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Analyzing Pedagogical Routines in the Upper Secondary School Teacher's Discourse Using the Commognitive Aproach[1]

Inés Gallego-Sánchez
Dr., Department of Didactics of Mathematics, Universidad de Sevilla, Spain, *inesgal@us.es*

Antonio González
Department of Didactics of Mathematics, Universidad de Sevilla, Spain, *gonzalezh@us.es*

José María Gavilán-Izquierdo
Department of Didactics of Mathematics, Universidad de Sevilla, Spain, *gavilan@us.es*

In this work, we investigated through a case study the pedagogical discourse of the upper secondary school teacher when introducing the derivative concept. The subject was selected considering her experience and expertise in the field of mathematics education. Eleven class sessions were audio and video recorded, and three of them were transcribed verbatim for analysis. The theoretical framework used in the analysis was the sociocultural theory of commognition (Sfard, 2008). Specifically, we focused on identifying one of the properties of discourse, routines, which are repetitive patterns that can be inferred by observing the rest of the properties (i.e., word use, visual mediators, and narratives). Thus, some subtypes of explanation and motivation routines that complement those appearing in the work by Viirman (2015) were identified and classified*:* exemplification, use of personified language, use of paraphrases and synonyms, promotion of autonomy in learning, variation, outline, location, linking concepts and reference to difficulty. Likewise, we distinguished among them which are more typical of the transition stage to university than of previous courses and vice versa, as well as which of them can be used by teachers to help students to be successful in their transition to university.

Keywords: commognition, pedagogical discourse, routines, transition, upper secondary school

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INTRODUCTION

Research on university levels is of growing interest (Biza et al., 2016). One of the relevant topics, on which our work focuses, is the secondary-tertiary transition, specifically the gap between school mathematics and university mathematics and its teaching. For Clark and Lovric (2009), the transition to university has three phases, and one of them corresponds to upper secondary education. This phase consists of two courses, but, according to Schüler-Meyer (2019), “the penultimate school year is the only option for school-based transition courses, as the last year of upper secondary education is dedicated to the final examination” (p. 169). Thus, also according to Gueudet (2008), the first year of upper secondary can be considered within the transition stage. All the aforementioned studies point out the difficulties of students in the transition to academic mathematics and the need for teachers to address this issue.

In recent years, sociocultural theories have gained ground in mathematics education research. One of the most widely used is the theory of commognition (Sfard, 2008). The word “commognition” derives from the words “communication” and “cognition” and studies discourse. Thinking in this theory is also considered as (intrapersonal) discourse and learning is characterized as a change in discourse.

Context And Review of Literature

There are several recent studies that investigate the discourse of teachers, students, or teachers and students jointly, using the theory of commognition. Most of these studies point out that the characteristics of the teacher’s discourse influence the students’ discourse and, therefore, their learning. Specifically, Güçler (2013) analyzes the mathematical discourse of the teacher and that of undergraduate students about the concept of limit and points out that “the discrepancies among participants’ discourses signal communicational breakages” (p. 439). Among the works that focus on students’ discourses, Fernández-León et al. (2021) identify routines in the mathematical discourse of undergraduate students when defining 3D solids, and Toscano et al. (2019) study the discourse of pre-service primary teachers when solving didactic-mathematical tasks. In the latter the authors distinguish two types of discourse: The discourse as students and a discourse closer to their future role as teachers. Schüler-Meyer (2019, 2020) studies the transition to university in upper secondary school students through their discourse about number theory and convergence of sequences. Viirman (2014, 2015) studies routines in the mathematical and pedagogical discourse of first-year university teachers, and Gavilán-Izquierdo and Gallego-Sánchez (2021) also study the transition stage in upper secondary school, identifying and classifying mathematical routines in the teacher’s discourse that complement those in the work by Viirman (2014). Some authors also consider written discourse, such as Morgan (2016), who analyzes high-stakes examinations focusing on agency, and Thoma and Nardi (2018), who study the commognitive conflicts caused by differences between secondary and tertiary discourses in first-year university exams.

One of the fundamental concepts in the transition stage to university is the derivative (Al-Zoubi & Suleiman, 2021; Marzuki et al., 2021; Thomas et al., 2015). This concept appears for the first time in the Spanish curriculum in the penultimate year before university (first phase of the transition) and encompasses the concepts of derivative of a function at a point and derivative function. It also has different forms of representation: Graphical, symbolic, real situations, and multiple applications in areas other than mathematics.

