij}^k = 1, && \forall i \in I, k \in K, && (2.3)\\
& x_{ij}^k \leq y_j^k, && \forall i \in I, j \in J, k \in K, && (2.4)\\
& y_j^k \leq n_j^k && \forall j \in J, k \in K, && (2.5)\\
& \sum_{i\in I} w_i^k x_{ij}^k \leq Q \cdot n_j^k && \forall j \in J, k \in K, && (2.6)\\
& \sum_{k\in K} n_j^k \leq N_j && \forall j \in J, && (2.7)\\
& x_{ij}^k \geq 0, y_j^k \in \{0,1\}, n_j^k \in \mathcal{N}^+ && \forall i \in I, j \in J, k \in K. && (2.8)
\end{aligned}
$$

The objective function 2.1 minimizes the cost of containers that should be installed. Note that when radii R^k decrease, the number of containers that can be grouped in the same location will then increase. Constraints 2.2 ensure that all demand is covered by the set of locations to be determined. Constraints 2.3 establish that the sum of coverage percentages for every demand point from the container must be equal to 1. Constraints 2.4 imply that if a location is activated, then at least one container must be installed at it. Constraints 2.5 guarantee that if a location is not activated, then no demand point can be covered by it. Constraints 2.6 imply that demand points that may be served from location j cannot exceed its capacity. Constraints 2.7 establish an upper bound on the number of containers that can be located at each site. Constraints 2.8 indicate the nature of the variables used in the model.
Maintaining the above described constraints, an additional criterion, consisting of minimizing user travel costs, can be incorporated into the objective by combining it with the previously considered minimization of costs in the deployment of the containers. The expression that follows formulates this purpose:

$$
z_2 \equiv \min \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} C_{ij}^k y_j^k + \sum_{j\in J}\sum_{k\in K} \beta_j^k n_j^k \tag{2.9}
$$

Both models 2.1–2.8 and 2.9-2.8 are of combinatorial nature and can be considered as instances of a Partial Set Covering problem (Daskin and Owen, 1999). The partial set covering model is NP-hard since it is a generalization of the traditional location set covering problem, which is NP-hard.
In Cormen et al. (2022), the problem is discussed in detail and its complexity is proved. This fact justifies the use of algorithms that provide a good heuristic solution.

A model similar to the one previously proposed has been investigated in Barrena et al. (2017), in which heuristics were designed in a computationally feasible way and consistent with the approach. Tests carried out on randomly generated data have shown that a simple heuristic of Overflowing Deviated to Immediate Neighbourhood (ODIN) yields the best results if the inter-location spacing between adjacent containers is not excessively large. Taking these precedents into account, we propose the heuristic ODIN for solving our optimization model in order to determine the most effective deployment of waste containers along the street network.

Heuristic ODIN

1. **Sort** the points that generate urban waste according to their production, from highest to lowest levels and **re-label** them.

2. **While** there exists a generator point i whose collection requirement exceed the established upper limit N_i ; i.e. $\sum_{k \in K} n_j^k \geq N_i$ **do**

   2.1 **Identify** the set of nodes $Prox(i;k)$ whose distances to node i are less than R^k (excluding node i) .

   2.2 **Sort** the nodes $j \in Prox(i;k)$ from the lowest to the highest levels according to ascending values of β_j^k.

   2.3 **For** each $j \in Prox(i;k)$ **do**

      2.3.1 **While** $\sum_{k \in K} n_i^k - N_i > 0$ and $N_i - \sum_{k \in K} n_j^k > 0$ **do**
      Decrease units from n_i^k and Increase them in n_j^k.

3. **If** all the generating points satisfy condition $\sum_{k \in K} n_i^k \leq N_i$ **then** a solution to the problem has been obtained. **Otherwise,** a modification of parameters R^k or N_i is required.

4. **End.**

2.2.2 Computational Experience

Our model has been tested on a laboratory example, described by a graph representing a part of the street system in the city of Seville. The historic centre of Seville is one of the largest in Europe, along with those of Venice and Genova. It has an approximately circular configuration and an area of $3.94km^2$. The streets of this sector of Seville are predominantly narrow and one-way. The most common type of dwellings is the single-family house or the multi-family building with a shared courtyard. In both housing models, there is hardly any space available for the establishment of selective garbage containers in the interior. Conversely, citizens have traditionally orientated the common areas, both in the insides and doorways, towards decorations in tile and other traditional elements (gardening pots, etc.) that enhance the beauty of the place.
This unavailability of physical space for locating fixed containers forces to a widespread use of a small container (with a maximum capacity for 4 bags of

garbage), mobile (provided with rear wheels and a handle to be dragged by the user), and polyvalent (without the possibility of classifying organic waste, glass, cardboard/plastic containers or paper), which serves the community of neighbours throughout each day.
The daily collection of waste from these multi-purpose containers is currently established by pre-designed truck routes, whose passing times are constrained to temporary windows that are known by the building neighbours. This collecting service is currently non-selective due to the above mentioned difficulties: widespread narrowness of streets to permanently locate containers in surface, aesthetic/tradition constraints, space limitations within dwellings that discourage the multiplicity of collecting instruments.
A computational experience has been carried out on a laboratory scenario composed of 39 dwellings (sites identified as elements of node set I). Location of nodes along the street network and the values of internode distances are illustrated in Fig. 2.3

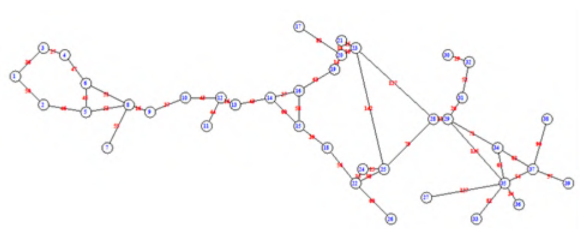

Figure 2.3: A network of thirty nine nodes.

The amount of daily produced waste is randomised within the interval [400 kg, 16000 kg]. By considering homogeneous containers of capacity 500kg, it is possible to initially assign to each residential place a container cluster (located at this same site) whose amount varies between 1 and 32. In order to provide the experiment a connection with a real context, a limitation on the number of containers that share the same geographical location has been set to 10. Additionally, a unit cost per locating a container at each site j must be considered. In the experiment, it has been considered as randomised within the interval $[0, 10]$, by representing the monthly cost in euros incurred on locating the container at this specific site.
In a first scenario, different types of garbage have not been distinguished. In Fig. 2.4, the container clusters, which are needed to guarantee the collection of all urban waste by means of user displacements to the deposit point nearer than $R = 100m$, are represented by circles of variable radius between 1 and 10.

If no optimization procedure was applied, the 238 containers needed to collect all the global garbage produced (see locations in Fig. 2.4), would involve a cost of 1091 euros per month.
On the other hand, if we assume a solidary behaviour of the clients when taking

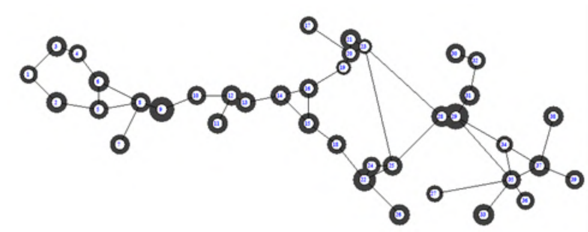

Figure 2.4: A garbage collection system with 39 nodes and a cost of 1091 euros per month.

their garbage to the container assigned to them within a pre-established proximity radius (not necessarily to the closest container to their place of residence), then a more efficient distribution of the containers can be obtained by means of the optimization model previously proposed. This optimized distribution is characterized by:

- A smaller number of clusters: the number of initial groups (39) has been reduced to 29.
- The compensation cost of the maintenance of the 238 containers on the street, is reduced by 14.48%. The monthly cost associated to the solution shown in Fig. 2.5 is now 933 euros.

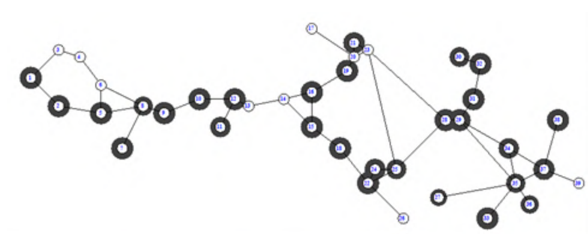

Figure 2.5: A garbage collection system with 29 nodes and a cost of 933 euros per month.

In a second scenario, we consider a selective collection of solid waste consisting of three different types (a situation like the one shown in Fig. 2.6).
The total amount of waste generated at each node coincides with that of the first scenario, but its distribution in each of the three types of waste considered is random. The requirement to separately store the waste yields an increment in the number of containers needed to carry out the collection of waste. This increment with respect to the first scenario, which in the experiment is equal to 17.65% (280 containers now, versus 238 in scenario 1), leads to a redistribution of the containers. A new application of the ODIN heuristic, maintaining the values established for the parameters R^k and N_j, provides the distribution of

Figure 2.6: An example of three different types of solid waste collection.

containers illustrated in Fig. 2.7. In this case, a more efficient distribution of the containers is also achieved after applying the algorithm, since those can be grouped in 33 places, instead of the initial 39 nodes.

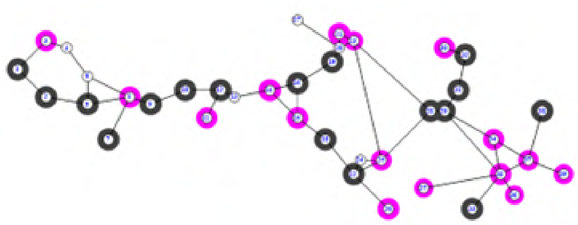

Figure 2.7: An example of three different types of solid waste collection.

If we compare the solution obtained in scenario 2 with the proposal for scenario 1, we can observe that 17 nodes (marked with the magenta colour) have had to change their allocation of containers due to the increment caused by the selective collection of solid waste.

2.2.3 Conclusions

A methodology for the deployment of containers for selective collection of urban solid waste has been proposed in this work. The mathematical optimization model formulated for this purpose has been identified as a version of the Partial Set Covering problem, whose computational complexity motivates the use of heuristics to face large real-life scenarios. Following that recommendation, a greedy algorithm of overflowing deviated to the immediate neighbourhood has

been developed to solve the proposed mathematical programming model.
To illustrate the performance of the developed methodology, a computational experience has been carried out on an urban system composed of 39 nodes with randomised data inspired in a zone belonging to the area of Seville (Spain). Two different scenarios are considered. The first scenario maintains the current non-selective collection of urban solid waste and, after the optimization procedure, the compensation cost is reduced by 14.48% and the number of clusters is reduced from 39 to 29 (25.65% decrease), thus facilitating the subsequent phases of service programming and collection route design. The second scenario incorporates the selective collection, thus yielding an increment on the number of containers but, even though, the number of nodes is reduced to 33 (15.38% decrease). The evaluation of two generated scenarios illustrates then that the methodology meets the objective of efficiently designing a deployment of containers for selective collection of urban solid waste.

2.3 Solidarity Behaviour for Optimizing the Waste Selective Collection

2.3.1 Model Formulation

We assume the previous mathematical description, associated to the characteristics of the problem, that consists of a connected graph $G = (V, A)$, composed of a node set V (portals) and an arcs set A (directed links, i.e. arcs, representing street sections). The arcs of set A connect the nodes belonging to set V, so that the existence of a shortest path in terms of distance or travel time between each pair of points of V is guaranteed. Let us suppose that set V contains nodes where urban waste is generated (set I) as well as points where it is possible to locate the containers to deposit them (set J). We will additionally assume that the following inclusion sequence is maintained: $J \subset I \subset V$. Note that any node i of V located at the entrance gate of a building could be identified as a generating point of waste; in that case, node i would belong to set I. Alternatively, node i could simply be a feasible site along the street, where the waste container could temporarily be located (in that case, node i would also belong to set J).
The following notation is used in order to formulate our location model for the waste containers:

- I: set of demand nodes ($i \in I$). There is a population p_i associated to each demand point $i \in I$.
- J: set of possible location nodes to locate waste containers ($j \in J$). There is an upper bound (cap_j, $j \in J$) in terms of capacity associated to each candidate point $j \in J$.
- K: set of main types of solid waste generated in the urban area (for instance, cardboard, plastic, organic waste, scrap metal, etc.) ($k \in K$).

Additionally, we assume a compensation cost β_j^k associated with the economic value that the municipal cleaning company would be willing to pay to maintain a container of modality k in node j during the planning horizon. This means that the cleaning company should have previously negotiated a payment reduction with the inhabitants nearby node j, due to the inconveniences generated by permanently establishing the container of type k close to their dwellings. In this way, if node j is located in the public domain far from any building, the compensation cost could be considered to be 0; and, on the other hand, if the location of the container is technically unfeasible due to the proximity of a place of residence, this cost could be associated to an infinite value.
The parameters involved in our optimization model are the following:

- Each node i has a known weight w_i^k (which can be identified with the amount of waste in kg or dm^3 generated in node i of waste modality k, i.e., organic material, glass, packaging or paper units) associated.
- The shortest distances between nodes of set V, along network G, have previously been determined and recorded in the matrix $D = (d_{ij})$, $d_{ij} \geq 0$.

Residents associated to node i would experience a displacement cost (discomfort) C_{ij}^k when having to take their type k waste to the container located at point j. This discomfort should be limited by means of including a restriction on the maximum allowed walking distance for the users. In practice, this restriction can be modelled by the assignation of a feasible coverage radius from point i. A point i can be considered covered by another point j if the distance between them does not exceed a radius of displacement R^k.
Observe that this radius is type-dependent since some types of waste may have less collecting points if its use is not as extended as others. Customers may be willing to walk longer or shorter to deposit the waste depending on its type.
Let us assume users have solidarity behavior, that is, that each customer is willing to use any container, as long as a maximum walking distance from their residence to the assigned container is not exceeded. That container might not be the closest, but this must lie within a predefined radius. In our model, a portion of inhabitants associated with node i could take their garbage to the container j and another portion of the population of the same node would be willing to take out their garbage to another unfilled container j that is not excessively distant. This solidary behaviour of the clients would allow an efficient deployment of the containers in the area under analysis, reducing their total number and grouping them in the points of lowest cost.

2.3.2 Modifications of Solving Algorithm

A model similar to the one previously proposed in Section 2.2.1 has been investigated by Barrena et al. (2017), in whose work heuristics were designed in a computationally feasible way and consistent with the approach. Tests carried out on randomly generated data have shown that a simple heuristic of Overflowing Deviated to Immediate Neighbourhood (ODIN) yields the best results if the inter-location spacing between adjacent containers is not excessively large. Taking these precedents into account, we propose the three-phases heuristic ODIN for solving our optimization model in order to determine the most effective deployment of waste containers along the street network.
We propose a solving algorithm which is divided into three parts. The first phase, ODIN1, is a slightly modified version of the algorithm ODIN presented by Barrena et al. (2019). ODIN1 does not requires an initial solution and this yields a feasible solution which tends to minimize the objective function. This is done by reallocating containers that cannot be installed at their demand points to the cheapest (in terms of compensation cost) available location within radius R^k. Having into account the customer solidarity behaviour, we also propose an extension (ODIN2 and ODIN3) of this algorithm in order to minimize the number of containers at each node and to allocate them, respectively, when this change helps reducing the objective function. Allocation is then done in order to minimize the cost as well as to reduce the number of stops in subsequent phases of waste collection

Heuristic ODIN1

1. **Sort** the points $i \in I$ that generate urban waste according to their production, from highest to lowest levels and **re-label** them.

2. **Assign** the required number of containers of type k to each node $i \in I$, that is, $n_i^k = \left\lceil \frac{w_i^k}{Q} \right\rceil$.

3. **While** there exists a generator point i whose collection requirement exceed the established upper limit N_i (i.e. $\sum_{k \in K} n_j^k \geq N_i$) or which does not belong to the set of possible location nodes (that is, $i \in I \setminus J$) **do**

   3.1 **Identify** the set of nodes $Prox(i;k)$ whose distances to node i are less than R^k (excluding node i) .

   3.2 **Sort** the nodes $j \in Prox(i;k)$ from the lowest to the highest levels according to ascending values of β_j^k.

   3.3 **For** each $j \in Prox(i;k)$ **do**

   3.3.1 **While** $\sum_{k \in K} n_i^k - N_i > 0$ and $N_i - \sum_{k \in K} n_j^k > 0$ **do**
   Decrease units from n_i^k and Increase them in n_j^k.

4. **If** all the generating points satisfy condition $\sum_{k \in K} n_i^k \leq N_i$ **then** a solution to the problem has been obtained. **Otherwise,** a modification of parameters R^k or N_i is required.

5. **End.**

This ODIN1 algorithm reallocates containers when demand cannot be attended at a certain node and add a container for this unattended demand at another location. However, in some occasions, there may be non-used capacity of the existing containers, and there is therefore no need to add a new one to attend demand. This give raise to propose a second part for this algorithm, aiming at a more efficient use of containers. This minimizes, not only compensation cost due to locations, but also the number of containers.

Heuristic ODIN2

In this second phase of the solution algorithm, a more efficient use of the containers is recommended. If there is enough non-used space at containers of type k at location j, then the waste of type k is reassigned to them in order to save number of containers.

1. **For** each location $j \in J$ we first **calculate** the number r_j^k of containers of type k required to attend the demand from node j and from all others nodes whose demand is partially assigned to j (that is, for all i such that $x_{ij}^k \neq 0$).

$$r_j^k = \left\lceil \frac{\sum_{i \in I} w_i^k x_{ij}^k}{Q} \right\rceil$$

2. **If** the number of containers needed is less than the number of containers obtained from ODIN1 (that is, if $r_j^k < n_j^k$), **then** diminish n_j^k and update its value to r_j^k .

Heuristic ODIN3

Once all the demand is attended with the minimum number of containers (solution obtained from ODIN1 and ODIN2), we propose to redistribute them in this third phase of the algorithm. Redistributing containers to cheaper locations may help to reduce the cost, and also to reduce the number of locations with containers. Reducing the number of locations with containers will facilitate the subsequent phases of waste collection and transportation since the number of stops is reduced. This phase makes more sense in scenarios in which there is a big proportion of generating nodes which are also possible container location nodes (that is, if set J is large). In these cases, it may happen that an excessive number of locations is activated and it is important to reduce them for operational tasks.

1. **Sort** $j \in J$ such that $n_j^k \neq 0$ from the highest to the lowest value of compensation the cost β_j^k and re-label them.

2. Consider the location nodes $j \in J$ that only attend demand from its own node (that is, $\sum_{i \neq j} x_{ij}^k = 0$).

3. **If** there exists a location node $j^* \in Prox(j,k) \cap J$ such that its compensation cost is lower than the one in ($\beta_{j^*}^k < \beta_j^k$) and that can allocate more containers (that is, if $\sum_k n_{j^*}^k < N_{j^*}$) **then** increase $n_{j^*}^k$ to $\max\{\sum_k n_{j^*}^k + n_j^k, N_{j^*}\}$ and decrease n_j^k accordingly.

4. **Go to** ODIN2 and **Iterate** until the stopping criterion is reached (when the improvement in the objective function is less or equal than a small value β).

2.3.3 Computational Experience

Our model has been tested on a graph representing a part of the street system in the city of Seville. In particular, the computational experience has been carried out on an urban area which contains a street network with 46 dwelling points (sites identified as elements of node set I). Location of nodes along the street network and internode distances are illustrated in Figure 2.8.

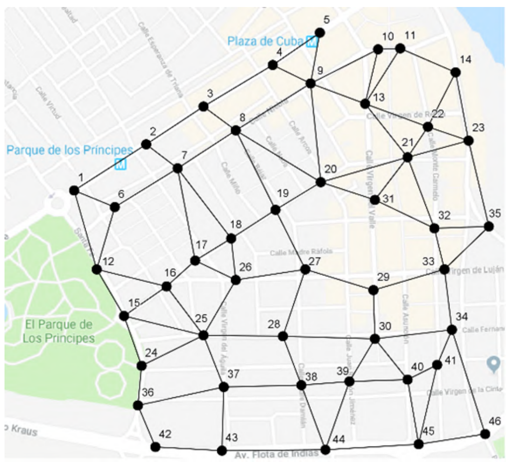

Figure 2.8: A network of 46 nodes depicting an area of Seville.

The amount of daily produced waste has been randomised within the interval $[400kg, 5000kg]$. By considering homogeneous containers of capacity $500kg$, it is possible to initially assign to each residential place a container cluster (located at this same site) whose amount varies between 1 and 10. In order to adapt to a real context, a limitation on the number of containers that share the same geographical location has been set to 10. Additionally, a monthly unit cost for locating a container at each site j must be considered. In the experiment, this cost has been considered as random within the interval $[0, 10]$ measured in euros. In the baseline scenario where no optimization procedure was applied, the 236 containers needed to collect all the produced garbage would involve a cost of 1149 euros per month.
In Figure 2.9, the container clusters, which are needed to guarantee the collection of all urban waste by means of user displacements to the deposit point nearer than $R = 100m$, are represented by circles of variable radius between 1 and 10. The radius of each circle is proportional to the size of the corresponding container group.

On the other hand, we also consider a first scenario assuming customer solidarity behaviour, that is, that customers are willing to carry their garbage to their specifically assigned containers (not necessarily to the closest container to their

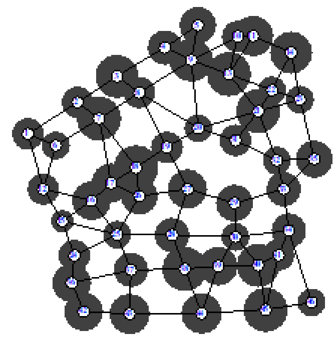

Figure 2.9: Container distribution with 46 nodes.

place of residence), within a pre-established proximity radius of $R = 100m$. In this scenario, a more efficient distribution of the containers can be obtained by means of the optimization model previously proposed (ODIN 1-3). Two sub-scenarios have been analysed according to size (N) of the container group at the same point. For $N = 6$, the proposed methodology yields the following results:

- The number of initial container groups (46) can be reduced to 43.
- The compensation cost of the maintenance of the 236 containers on the street, is reduced by 10.01 percent. The monthly cost associated to the solution shown in Figure 2.10 is now 1034 euros.