The objective of this work is to characterize the teacher's pedagogical discourse about the derivative concept and identify some of its features that can facilitate the transition for students from school to university mathematical discourse.

Theoretical Framework

The theory of commognition (Sfard, 2008) is used to analyze data. This is a sociocultural theory that studies discourse and characterizes learning as a change in discourse. According to Sfard (2008), discourse can be studied by examining its four characteristic properties:

Word use. This refers to the use of mathematical words, such as trigonometry, or the use of common language words with mathematical meaning, such as function.

Visual mediators. They are "providers of the images through which speakers identify the object of their speech and coordinate their communication" (Sfard, 2008, p. 147). Examples of visual mediators are formulas, graphs, or manipulatives.

Narratives. They are sentences that describe objects, relationships between objects or processes with objects, and can be approved or rejected. If they are approved by the participants in the discourse they are called "endorsed narratives". An example is the statement "Function x^3 is strictly increasing".

Routines. "They are characteristic repetitive patterns of a given discourse" (Sfard, 2008, p. 134), and can be inferred by looking at the other properties of discourse. Some typical examples are defining, calculating, or proving mathematical statements.

The above properties were introduced by Sfard (2008) for mathematical discourse analysis. Some researchers used them for analyzing students' mathematical discourse (Fernández-León et al., 2021; Heyd-Metzuyanim et al., 2016; Schüler-Meyer, 2020) and other authors used them to characterize teachers' mathematical discourse (Gavilán-Izquierdo & Gallego-Sánchez, 2021; Güçler, 2013; Viirman, 2014). Viirman (2015) considered these properties in the teacher's pedagogical discourse, establishing analogies and differences with the properties of mathematical discourse. According to him, the use of words in pedagogical discourse contains in addition to mathematical words and common language words with mathematical meanings, words specific to didactics such as for example "important", "difficult", or "doubt". As for the visual mediators, Viirman (2015) indicates that they are the same as in the mathematical discourse, but with a didactic purpose. On the other hand, examples of didactic narratives could be explanations or questions, according to Viirman (2015).

Viirman (2015) studied and classified pedagogical routines in the discourse of first-year university teachers when they introduced the concept of function. He found the following three types:

Explanation routines. They are used to explain mathematical activities and statements and are divided into five subtypes: *Reference to known mathematical facts, summary, and repetition,* use of *different representations* (e.g., graphical, algebraic), reformulation in *everyday language*, and use of *concretization and metaphor*.

Motivation routines. They "serve two motivational purposes: Providing motives for mathematical activities undertaken and facilitating student motivation and interest" (Viirman, 2015, p. 1173). They are divided into four subtypes: Reference to *utility* (intra-mathematical or in other contexts outside of mathematics), *nature of mathematics* (e.g., the discussion of the need for a mathematical definition), *humour* (which promotes student engagement and interest), or *result focus.*

Question posing routines. They are grouped into four categories, according to the role they play. The first are *control* or comprehension questions (e.g., "Do you follow me?"), that usually appear after an important or complicated concept has been introduced or before moving on from one topic to another. These types of questions are usually unanswered. Another category is asking for *facts* that are supposed to be known and whose answer is straightforward. Another type is *enquiries*, which require the students to reflect on mathematics and develop mathematical narratives. Finally, there are *rhetorical* questions, which "direct the learner's attention to specific steps in reasoning or to certain aspects of mathematics worthy of reflection" (Viirman, 2015, p. 1176). These questions can be taken as a model of mathematical reasoning by students, as they show the questions "you would have to ask yourself if you were doing the reasoning yourself" (Viirman, 2015, p. 1178).

Specifically, the objective of this study is to describe, analyze and classify, using the theory of commognition (Sfard, 2008), the pedagogical routines in the discourse of the upper secondary school teacher that complement those in the work by Viirman (2015). Besides, we aim to identify the characteristics of this discourse at this stage that can help students to successfully start university.

METHOD

In this section, the participants and context of our study, the collection of data and how the analysis have been carried out are described.