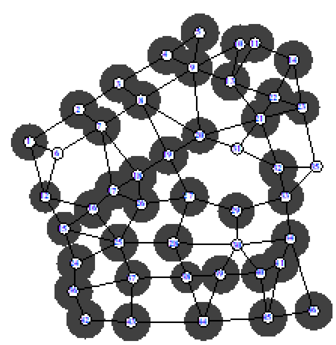

Figure 2.10: Container distribution with 43 nodes.

For $N = 8$, the proposed methodology yields the following results:

- The number of initial container groups (46) can be reduced to 33.
- The compensation cost of the maintenance of the 236 containers on the street, is reduced by 29.42 percent. The monthly cost associated to the

solution shown in Figure 2.11 is now 811 euros.

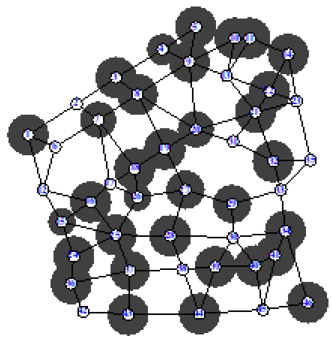

Figure 2.11: Container distribution with 33 nodes.

In a second scenario, we consider a selective collection of solid waste by using three different types of containers (a situation like the one shown in the Figure 2.12).
For this case, the total amount of waste generated at each node coincides with

Figure 2.12: An example of three different types of solid waste collection.

that of the first scenario, but its distribution in each of the three types of waste considered has been randomly generated. The requirement to separately store the waste yields an increment in the number of containers needed to carry out the collection of waste. This increment with respect to the first scenario, which in the experiment is equal to 22.03% (288 containers now, versus 236 in scenario 1), leads to a redistribution of the containers. A new application of the ODIN heuristics, maintaining the values established for the parameters R^k and N_j, provides a more efficient distribution of the containers, since those ones can be grouped in 41 places, instead of the initial 46 nodes.

2.3.4 Conclusions

A methodology for the deployment of containers for selective collection of urban solid waste has been proposed in this work. The mathematical optimization model formulated for this purpose has been identified as a version of the Partial Set Covering problem, whose computational complexity motivates the use of heuristics to face large real-life scenarios. Following that recommendation, a three-phase greedy algorithm of overflowing deviated to the immediate neighbourhood has been developed to solve the proposed mathematical programming model. This algorithm takes into account the characteristics of the problem and it specially considers the customer solidarity behaviour.

In order to illustrate the performance of the developed methodology, a computational experience has been carried out on an urban system composed of 46 nodes with randomised data based on a zone belonging to the area of Seville (Spain). Apart from the baseline scenario, two different scenarios are considered by assuming that customers have solidarity behaviour, as they commit to deposit their waste in containers that are not necessarily the closest to their homes.

The first scenario maintains the current non-selective collection of urban solid waste and considers different options by varying the size of the container group at the same point. After the optimization procedure for the biggest size considered, the compensation cost is reduced by 14.48% and the number of clusters is reduced from 46 to 33 (decrease of 29.42%), thus facilitating the subsequent phases of service programming and collection route design. The second scenario incorporates the selective collection, thus yielding an increment on the number of containers but, even though, the number of nodes is reduced to 41 (decrease of 10.86%). The evaluation of two generated scenarios illustrates then that the methodology meets the objective of efficiently designing a deployment of containers for selective collection of urban solid waste.

We must conclude that the optimization of sites, where community storage facilities should be located highly, depends on the customer behaviour. The number of containers and therefore the cost associated with their location and transportation can be significantly reduced if a community is willing to co-operate by carrying their waste to the appropriate containers within a predefined radius, even if eventually these are not the nearest to their residence. A solidarity co-operation of the costumers is assumed in this section with the goal of reducing the number of collection points.

2.4 A heuristic for the deployment of collecting routes for urban recycle stations (eco-points)

2.4.1 Model development

In order to determine optimal routes for mobile multi-block containers, a strongly connected graph $G = (V, A)$ is assumed, composed of a node set V and an arc set A (directed edges representing street sections), such that the existence of a shortest path in terms of distance (or travel time) between each pair of points of V is guaranteed inside G. Let us suppose that set V contains the set J of nodes where waste is placed by the users in order to be collected in mobile multi-block containers. Each container is divided into sections (bins or blocks) and is associated to one route, to be determined by the optimisation model, that starts and ends at the same depot point (O) whose location is assumed to be fixed. Hence, the same index can be employed to simultaneously represent each container and its corresponding route.
The following notation is used in our formulation:
I: Set of routes for containers ($i \in I$). We assume that all vehicles are identical and have the same transport capacity, which corresponds to one container. All containers are homogeneous and contain $|L|$ blocks of capacity c. Let C be the total capacity of each container ($C = c \cdot |L|$).
J: Set of locations to be visited within the city ($j, j\prime \in J \subseteq V$). Depot point O is assumed to belong to this set J. Distances across network G between nodes j and $j\prime$are known and recorded in the $D = (djj\prime)$ matrix. Term $d_{jj\prime}$ indicates the minimum cost of travelling from point j to point $j\prime$. Once all the shortest paths between pairs of nodes of the set J have been established, a graph can be used as a new solution space whose set of vertices is J and where each arc (j, j') is weighted by $d_{jj\prime}$. Let $A(J)$ be the set of these direct arcs between pairs of points in J. Analogously, we will denote $A(S)$ as the set of arcs that connect pairs of points belonging to subset $S \subseteq V$.
K: Set of waste modalities ($k \in K$).
Quantity of waste modality k produced at point j is represented by means of the parameter $w_j^k \geq 0$.
Therefore:

$\sum_{k \in K} w_j^k$: Indicates the total waste generated at point $j \in J$.

$\sum_{j \in J} w_j^k$: Indicates the total waste of modality $k \in K$ produced within the city.

$\sum_{k \in K} \sum_{j \in J} w_j^k$: Evaluates the total amount of waste produced within the

city for all waste modalities.

Moreover, the following variables are required in the model:
y_i^k: Binary variable that takes value 1 if route $i \in I$ is employed for collecting waste of modality $k \in K$, and is equal to 0 otherwise.
x_{ij}^k: Binary variable that takes value 1 if location $j \in J$ is visited by route $i \in I$ and waste of modality $k \in K$ is collected, and is set to 0 otherwise.
$z_{jj\prime}^i$: Binary variable that takes value 1 if the connection $(j, j\prime)$ is used for container $i \in I$ along its route, and takes value 0 otherwise.
n_i^k : Integer variable that indicates the number of k-blocks ($k \in K$) packed in container $i \in I$.
Note that, with these variables, the waste volume of those shipments that visit location j with the purpose of collecting waste from modality k can be expressed by means of. $\sum_{i \in I} c \cdot n_i^k \cdot x_{ij}^k$

The objective function can be treated in three different ways depending on the problem.

1. When solving the classic BP, the objective is to minimise the number of containers used.

$$\min Z_1 := \sum_{i \in I} \sum_{k \in K} y_i^k$$

2. When solving the classic VRP, the objective is to minimise the total distance travelled.

$$\min Z_2 := \sum_{i \in I} \sum_{(j,j\prime) \in A(J)} d_{jj\prime} \cdot z_{jj\prime}^i$$

3. We propose applying a convex combination of both objectives with a parametric coefficient $\lambda \in (0, 1)$ to be calibrated by municipal waste collection services.

$$\min Z_3 := (1 - \lambda) \cdot \sum_{i \in I} \sum_{k \in K} y_i^k + \lambda \cdot \sum_{i \in I} \sum_{(j,j\prime) \in A(J)} d_{jj\prime} \cdot z_{jj\prime}^i$$

The nature of the variables used in the model allows us to build suitable programs to tackle different objectives. In our case, the following integer programming model inspired by the BP and VR schemes determines the deployment of routes for mobile eco-points for the selective collection of urban solid waste.

$$\min z_3 \equiv (1-\lambda)\cdot \sum_{i\in I}\sum_{k\in K} y_i^k + \lambda \cdot \sum_{i\in I}\sum_{(j,j\prime)\in A(J)} d_{jj\prime}\cdot z_{jj\prime}^i$$

$$\text{s.t.}\quad \sum_{k\in K} y_i^k \geq 1, \qquad \forall i\in I, \tag{2.10}$$

$$\sum_{i\in I}\sum_{k\in K} x_{ij}^k \geq 1, \qquad \forall j\in J, \tag{2.11}$$

$$y_i^k \leq n_i^k \leq |L|\cdot y_i^k, \qquad \forall i\in I, k\in K, \tag{2.12}$$

$$\sum_{k\in K} n_i^k \leq |L|, \qquad \forall i\in I, \tag{2.13}$$

$$\sum_{j\in J} w_j^k x_{ij}^k \leq c\cdot n_i^k, \qquad \forall i\in I, k\in K, \tag{2.14}$$

$$\sum_{i\in i} c\cdot n_i^k \cdot x_{ij}^k \geq w_j^k, \qquad \forall j\in J (j\neq O), k\in K, \tag{2.15}$$

$$\sum_{j\prime\in J|j\prime\neq O} z_{Oj\prime}^i = 1 \qquad \forall i\in I, \tag{2.16}$$

$$\sum_{j\in J|j\prime\neq O} z_{jO}^i = 1 \qquad \forall i\in I, \tag{2.17}$$

$$\sum_{j\prime\in J|(j,j\prime)\in A(J)} z_{jj\prime}^i - \sum_{j\prime\in J|(j\prime,j)\in A(J)} z_{j\prime j}^i = 0,\ \forall j\in J(j\neq O), i\in I, \tag{2.18}$$

$$y_i^k \geq x_{ij}^k;\ \sum_{j\prime\in J|(j\prime,j)\in A(J)} z_{j\prime j}^i \geq x_{ij}^k, \quad \forall j\in J(j\neq O), i\in I, k\in K, \tag{2.19}$$

$$\sum_{j\prime\in J|(j,j\prime)\in A(J)} z_{jj\prime}^i \leq |S|-1,\ \forall i\in I, \{S : S\subseteq J, O\notin S, |S|\geq 2\}, \tag{2.20}$$

$$x_{ij}^k, y_j^k, z_{jj\prime}^i \in \{0,1\}, n_j^k \in \{1,2,\ldots,|L|\}, \quad \forall i\in I, j\in J, k\in K. \tag{2.21}$$

Constraints (2.10) ensure that every route must collect at least one kind of waste. Constraints (2.11) establish that each location must be visited at least once. Constraints (2.12)–(2.13) guarantee consistency and an upper limit of the number of waste blocks of modality k within each container. Constraints (2.14) ensure that the capacity of collecting type-k waste in shipment i is sufficient to satisfy the demand generated at nodes j that are visited. Constraints (2.15) establish that the collection capacity of the total shipments that pass through point j is sufficient to collect waste of each modality generated at that location. Constraints (2.16)–(2.18) guarantee the flow conservation at nodes, this is the classic constraint of the VRP. Constraints (2.19) connect the decision variables used in the BP and VR blocks of the model. Constraints (2.20) are subtour elimination constraints. Constraints (2.21) indicate the nature of the variables used in the model.

2.4.2 Algorithm for solving the model

In the BP problem, items of different volumes must be packed into a finite number of bins (or containers), each of a fixed given volume, in order to minimise the number of bins used. The VRP addresses the determination of the optimal set of routes for a fleet of vehicles, in order to serve a given set of customers. Both models are of combinatorial nature and, in computational complexity theory, are classified as NP-hard problems. This fact justifies the use of algorithms that provide a good heuristic solution for the combined model. Furthermore, our proposed optimisation model is non-linear, as can be seen in Constraints (2.15). Therefore, in order to solve this complex problem, we propose an algorithm that simultaneously configures the containers and designs the routes that provide a good solution to our original problem. When analysing the set of restrictions in the model, the existence of quasi-separability between them can be appreciated, both in relation to the variables involved and the purpose pursued. Blocks (2.10)–(2.15) are aimed to establish the configuration of the multiple bin container, while blocks (2.16)–(2.18) determine the most appropriate route to be established taking into account the existing demand for waste to be collected according to their types. As was previously pointed out, Constraints (2.19) connect the decision variables used in the BP and VR blocks of the model.
If Constraints (2.15) can be algebraically manipulated, they can be expressed as follows:

$$\sum_{k\in K}\left(\sum_{i\in i} n_i^k \cdot x_{ij}^k\right) \geq \sum_{k\in K} \frac{w_j^k}{c}$$

Note that the following quotient determines the number of bin blocks required to collect the generation of k-waste at location j:

$$q_j^k = \left\lceil \frac{w_j^k}{c} \right\rceil$$

A matrix of $|J|\cdot|K|$ elements, whose individual components are the coefficients q_j^k , can be calculated according to the input data set. Note that:

- If $q_j^k > 0$, then node j must be visited at least once for collecting the k-waste generated at location j. More specifically,

- If $0 < q_j^k \leq |L|$, then node j does not need to be visited more than once in order to collect all the k-waste generated at location j. The number of bins needed to collect all the waste of modality k located at point j could be concentrated in a single shipment $i*$, thus adapting the configuration of the container to the characteristics of the point to be visited (Adapted configuration strategy, Option 2), or alternatively, it could be divided into several shipments that would repeatedly decrease the amount of waste to be collected. Among the multiple possible options for configuring the containers that would carry out these shipments is the one in which each type of waste is represented with a single bin in the container configuration (Fixed configuration strategy, Option 1).

- If $q_j^k > |L|$ then node j must necessarily be visited more than once in order to collect all the k-waste generated at location j. In fact, quotient $\left\lceil \frac{q_j^k}{|L|} \right\rceil$ indicates the minimum number of required visits to carry out at location j. Since the total waste generated at location j must be collected, according to Constraints (2.15), we must assume that if the demand of collecting the k-waste generated at location j is not satisfied by the visit of a first container, then a sequence of iterative visits must be implemented.

In the first phase of our algorithm, all the nodes j that have the entire universe of specific residues will be covered. Hence, a container with a completely diversified configuration (that is, each block collects a different modality of residues from the rest of the blocks that configure the container) will travel from the depot node to node j and will return following the shortest path.
In the second phase of our algorithm, there are only nodes where certain types of waste are missing. The choice is now between two strategies when configuring a multi-block container: either to use identical containers, in which there are no repeated blocks in their configuration; or to use heterogeneous containers, where certain block modalities have a greater presence, at the expense of others, when the configuration of the container is decided. Therefore, the efficiency of the algorithm that solves the problem depends on the correct choice in the order of action of two simple strategies:
Option 1 (see Fig. 2.13): To remove the plurality of blocks by using a single configuration for all collecting containers and extend their routes by visiting other nodes, until the bin blocks are filled with the corresponding specific materials.
Option 2 (see Fig. 2.14): To adapt the waste bin configuration of the visiting container to the characteristics of the node, since there is no total variety of waste modalities at that point. In the following heuristic, both strategies are

present.

- If solely the main program is used, by ignoring the call to the subroutine in step 4.1, then Option 1 would actually be applied, in which the con-

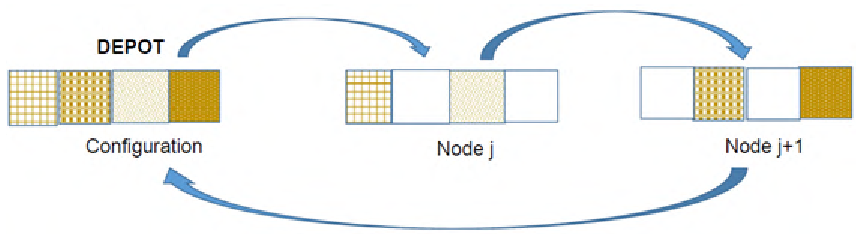

Figure 2.13: Route for visiting nodes following Option 1 as the configuration strategy.

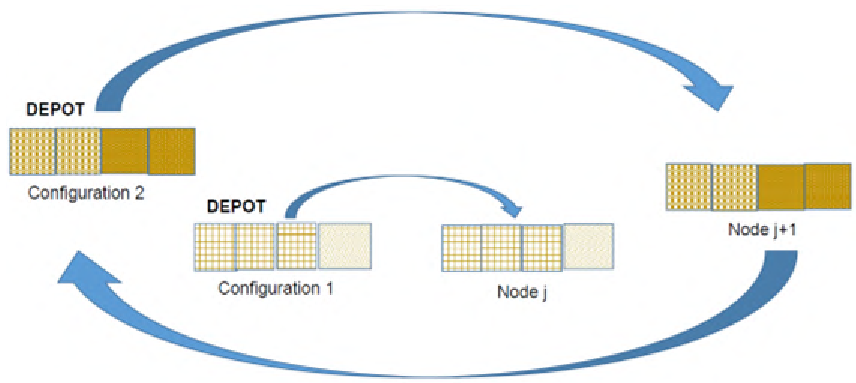

Figure 2.14: Two routes for visiting nodes with different container configurations adapted to the existing demand (Option 2).

tainer configuration is homogeneous for all vehicles. This situation would be applicable when the decision maker has only partial information on the demand at each point, that is, he/she knows that there is waste to be collected, but is unaware of its distribution in modalities, and hence optimising the bin packaging is not a prerequisite.

- If, alternatively, the use of the subroutine in step 4.1 is forced, then a configuration would be obtained of the multi-block container adapted to the existing demand in the nodes to be visited. By applying this second strategy (Option 2), the search for shorter vehicle routes would be combined with the design of the most suitable packing of blocks for the container. If the decision maker were in possession of all the information on the demand at each point (that is, the distribution of the types of waste to be collected), he/she could optimise the container packaging before starting the route of the collecting containers.

HEURISTIC_1 (main program)

1. **Sort** node set J according to the shortest distance from depot O.
2. **While,** at each node j, there exists all the universe of specific waste:
   2.1 **Generate** a route for a new visiting multi-block container i that follows a shortest path from depot O to node j along the street network.
   2.2 **Decrease** units from the unsatisfied demand at the visited node j.
3. **Identify** those nodes where some specific type of waste remains pending of collecting, but not for all types. **Let** T be this node set.
4. **While** there exist nodes j in T do
   4.1 **Select** the most appropriate configuration for a new multi-block bin i (by using subroutine HEURISTIC_2).
   4.2 **Generate** a route following a shortest path from depot O to node j along the street network.
   4.3 **Decrease** units of unsatisfied demand at each node j, according to the characteristics of the visiting multi-block bin.
   4.4 **If** the global demand of collecting specific waste has been satisfied, **Remove** node j from set T.
5. **End.**

HEURISTIC_2 (subroutine to decide configuration of container i)
For $k = 1$ to $|K|$ **do**

1. **If** $q_j^k \geq |L|$ **then** node j must be visited by means of a container i such that $n_i^k = |L|, \quad \forall k \in K$. **Return.**
2. **If** $\sum_{k \in K} q_j^k \geq |L|$ **then** node j must be visited by means of a container i such that $\sum_{k \in K} n_i^k = |L|$. **Return.**
3. **If** $\sum_{k \in K} q_j^k < |L|$ **then** node j must be visited by means of a container i such that $n_i^k = q_j^k \quad \forall k \in K$. **Return.**

In step 4.2, when determining the route employed to visit a group of nodes in the same shipment, the order of visits that produces the shortest distance travelled has been selected.
The tests carried out in the following section with randomly generated data have shown that Option 2 involves the generation of shorter collecting routes, while maintaining the number of routes required to guarantee the collection of all waste. Therefore, the use of Option 2 is preferable, whenever possible.

2.4.3 Results and discussion

In order to validate the proposed optimisation model, the Sioux Falls graph has been considered for different instances, and the number of nodes and the quantity of waste produced at each node are varied. In Fig. 2.15, the Sioux Falls network with 24 nodes and 38 edges (76 directed arcs) is shown, where one depot is located at node 1 and the waste generating points are the points labelled 2–18. Table 2.1 shows the results obtained when the problem of op-

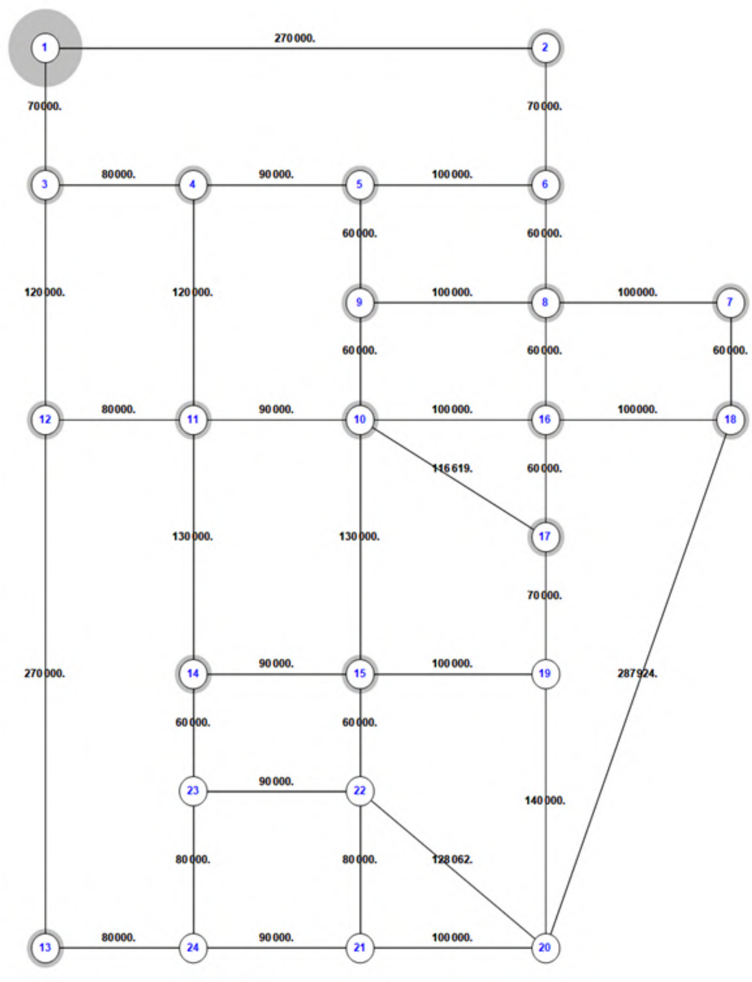

Figure 2.15: The Sioux Falls network with 24 nodes and 38 edges (76 directed arcs).

timally deploying waste collection routes is implemented for several instances. The objective function considered is the global distance travelled along the total

Number	Objective function	CPU (seconds)	Heuristic 1	%	Heuristic 2	%
10	$7.020 * 10^6$	3.45	$9.740 * 10^6$	38,75	$7.180 * 10^6$	2,28
11	$1.076 * 10^7$	4.82	$1.282 * 10^7$	30,02	$1.016 * 10^7$	3,04
12	$9.260 * 10^6$	4.73	$1.514 * 10^7$	63,5	$9.680 * 10^6$	4,54
13	$1.080 * 10^7$	5.90	$1.552 * 10^7$	43,7	$1.114 * 10^7$	3,15
14	$1.240 * 10^7$	19.77	$1.924 * 10^7$	55,16	$1.290 * 10^7$	4,03
15	$1.338 * 10^7$	76.79	$1.974 * 10^7$	48,42	$1.352 * 10^7$	1,05
16	-	7200 (3.08% Gap)	$1.986 * 10^7$	-	$1.692 * 10^7$	-
18	-	Out of memory	$2.294 * 10^7$	-	$1.907 * 10^7$	-
24	-	Out of memory	$4.124 * 10^7$	-	$3.301 * 10^7$	-

Table 2.1: Results obtained from the computational experience.

of routes required to collect the totality of randomly generated waste.

It can be observed that, for small size problems, the solver achieves the optimal solution, improving the solutions obtained through Heuristic Option 1 (fixed configuration for all containers) and 2 (container configuration adapted to demand). However, when the network size increases, the problem cannot be solved because of a problem of memory. As the experiments carried out have shown, when the exact model fails, both heuristic strategies succeed in finding solutions to the problem in a very short time. In particular, Strategy 2, which combines the BP and VR optimisation models, proves to be more efficient, as can be observed in Table 2.1. Computational experience was solved by means of the GuRoBi 9.1.2 in Python solver on a laptop with 16 GB of RAM and an Intel processor i7-1165G7 (with a 64-bit Windows 10 professional operating system and limiting the execution time to 2 h).

As is shown in Table 2.1, the difference in quality between the solutions provided by each of the two strategies is significant. The solutions generated after the application of Strategy 1 (which does not adapt the configuration of its compartments to the demand of the points to be visited) are poor, since they exceed the values obtained when the exact model is applied by more than 30%. In contrast, the solutions produced by Strategy 2 for the generation of collection routes with vehicle configurations adapted to demand are very close to those provided by the exact model: less than 5% in all instances of the experiment. Additionally, in order to exhaustively compare the efficiency of the two heuristic strategies developed, a computational experience has been carried out in a laboratory scenario composed of 39 nodes (sites identified as elements of node set J). The locations of nodes along the street network are illustrated in Fig. 2.16

From among the 39 nodes, 7 points have been selected (nodes labelled 1, 11, 19, 22, 29, 33, and 39) where the demand for a selective collection of waste is located. The node labelled with number 17 represents the depot from where the routes of the mobile eco-points start. Urban waste to be collected is sorted

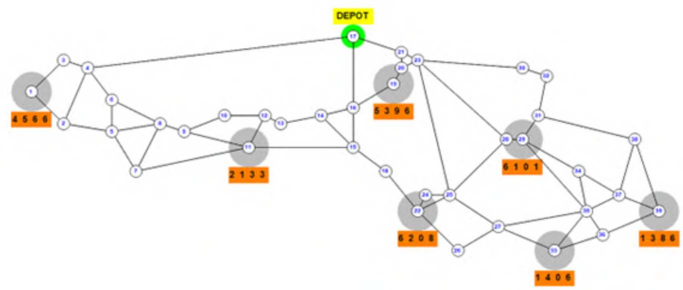

Figure 2.16: Network with 39 nodes and 7 waste collecting points.

into 4 categories and the quantity generated at each demand node is, for each category, a random integer number in the interval [1 kg, 9 kg]. The vehicle's collection capacity is limited to 4 kg in total and its configuration may follow a homogeneous type (that is, 4 different blocks, each with the capacity to collect 1 kg), or an adapted type (where blocks can be grouped in order to adapt it to the characteristics of the demand).
The first option is to use vehicles with the homogeneous configuration [1,1,1,1] in order to visit nodes where the demand has one representation of each modality. If at one node there are no units of a certain type, then the vehicle should prolong its route to visit other nodes and, hence, complete its collection capacity before returning to the depot. The results in this case are illustrated in Fig. 2.17. Node 1 has the demand [0,1,1,2]; after this node is visited by the vehicle of homogeneous configuration, the vehicle would store the distribution [0,1,1,1] and the subsequent demand that would remain at the node would be [0,0,0,1]. Therefore, this vehicle could continue its route in order to complete its loading capacity before returning to the depot; for example, visiting node 22 whose demand distribution is [0,2,1,1]. After visiting node 22, the vehicle would store [1,1,1,1] and then could return to the depot, whereas the demand of node 22 would now be [0,1,1,1].
A new vehicle of homogeneous configuration [1,1,1,1] could then sequentially revisit nodes 1 and 22, culminating the satisfaction of total demand with a second route. Applying this strategy, the cost of waste collection would be proportional to the total distance travelled by the vehicles on both routes.
Alternatively, another strategy could be applied, where the distribution of the 4 blocks that the vehicle can transport is configured prior to starting the route from the depot. The chosen configuration would be determined by the existing demand at the nodes that are to be visited. In the previously described case, where node 1 has the demand [0,1,1,2], it would be possible to dispatch a vehicle with the same configuration, since the sum of blocks (2 + 0+1 + 1) is exactly 4. In this way, the vehicle could travel directly from the depot to node 1 on both the outward and return journey, without deviating from the shortest route.

In Fig. 2.18, it can be observed that, subsequent to the vehicle visit, the demand

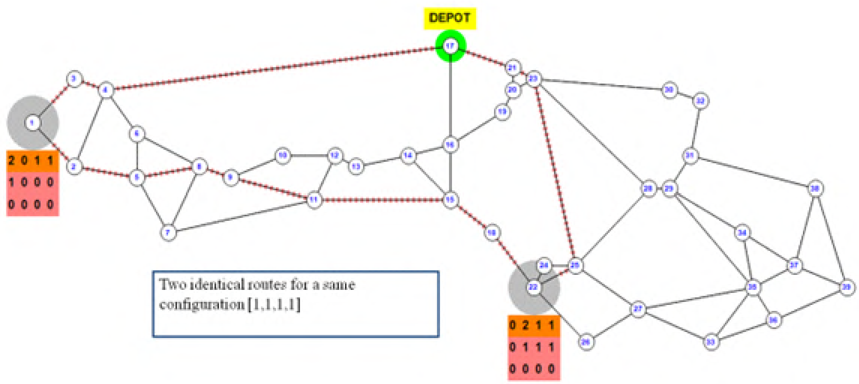

Figure 2.17: Routes for the homogeneous configuration [1,1,1,1].

located at node 1 is cancelled ([0,0,0,0]). Analogously this would occur with node 22 whose initial demand was [0,2,1,1]. Therefore, the application of this second strategy apparently enables the shortest routes to be taken more frequently by the mobile eco-points between the depot node and the demand points.

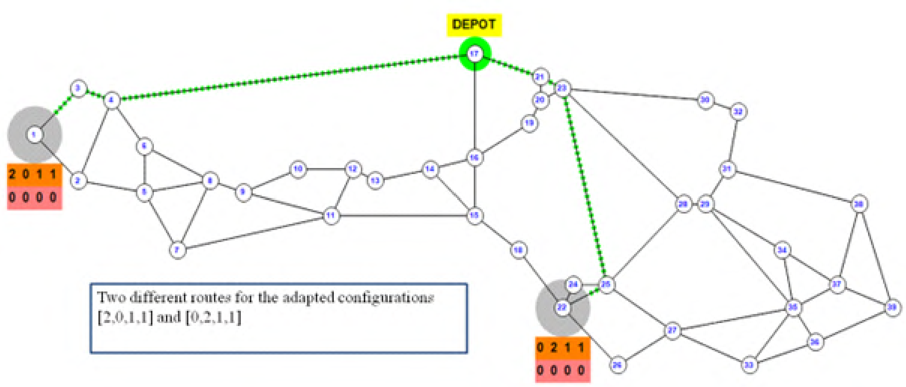

Figure 2.18: Routes for adapted configurations of containers.

A computational experience, consisting of performing 30 experiments on the network in Fig. 2.16 has been carried out in the aforementioned terms in order to be able to compare the effectiveness of the two strategies, which are named fixed (Option 1) and adapted (Option 2), respectively. Table 2.2 shows the results obtained.

The columns of Fixed configuration strategy show the numbers of routes and total kilometres that a vehicle must travel to cover all the demand with this strategy (Option 1). The columns of Adapted configuration strategy show the same data with the adapted strategy (Option 2). Finally, the columns of Absolute improvement and Relative improvement show the difference of the adapted strategy against the fixed strategy in terms of absolute cost and relative cost. In all the experiments carried out, the adapted configuration strategy has improved the total cost invested in the determination of routes with respect to the results obtained under the fixed configuration strategy. The improvement is above 19% on average. In the number of vehicle routes dispatched from the depot, the improvement remains inconclusive. As can be observed in Table 2.2, the number of routes is similar, regardless of the strategy used.

2.4.4 Conclusions

A methodology for the deployment of mobile multi-block containers for the selective collection of urban solid waste is proposed in this section. The mathematical optimisation model formulated for this purpose is identified as a combined version of the BP problem and the VR problem, whose computational complexity justifies the use of heuristics to face real-life scenarios on a large scale. Following that recommendation, a greedy algorithm has been developed to solve the proposed mathematical programming model. Two strategies are identified for the design of the configurations of the mobile multi-block containers that visit the demand nodes. The Sioux Falls network has been applied to show how the state-of-the-art solvers are incapable of solving medium-size instances, although both heuristic strategies provide solutions for large size instances. In particular, the strategy consisting of the generation of collection routes with vehicle configurations that are adapted to demand provides solutions of a very high standard. In order to ascertain the most efficient strategy for the implementation of a solving algorithm, a computational experience has been carried out on a laboratory instance. Results show that priority must be granted to the ability to adapt the configuration of the mobile multiblock container to the characteristics of the node to be visited.

This work provides useful information to environmental engineers and operators in the field of waste management, in the form of recommendations for possible performance enhancement modifications in the management of the logistics of urban waste collection.

	Fixed configuration strategy		Adapted configuration strategy		Absolute improvements		Relative improvements	
n	# Routes	# Km	# Routes	# Km	# Routes	# Km	# Routes	# Km
1	37	22587	37	19910	0	2677	0.00%	11.85%
2	32	24989	32	17505	0	7484	0.00%	29.95%
3	26	17959	27	15408	-1	2551	-3.85%	14.20%
4	29	18282	29	16743	0	1539	0.00%	8.42%
5	31	23388	31	18272	0	5116	0.00%	21.87%
6	34	25189	34	19476	0	5713	0.00%	22.68%
7	31	22758	30	17646	1	5112	3.23%	22.46%
8	37	24855	36	19972	1	4883	2.70%	19.65%
9	39	23740	39	21250	0	2490	0.00%	10.49%
10	29	20620	30	17749	-1	2871	-3.45%	13.92%
11	35	22384	35	19251	0	3133	0.00%	14.00%
12	35	23778	34	18737	1	5041	2.86%	21.20%
13	33	23559	33	16722	0	6837	0.00%	29.02%
14	38	26480	38	22413	0	4067	0.00%	15.36%
15	27	16832	27	15083	0	1749	0.00%	10.39%
16	36	27400	36	19980	0	7420	0.00%	27.08%
17	33	24442	33	19117	0	5325	0.00%	21.79%
18	35	21937	34	17108	1	4829	2.86%	22.01%
19	28	19032	28	15556	0	3476	0.00%	18.26%
20	32	23994	31	18744	1	5250	3.13%	21.88%
21	29	21940	29	17738	0	4202	0.00%	19.15%
22	25	17548	25	14478	0	3070	0.00%	17.49%
23	31	21119	31	18045	0	3074	0.00%	14.56%
24	33	22022	32	16820	1	5202	3.03%	23.62%
25	29	23679	28	15818	1	7861	3.45%	33.20%
26	35	22148	34	18306	1	3842	2.86%	17.35%
27	36	23894	36	19265	0	4629	0.00%	19.37%
28	24	17574	24	11680	0	5894	0.00%	33.54%
29	25	16908	25	13564	0	3344	0.00%	19.78%
30	35	22835	36	20814	-1	2021	-2.86%	8.85%

Table 2.2: Results obtained from the computational experience.

2.5 Chapter conclusions

In this chapter we have presented three different problems on the location and routing of containers (fixed and mobile) for the selective collection of urban solid waste. In section 2.2, we have modeled the determination of routes for the selective collection of urban solid waste within the historic centers where the distances between the installation and the client must be considered in the collection design system, as well as other considerations such as the size of the groups of containers, their capacities according to the nearest population and the installation cost of those containers in specific sites along the streets . In section 2.3, we have developed the work presented in the previous section assuming that customers are willing to have a solidarity behaviour, which consists of using a container assigned to them within a pre-stablished proximity radius, although the assigned container may not be necessarily the closest to their residence. For this scenario, a more efficient deployment of containers for selective collection of urban solid waste can be obtained. To illustrate the performance of the developed methodology, a computational experience has been carried out on a network with randomized data based on a zone belonging to city of Seville (Spain). In section 2.4, we have worked on the location and number of ecopoint containers, the determination of the fleet size for picking up the collected waste, and the design of itineraries are all intertwined, but present computationally difficult problems, and therefore must be solved in a sequential way. The mathematical optimization model formulated for this purpose has been identified as a combined version of BP problem and the VR problem, whose computational complexity motivates the use of heuristics to face large real-life scenarios. Following that recommendation, a greedy algorithm has been developed to solve the proposed mathematical programming model. Two strategies have been identified for designing the configurations of the mobile multi-block containers that will visit the demand nodes. The results obtained from the numerical simulations show the validation of the proposed methodology carried out for the Sioux Falls network benchmark and the specific real case study.

Chapter 3

Efficient strategies based on waiting time for operators/users in transport networks

3.1 Introduction

One of the most common objectives in the optimization problems in the transport of people is to minimize the total travel time of the users. In this chapter, we present two transport optimization problems whose objective is to minimize the travel time of users for trains, in the first problem, and for private vehicles, in the second problem.

In the first of the problems we have worked on the strategy called the Skip-Stop (or limited-stop) which is a mechanism that some transport companies follow to reduce travel times without the need to increase the fleet of vehicles. It consists of privileging a majority of passengers by offering shorter travel times, after having previously selected a group of low activity stations, where the trains will not stop to leave / pick up passengers.

Distinguishing between express and local stations, it appears first in the Northwestern Elevated of Chicago by July 1900. The skip-stop service was also first developed for the Chicago Metro system in 1947, and later implemented in Philadelphia and New York (see, Chicago-L.org. North Side Express Operations (n.d.)).

In 1947, system of express and local schedules provided by the Chicago Transit Authority (CTA) had become a nuisance for users, because the really available services for riders were hard to comprehend. In order to stop the fall in demand that was being happening, the CTA planned a clever way of running express service on its two-track lines. This proposal was known as the A/B skip-stop plan (see, Chicago-L.org. A/B Skip-Stop Express Service (n.d.)). As illustrative examples, the skip-stop operation mode has been used (and/or is being used) in practice in Santiago de Chile since 2001 (Freyss et al., 2013), in Bogota's Transmilenio system (Leiva et al., 2010), in Los Angeles's Metro Rapid system (Zhang et al., 2017), in Singapur's Transit Link (Chen et al., 2015b), and in the bus line that connects the East zone of Seville area with its historic center (Transportes Urbanos de Sevilla S. A. M. (TUSSAM) (n.d.)). Figure 3.1 shows 4 train services that run along a railway corridor with 7 stations. The services appear classified in types A and B and the stations, in types A, B and A/B. The horizontal sections in the polygonal lines represent the stopping time of the trains at the stations.

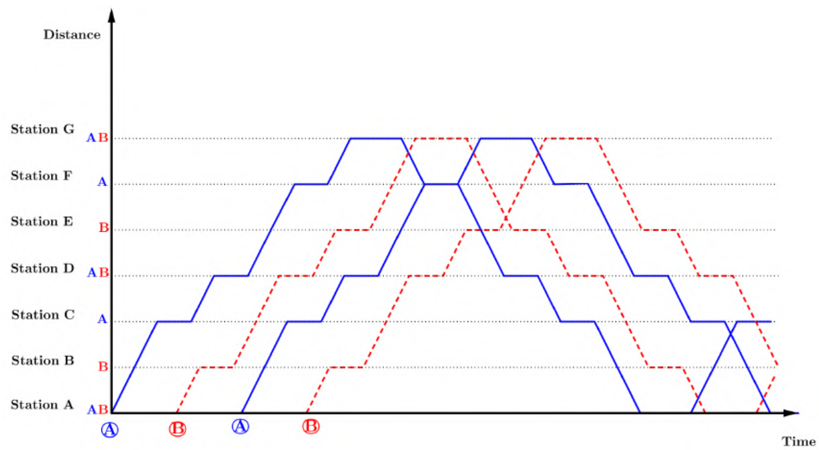

Figure 3.1: An A/B skip–stop plan for a set of four trains and seven stations.

The travel time between stations along a railway line consists of five components, usually identified as phases of acceleration, constant speed, inertia, braking and downtime. Several studies have shown that skip–stop operations can:

- Improve passenger waiting and in-vehicle times.
- Save operating costs (note that skipping stops allows vehicles to return to their depots in a shorter period of time; as a result, vehicles can be reused sooner).
- Reduce fuel (or power), as a result of not accelerating or decelerating at skipped stations.

Nevertheless, the prevalent purpose of introducing stop–skip patterns is not actually to save time, as discussed in Lee et al. (2014); Feng et al. (2013). The main aim is to better distribute passenger loads during peak periods, when trains are at their highest capacity level. The skip–stop services are especially suitable for those transit routes with unbalanced demand, so the stops with higher demand would be served by more vehicles, in order to improve the overall serviceability of the route.

The idea consists of trains stopping at stations so that there's more of an opportunity for folks to get on the trains at those stops. In order to avoid confusions and misunderstandings to passengers when skip–stop services are implemented, different means are usually used, like providing information boards and verbal indications at the stations.

As the Directive (EU) 2016/797 of the European Parliament and of the Council of 11 May 2016 on the interoperability of the rail system within the European Union (Official journal of the european union 26.5.2016. (n.d.)) points out, passengers must be provided with easily understandable travel information about rules applicable to them both in railway stations and in trains. For example, the two types of buses that operate in the east of Seville and share the same route (express line and normal line) are differentiated by signals on the front panel display of the vehicle. In the same way, the trains of their two respective lines that operate with skip–stop patterns in Santiago de Chile are also visually identified with green and red signals; hence, passengers know in advance what colour they should choose. Additionally, this information is also provided on screens at the station platforms, as well as through the public address system when trains are going to arrive at the stations.

Regarding the stop–skipping patterns for a one-way single track, the fundamental approaches are divided into deterministic (see, for instance, Mesa et al., 2009; Freyss et al., 2013) and stochastic approaches (Sun and Hickman, 2005). The deterministic form is derived from the description and analysis given by Vuchic (1973) in which stations along a line are classified into three groups, A, B and AB. The consideration of only two types of stations simplifies the degree of diversity, and travellers can more easily memorize the options to be able to configure their own routes between nodes of the transit network.

The trains in line A stop at the A and AB stations, while the trains belonging to line B stop at the B and AB stations. When they intend to alight at a B station, passengers boarding at an A station will need to transfer at an AB station onto line B. Thus, this disadvantage might affect the attractiveness of stop–skipping schedules.

The skip–stop operation scheme has been widely applied in bus transit services. Eberlein et al. (1998) proposed a real-time deadheading strategy to determine the dispatching time, deadhead vehicles and skip stations to minimize the total passenger cost. A heuristic algorithm was used to solve the model for operating the MBTA Green line. Sun and Hickman (2005) focused on the real-time stop–skipping control problem and presented an enumeration method with fast solving speed. Yu et al. (2012) studied the service reliability of a route in the city of Dalian and an optimized deadheading strategy for a part of the route by means of a heuristic algorithm. The advantages and disadvantages of four kinds of operating strategies were analysed in Fu and Liu (2003), and a nonlinear integer programming model was developed to solve the real-time dynamic transit operation problem, in a setting where the benefits of the operators and passengers were balanced. The skip–stop operation mode on rail transit lines has been described in the literature; for instance, see Vuchic (1976); Vuchic (2005).

Different mathematical tools have been used to solve the skip–stop service problems:

- Dynamic programming was adapted for this purpose in Ghoneim and Wirasinghe (1986); Nemhauser (1969).
- Greedy algorithms have been investigated for solving multiple train problems in Assad (1982).
- Fuzzy mathematical programming was the method used in Chang et al. (2000) for the Taiwan's highspeed rail.
- Formulations in terms of nonlinear integer programming were proposed in Fu et al. (2003); Larrain et al. (2010); Leiva et al. (2010); Wang et al. (2018) for solving dynamic versions of the skip–stop service problem.
- Metaheuristic genetic algorithms (GAs) have extensively been used for solving skip–stop scheduling problems. See, for instance, Sun et al. (2008); Niu (2012); Liu et al. (2013); Lin and Ku (2014); Chen et al. (2015b).
- Other metaheuristics such as Tabu search method (Cao et al., 2014) or bee colony algorithm (Chen et al., 2015b) have also been used for this context.

Matheuristics are heuristic algorithms made by the interoperation of metaheuristics and mathematical programming (MP) techniques (Boschetti et al., 2009). Matheuristics are optimization algorithms inspired in (or derived from) a mathematical model. An essential feature of the matheuristics is the implementation in some part of the solution procedure of characteristics or properties derived from a mathematical model. Metaheuristics topic has attracted the interest of researchers, as shown in the publication of monographs in journal special issues (Maniezzo et al., 2009, 2010).

We propose, in this chapter, to determine a skip–stop scheme through a three-phase methodology. In the first, we find the optimal strategy of skipping stops for a given train fleet and, in the second phase, we determine, by means of a matheuristic procedure, the optimal allocation for train itineraries. For this last purpose, we will develop the concept of proximity between configurations of train itineraries and, in accordance with Hall's method (Hall, 1970), design a matheuristic that optimizes the skip–stop strategy. In Section 3.2.1, a methodology of three phases for determining an optimal skip–stop scheme for train schedules is introduced. The first phase consists of formulating a nonlinear integer programming inspired in the multiple knapsack problem (MKP). The second phase is a matheuristic procedure adapted from the Hall's method. The third phase is a greedy algorithm of comparing and replacing. A computational experience, which illustrates the proposed procedure, is carried out in Section 3.2.2. Finally, conclusions are summarized in Section 3.2.3.

Routing in road networks on geographic maps is a problem of great practical interest (Preuss and Syrbe, 1997). Internet applications such as Google Maps, HERE WeGo, Baidu or Yandex are daily being used by a lot of users. The problem solved by those applications is the computation of the fastest route between a source point and a target point, both belonging to a connected network. In the underlying abstract model streets are assumed to be arcs which are adequately weighted in accordance to the corresponding travel times. In this way, determining the fastest route can be formalized as the classic point-to-point shortest path problem.

From a theoretical point of view, the problem of finding a shortest path from one node to another in a graph with fixed lengths (or fixed travel times) on its arcs is satisfactorily solved. In route planning problems for a single objective, Dijkstra's algorithm (Dijkstra, 1959) is preferably used to obtain the solution to the problem of determining the shortest path between two nodes of a connected graph. Its algorithmic complexity is O (m+n log n), where n is the number of nodes and m the number of arcs of the underlying graph (Leiserson et al., 1994). Unfortunately, this idealized view does not fit the real model because of transit times can widely vary during peak hours due to the existing traffic intensity. This time-dependence along the arcs can occur both in the determination of optimal public transport routes (bus, metro or dense networks of commuter train) and in private transport routes (own motorized vehicle, bicycle or similar). According to the report by Cookson and Pishue (2017), congestion in city centres causes massive losses (400 billion dollars per year in the USA). Consequently, literature on vehicle routing problems considering time-dependent travel times (TDVRP) has been increasing last years (see Gendreau et al., 2015; Cattaruzza et al., 2017; Vidal et al., 2021; Strasser et al., 2021).

In general, routing is the process of selecting the "best" paths in a graph $G = (V, A)$, where V is a set of nodes and A is a set of directed arcs. Most studies on routing problems have been carried out under the assumption that all the information needed to formulate such routing problems is time invariant (Toth and Vigo, 2014). In many practical applications, this assumption is generally not satisfied since travel times can vary exogenously due to traffic congestion, weather conditions, etc., or endogenously, depending on the decisions freely adopted by the driver, like modifying the speed of the vehicle at its discretion (for example, to adjust fuel consumption) or altering the travel time by including rest periods in driving. A classification of time-dependent routing problems with respect to various criteria can be seen in Pillac et al. (2013). Ichoua et al. (2003) propose that time-dependent driving speeds along network arcs can be modelled by piecewise constant vehicle speed functions. Under such an assumption, it is possible to ensure that there is a single fastest route between any pair of locations. Ghiani and Guerriero (2014) advise that the fastest route between two points in a network, where the travel times along their arcs change with time, must preserve the validity of the FIFO property, in the sense that leaving earlier a node cannot generate a later arrival. Rincon-Garcia et al.

(2018) highlighted that, in practice, time-dependent speed profiles are specific to each street or region and that the biggest difficulty in implementing vehicle routing software is to properly manage time-dependent trips according to the reality perceived by the user. Recently, Zeitz (2023) studies the complexity of fastest route problem on time-dependent networks with non-FIFO travel time functions, where travel time functions are piecewise linear on a sequence of breakpoints with integer coordinates. Author shows that the formulation of this problem is strongly NP-hard.

Computing shortest paths on real-time state in a transit network is carried out by pulling real-time data from an Application Programming Interface (API). Transit agencies have been developing online APIs to estimate real-time positions and arrival for their covered vehicles. Over a web-based or smartphone interface, a user enters a geo-coded origin and destination (O-D), and the algorithm must respond by returning shortest path based on the real-time state of the transit network. Recently, the industry is evolving towards an open model where the public agencies are making real-time bus data available on the web, allowing third party developers to use this data to provide transit information via web-based services (Jariyasunant et al., 2011). As illustration, Figure 3.2 shows the evolution of the percentage levels of hourly congestion in the city of Seville (Spain) throughout a day of the month of January in the years 2019 (sky blue colour), 2020 (blue) and 2021 (pink) day (source: www.tomtom.com/traffic-index/seville-traffic/, updated: 27/12/2022, 11:45 am).

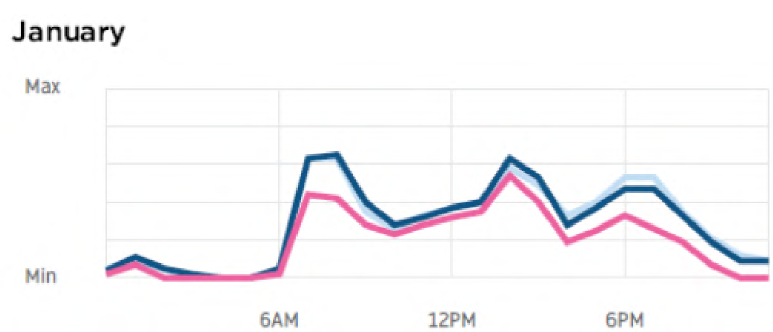

Figure 3.2: Evolution of the average intensity in the city of Seville throughout a day of the month of January in the years 2019-2021.

This figure empirically demonstrates the existence of a pattern in the daily evolution of traffic difficulty, which justifies the use of constant speed profiles for sections of a deterministic nature (piecewise constant functions).

Routing calculation offered by Google Maps, for instance, allows users to know an estimated time of arrival at the selected destination along the itinerary. Nevertheless, the procedure followed to obtain this prediction offered by commercial web applications is subject to business confidentiality.

In order to determine a fastest route in a network whose arcs are time-dependent, the logical approach might be to model the transit network as a graph, although time expanded (Schulz et al., 2000) or time dependent (Nachtigall, 1995) and run one of the widely used shortest path algorithms on it (Dijkstra, 1959). There has been research on speed up techniques to Dijkstra's Algorithm (Delling et al., 2009) as well as heuristic algorithms developed (Desaulniers and Villeneuve, 2000). There is also literature on routing in transit networks formulated as shortest path problems in stochastic and dynamic networks (Hall, 1986; Karaçan et al., 2003). Feder et al. (2007) addresses the routing problem when edge costs are approximately known but can be made more precise at run-time at a cost, a parallel to the problem of determining the wait and travel times at bus stops by accessing a real-time API. Likewise, Pallottino and Scutella (2003) developed an algorithm for transit graphs where the edge costs are known, but subject to small changes (i.e., updates in real-time information). Bast et al. (2007) developed an algorithm for routing in road networks. Their algorithm, called Transit Node routing is currently the fastest static routing technique available.

Determining a shortest itinerary in a network whose arcs are time-dependent can result in a diversity of optimal routes for a same origin-destination pair based on different departure times. Assuming the availability of the estimated data of the time required to travel along each section of the street network, once the departure time has been previously set, we propose in this work an efficient algorithm for obtaining faster routes on time-dependent arcs, in such a way that the sum of driving times is minimized, which in parallel allows improving fuel consumption and reducing associated polluting emissions. The possibility of introducing waiting periods in the nodes to optimize the total time spent on the trip has also been considered in the design of the proposed procedure. An experimental evaluation is carried out to show the effectiveness of the provided algorithm. Section 2 analyses Dijkstra's Algorithm designed to solve the problem of obtaining optimal paths in networks with not time-dependent arcs. Section 3 adapts the algorithm of Dijkstra to solve this problem with arc lengths that depend on time when the trip through the arc starts. Finally, conclusions are addressed in Section 4.

3.2 A matheuristic for optimizing skip-stop operation strategies in rail transit lines

3.2.1 Methodology

Let I be the set of stations of a railway corridor and let S be a train service set. We define the binary variable y_i^s; $i \in I, s \in S$. If $y_i^s = 1$, then station $i \in I$ is visited by transit service $s \in S$. This variable y_i^s will allow us to construct the solution vector: $Y = (0/1, 0/1, \ldots, 0/1)$. In order to deal with the demand the binary variables x_{ij}^s $(i, j \in I, s \in S)$, which take value 1 when the train s stops at both stations i and j, are defined.

Assume that travel demand from station i to station j depends on the time window considered and, subsequently, on the train s that transits during the time period considered. Hence, we can assume a preliminary study in which the potential demand from station i to j when train s passes through i has been estimated. This demand will be an initial data that we will denote by p_{ij}^s; $i, j \in I, s \in S$. This value will be associated to the population available to board train s at station i with destination j if the number of intermediate stops was 0. Define a new variable n_{ij}^s; $i, j \in I, s \in S$, that will be the number of effective intermediate stops between stations i and j for train service s. If stations i and j were consecutive stops along the railway corridor, then $n_{ij}^s = 0, \forall s \in S$. Otherwise, n_{ij}^s will be an integer value, superiorly bounded by the real number of intermediate stations between i and j. Now we can estimate the real number of travellers that will depend on the potential demand p_{ij}^s; $i, j \in I, s \in S$, and the number of intermediate stops n_{ij}^s; $i, j \in I, s \in S$. The greater number of intermediate stops introduced, the fewer number of travellers will be interested in the train service. We propose to use the following mathematical expression:

$$w_{ij}^s = \frac{p_{ij}^s}{\lambda \cdot n_{ij}^s + 1} \quad \forall i, j \in I, j > i \text{ [where parameter } \lambda > 0 \text{ must be calibrated]}$$

Therefore, if $n_{ij}^s = 0$ then the real demand (w_{ij}^s) coincides with the initial value p_{ij}^s. Any other positive value of n_{ij}^s will suppose a decrease in the effective number of travellers with respect to the initial value.
By means of the following linear integer programming model, we formulate the problem of maximizing the number of passengers for a transit line in which an indeterminate number of intermediate stops along the line can be omitted.
Objective and constraints: Maximize the number of passengers boarding trains at stations.

$$\max \quad \sum_{s \in S} \sum_{\substack{i,j \in I \\ j > i}} w_{ij}^s \cdot x_{ij}^s \tag{3.1}$$

$$\text{s.t.} \quad w_{ij}^s = \frac{p_{ij}^s}{\lambda \cdot n_{ij}^s + 1}, \forall i, j \in I, j > i \tag{3.2}$$

$$\left(\sum_{\substack{j \in I \\ j > i}} w_{ij}^s - \sum_{\substack{k \in I \\ i > k}} w_{ki}^s \right) \cdot y_i^s \leq c^s, \forall i \in I, \forall s \in S \tag{3.3}$$

$$x_{ij}^s \leq y_i^s; x_{ij}^s \leq y_j^s, \forall i, j \in I, j > i, \forall s \in S \tag{3.4}$$

$$x_{ij}^s, y_i^s \in \{0, 1\}; n_{ij}^s \in \mathcal{Z}^+ \cup \{0\}, \forall i, j \in I, \forall s \in S. \tag{3.5}$$

The objective function 3.1 maximizes the number of passengers for a transit line using a generalized multiple knapsack model; note that each train can be assumed like a backpack that may or may not pick up the demand for OD pair trips in their corresponding temporary windows. Constraints 3.2 identify the actual demand according to the number of intermediate stations. Constraints 3.3 prevent the capacity c^s of train s from being exceeded when it stops at each

station $i \in I$. Constraints 3.4 imply that if it is decided to pick up travellers from an origin–destination pair, the respective service s will have to stop at both stations. Constraints 3.5 indicate the nature of the variables used in the model.
The KP is a classic problem of combinatorial optimization that has been widely studied for more than a century (see, for example, Martello and Toth, 1980). It consists in selecting objects with the objective of filling a knapsack so that they provide the greatest profit without exceeding the storage capacity of the own knapsack. The MKP is a generalization of the standard KP where, instead of considering only a knapsack, it is about filling several knapsacks of different capacities.
The problem of MKP is strongly NP-complete and, due to its computational complexity, the need for using heuristic algorithms for generating good solutions is justified (see, for instance, Kellerer et al., 2004). Previous model can be considered a variant of the MKP model, where each service may be assimilated with a different knapsack that stores passengers boarding the train from stations, as long as the capacity of the vehicle allows it.

FIRST PHASE: Taking these precedents into account, we propose the heuristic shown in Table 3.1 for solving the optimization problem in order to determine the most effective deployment of skip–stop services along the rail corridor.
Once the model is solved, we will obtain a set of optimal solutions (train services) that will indicate the stops that each train must make in its corresponding service, in order to globally maximize the number of passengers in the transit system. The solutions obtained can be very different from each other. But remember that we do not want each service to be different from one another in general but to divide the services into type A and type B; so in most cases, the optimal solutions obtained from the previous optimization model would not be the final solution to our problem, because our skip–stop scheme must only contain two different types of services. That is why we must develop a second phase. For that purpose, we propose a heuristic which transforms the optimal solutions of the KP into the best possible configuration for our skip–stop problem.

Table 3.1: **Heuristic 1.**

1. **For each** $s \in S$ **do**

   a. **Set** $Y^s = (1, 1, \dots, 1) = (y_i^s)$.

   b. **Read matrix** (p_{ij}^s).

   c. **Compute matrix** (w_{ij}^s)

   d. **Set** $Q(s) = \sum_{i \in I} \sum_{\substack{j \in I \\ j > i}} w_{ij}^s \cdot x_{ij}^s$

   e. **For each** $l \in I$ **do**

      i. **While** $\sum_{s \in S} y_l^s \geq 1$[*] **and** $\sum_{i \in I} y_i^s \geq 2$ [**]

         1. **Set** $y_l^s = 0$ [parameters n_{ij}^s will change]
         2. **Re-compute matrix** (w_{ij}^s)
         3. **Set** $R(s) = \sum_{i \in I} \sum_{\substack{j \in I \\ j > i}} w_{ij}^s \cdot x_{ij}^s$
         4. **If** $R(s) > Q(s)$ **then** $Q(s) = R(s)$ **else** $y_l^s = 1$

- Prerequisite [*] means that there must be at least one train that stops at station l.

- Condition [**] means that there must be at least two stations where each service s is. forced to stop.

SECOND PHASE: The solutions obtained in the first phase are binary sequences where 1 in the ith position means train stops at stop i and 0, otherwise. From Hamming/rectangular/Euclidean metrics, we can calculate the matrix W of inter-distances between pair of binary sequences. In this way, we can classify the sequences according to a concept of proximity. This proximity is one-dimensional in nature. Therefore, we can construct a W matrix of inter-distances (from any metric, like Hamming or rectangular or euclidean) between pairs of service sequences and, based on the method published by Hall (1970) (where a spatial interpretation of maximum eigenvectors of the matrix $B = D–W$ is made) , we will obtain the relative position on the OX axis of the representative points. This relative position will allow us to establish a classification of trains and stations in types A and B. Matheuristic shown in Table 3.2 is inspired in the above-mentioned work.

Table 3.2: **Heuristic 2.**

STEP 1: **Set** $W = (w : ij)$ with $w_{ij} = y_i^j$.

STEP 2: **Compute** $D = d_{ij}$ as a diagonal matrix such that:

$$d_{ij} = 0, \text{ if } i \neq j$$

$$d_{ij} = \sum_{k=1}^{n} w_{ki}, \text{ if } i = j$$

STEP 3: **Compute** $B = D - W$.

STEP 4: **Compute** the set of eigenvalues of B and **Take only** the maximum α_{max}.

STEP 5: **Compute** v_{max} eigenvector associated to α_{max}.

THIRD PHASE: Note that coordinates of eigenvector v_{max} are values included in interval $[-1, 1]$. The ith point corresponding to ith coordinate of v_{max} indicates the relative position of the ith train service within interval $[-1, 1]$. The observed proximity between points will allow us to classify both trains and stations in types A and B. For this third phase, we propose the application of the greedy algorithm of comparing and replacing shown in Table 3.3

Table 3.3: **Heuristic 3.**

STEP 1: **Denote** by i and j the trains corresponding to the two points farthest from each other in the previous distribution. Let train i be included in type A and train j in type B.

STEP 2: **For** For each station k **do**

- **If** train i (type A) stops (i.e. is equal to 1 in kth coordinate) **and** train j doesn't stop (=0 in k) **then** INCLUDE station k in type A set.
- **If** train $i = 0$ and train $j = 1$ in kth coordinate **then** INCLUDE station k in type B set.
- **If** train $i = 1$ and train $j = 1$ in kth coordinate **then** INCLUDE station k in type AB set.

STEP 3: **For** each intermediate train m **do**

- **Compute** the number of coincident coordinates with respect to trains i and j.
- **Choose,** between i and j, the train where a higher number of coincidences was reached with m (assume, for example, i).
- **Force** coincidences in the binary sequences (changing values 0 to values 1) until trains i and m belong to the same type.

3.2.2 Computational experience

In order to illustrate the developed methodology, let us suppose a railway corridor with five stations where four trains circulate.
PHASE I. Assume that, as a result of the optimization procedure, the optimal sequence of skip–stop operations for the four trains is represented by means the sequences:

$$s = 1 : (1, 1, 0, 0, 1)$$

$$s = 2 : (0, 1, 0, 1, 0)$$

$$s = 3 : (0, 1, 1, 1, 0)$$

$$s = 4 : (0, 0, 1, 1, 1)$$

From these data, the distance matrix W between each pair of sequences can be built. For Hamming's distance, the rows of matrix W are the following:

PHASE II. According to the above-mentioned methodology, let us build from W, matrices D and B.

$$
\begin{matrix} (1,1,0,0,1) \\ (0,1,0,1,0) \\ (0,1,1,1,0) \\ (0,0,1,1,1) \end{matrix} \qquad \Rightarrow W = \begin{pmatrix} 0 & 3 & 4 & 4 \\ 3 & 0 & 1 & 3 \\ 4 & 1 & 0 & 2 \\ 4 & 3 & 2 & 0 \end{pmatrix}
$$

$$
\begin{pmatrix} 11 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 9 \end{pmatrix}; \quad B = D - W = \begin{pmatrix} 11 & -3 & -3 & -4 \\ -3 & 7 & -1 & -3 \\ -3 & -1 & 7 & -2 \\ -4 & -3 & -2 & 9 \end{pmatrix}
$$

The eigenvalues of matrix B, ordered from highest to lowest, are $14.8482, 11.3273,$ 7.82446 and 0. Note that the eigenvalue 0 is always sold by the construction of the matrix B. The meaning of each eigenvalue is the value of the objective function, as is referred in Hall (1970). Since our interest is the maximization of inter-distances, for better appreciating the existing differences between binary sequences, we select the maximum eigenvalue (14.8482) and calculate its associated eigenvector. The coordinates of this eigenvector provide us the position on the OX axis of the four points representative of the binary sequences. The eigenvector corresponding to the highest eigenvalue 14.8482 is:

$$
(-0.848468, 0.128783, 0.312184, 0.407501)
$$

which indicates the relative positions of train sequences 1, 2, 3 and 4 on the OX axis. In Figure 3.3, these four points are graphically represented in interval $[-1, 1]$.

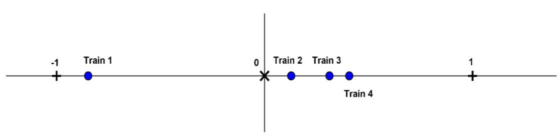

Figure 3.3: Relative positions in segment $[-1, 1]$ of four binary sequences associated to trains.

PHASE III. Note that the transit services, corresponding to the two points farthest from each other in Figure 3.3, are sequences 1 and 4. Let train 1 be included in type A and train 4 in type B.
According to this initial assignment of trains type A and type B, the stations are classified as follows:

$$\begin{array}{l}\text{Train 1= (1,1,0,0,1)}\\ \text{Train 2=(0,1,0,1,0)}\\ \text{Train 3=(0,1,1,1,0)}\\ \text{Train 4=(0,0,1,1,1)}\end{array} \quad \equiv \quad \begin{array}{c}\text{Type A}\\ \text{Type ?}\\ \text{Type ?}\\ \text{Type B}\end{array} \Rightarrow \begin{array}{l}\text{Station 1: Type A}\\ \text{Station 2: Type A}\\ \text{Station 3: Type B}\\ \text{Station 4: Type B}\\ \text{Station 5: Type A/B}\end{array}$$

When the intermediate trains are compared with their prototypes, some modifications are required. Since the point corresponding to service 2 is closer to the point corresponding to service 4 than the point corresponding to service 1 (see Figure 3.3), we must assign type B to service 2. To achieve a total match between services 2 and 4, it is necessary to modify both the configurations of the sequences of 2 and 4.

$$\begin{array}{l}\text{Train 1= (1,1,0,0,1)}\\ \text{Train 2=(0,1,1,1,1)}\\ \text{Train 3=(0,1,1,1,0)}\\ \text{Train 4=(0,1,1,1,1)}\end{array} \quad \equiv \quad \begin{array}{c}\text{Type A}\\ \text{Type B}\\ \text{Type ?}\\ \text{Type B}\end{array} \Rightarrow \begin{array}{l}\text{Station 1: Type A}\\ \text{Station 2: Type A/B}\\ \text{Station 3: Type B}\\ \text{Station 4: Type B}\\ \text{Station 5: Type A/B}\end{array}$$

Similar reasoning can be applied to the point corresponding to service 3. There are initially four coordinates that coincide when we compare the binary sequences associated with services 3 and 4. To achieve total match, we modify one more coordinate of service 3

$$\begin{array}{l}\text{Train 1= (1,1,0,0,1)}\\ \text{Train 2=(0,1,1,1,1)}\\ \text{Train 3=(0,1,1,1,1)}\\ \text{Train 4=(0,1,1,1,1)}\end{array} \quad \equiv \quad \begin{array}{c}\text{Type A}\\ \text{Type B}\\ \text{Type B}\\ \text{Type B}\end{array} \Rightarrow \begin{array}{l}\text{Station 1: Type A}\\ \text{Station 2: Type A/B}\\ \text{Station 3: Type B}\\ \text{Station 4: Type B}\\ \text{Station 5: Type A/B}\end{array}$$

Finally, it is possible to determine an optimal classification of trains and stations for the implementation of a skip–stop strategy.

3.2.3 Conclusions

The skip–stop operation represents a low-cost approach to improve the operation speed into transit networks without additional investments in infrastructure are required. A three-phase methodology for determining an optimal skip–stop scheme for train schedules has been introduced. Matheuristic procedure includes the formulation of a nonlinear integer programming inspired in the MKP, the application of an algorithm adapted from Hall's method and, finally, the implementation of a greedy algorithm of comparing and replacing. To illustrate the performance of the proposed procedure, the methodology has been applied to a

laboratory case.
As a future research, we propose to generalize the mathematical model by considering the possibility of trains not stopping at stations and, additionally, by taking transshipment into account, as feature which affects the passenger's behaviour.

3.3 An algorithm for finding optimal routes in transit networks with time-dependent arcs and waiting times at nodes

3.3.1 Modelling the problem

Following the notation previously used in the work of Dreyfus (1969), let $G = (V, A)$ a graph where V is a finite set of n nodes or vertices, and A is a finite set of m arcs that connect the nodes. Each arc can be denoted as the ordered pair (i, j) when it corresponds to the pair of vertices i and j. Let O (origin or initial node) and D (destination or terminal vertex) be two given nodes of G.
A path p from O to D in G is defined as the alternating sequence of vertices and arcs: $p = \{O = v_0, a_1, v_1, a_2, v_2, \ldots, a_k, v_k = D\}$, such that:

- $a_i \in A, \forall i = 1, \ldots, k; v_i \in V, \forall i = 1, \ldots, k-1$.
- $a_i = (v_{i-1}, v_i) \in A, \forall i = 1, \ldots, k$.
- $O, D \in V\{v_1, v_2, \ldots, v_{k-1}\}$.

The cost associated with arc (i, j) is defined as a positive real number c_{ij}. The cost of the path p will result the accumulated sum of the costs of the arcs that compose it:

$$c(p) = \sum_{(i,j)\in p} c_{ij}$$

In order to find the fastest path between points O and D, where the travel time through each arc (i, j) depends on the start time of point i, can be formulated as follows: Let us denote by $f_i(t)$ the minimum invested travel time until reaching destination D starting from point i at time t. Moreover, let $d_{ij}(t)$ be a positive value that represents the travel time invested when travelling arc (i, j) starting from point i at time t.
The recurring scheme:

$$\begin{cases} f_i(t) = \min_{j\neq i}[d_{ij}(t) + f_j(t + d_{ij}(t))] \\ f_D(t) = 0 \end{cases}$$

allows us to design an iterative algorithm that, based on the procedure devised by Dijkstra (1959), provides the fastest route, in this context of travel times in time-dependent arcs, to obtain $f_O(t)$.
Originally, Dijkstra's Algorithm is designed to solve the problem of obtaining optimal paths in networks with not time-dependent arcs. An encoding of this algorithm has presented in Ortega et al. (2022) as below.

Dijkstra's Algorithm (not time-dependent arcs)

1. **Read** graph $G = (V, A)$, adjacency matrix **Ady** and matrix D of displacement costs between pairs of adjacent points.

2. **Build** vector f that will store the minimum displacement costs to the origin from each node of G (initially, only the nodes adjacent to the origin will not have infinite value).

3. **Build** vector p that stores the successor nodes to each node of G in the optimal path (initially, only nodes adjacent to the origin can be determined).

4. **Initialize** set S of nodes explored by including only the origin point. **Initialize** set $LIST = G \setminus S$.

5. **While** $LIST$ is not empty:

   5.1 **Identify** index j^*, $j^* := Arg[min\{f(j) : j \in LIST\}]$.

   5.2 **Remove** index j^* from LIST.

   5.3 **For** each successor k of j^* included in $LIST$:
   If $f(k) > f(j^*) + d_{j^*k}(0)$ **then**

      a. **Update** $f(k) := f(j^*) + d_{j^*k}(0)$.

      b. **Update** $p(k) := j^*$.

6. **End.**

The following two figures graphically reproduce the same example used in Wen et al. (2014a) to illustrate the operative limitations of Dijkstra's algorithm when the network is time dependent. In Figure 3.4 a static situation is considered, where the weights of the arcs do not vary with time.

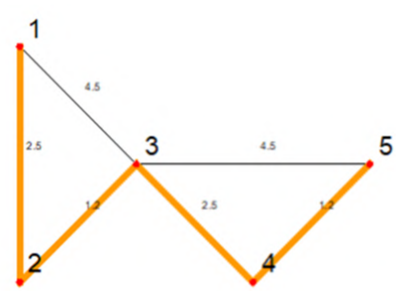

Figure 3.4: Fastest path between nodes 1 and 5 using Dijkstra's algorithm.

As illustrated Figure 3.4, the solution to the problem is the path $1-2-3-4-5$, and the time spent is $7.4(2.5 + 1.2 + 2.5 + 1.2)$ minutes, as can be verified by applying the step-by-step algorithm. Let now assume that arc $(3, 5)$ is time dependent as is shown in Figure 3.5.

1. When arc $(3, 5)$ is travelled, starting at node 3 in a time t included in the interval $[0, 4]$, the total time needed to complete arc $(3, 5)$ is 4.5minutes, as in the previous static case.

2. However, when the trip through the arc $(3, 5)$ starts after time 4, then the travel time needed to traverse arc $(3, 5)$ decreases to the value of 1.3 minutes. In that case, as shown in Figure 3.5, the fastest path between nodes 1 and 5 would be the sequence $1 - 3 - 5$, with a total invested time corresponding to $5.8(4.5 + 1.3)$ minutes.

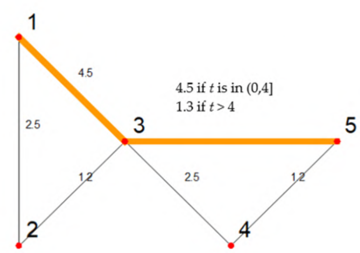

Figure 3.5: Fastest path between nodes 1 and 5 with time-dependent arcs and no waiting at nodes.

As was stated in Wen et al. (2014a), Dijkstra's algorithm cannot identify the shortest paths for this context of networks with arcs whose lengths may depend on time. Dijkstra's algorithm, as coded above, is also impractical for managing other user strategies, such as setting wait times on some nodes to benefit from shorter travel times after waiting.

Extending the notation used so far, let $d_{ij}(t)$ be the travel time required to complete arc (i, j) when the start time is t at node i. As will later be detailed, the variation form of $d_{ij}(t)$ will correspond to a stepped function that changes (increasing or decreasing) its value at certain instants and remains at that constant level until the next milestone.

In order to adapt the original algorithm of Dijkstra to solve the problem of determining shortest paths in networks whose arc lengths can depend on the time when the trip along each arc is started, we will make a series of modifications to the original network.

First, we will immerse the planar network in three-dimensional space by raising an axis orthogonal to the base plane for each of the nodes of the network. These

axes will indicate the temporal progress at their respective node. An arc (i, j), starting at level 0, indicates that no wait has been carried out at node i by the user before continuing the journey to node j. On the other hand, the arc (i, j) should start at a certain level t_k indicating that the user has made a wait in node i whose duration is t_k. The vertical distance between nodes i (level 0) and i (level t_k) should be interpreted as waiting time, since geographically there is no change in position. According to the previous assertion, the arc (i, j) that starts at level 0 will be weighted with the value $d_{ij}(0)$, while the arc (i, j) that starts at level t_k will have the value $d_{ij}(t_k)$ as the travel cost.
The descriptive data of this new network context includes the functional characterization of those arcs (i, j) whose travel time $d_{ij}(t)$ changes according to the exit time of node i. We will assume that the form of variation of $d_{ij}(t)$ corresponds to a piecewise-constant (step) function that changes its value at certain instants (increasing or decreasing) and staying at that level until the next milestone. In Figure 3.6 has graphically been represented an example of a step function that indicates that the time required to traverse the arc (i, j) is 5, if the start time of that trip takes place inside interval $[0, 7]$; 9, if the trip start takes place within $[7, 10]$; and so, on until the daily interval $[0, 24]$ is completed.

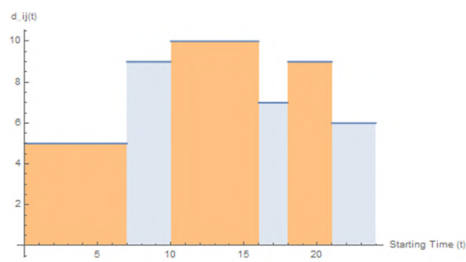

Figure 3.6: Description of $d_{ij}(t)$ as a piecewise-constant function.

According to Ortega et al. (2022), the number of times it would be necessary to replicate a node i along its vertical axis (indicating the consumption of waiting time) will be a function of the set of vertices j accessible from node i, denoted by $Succ(i)$, and the number of piecewise segments $n_i(j)$ that have been considered for explain the behaviour of the function $d_{ij}(t)$. Therefore, the number of replications of node i will be, in principle:

$$\sum_{j \in Succ(i)} n_i(j)$$

Actually, if we carry out a control of the time progress in each of the vertical axes that start from each node of the network, the number of replicated nodes can be substantially reduced. In fact, it will only make sense to consider the incorporation of the node associated with a travel time $d_{ij}(t_w)$,, obtained after

a waiting time t_w at node i, if the sum of both times t_w and $d_{ij}(t_w)$, improves, i.e., is less than the set value without the need to enter waits. This reasoning will be explicitly included when coding the algorithm.
The following example, that will be explained step by step, will illustrate the proposed methodology. Consider the planar network shown in Figure 3.7 (8 nodes and 13 arcs), which has previously been embedded in three-dimensional (3D) space. Suppose that origin and destination nodes are respectively labelled 1 and 8. At the midpoint of each arc of the graph, the numerical value (in red) corresponds to the time needed to traverse such arc when in absence of time dependency. In this static context, the fastest path between origin and destination points for these weights is the sequence $\{1, 3, 5, 7, 8\}$ (highlighted in orange) with a total length of 14.5.

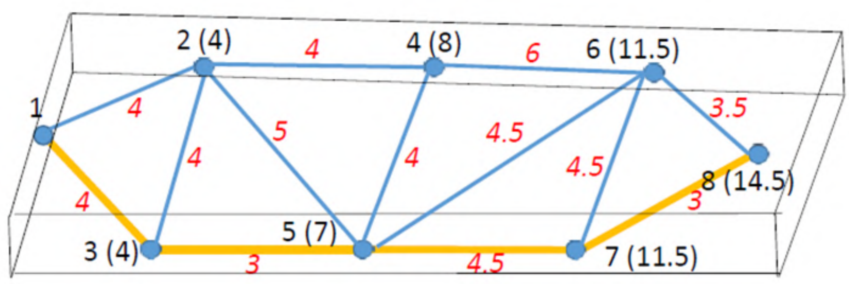

Figure 3.7: Instance of a planar network embedded in 3D-space.

Starting from each node of the network, vertical axes will be established where the new nodes to be replicated can be located. In order to have a reliable control of the time when we analyse the possible alternatives, we will relocate the intermediate nodes (that is, nodes $2-7$) to the heights that correspond on their respective axes with the shortest access time from the origin. Hence, in Figure 3.8, the intermediate nodes have been relocated to the corresponding level with the time necessary to reach them from the origin. The origin (1) and destination (8) nodes have not changed their location.
Suppose now that two of the three arcs starting from node 4 were time dependent. Specifically, from $t = 5$, in the arc (4.5) its travel time decreases from 4 to 1.5. For that same mark of $t = 5$, the arc $(4, 6)$ changes its travel time from 6 to 3. Similarly, suppose that the travel time of the arc $(5, 4)$ also changes as indicated in Figure 3.9.

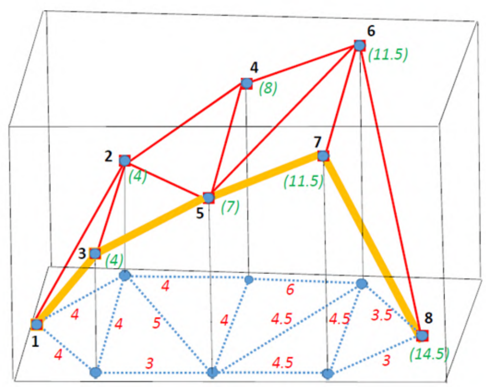

Figure 3.8: Network with replicated nodes in 3D-space.

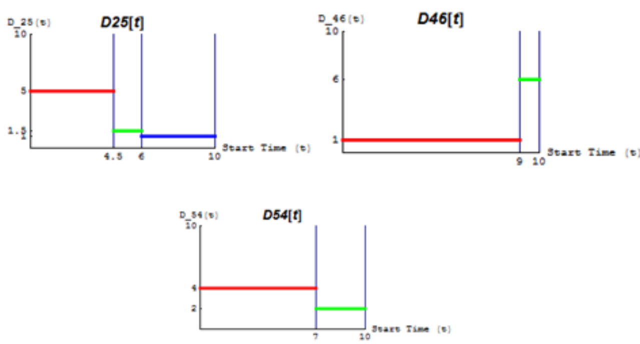

Figure 3.9: Graphical display of functions $d_{25}(t)$, $d_{46}(t)$ and $d_5 4(t)$.

Since node 2 is reachable from the origin point (1) in a time $t = 4$, the user will have two options if node 2 were part of the optimal route to the destination point (8):

1. Explore the already existing arcs starting from node 2, as possible continuation alternatives with the objective of building an efficient candidate route. This option does not involve the inclusion of new additional nodes nor modifications in the weights of the outgoing arcs from node 2.

2. Establish a strategic stop at node 2, with the expectation of obtaining a more competitive travel time on some outgoing arc. This second option forces node 2 to be replicated at the higher vertical level that results after adding the waiting time to the already existing arrival time at the node 2. The new node 9, on the same vertical axis of node 2, represents the option to spend 0.5 units of waiting time at node 2 before choosing to continue towards node 5; consequently, the arc $(2, 9)$ is weighted with the value 0.5. Similarly, the new node 10, located along the same vertical axis, represents

a waiting time of 1.5 units with reference to node 9. Note that both nodes 9 and 10 have been introduced as logical consequence of the configuration of function $d_{25}(t)$, which describes two level changes at starting times $t = 4.6$ and $t = 6$ along arc $(2, 5)$. Furthermore, two new arcs must be added to the 3D-graph, which will be weighted with the new travel times required to reach node 5 after applying a strategic waiting time at node 2. The new arc $(9, 5)$ must consistently end at a specific point located along the vertical axis corresponding to point 5, whose height coincides with the arrival time to such node 5 from origin point 1. In the case of arc $(9, 5)$ it will be necessary to introduce a new point, labeled 12 in Figure 3.10, because the access time to node 5 has changed (improving). In the case of the arc $(10, 5)$ it will not be necessary to introduce a new node on the vertical of node 5, since the access time does not change with respect to the previously existing one without applying the wait strategy.

Figure 3.10 illustrates the resulting final graph where the adapted Dijkstra's algorithm, coded in Section 3.3.2, can be applied to find the optimal connection route from point 1 to point 8. The consideration of functions $d_{25}(t), d_{46}(t)$ and $d_{54}(t)$, such that they were described in Figure 3.9, forces the introduction of new nodes $9, 10, 11$ and 12, as well as new arcs $(2, 9), (9, 10), (12, 5), (11, 4)$ - that represent waiting time options - $(9, 12), (9, 4), (10, 5), (10, 4), (12, 11), (12, 6)$ and $(12, 7)$ - that represent travelling times -. The sequence $\{1, 2, 9, 12, 11, 6, 8\}$, with a total length of 12.5, is the optimal route. The presence of node 9 in the optimal node sequence indicates that, in this case, the strategy of setting a waiting time at node 2 has been successful.

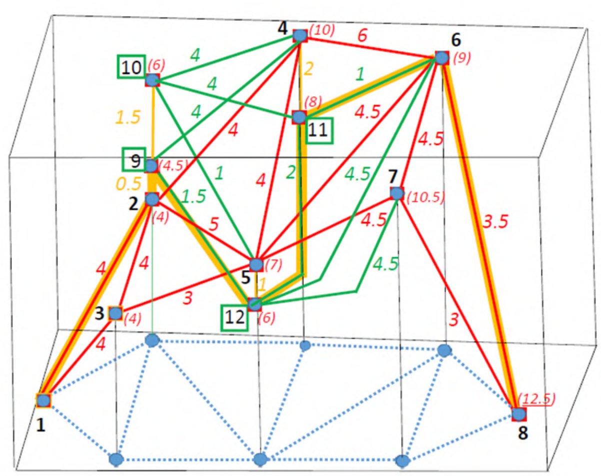

Figure 3.10: Optimal route connecting points 1 and 8 in 3D-space.

3.3.2 An adapted algorithm

The algorithm presented below includes the necessary modifications to be able to determine the fastest paths between pairs of nodes within a network, with the weights of their arcs being subject to variations according to known or forecast schedules in advance.

Adapted Dijkstra's Algorithm with time-dependent arcs

1. **Read** graph $G = (V, A)$, adjacency matrix **Ady** and matrix D of displacement costs between pairs of adjacent points.

2. **Build** vector f that will store the minimum displacement costs to the origin from each node of G (initially, only the nodes adjacent to the origin will not have infinite value).

3. **Build** vector p that stores the successor nodes to each node of G in the optimal path (initially, only nodes adjacent to the origin can be determined).

4. **Initialize** set S of nodes explored by including only the origin point. **Initialize** set $LIST = G \setminus S$.

5. **While** $LIST$ is not empty:

   5.1 **Identify** index j^*, $j^* := Arg[min\{f(j) : j \in LIST\}]$.

   5.2 **If** all output arcs from node j^* do not change in time **then**

   5.2.1 **Remove** index j^* from LIST.

   5.2.2 **For** each successor k of j^* included in $LIST$:
   If $f(k) > f(j^*) + d_{j^*k}(0)$ **then**
   a. **Update** $f(k) := f(j^*) + d_{j^*k}(0)$.
   b. **Update** $p(k) := j^*$.

   5.3 **else For each** output arc from **node** j^* towards some not visited node that changes over time and for each modification of the weight of this arc:

   5.3.1 **Replicate node** j^* **(let be** $j^{*'}$**)**, updating set V.

   5.3.2 **Add** an arc from j^* to $j^{*'}$, weighted with the required wait time at node j^*.

   5.3.3 **Repeat** for the new node $j^{*'}$ the same out connections that node j^* already had (with the corresponding new weights).

   5.3.4 **Add** an arc from j^* to $j^{*'}$, weighted with the required wait time at node j^*.

   5.3.5 **Update** set A according to 5.3.2 and 5.3.3.

   5.3.6 **Include** node $j^{*'}$ in set LIST.

6. **End.**

The complexity of this algorithm, that maintains the same structure of the original Dijsktra's algorithm, depends precisely on the number of arcs whose weights are subject to time-dependent changes, as well as the number of changes to be incorporated along the time horizon for each arc. Note that the section labelled 5c is only activated when time-dependent output arcs are detected.

3.3.3 Conclusions

In this subsection, the problem of determining the fastest paths with time-dependent arcs in transport networks has been analysed. There are numerous existing bibliographical contributions related to this subject, mainly due to the relevance that this issue acquires in logistics and travel planning for all types of users. Specifically, the methodology presented in Wen et al. (2014a), where two heuristic methods were proposed to solve the least cost path problem between a pair of nodes with a time-varying road network and a congestion charge. The tool developed by these authors was based on modifications of Dijkstra's algorithm, where a wait ban had been established on the nodes.
The fastest path algorithmic search technique with time-dependent arcs introduced in this contribution follows this same methodological line of adaptation of Dijkstra's algorithm to this context, which guarantees a high level of efficiency for the calculation of solutions. On the other hand, our contribution does allow the incorporation of strategic waiting times by extending the structure of the original connection graph, growing both in nodes and in arcs. It would be possible, however, to limit this growth of the solution container graph, having a reliable control of the time when the possible alternatives are analysed.

3.4 Chapter conclusions

In this chapter we have presented two different problems on transport optimization problems whose common interest is the management of waiting times from perspectives of operator or users and we have proposed a new matheuristic/algorithm respectively to solve its.
In section 3.2 we have worked on skip–stop strategies to reduce travel time of particular train services by not stopping (skipping) at less densely populated stations. This decision of omitting some stops reduces the travel time for the users within the vehicle and increases the speed of operation, favouring the provision of new transit services where are more necessary. In this work, the best A/B stop–skip patterns for a set of transit services along a railway corridor has been determined by means a three-phase methodology that includes the formulation of a nonlinear integer programming inspired in the multiple knapsack problem and the application of a heuristic algorithm based on mathematical properties (matheuristic).
In section 3.3 we have worked on an extension of the Shortest Path Problem that initially consists of finding a path with a minimum travel cost from one origin to one destination through a connected network. It is an important and well-

known problem, due to its wide range of applications in means of transport. The determination of the shortest routes in a network whose arcs depend on time (as consequence of traffic congestion, weather conditions, possible incidents, etc.) can result in a diversity of optimal routes for the same origin-destination pair based on different departure times. In this section has been shown that shortest path search techniques can continue to be valid if the original graphs are suitably extended, by duplicating nodes and arcs, and the solving algorithm is conveniently adapted to deal with networks whose arcs are time-dependent and/or the introduction of waiting times at nodes is allowed. The theoretical development has been illustrated with an example to clarify the concepts used throughout the article and to show the efficiency of the provided algorithm.

Chapter 4

Conclusions and future research lines

Below, we briefly summarize the main achievements and discuss possible future lines of research for each chapter. Chapter 1 presents three optimization problems related to improving people's quality of life. In the first contribution, a new criterion is provided for the design of rapid transit lines, so that the criterion of greater population does not have as decisive a role as it occurs in most existing models in the literature, but rather other factors be taken into account such as workplaces, the existence of tourist attractions or educational centers, among many others. In this way, we have proposed that those areas that present more imbalances in these factors come together to favor greater territorial cohesion, which should produce a reduction in the need to make forced trips. There are therefore two repercussions of the use of this new criterion for the design of urban transit lines in a metropolitan context: to cover the demand for trips for reasons not foreseen in the classic models of transport network planning and, furthermore, to reduce the existing territorial differences by uniting them through a rapid transit connection.

In the second of the discussed problems, we initially give a description of the park-and-ride service location problem and then model a user demand pattern that distinguishes different levels of information regarding the availability of facilities. of parking. Based on the assumption that users are aware of the number of vacant spaces currently offered by the city's parking facilities, consulted online through an internet device, it could be estimated, with knowledge of the historical evolution From the behavior of this data, the availability of parking spaces at the time of arrival at the car park, taking into account the characteristics of the traffic in the transit network. We develop, for this context, a mathematical programming formulation where the objective function integrates several cost attributes with the objective of determining efficient routes through a multimodal network; Among them, time-dependent transit times, minimization of parking costs and an additional attractiveness criterion related to the risk of finally not having a parking space available at the time of arrival at the chosen facility have been considered. This formulation is novel compared to other models previously published in the literature on this topic, because it considers restrictions that reasonably limit the travel time and the attractiveness of the candidate parking lots. We show that the problem can be quickly solved by using ad-hoc polynomial time algorithms, such as the modified Dijkstra shortest path algorithm. The results of a computational experiment based on data from the city of Seville (Spain) are reported, which empirically show the sensitivity of the model to the input parameters.

Finally, in this same chapter, we have solved the problem of locating charging stations for electric vehicles, partially transforming the old gas stations of the existing network, using typically complementary criteria: government point of view and perspective of the concessionary company. One of these objectives has been that, for each existing charging point, there is at least another nearby charging point, to prevent incidents. A conditional coverage model has been introduced into the formulation so that if the selected power station were to fail, the user could find another station a short distance away. Complementarily, the objective of companies to capture the largest number of potential customers has also been taken into account, maximizing the benefit without exceeding the existing capacity. Both objectives have been combined, giving rise to a bilevel formulation, which has been solved using various methodologies.

As future work, progress can be made in achieving new results in each of these lines of research. In the first of these lines, we could propose the formulation of a model to decide the layout of a new rapid transit line, starting from the assumption that there are others already determined. Likewise, and as a preliminary step, a methodology could be developed to divide the city into regions so that the result would be as homogeneous as possible in terms of the different criteria that would intervene in the configuration of the connection network. In the second of the lines, one could choose to incorporate a probabilistic approach to model the uncertainty along time of the number of available parking spaces that the user will find at the end of his journey by vehicle. Finally, in the third line of work, it would be interesting to incorporate technical criteria on the electrical voltage available at some (electric-)gas stations that could limit the capacity of charging points offered in certain locations.

Chapter 2 presents three problems on waste management. We have optimized the location of containers for the selective collection of urban waste in the first two thesis sections, minimizing the cost, as a generic criterion. In the first of the problems dealt with, we have assumed that users behave by minimizing the distance to the container used to deposit their waste, while in the second problem, we have considered that users do not necessarily have to go to the nearest container, but rather they can do it to a container that is within a predefined radius. We have seen through different computational experiences that this second problem improves the solution obtained with respect to the first. Finally, in the third problem we have verified that the previous design of the configuration of the multiple containers, which is going to be transported by the vehicle, significantly influences the determination of the optimal route for the selective collection of the different types of garbage generated in its respective places of generation. The mathematical model developed has been in the Bin Packing (BP) Problem and the Vehicle Routing (VR) Problem, two well-studied classic problems that are highly complex to solve using exact algorithms. We have designed a heuristic to be able to solve real problems on a large scale. We proposed two different strategies, the first, establishing a fixed configuration for all containers and, in the second, using a configuration adapted to the characteristics of the demand for waste to be collected. This second strategy has given us better results (least cost routes) in the computational experiments carried out. As future work we propose to apply the methodology, that has been shown to be efficient as a strategy in the deployment of ecopoints, to the daily case of selective collection of habitual waste. Likewise, we understand that it would be interesting to introduce, in the frequency of selective collection routes, certain climatic characteristics of the places (excessive heat to which the containers of certain materials may be exposed), as well as a predominant profile in certain areas of the city (greater density of bars that produce glass waste, etc.).

Finally, in chapter 3, two problems are presented in which the management of waiting times can favour obtaining efficient results, either for the whole system, or for the final interest of the user. In the first one, the problem of classifying the trains of a transport line into two types of trains A and B has been dealt with, so that the trains of type A have the possibility of stopping at the stations of type A or AB, while that type B trains can do it in type B or AB trains. Imposing the condition that all stations must be covered by at least one type of train, it is intended to determine said classification so that the average travel time of users is efficiently reduced. To solve this problem, we have proposed a mathematical methodology consisting of three phases. In the first, an optimization model is applied to the problem of establishing where each train would have to stop in order to maximize the trip coverage of demanding passengers, taking into account that the demand of these users decreases as their waiting time increases at station platform. The second phase consists of, with the Hamming distance, seeing which two trains are further away (they differ in more stops) in order to lead the configurations of the routes of the rest of the trains, which must finally be type A or type B.

The second of the problems addressed in this chapter is the design of an algorithm to solve the shortest path between two points of a transport network, where the cost of traversing its arcs may depend on the time of the start of the journey. Additionally, it includes the possibility that the user can strategically enter waiting times in the nodes, with the purpose of taking advantage of a better situation in traffic conditions that allows him to reach his destination using less time. Inspired by the idea initially proposed by Wen et al. (2014a), a decision graph is built from the geographic graph of connections, on which the solution of the problem is obtained using a modification of Dijkstra (1959) algorithm. Since the extension of the original graph is controlled in its growth (in number of new nodes and in number of new arcs), the approach efficiency in obtaining solutions is guaranteed.

List of Abbreviations

ALNS	Adaptive Large Neighbourhood Search
API	Application Programming Interface
B&B	Branch and Bound
BKP	Bounded Knapsack Problem
BP	Bin Packing
BPP	Bin Packing Problem
BRT	Bus Rapid Transit
BSA	Backtracking Search Algorithm
CARP	Capacitated Arc Routing Problem
CCP	Conditional Covering Problem
CFRLM	Capacitated Flow Refueling Location Model
CKP	Continuous Knapsack Problem
CS	Charging Station
CTA	Chicago Transit Authority
CVRP	Capacitated Vehicle Routing Problem
DES	Destination
EU	European Union
EV	Electric Vehicle
FIFO	First In, First Out
FPTAS	Fully Polynomial Time Approximation Schemes
FRLM	Flow Refueling-Location Model
FRLP	Flow Refueling-Location Problem
GA	Genetic Algorithm
GINA	Greedy Insertion of Nodes along an Alignment
ILP	Integer Linear Programming
INE	National Institute of Statistics
ITS	Intelligent Transport Systems
KP	Knapsack Problem
MCVRP	Multi-Compartment Vehicle Routing Problem
MdKP	Multidimensional Knapsack Problem
MILP	Mixed Integer Linear Programming
MIP	Mixed Integer Programming
MKP	Multiple Knapsack Problem

MP	Mathematical Programming
MSW	Municipal Solid Waste
NP	Nondeterministic Polynomial time
OD	Origin-Destination
ODIN	Overflowing Deviated to Immediate Neighbourhood
QKP	Quadratic Knapsack Problem
RKP	Rectangular Knapsack Problem
SQKP	Symmetric Quadratic Knapsack Problem
SSP	Subset Sum Problem
TDVRP	Time-Dependent Travel Times Vehicle Routing Problem
TSP	Travelling Salesman Problem
UNFCCC	United Nations Framework Convention on Climate Change
UKP	Unbounded Knapsack Problem
VR	Vehicle Routing
VRP	Vehicle Routing Problem

List of Figures

List of Tables

Bibliography

The future of the eu automotive sector — think tank — european parliament. URL https://www.europarl.europa.eu/thinktank/en/document/IPOL_STU(2021)695457.

Official journal of the european union 26.5.2016. URL https://www.era.europa.eu/activities/technical-specifications-interoperability_en.

Executive summary – global ev outlook 2022 – analysis - iea. URL https://www.iea.org/reports/global-ev-outlook-2022/executive-summary.

Buscador de estaciones de servicio repsol. URL https://www.repsol.es/buscador-eess-y-puntos-de-recarga/.

Knapsack problem - google académico. URL https://scholar.google.com/scholar?q=knapsack+problem&hl=es&as_sdt=0%2C5&as_ylo=2022&as_yhi=2022.

M. Akhtar, M. Hannan, R. Begum, H. Basri, and E. Scavino. Backtracking search algorithm in cvrp models for efficient solid waste collection and route optimization. *Waste Management*, 61:117–128, 2017.

L. Alçada-Almeida, J. Coutinho-Rodrigues, and J. Current. A multiobjective modeling approach to locating incinerators. *Socio-Economic Planning Sciences*, 43(2):111–120, 2009.

A. S. Ameneiro. Proyecto de aparcamiento inteligente en el parque científico y tecnológico cartuja, 2019. URL https://www.diariodesevilla.es/sevilla/aparcamiento-inteligente-PCT-Cartuja_0_1325567588.html.

E. Angelelli and M. G. Speranza. The periodic vehicle routing problem with intermediate facilities. *European journal of Operational research*, 137(2):233–247, 2002.

O. Apaydin and M. T. Gonullu. Route time estimation of solid waste collection vehicles based on population density. *Global Nest Journal*, 13:162–169, 6 2011. ISSN 11084006. doi: 10.30955/GNJ.000739.

A. Aremu. In-town tour optimization of conventional mode for municipal solid waste collection. *Nigerian Journal of Technology*, 32(3):443–449, 2013.

R. Arnott and P. Williams. Cruising for parking around a circle. *Transportation Research Part B: Methodological*, 104:357–375, 2017. ISSN 0191-2615. doi: https://doi.org/10.1016/j.trb.2017.07.009. URL `https://www.sciencedirect.com/science/article/pii/S0191261516309882`.

C. Artigues, M.-J. Huguet, F. Gueye, F. Schettini, and L. Dezou. State-based accelerations and bidirectional search for bi-objective multi-modal shortest paths. *Transportation Research Part C: Emerging Technologies*, 27:233–259, 2013.

J. Asamer, M. Reinthaler, M. Ruthmair, M. Straub, and J. Puchinger. Optimizing charging station locations for urban taxi providers. *Transportation Research Part A: Policy and Practice*, 85:233–246, 2016.

A. A. Assad. A class of train-scheduling problems. *Transportation Science*, 16:281–310, 8 1982. ISSN 00411655. doi: 10.1287/TRSC.16.3.281. URL `https://pubsonline.informs.org/doi/abs/10.1287/trsc.16.3.281`.

E. Balas and E. Zemel. An algorithm for large zero-one knapsack problems. *operations Research*, 28(5):1130–1154, 1980.

N. J. Bandara, J. P. A. Hettiaratchi, S. Wirasinghe, and S. Pilapiiya. Relation of waste generation and composition to socio-economic factors: a case study. *Environmental monitoring and assessment*, 135(1):31–39, 2007.

E. Barrena, D. Canca, and F. A. Ortega. Improving circulation plans of rolling stock in railway networks by means of the optimal location of depots. *XXIII EURO Working Group on Locational Analysis, Malaga, Spain*, 2016.

E. Barrena, D. Canca, and F. A. Ortega. Routing vehicle fleets during disaster relief. In *Proceedings of the VIII International Workshop on Locational Analysis and Related Problems (2017). Edited by Marta Baldomero-Naranjo, Inmaculada Espejo-Miranda, Luisa I. Martínez-Merino, Antonio M. Rodríguez-Chía, Diego Ruiz-Hernández*, pages 15–16, 2017.

E. Barrena, D. Canca, F. A. Ortega, and R. Piedra-de-la Cuadra. Optimizing container location for selective collection of urban solid waste. *Waste Management and the Environment IX*, 231:1, 2019.

R. Barry, M. Ralph, and M. E. Hanna. Quantitative analysis for management. *Eight Edition, Prentice Hall: Upper Saddle River, New Jersey*, pages 562–599, 2003.

H. Bast, S. Funke, P. Sanders, and D. Schultes. Fast routing in road networks with transit nodes. *Science*, 316(5824):566–566, 2007.

H. Becker and L. S. Buriol. An empirical analysis of exact algorithms for the unbounded knapsack problem. *European Journal of Operational Research*, 277(1):84–99, 2019.

T. Belgacem and M. Hifi. Sensitivity analysis of the optimum to perturbation of the profit of a subset of items in the binary knapsack problem. *Discrete Optimization*, 5(4):755–761, 2008.

J. Beliën, L. D. Boeck, and J. V. Ackere. Municipal solid waste collection and management problems: A literature review. *Transportation Science*, 48(1): 78–102, 2014. ISSN 00411655, 15265447. URL `http://www.jstor.org/stable/43666997`.

R. Bellman. On a routing problem. *Quarterly of applied mathematics*, 16(1): 87–90, 1958.

I. Benenson, K. Martens, and S. Birfir. Parkagent: An agent-based model of parking in the city. *Computers, Environment and Urban Systems*, 32(6):431–439, 2008. ISSN 0198-9715. doi: https://doi.org/10.1016/j.compenvurbsys.2008.09.011. URL `https://www.sciencedirect.com/science/article/pii/S0198971508000689`. GeoComputation: Modeling with spatial agents.

R. Benkoczi, B. Bhattacharya, Y. Hu, C.-H. Lin, Q. Shi, and B.-F. Wang. Efficient algorithms for the conditional covering problem. *Information and Computation*, 219:39–57, 2012.

N. Berkeley, D. Bailey, A. Jones, and D. Jarvis. Assessing the transition towards battery electric vehicles: A multi-level perspective on drivers of, and barriers to, take up. *Transportation Research Part A: Policy and Practice*, 106:320–332, 12 2017. ISSN 09658564. doi: 10.1016/j.tra.2017.10.004.

L. Bertazzi, M. G. Speranza, and W. Ukovich. Minimization of logistic costs with given frequencies. *Transportation research part B: Methodological*, 31(4): 327–340, 1997.

B. Bhatta. Causes and consequences of urban growth and sprawl. *Advances in Geographic Information Science*, pages 17–36, 2010. doi: 10.1007/978-3-642-05299-6_2. URL `https://link.springer.com/chapter/10.1007/978-3-642-05299-6_2`.

X. Bing, M. de Keizer, J. M. Bloemhof-Ruwaard, and J. G. van der Vorst. Vehicle routing for the eco-efficient collection of household plastic waste. *Waste management*, 34(4):719–729, 2014.

P. Bonsall and I. Palmer. Modelling drivers' car parking behaviour using data from a travel choice simulator. *Transportation Research Part C: Emerging Technologies*, 12(5):321–347, 2004.

M. A. Boschetti, V. Maniezzo, M. Roffilli, and A. Bolufé Röhler. Matheuristics: Optimization, simulation and control. In *International workshop on hybrid metaheuristics*, pages 171–177. Springer, 2009.

G. Bruno, M. Gendreau, and G. Laporte. A heuristic for the location of a rapid transit line. *Computers & Operations Research*, 29:1–12, 1 2002. ISSN 0305-0548. doi: 10.1016/S0305-0548(00)00051-4.

K. Buhrkal, A. Larsen, and S. Ropke. The waste collection vehicle routing problem with time windows in a city logistics context. *Procedia-Social and Behavioral Sciences*, 39:241–254, 2012.

V. Cacchiani, M. Iori, A. Locatelli, and S. Martello. Knapsack problems-an overview of recent advances. part i: Single knapsack problems. *Computers & Operations Research*, page 105692, 2022a.

V. Cacchiani, M. Iori, A. Locatelli, and S. Martello. Knapsack problems-an overview of recent advances. part ii: Multiple, multidimensional, and quadratic knapsack problems. *Computers & Operations Research*, page 105693, 2022b.

D. Canca and E. Barrena. The integrated rolling stock circulation and depot location problem in railway rapid transit systems. *Transportation Research Part E: Logistics and Transportation Review*, 109:115–138, 2018.

Z. Cao, Z. Yuan, and D. Li. Estimation method for a skip-stop operation strategy for urban rail transit in china. *Journal of Modern Transportation*, 22:174–182, 9 2014. ISSN 21960577. doi: 10.1007/S40534-014-0059-6.

I. Capar and M. Kuby. An efficient formulation of the flow refueling location model for alternative-fuel stations. *Iie Transactions*, 44(8):622–636, 2012.

S. Carrese, V. Cuneo, M. Nigro, R. Pizzuti, C. F. Ardito, and G. Marseglia. Optimization of downstream fuel logistics based on road infrastructure conditions and exposure to accident events. *Transport Policy*, 2019.

D. Cattaruzza, N. Absi, D. Feillet, and J. González-Feliu. Vehicle routing problems for city logistics. *EURO Journal on Transportation and Logistics*, 6(1): 51–79, 2017.

T. M. Chan. Approximation schemes for 0-1 knapsack. In *1st Symposium on Simplicity in Algorithms (SOSA 2018)*. Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik, 2018.

Y. H. Chang, C. H. Yeh, and C. C. Shen. A multiobjective model for passenger train services planning: application to taiwan's high-speed rail line. *Transportation Research Part B: Methodological*, 34:91–106, 2 2000. ISSN 0191-2615. doi: 10.1016/S0191-2615(99)00013-2.

E. Chaniotakis and A. J. Pel. Drivers' parking location choice under uncertain parking availability and search times: A stated preference experiment. *Transportation Research Part A: Policy and Practice*, 82:228–239, 2015. ISSN 0965-8564. doi: https://doi.org/10.1016/j.tra.2015.10.004. URL `https://www.sciencedirect.com/science/article/pii/S0965856415002542`.

S. S. Chaudhry, I. D. Moon, and S. T. McCormick. Conditional covering: greedy heuristics and computational results. *Computers & operations research*, 14 (1):11–18, 1987.

S. Chen, Y. Shi, X. Chen, and F. Qi. Optimal location of electric vehicle charging stations using genetic algorithm. In *2015 17th Asia-Pacific Network Operations and Management Symposium (APNOMS)*, pages 372–375. IEEE, 2015a.

X. Chen, B. Hellinga, C. Chang, and L. Fu. Optimization of headways with stop-skipping control: a case study of bus rapid transit system. *Journal of Advanced Transportation*, 49:385–401, 4 2015b. ISSN 2042-3195. doi: 10.1002/ATR.1278. URL `https://onlinelibrary.wiley.com/doi/full/10.1002/atr.1278`.

Z. Chen, J. C. Xia, B. Irawan, and C. Caulfied. Development of location-based services for recommending departure stations to park and ride users. *Transportation Research Part C: Emerging Technologies*, 48:256–268, 2014.

Chicago-L.org. Chicago-l.org. north side express operations, a. URL `https://www.chicago-l.org/operations/lines/route_ops/north.html`.

Chicago-L.org. Chicago-l.org. a/b skip-stop express service, b. URL `https://www.chicago-l.org/operations/lines/route_ops/A-B.html`.

F. Chu, N. Labadi, and C. Prins. Heuristics for the periodic capacitated arc routing problem. *Journal of Intelligent Manufacturing*, 16(2):243–251, 2005.

L. C. Coelho and G. Laporte. A branch-and-cut algorithm for the multi-product multi-vehicle inventory-routing problem. *International Journal of Production Research*, 51(23-24):7156–7169, 2013.

M. Coffey and A. Coad. Collection of municipal solid waste in developing countries, united nations human settlements programme (un-habitat), 2010.

E. G. Coffman, J. Csirik, G. Galambos, S. Martello, and D. Vigo. Bin packing approximation algorithms: survey and classification. In *Handbook of combinatorial optimization*, pages 455–531. 2013.

K. L. Cooke and E. Halsey. The shortest route through a network with time-dependent internodal transit times. *Journal of mathematical analysis and applications*, 14(3):493–498, 1966.

G. Cookson and B. Pishue. Inrix global traffic scorecard–appendices. *INRIX research*, 2017.

A. Corberán and C. Prins. Recent results on arc routing problems: An annotated bibliography. *Networks*, 56(1):50–69, 2010.

T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein. *Introduction to algorithms*. MIT press, 2022.

L. A. Coward and N. A. Salingaros. The information architecture of cities:. *Sage PublicationsSage CA: Thousand Oaks, CA*, 30:107–118, 2016. doi: 10.1177/0165551504041682. URL `https://journals.sagepub.com/doi/10.1177/0165551504041682`.

C. L. da Silva. Proposal of a dynamic model to evaluate public policies for the circular economy: Scenarios applied to the municipality of curitiba. *Waste Management*, 78:456–466, 2018.

G. B. Dantzig. Discrete-variable extremum problems. *Operations research*, 5 (2):266–288, 1957.

S. Das and B. K. Bhattacharyya. Optimization of municipal solid waste collection and transportation routes. *Waste Management*, 43:9–18, 2015.

M. S. Daskin and S. H. Owen. Two new location covering problems: The partial p-center problem and the partial set covering problem. *Geographical Analysis*, 31(3):217–235, 1999.

V. G. Deineko and G. J. Woeginger. A well-solvable special case of the bounded knapsack problem. *Operations Research Letters*, 39(2):118–120, 2011.

D. Delling, T. Pajor, and D. Wagner. Engineering time-expanded graphs for faster timetable information. *Robust and Online Large-Scale Optimization*, 5868:182–206, 2009.

M. Delorme, M. Iori, and S. Martello. Bin packing and cutting stock problems: Mathematical models and exact algorithms. *European Journal of Operational Research*, 255(1):1–20, 2016.

G. Desaulniers and D. Villeneuve. The shortest path problem with time windows and linear waiting costs. *Transportation Science*, 34(3):312–319, 2000.

E. Dijkstra. A note on two problems in connexion with graphs. *Numerische Mathematik*, 1:269–271, 1959.

N. B. Dimitrov, D. Solow, J. Szmerekovsky, and J. Guo. Emergency relocation of items using single trips: Special cases of the multiple knapsack assignment problem. *European Journal of Operational Research*, 258(3):938–942, 2017.

S. E. Dreyfus. An appraisal of some shortest-path algorithms. *Operations research*, 17(3):395–412, 1969.

B. Du and D. Z. Wang. Continuum modeling of park-and-ride services considering travel time reliability and heterogeneous commuters–a linear complementarity system approach. *Transportation Research Part E: Logistics and Transportation Review*, 71:58–81, 2014.

H. Dufourd, M. Gendreau, and G. Laporte. Locating a transit line using tabu search. *Location Science*, 4:1–19, 5 1996. ISSN 0966-8349. doi: 10.1016/S0966-8349(96)00008-3.

H. Dyckhoff. A typology of cutting and packing problems. *European Journal of Operational Research*, 44(2):145–159, 1990.

X. J. Eberlein, N. H. Wilson, C. Barnhart, and D. Bernstein. The real-time deadheading problem in transit operations control. *Transportation Research Part B: Methodological*, 32:77–100, 2 1998. ISSN 0191-2615. doi: 10.1016/S0191-2615(97)00013-1.

H. Eiselt and V. Marianov. Location modeling for municipal solid waste facilities. *Computers & Operations Research*, 62:305–315, 2015. ISSN 0305-0548. doi: https://doi.org/10.1016/j.cor.2014.05.003. URL `https://www.sciencedirect.com/science/article/pii/S0305054814001324`.

D. D. Eisenstein and A. V. Iyer. Garbage collection in chicago: A dynamic scheduling model. *Management Science*, 43(7):922–933, 1997. doi: 10.1287/mnsc.43.7.922. URL `https://doi.org/10.1287/mnsc.43.7.922`.

E. Falkenauer. A hybrid grouping genetic algorithm for bin packing. *Journal of heuristics*, 2(1):5–30, 1996.

T. Feder, R. Motwani, L. O'Callaghan, C. Olston, and R. Panigrahy. Computing shortest paths with uncertainty. *Journal of Algorithms*, 62(1):1–18, 2007.

S. Feng, Z. Wen-Tao, Y. Ying, and W. Dian-Hai. Optimal skip-stop schedule under mixed traffic conditions for minimizing travel time of passengers. *Discrete Dynamics in Nature and Society*, 2013, 2013. ISSN 10260226. doi: 10.1155/2013/930562.

I. Flinsenberg. Route planning algorithms for car navigation. 2004.

L. S. Franca, G. M. Ribeiro, and G. d. L. D. Chaves. The planning of selective collection in a real-life vehicle routing problem: A case in rio de janeiro. *Sustainable Cities and Society*, 47:101488, 2019.

A. Fréville. The multidimensional 0–1 knapsack problem: An overview. *European Journal of Operational Research*, 155(1):1–21, 2004.

A. Fréville and S. Hanafi. The multidimensional 0-1 knapsack problem—bounds and computational aspects. *Annals of Operations Research*, 139(1):195–227, 2005.

M. Freyss, R. Giesen, and J. C. Muñoz. Continuous approximation for skip-stop operation in rail transit. *Transportation Research Part C: Emerging Technologies*, 36:419–433, 11 2013. ISSN 0968-090X. doi: 10.1016/J.TRC.2013.07.004.

L. Fu and Q. Liu. Real-time optimization model for dynamic scheduling of transit operations:. *https://doi.org/10.3141/1857-06*, pages 48–55, 1 2003. ISSN 03611981. doi: 10.3141/1857-06. URL `https://journals.sagepub.com/doi/10.3141/1857-06`.

L. Fu, Q. Liu, and P. Calamai. Real-time optimization model for dynamic scheduling of transit operations. *Transportation Research Record*, page 1857, 2003.

R. García and A. Marín. Parking capacity and pricing in park'n ride trips: a continuous equilibrium network design problem. *Annals of operations research*, 116(1):153–178, 2002.

M. R. Garey. A guide to the theory of np-completeness. *Computers and intractability*, 1979.

M. R. Garey and D. S. Johnson. Approximation algorithms for bin packing problems: A survey. In *Analysis and design of algorithms in combinatorial optimization*, pages 147–172. Springer, 1981.

M. Gendreau, G. Laporte, and J. A. Mesa. Locating rapid transit lines. *Journal of Advanced Transportation*, 29:145–162, 6 1995. ISSN 2042-3195. doi: 10.1002/ATR.5670290202. URL https://onlinelibrary.wiley.com/doi/full/10.1002/atr.5670290202https://onlinelibrary.wiley.com/doi/abs/10.1002/atr.5670290202https://onlinelibrary.wiley.com/doi/10.1002/atr.5670290202.

M. Gendreau, G. Ghiani, and E. Guerriero. Time-dependent routing problems: A review. *Computers & operations research*, 64:189–197, 2015.

G. Ghiani and E. Guerriero. A note on the ichoua, gendreau, and potvin (2003) travel time model. *Transportation Science*, 48(3):458–462, 2014.

G. Ghiani, D. Laganà, E. Manni, and C. Triki. Capacitated location of collection sites in an urban waste management system. *Waste management*, 32(7):1291–1296, 2012.

N. S. Ghoneim and S. C. Wirasinghe. Optimum zone structure during peak periods for existing urban rail lines. *Transportation Research Part B: Methodological*, 20:7–18, 2 1986. ISSN 0191-2615. doi: 10.1016/0191-2615(86)90032-9.

G. Ghosh. Land use planning for ev infrastructure. *WIT Transactions on The Built Environment*, 212:61–71, 2022.

P. Gilmore and R. E. Gomory. Multistage cutting stock problems of two and more dimensions. *Operations research*, 13(1):94–120, 1965.

P. Gilmore and R. E. Gomory. The theory and computation of knapsack functions. *Operations Research*, 14(6):1045–1074, 1966.

S. D. Gleave. Park and ride, best practice review (final report). *City of Edmonton.*, 2017.

C. Guéret and C. Prins. A new lower bound for the open-shop problem. *Annals of Operations Research*, 92:165–183, 1999.

F. Guo, J. Yang, and J. Lu. The battery charging station location problem: Impact of users' range anxiety and distance convenience. *Transportation Research Part E: Logistics and Transportation Review*, 114:1–18, 2018.

K. M. N. Habib, C. Morency, and M. Trépanier. Integrating parking behaviour in activity-based travel demand modelling: Investigation of the relationship between parking type choice and activity scheduling process. *Transportation Research Part A: Policy and Practice*, 46(1):154–166, 2012.

K. Hall. An r-dimensional quadratic placement algorithm. *Management Science*, 17:219–229, 1970. ISSN 00251909. doi: 10.1287/MNSC.17.3.219. URL https://pubsonline.informs.org/doi/abs/10.1287/mnsc.17.3.219.

R. W. Hall. The fastest path through a network with random time-dependent travel times. *Transportation science*, 20(3):182–188, 1986.

H. W. Hamacher, S. Ruzika, and S. A. Tjandra. Algorithms for time-dependent bicriteria shortest path problems. *Discrete optimization*, 3(3):238–254, 2006.

H. Han and E. Ponce Cueto. Waste collection vehicle routing problem: literature review. *PROMET-Traffic& Transportation*, 27(4):345–358, 2015.

S. Hanafi, A. Freville, and P. Vaca. Municipal solid waste collection: An effective data structure for solving the sectorization problem with local search methods. *INFOR: Information Systems and Operational Research*, 37(3):236–254, 1999. doi: 10.1080/03155986.1999.11732383. URL https://doi.org/10.1080/03155986.1999.11732383.

P. E. Hart, N. J. Nilsson, and B. Raphael. A formal basis for the heuristic determination of minimum cost paths. *IEEE transactions on Systems Science and Cybernetics*, 4(2):100–107, 1968.

J. He, H. Yang, T.-Q. Tang, and H.-J. Huang. An optimal charging station location model with the consideration of electric vehicle's driving range. *Transportation Research Part C: Emerging Technologies*, 86:641–654, 2018.

D. A. Hensher and J. King. Parking demand and responsiveness to supply, pricing and location in the sydney central business district. *Transportation Research Part A: Policy and Practice*, 35(3):177–196, 2001.

S. Hess and J. W. Polak. An analysis of parking behaviour using discrete choice models calibrated on sp datasets. 2004.

P. A. L. Hidalgo, M. Ostendorp, and M. Lienkamp. Optimizing the charging station placement by considering the user's charging behavior. In *2016 IEEE International Energy Conference (ENERGYCON)*, pages 1–7. IEEE, 2016.

M. Hifi, H. Mhalla, and S. Sadfi. Sensitivity of the optimum to perturbations of the profit or weight of an item in the binary knapsack problem. *Journal of Combinatorial Optimization*, 10(3):239–260, 2005.

M. Hifi, H. Mhalla, and S. Sadfi. An adaptive algorithm for the knapsack problem: perturbation of the profit or weight of an arbitrary item. *European Journal of Industrial Engineering*, 2(2):134–152, 2008.

J. Holguín-Veras, W. F. Yushimito, F. Aros-Vera, and J. J. Reilly. User rationality and optimal park-and-ride location under potential demand maximization. *Transportation Research Part B: Methodological*, 46(8):949–970, 2012. ISSN 0191-2615. doi: https://doi.org/10.1016/j.trb.2012.02.011. URL `https://www.sciencedirect.com/science/article/pii/S0191261512000367`.

G. Homsi, J. Jordan, S. Martello, and M. Monaci. The assignment and loading transportation problem. *European Journal of Operational Research*, 289(3): 999–1007, 2021.

J. A. Hoogeveen, H. Oosterhout, and S. L. van de Velde. New lower and upper bounds for scheduling around a small common due date. *Operations research*, 42(1):102–110, 1994.

J. A. Horne and J. Cole. Dynamic programming algorithms for the conditional covering problem on path and extended star graphs. *Networks: An International Journal*, 46(4):177–185, 2005.

E. Horowitz. *Fundamentals of computer algorithms*. Galgotia publications, 1978.

R. Hössinger, P. Widhalm, M. Ulm, K. Heimbuchner, E. Wolf, R. Apel, and T. Uhlmann. Development of a real-time model of the occupancy of short-term parking zones. *International Journal of Intelligent Transportation Systems Research*, 12(2):37–47, 2014.

T. Hu, L. Landa, and M.-T. Shing. *The unbounded knapsack problem*. Springer, 2009.

Y. Huang and K. M. Kockelman. Electric vehicle charging station locations: Elastic demand, station congestion, and network equilibrium. *Transportation Research Part D: Transport and Environment*, 78:102179, 2020.

Y. Huang, S. Li, and Z. S. Qian. Optimal deployment of alternative fueling stations on transportation networks considering deviation paths. *Networks and Spatial Economics*, 15(1):183–204, 2015.

A. Ibeas, L. Dell'Olio, M. Bordagaray, and J. d. D. Ortúzar. Modelling parking choices considering user heterogeneity. *Transportation Research Part A: Policy and Practice*, 70:41–49, 2014.

S. Ichoua, M. Gendreau, and J.-Y. Potvin. Vehicle dispatching with time-dependent travel times. *European journal of operational research*, 144(2): 379–396, 2003.

S. T. Islam, Z. Liu, M. Sarvi, and T. Zhu. Exploring the mode change behavior of park-and-ride users. *Mathematical Problems in Engineering*, 2015, 2015.

A. Jank, W. Müller, I. Schneider, F. Gerke, and A. Bockreis. Waste separation press (wsp): A mechanical pretreatment option for organic waste from source separation. *Waste Management*, 39:71–77, 2015.

J. Jariyasunant, E. Mai, and R. Sengupta. Algorithm for finding optimal paths in a public transit network with real-time data. *Transportation research record*, 2256(1):34–42, 2011.

C. Jin. An improved fptas for 0-1 knapsack. In *46th International Colloquium on Automata, Languages, and Programming (ICALP 2019)*, volume 132, page 76. Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik, 2019.

W. Jing, K. An, M. Ramezani, and I. Kim. Location design of electric vehicle charging facilities: A path-distance constrained stochastic user equilibrium approach. *Journal of Advanced Transportation*, 2017, 2017.

S. Kaplan and S. Bekhor. Exploring en-route parking type and parking-search route choice: decision making framework and survey design. In *Proceedings of the 2nd International Choice Modelling Conference*, 2011. International Choice Modelling Conference 2011 ; Conference date: 04-07-2011 Through 06-07-2011.

O. E. Karaçan, M. C. Pinar, and H. Yaman. The robust shortest path problem with interval data. 2003.

M. Karaliopoulos, K. Katsikopoulos, and L. Lambrinos. Bounded rationality can make parking search more efficient: The power of lexicographic heuristics. *Transportation Research Part B: Methodological*, 101:28–50, 2017. ISSN 0191-2615. doi: https://doi.org/10.1016/j.trb.2017.03.008. URL `https://www.sciencedirect.com/science/article/pii/S0191261516306683`.

R. M. Karp. Reducibility among combinatorial problems. In *Complexity of computer computations*, pages 85–103. Springer, 1972.

S. Kataoka and T. Yamada. Upper and lower bounding procedures for the multiple knapsack assignment problem. *European Journal of Operational Research*, 237(2):440–447, 2014.

A. Kate and I. Goldberg. Generalizing cryptosystems based on the subset sum problem. *International Journal of Information Security*, 10(3):189–199, 2011.

H. Kellerer and V. A. Strusevich. Fully polynomial approximation schemes for a symmetric quadratic knapsack problem and its scheduling applications. *Algorithmica*, 57(4):769–795, 2010.

H. Kellerer, U. Pferschy, and D. Pisinger. *Multidimensional Knapsack Problems*, pages 235–283. Springer Berlin Heidelberg, Berlin, Heidelberg, 2004. ISBN 978-3-540-24777-7. doi: 10.1007/978-3-540-24777-7_9. URL `https://doi.org/10.1007/978-3-540-24777-7_9`.

D. Khan and S. R. Samadder. Municipal solid waste management using geographical information system aided methods: A mini review. *Waste management & research*, 32(11):1049–1062, 2014.

B.-I. Kim, S. Kim, and S. Sahoo. Balanced clustering algorithms for vehicle routing problems. In *Proceedings of the Institute of Industrial Engineers annual conference. Houston, USA*, 2004.

B.-I. Kim, S. Kim, and S. Sahoo. Waste collection vehicle routing problem with time windows. *Computers & Operations Research*, 33(12):3624–3642, 2006.

J.-G. Kim and M. Kuby. The deviation-flow refueling location model for optimizing a network of refueling stations. *international journal of hydrogen energy*, 37(6):5406–5420, 2012.

J.-G. Kim and M. Kuby. A network transformation heuristic approach for the deviation flow refueling location model. *Computers & Operations Research*, 40(4):1122–1131, 2013.

P. J. Kolesar. A branch and bound algorithm for the knapsack problem. *Management science*, 13(9):723–735, 1967.

M. Kuby and S. Lim. The flow-refueling location problem for alternative-fuel vehicles. *Socio-Economic Planning Sciences*, 39:125–145, 6 2005. ISSN 00380121. doi: 10.1016/J.SEPS.2004.03.001.

M. Kuby and S. Lim. Location of alternative-fuel stations using the flow-refueling location model and dispersion of candidate sites on arcs. *Networks and Spatial Economics*, 7(2):129–152, 2007.

S. Laabadi, M. Naimi, H. El Amri, B. Achchab, et al. The 0/1 multidimensional knapsack problem and its variants: a survey of practical models and heuristic approaches. *American Journal of Operations Research*, 8(05):395, 2018.

M. Labbé and P. Marcotte. Bilevel network design. *Network Design with Applications to Transportation and Logistics*, page 10, 2021. doi: 10.1007/978-3-030-64018-7_9i. URL https://hal.inria.fr/hal-01937014.

A. Labelle, A. Langevin, and J. Campbell. Sector design for snow removal and disposal in urban areas. *Socio-Economic Planning Sciences*, 36(3):183–202, 2002. ISSN 0038-0121. doi: https://doi.org/10.1016/S0038-0121(01)00024-6. URL https://www.sciencedirect.com/science/article/pii/S0038012101000246.

G. Laporte, J. A. Mesa, and F. A. Ortega. Assessing topological configurations for rapid transit networks. *tudies in Locational Analysis*, 7:105–121, 1994. URL https://trid.trb.org/view/448537.

G. Laporte, J. A. Mesa, and F. A. Ortega. Assessing the efficiency of rapid transit configurations. *Top 1997 5:1*, 5:95–104, 6 1997. ISSN 1863-8279. doi: 10.1007/BF02568532. URL `https://link.springer.com/article/10.1007/BF02568532`.

G. Laporte, J. A. Mesa, and F. A. Ortega. Determinación del alineamiento de máxima cobertura de viaje en el diseño de sistemas urbanos de transporte. pages 2179–2188, 2002.

G. Laporte, R. Musmanno, and F. Vocaturo. An adaptive large neighbourhood search heuristic for the capacitated arc-routing problem with stochastic demands. *Transportation Science*, 44(1):125–135, 2010.

G. Laporte, J. A. Mesa, F. A. Ortega, and F. Perea. Planning rapid transit networks. *Socio-Economic Planning Sciences*, 45:95–104, 9 2011. ISSN 0038-0121. doi: 10.1016/J.SEPS.2011.02.001.

H. Larrain, R. Giesen, and J. C. Muñoz. Choosing the right express services for bus corridor with capacity restrictions. *Transportation Research Record*, 2197:63–70, 2010. doi: 10.3141/2197-08. URL `https://doi.org/10.3141/2197-08`.

J.-B. Lee. Review on environmentally-friendly vehicle charging station location and size problems. *KSCE Journal of Civil Engineering*, 26:3741–3751, 2022. ISSN 1976-3808. doi: 10.1007/s12205-022-0214-2. URL `www.springer.com/12205`.

Y.-J. Lee, S. Shariat, and K. Choi. Optimizing skip-stop rail transit stopping strategy using a genetic algorithm. *Journal of Public Transportation*, 17:7, 6 2014. ISSN 1077-291X. doi: http://doi.org/10.5038/2375-0901.17.2.7. URL `https://digitalcommons.usf.edu/jpt/vol17/iss2/7`.

C. E. Leiserson, R. L. Rivest, T. H. Cormen, and C. Stein. *Introduction to algorithms*, volume 3. MIT press, 1994.

C. Leiva, J. C. Muñoz, R. Giesen, and H. Larrain. Design of limited-stop services for an urban bus corridor with capacity constraints. *Transportation Research Part B: Methodological*, 44:1186–1201, 12 2010. ISSN 0191-2615. doi: 10.1016/J.TRB.2010.01.003.

S. Li and Y. Huang. Heuristic approaches for the flow-based set covering problem with deviation paths. *Transportation Research Part E: Logistics and Transportation Review*, 72:144–158, 2014.

S. Li, Y. Huang, and S. J. Mason. A multi-period optimization model for the deployment of public electric vehicle charging stations on network. *Transportation Research Part C: Emerging Technologies*, 65:128–143, 2016.

D. Y. Lin and Y. H. Ku. Using genetic algorithms to optimize stopping patterns for passenger rail transportation. *Computer-Aided Civil and Infrastructure Engineering*, 29:264–278, 4 2014. ISSN 1467-8667. doi: 10.1111/MICE.12020. URL https://onlinelibrary.wiley.com/doi/full/10.1111/mice.12020https://onlinelibrary.wiley.com/doi/abs/10.1111/mice.12020https://onlinelibrary.wiley.com/doi/10.1111/mice.12020.

Z. Liu, Y. Yan, X. Qu, and Y. Zhang. Bus stop-skipping scheme with random travel time. *Transportation Research Part C: Emerging Technologies*, 35:46–56, 10 2013. ISSN 0968-090X. doi: 10.1016/J.TRC.2013.06.004.

E. Lorentzen, P. Haugneland, C. Bu, and E. Hauge. Charging infrastructure experiences in norway-the worlds most advanced ev market. 2017.

V. Lotfi and I. D. Moon. Hybrid heuristics for conditional covering problems. *International Journal of Modelling and Simulation*, 17(3):185–190, 1997.

A. Lozano and G. Storchi. Shortest viable path algorithm in multimodal networks. *Transportation Research Part A: Policy and Practice*, 35(3):225–241, 2001.

A. Lozano and G. Storchi. Shortest viable hyperpath in multimodal networks. *Transportation Research Part B: Methodological*, 36(10):853–874, 2002.

B. J. Lunday, J. C. Smith, and J. B. Goldberg. Algorithms for solving the conditional covering problem on paths. *Naval Research Logistics (NRL)*, 52 (4):293–301, 2005.

W. Makhlouf, M. Kchaou-Boujelben, and C. Gicquel. A bi-level programming approach to locate capacitated electric vehicle charging stations. In *2019 6th International Conference on Control, Decision and Information Technologies (CoDIT)*, pages 133–138. IEEE, 2019.

J. W. Male and J. C. Liebman. Districting and routing for solid waste collection. *Journal of the Environmental Engineering Division*, 104(1):1–14, 1978. doi: 10.1061/JEEGAV.0000720. URL https://ascelibrary.org/doi/abs/10.1061/JEEGAV.0000720.

V. Maniezzo, S. Voss, and P. Hansen. Special issue on mathematical contributions to metaheuristics. *Journal of Heuristics*, 15, 2009. URL http://www.csr.unibo.it/~maniezzohttp://www1.uni-hamburg.de/IWI/http://www.hec.ca/profs/pierre.hansen.html.

V. Maniezzo, T. Stützle, and S. Voß. Matheuristics: Hybridizing metaheuristics and mathematical programming. *Annals of Information Systems. Springer Verlag*, 10:1–282, 2010. URL http://scholar.google.com/scholar?hl=en&btnG=Search&q=intitle:Matheuristics#1http://link.springer.com/10.1007/978-1-4419-1306-7.

R. Margalef. Teoría de los sistemas ecológicos. *Publicacions de la Universitat de Barcelona. Barcelona*, 1991.

G. Marseglia, C. M. Medaglia, F. A. Ortega, and J. A. Mesa. Optimal alignments for designing urban transport systems: Application to seville. *Sustainability*, 11(18):5058, 2019.

S. Martello and P. Toth. An upper bound for the zero-one knapsack problem and a branch and bound algorithm. *European Journal of Operational Research*, 1 (3):169–175, 1977.

S. Martello and P. Toth. Solution of the zero-one multiple knapsack problem. *European Journal of Operational Research*, 4:276–283, 4 1980. ISSN 0377-2217. doi: 10.1016/0377-2217(80)90112-5.

S. Martello and P. Toth. *Knapsack problems: algorithms and computer implementations*. John Wiley & Sons, Inc., 1990.

G. Martinho, A. Gomes, P. Santos, M. Ramos, J. Cardoso, A. Silveira, and A. Pires. A case study of packaging waste collection systems in portugal–part i: Performance and operation analysis. *Waste Management*, 61:96–107, 2017.

E. Martins. On a multicriteria shortest path problem. *European Journal of Operational Research*, 16(2):236–245, 1984.

S. Martins, M. Ostermeier, P. Amorim, A. Hübner, and B. Almada-Lobo. Product-oriented time window assignment for a multi-compartment vehicle routing problem. *European Journal of Operational Research*, 276(3):893–909, 2019.

G. B. Mathews. On the partition of numbers. *Proceedings of the London Mathematical Society*, 1(1):486–490, 1896.

M. Mes, M. Schutten, and A. P. Rivera. Inventory routing for dynamic waste collection. *Waste management*, 34(9):1564–1576, 2014.

J. A. Mesa and F. A. Ortega. Park-and-ride station catchment areas in metropolitan rapid transit systems. *Mathematical Methods on Optimization in Transportation Systems,*, pages 81–93, 2001. doi: 10.1007/978-1-4757-3357-0_5.

J. A. Mesa, F. A. Ortega, and M. A. Pozo. Effective allocation of fleet frequencies by reducing intermediate stops and short turning in transit systems. *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, 5868 LNCS:293–309, 2009. ISSN 03029743. doi: 10.1007/978-3-642-05465-5_12. URL `https://link.springer.com/chapter/10.1007/978-3-642-05465-5_12`.

S. MirHassani and R. Ebrazi. A flexible reformulation of the refueling station location problem. *Transportation Science*, 47(4):617–628, 2013.

P. Modesti and A. Sciomachen. A utility measure for finding multiobjective shortest paths in urban multimodal transportation networks. *European Journal of Operational Research*, 111(3):495–508, 1998.

M. Mohsenizadeh, M. K. Tural, and E. Kentel. Municipal solid waste management with cost minimization and emission control objectives: A case study of ankara. *Sustainable Cities and Society*, 52:101807, 2020.

I. D. Moon and S. S. Chaudhry. An analysis of network location problems with distance constraints. *Management Science*, 30(3):290–307, 1984.

I. D. Moon and L. Papayanopoulos. Facility location on a tree with maximum distance constraints. *Computers & operations research*, 22(9):905–914, 1995.

F. A. Morales and J. A. Martínez. Analysis of divide-and-conquer strategies for the 0–1 minimization knapsack problem. *Journal of Combinatorial Optimization*, 40(1):234–278, 2020.

H. M. Mosammam, J. T. Nia, H. Khani, A. Teymouri, and M. Kazemi. Monitoring land use change and measuring urban sprawl based on its spatial forms: The case of qom city. *The Egyptian Journal of Remote Sensing and Space Science*, 20:103–116, 6 2017. ISSN 1110-9823. doi: 10.1016/J.EJRS.2016.08.002.

Y. Motoaki. Location-allocation of electric vehicle fast chargers—research and practice. *World Electric Vehicle Journal*, 10(1):12, 2019.

A. Musso and V. Vuchic. Characteristics of metro networks and methodology for their evaluation. *Transportation Research Record*, 1162:22–33, 1988. URL `http://onlinepubs.trb.org/Onlinepubs/trr/1988/1162/1162-004.pdf`.

K. Nachtigall. Time depending shortest-path problems with applications to railway networks. *European Journal of Operational Research*, 83(1):154–166, 1995.

U. Nations. Transforming our world: The 2030 agenda for sustainable development. *New York: United Nations, Department of Economic and Social Affairs*, 2015.

G. L. Nemhauser. Scheduling local and express service. *Transportation Science*, 3:164–175, 5 1969. ISSN 0041-1655. doi: 10.1287/TRSC.3.2.164.

M. Niestadt and A. Bjørnåvold. Briefing eprs — european parliamentary research service.

H. Niu. Determination of the skip-stop scheduling for a congested transit line by bilevel genetic algorithm. *New pub: Atlantis Press*, 4:1158–1167, 12 2012. ISSN 18756891. doi: 10.1080/18756891.2011.9727864.

T. Nuortio, J. Kytöjoki, H. Niska, and O. Bräysy. Improved route planning and scheduling of waste collection and transport. *Expert systems with applications*, 30(2):223–232, 2006.

U. S. C. O. of Technology Assessment. Facing america's trash: What next for municipal solid waste? Congress of the US Office of Technology Assessment, 1989.

F. A. Ortega, G. Marseglia, J. A. Mesa, and R. Piedra-de-la Cuadra. Algorithm for planning faster routes in urban networks with time-dependent arcs and the possibility of introducing waiting periods at nodes. *WIT Transactions on The Built Environment*, 212:25–36, 2022.

M. Ostermeier, T. Henke, A. Hübner, and G. Wäscher. Multi-compartment vehicle routing problems: State-of-the-art, modeling framework and future directions. *European Journal of Operational Research*, 292(3):799–817, 2021.

R. Pagany, L. Ramirez Camargo, and W. Dorner. A review of spatial localization methodologies for the electric vehicle charging infrastructure. *International Journal of Sustainable Transportation*, 13(6):433–449, 2019.

S. Pallottino and M. G. Scutella. A new algorithm for reoptimizing shortest paths when the arc costs change. *Operations Research Letters*, 31(2):149–160, 2003.

C. Paquay, S. Limbourg, and M. Schyns. A tailored two-phase constructive heuristic for the three-dimensional multiple bin size bin packing problem with transportation constraints. *European Journal of Operational Research*, 267(1): 52–64, 2018.

G. Paredes-Belmar, V. Marianov, A. Bronfman, C. Obreque, and A. Lüer-Villagra. A milk collection problem with blending. *Transportation Research Part E: Logistics and Transportation Review*, 94:26–43, 2016.

G. Paredes-Belmar, A. Bronfman, V. Marianov, and G. Latorre-Núñez. Hazardous materials collection with multiple-product loading. *Journal of cleaner production*, 141:909–919, 2017a.

G. Paredes-Belmar, A. Lüer-Villagra, V. Marianov, C. E. Cortés, and A. Bronfman. The milk collection problem with blending and collection points. *Computers and electronics in agriculture*, 134:109–123, 2017b.

G. Parkhurst and J. Richardson. Modal integration of bus and car in uk local transport policy: the case for strategic environmental assessment. *Journal of Transport Geography*, 10(3):195–206, 2002.

F. Perea, R. Ruiz, K. Katragjini, and G. d. S. de Optimización Aplicada. Integer programming, clustering, and local search approaches for grouping urban waste collection sites. *Boletin de Estadistica e Investigacion Operativa*, 32(3): 203–224, 2016.

R. Piedra-de-la Cuadra, G. Bruno, and F. A. Ortega. *Bilevel optimization for the location of charging stations in corridors*, pages 97–98. UA Editora

Universidade de Aveiro 1st Edition–September 2022, 2022. ISBN 978-972-789-799-5. URL `http://ewgla.web.ua.pt/wp-content/uploads/2022/09/AbstractsEWGLA2022.pdf`.

V. Pillac, M. Gendreau, C. Guéret, and A. L. Medaglia. A review of dynamic vehicle routing problems. *European Journal of Operational Research*, 225(1): 1–11, 2013.

D. Pisinger. An exact algorithm for large multiple knapsack problems. *European Journal of Operational Research*, 114(3):528–541, 1999.

D. Pisinger. A minimal algorithm for the bounded knapsack problem. *INFORMS Journal on Computing*, 12(1):75–82, 2000.

D. Pisinger. The quadratic knapsack problem—a survey. *Discrete applied mathematics*, 155(5):623–648, 2007.

D. Pisinger and A. Saidi. Tolerance analysis for 0–1 knapsack problems. *European Journal of Operational Research*, 258(3):866–876, 2017.

D. Pisinger and P. Toth. Knapsack problems. *Handbook of Combinatorial Optimization*, pages 299–428, 1998. doi: 10.1007/978-1-4613-0303-9_5.

T. Preuss and J.-H. Syrbe. An integrated traffic information system. In *Proc. 6th Int. Conf. Appl. Computer Networking in Architecture, Construction, Design, Civil Eng., and Urban Planning (europIA'97)*. Citeseer, 1997.

H. Qin, H. Guan, and Y.-J. Wu. Analysis of park-and-ride decision behavior based on decision field theory. *Transportation research part F: traffic psychology and behaviour*, 18:199–212, 2013.

R. Riemann, D. Z. Wang, and F. Busch. Optimal location of wireless charging facilities for electric vehicles: flow-capturing location model with stochastic user equilibrium. *Transportation Research Part C: Emerging Technologies*, 58:1–12, 2015.

N. Rincon-Garcia, B. J. Waterson, and T. J. Cherrett. Requirements from vehicle routing software: Perspectives from literature, developers and the freight industry. *Transport Reviews*, 38(1):117–138, 2018.

S. Rodrigues, G. Martinho, and A. Pires. Waste collection systems. part a: a taxonomy. *Journal of Cleaner Production*, 113:374–387, 2016.

S. Sahoo, S. Kim, B.-I. Kim, B. Kraas, and A. Popov. Routing optimization for waste management. *Interfaces*, 35(1):24–36, 2005. doi: 10.1287/inte.1040.0109. URL `https://doi.org/10.1287/inte.1040.0109`.

S. Salat and L. Bourdic. Urban complexity, scale hierarchy, energy efficiency and economic value creation. *The Sustainable City VII: Urban Regeneration and Sustainability*, 155:97–107, 2012. doi: 10.2495/SC120091. URL `https:`

//books.google.com/books?hl=es&lr=&id=wmk3mCrMDTQC&oi=fnd&pg=PA97&dq=Urban+complexity,+scale+hierarchy,+energy+efficiency+and+economic+value+creation&ots=YxdR-gHGsD&sig=WmdXyGJcFZwI56PBMbPRmrX3KIY.

J. Salmon. Systems optimization of charging infrastructure for electric vehicles. In *2016 Annual IEEE Systems Conference (SysCon)*, pages 1–8. IEEE, 2016.

F. Schulz, D. Wagner, and K. Weihe. Dijkstra's algorithm on-line: An empirical case study from public railroad transport. *Journal of Experimental Algorithmics (JEA)*, 5:12–es, 2000.

B. Schulze, M. Stiglmayr, L. Paquete, C. M. Fonseca, D. Willems, and S. Ruzika. On the rectangular knapsack problem: approximation of a specific quadratic knapsack problem. *Mathematical Methods of Operations Research*, 92(1):107–132, 2020.

M. T. Sebastiani, R. Lüders, and K. V. O. Fonseca. Evaluating electric bus operation for a real-world brt public transportation using simulation optimization. *IEEE Transactions on Intelligent Transportation Systems*, 17(10): 2777–2786, 2016.

C. Shannon. A mathematical theory of communication. *Bell System Technical Journal*, 27:379–423, 1948. URL https://ieeexplore.ieee.org/document/6773024.

D. C. Shoup. Cruising for parking. *Transport policy*, 13(6):479–486, 2006.

M. Sierra. *Towards an ecology of the form. Game theory and urban sustainability in the information age*. 2009.

J. Simon, A. Apte, and E. Regnier. An application of the multiple knapsack problem: The self-sufficient marine. *European Journal of Operational Research*, 256(3):868–876, 2017.

M. M. Solomon. Algorithms for the vehicle routing and scheduling problems with time window constraints. *Operations research*, 35(2):254–265, 1987.

L. H. Son and A. Louati. Modeling municipal solid waste collection: A generalized vehicle routing model with multiple transfer stations, gather sites and inhomogeneous vehicles in time windows. *Waste Management*, 52:34–49, 2016. ISSN 0956-053X. doi: https://doi.org/10.1016/j.wasman.2016.03.041. URL https://www.sciencedirect.com/science/article/pii/S0956053X16301258.

B. Strasser, D. Wagner, and T. Zeitz. Space-efficient, fast and exact routing in time-dependent road networks. *Algorithms*, 14(3):90, 2021.

A. Sun and M. Hickman. The real–time stop–skipping problem. *Journal of Intelligent Transportation Systems*, 9:2:91–109, 2005. ISSN 15472450. doi: 10.1080/15472450590934642. URL https://www.tandfonline.com/doi/abs/10.1080/15472450590934642.

C. Sun, W. Zhou, and Y. Wang. Scheduling combination and headway optimization of bus rapid transit. *Journal of Transportation Systems Engineering and Information Technology*, 8:61–67, 10 2008. ISSN 1570-6672. doi: 10.1016/S1570-6672(08)60039-2.

Z. Sun, W. Gao, B. Li, and L. Wang. Locating charging stations for electric vehicles. *Transport Policy*, 98:48–54, 2020.

É. Taillard, P. Badeau, M. Gendreau, F. Guertin, and J.-Y. Potvin. A tabu search heuristic for the vehicle routing problem with soft time windows. *Transportation science*, 31(2):170–186, 1997.

J. Teixeira, A. P. Antunes, and J. P. de Sousa. Recyclable waste collection planning—-a case study. *European Journal of Operational Research*, 158(3): 543–554, 2004.

R. G. Thompson and P. Bonsall. Drivers' response to parking guidance and information systems. *Transport Reviews*, 17(2):89–104, 1997. doi: 10.1080/01441649708716974. URL https://doi.org/10.1080/01441649708716974.

R. G. Thompson and A. J. Richardson. A parking search model. *Transportation Research Part A: Policy and Practice*, 32(3):159–170, 1998. ISSN 0965-8564. doi: https://doi.org/10.1016/S0965-8564(97)00005-0. URL https://www.sciencedirect.com/science/article/pii/S0965856497000050.

E. B. Tirkolaee, I. Mahdavi, and M. M. S. Esfahani. A robust periodic capacitated arc routing problem for urban waste collection considering drivers and crew's working time. *Waste Management*, 76:138–146, 2018.

P. Toth and D. Vigo. *Vehicle routing: problems, methods, and applications.* SIAM, 2014.

C. Q. Tran, D. Ngoduy, M. Keyvan-Ekbatani, and D. Watling. A user equilibrium-based fast-charging location model considering heterogeneous vehicles in urban networks. *Transportmetrica A: Transport Science*, 17(4):439–461, 2021.

D. V. Tung and A. Pinnoi. Vehicle routing-scheduling for waste collection in hanoi. *European Journal of Operational Research*, 125(3):449–468, 2000. URL https://EconPapers.repec.org/RePEc:eee:ejores:v:125:y:2000:i:3:p:449-468.

TUSSAM. Transportes urbanos de sevilla s. a. m. (tussam). URL https://www.tussam.es/.

C. Upchurch, M. Kuby, and S. Lim. A model for location of capacitated alternative-fuel stations. *Geographical Analysis*, 41(1):85–106, 2009.

S. van Ewijk and J. A. Stegemann. Recognising waste use potential to achieve a circular economy. *Waste Management*, 105:1–7, 2020.

T. Vidal, R. Martinelli, T. A. Pham, and M. H. Hà. Arc routing with time-dependent travel times and paths. *Transportation Science*, 55(3):706–724, 2021.

V. Vuchic. Skip-stop operation as a method for transit speed increase. *Traffic Quarterly*, 27:307–327, 1973. URL `https://repository.upenn.edu/ese_papers`.

V. Vuchic. Skip-stop operation: High speed with good area coverage. *Revue de I'UITP*, 2:105–128, 1976. URL `https://repository.upenn.edu/ese_papers/745`.

V. Vuchic. *Urban Transit: Operations, Planning and Economics*. 1 2005. ISBN ISBN: 978-0-471-63265-8. URL `https://www.worldtransitresearch.info/research/4139`.

F. B. Walker, D. B. Gana, and J. R. Fernández. Efectos de la entropia urbana en el coste energetico del trasporte. *Urbano*, 14:20–27, 2011.

D. Z. W. Wang, A. Nayan, and W. Y. Szeto. Optimal bus service design with limited stop services in a travel corridor. *Transportation Research Part E: Logistics and Transportation Review*, 111:70–86, 2018. ISSN 1366-5545. doi: https://doi.org/10.1016/j.tre.2018.01.007. URL `https://www.sciencedirect.com/science/article/pii/S1366554517302922`.

J. Wang, H. Wang, and X. Zhang. A hybrid management scheme with parking pricing and parking permit for a many-to-one park and ride network. *Transportation Research Part C: Emerging Technologies*, 112:153–179, 2020.

X. Wang, C. Yuen, N. U. Hassan, N. An, and W. Wu. Electric vehicle charging station placement for urban public bus systems. *IEEE Transactions on Intelligent Transportation Systems*, 18(1):128–139, 2016.

Y.-W. Wang and C.-C. Lin. Locating road-vehicle refueling stations. *Transportation Research Part E: Logistics and Transportation Review*, 45(5):821–829, 2009.

Y.-W. Wang and C.-R. Wang. Locating passenger vehicle refueling stations. *Transportation Research Part E: Logistics and Transportation Review*, 46(5): 791–801, 2010.

L. Wen, B. Çatay, and R. Eglese. Finding a minimum cost path between a pair of nodes in a time-varying road network with a congestion charge. *European Journal of Operational Research*, 236(3):915–923, 2014a.

M. Wen, G. Laporte, O. B. Madsen, A. V. Nørrelund, and A. Olsen. Locating replenishment stations for electric vehicles: application to danish traffic data. *Journal of the Operational Research Society*, 65(10):1555–1561, 2014b.

E. J. Willemse and J. W. Joubert. Constructive heuristics for the mixed capacity arc routing problem under time restrictions with intermediate facilities. *Computers & Operations Research*, 68:30–62, 2016.

Y. Xiang, J. Liu, R. Li, F. Li, C. Gu, and S. Tang. Economic planning of electric vehicle charging stations considering traffic constraints and load profile templates. *Applied Energy*, 178:647–659, 2016.

Y. Yang, N. Boland, and M. Savelsbergh. Multivariable branching: A 0-1 knapsack problem case study. *INFORMS Journal on Computing*, 33(4):1354–1367, 2021.

A. G.-O. Yeh and X. Li. Measurement and monitoring of urban sprawl in a rapidly growing region using entropy. *Photogrammetric engineering and remote sensing*, 67:83–90, 2001. ISSN 0099-1112.

J. S. Yeomans. Solid waste planning under uncertainty using evolutionary simulation-optimization. *Socio-Economic Planning Sciences*, 41(1):38–60, 2007.

B. Yu, Z. Yang, and S. Li. Real-time partway deadheading strategy based on transit service reliability assessment. *Transportation Research Part A: Policy and Practice*, 46:1265–1279, 10 2012. ISSN 0965-8564. doi: 10.1016/J.TRA.2012.05.009.

T. Zeitz. Np-hardness of shortest path problems in networks with non-fifo time-dependent travel times. *Information Processing Letters*, 179:106287, 2023.

H. Zhang, S. J. Moura, Z. Hu, and Y. Song. Pev fast-charging station siting and sizing on coupled transportation and power networks. *IEEE Transactions on Smart Grid*, 9(4):2595–2605, 2016.

P. Zhang, Z. Sun, and X. Liu. Optimized skip-stop metro line operation using smart card data. *Journal of Advanced Transportation*, 2017, 2017. ISSN 20423195. doi: 10.1155/2017/3097681.

T. Zhang. Community features and urban sprawl: the case of the chicago metropolitan region. *Land Use Policy*, 18:221–232, 7 2001. ISSN 0264-8377. doi: 10.1016/S0264-8377(01)00018-7.

H. Zheng, X. He, Y. Li, and S. Peeta. Traffic equilibrium and charging facility locations for electric vehicles. *Networks and Spatial Economics*, 17(2):435–457, 2017.

T. Zijlstra, T. Vanoutrive, and A. Verhetsel. A meta-analysis of the effectiveness of park-and-ride facilities. *European Journal of Transport and Infrastructure Research*, 15(4), 2015.