Participants and context

In this work, we used a qualitative-interpretative methodology through the implementation of a case study (Yin, 2003). The participant in this research was Anna (pseudonym), a mathematics graduate with more than 20 years of experience in secondary and upper secondary education. She was selected because of her experience and willingness to participate in the study. We analyzed her discourse in the first year of upper secondary school, in a high school of a medium-sized city. This course had 23 students aged 16 to 18. We chose to record the sessions in which she introduced the derivative concept because, as mentioned above, this is a fundamental concept in the secondary-tertiary transition. The explanation of the concept of the derivative occupied 11 sessions of 50 minutes each, and it should be noted that it was the students' first contact with this concept. The methodology of Anna's classes was mainly traditional, i.e., she explained the theoretical concepts on the blackboard interspersed with examples. Students participated occasionally, asking or answering questions. The researchers did not participate in the planning and development of the classes.

Data collection

The data of our research are the audio and video recordings of the 11 class sessions devoted to the derivative concept. They were taken by one of the researchers placed at the back of the class, focusing straight on the blackboard and Anna. The first three sessions, in which Anna introduced the main aspects of the derivative concept, were taken for analysis. These sessions were transcribed verbatim, triangulating the audio and video recordings for more accuracy. The transcriptions were accompanied by indications in brackets about the actions that could be seen in the video.

Analysis procedure

The analysis of the data was first done individually, then there were sharing sessions where the discrepancies were discussed until reaching a consensus. Specifically, the properties in the teacher's pedagogical discourse were identified: Use of words, visual mediators, narratives, and from these the pedagogical routines were inferred and classified. Due to space limitation, protocols that are examples of the types of routines present in Viirman's (2015) work are omitted. As in the work by Viirman (2021), when we infer a routine from a sequence of utterances, we base this inference solely on the function that these fulfil locally in the discourse, and we do not presume any intention on the part of the teacher.

FINDINGS

We have found several examples of the kinds of routines that appear in the work of Viirman (2015) and other examples of pedagogical routines, most of them of explanation, that complement Viirman's categories. Only the latter are presented here, due to space limitation.

Exemplification

This kind of routine can be considered as a subtype of explanation routines (Viirman, 2015). Exemplification routines can also be divided into various subtypes, depending on the type of examples used and their different functions. We show some subtypes and related protocols below.

-*Use of generic examples.*

Anna uses a generic example of function to introduce the (procedural) definition of derivative of a function at a point.

Let's begin with the concept of derivative. Let's take a perfect function in an interval, okay? Without any problem, okay? [She draws a function similar to the logarithm on the board]. Let's take a point 'a' and let's give it an increment […] Well, we have here points '(a, f(a))' and '(f(a), f(a+h))'. Let's draw the secant line, and let's calculate the slope of this secant line […] We are going to make this 'h' smaller and smaller, and now we are here, and we draw the secant line that passes through here and here, and we would calculate the average rate of change or the slope of the line, okay? We are getting nearer and nearer… If I put this point closer to 'a', the line that is secant, where does it go? […] The line that was secant is going to become the tangent line there [he draws it], tangent at 'a', okay?

In the next protocol Anna uses the classic generic example of the absolute value function (at point (0,0)) to introduce the definitions of angular point and lateral derivatives.

[…] This function is continuous here, but it breaks [Anna makes a gesture], what happens here? If we want to calculate the tangent line, what happens at that point? […] This function is continuous in x equal to zero, but it is not differentiable at that point because the lateral limits are different, okay? Well, these lateral limits are called lateral derivatives…

-Use of concrete examples

Concrete examples are introduced immediately after a definition to help students become familiar with the notation, the substitution of variables for values in the formula, and the application of the formula in simple cases.

Here it says compute the average rate of change between points 'a' and 'a+h' [Anna writes $ARC_{[a,\ a+h]}$], that is this quotient [she points to the formula] between points -2 and 0, [she writes $ARC_{[-2,\ 0])}$], okay? Then you must compute the image of 0 under 'f'; remember always in the second point, 'f' in 0 minus 'f' in -2 divided by the increment, between -2 and 0 [she writes it simultaneously] …

-Use of worked-out examples

Worked-out examples are mainly used to show the different steps of procedures. These examples usually appear shortly after a definition and a concrete example have been introduced (although sometimes the same example serves as a concrete example and as a worked-out example). They show students how a procedure is performed. The procedure is carried out by the teacher at the blackboard, usually followed by an explanation or comments for each step performed (Bills et al., 2006). In the protocols below, Anna states that the students must practice with many examples to understand it, what reveals that Anna thinks that operational conceptions precede structural ones (Sfard, 1991).

For example, if we are asked to compute the derivative at point 2 with the function f (x) = 3 'x' squared, a simple example, how much will 'f' prime in 2 be? Definition, please, limit of 'f' in '2+h' minus 'f' in '2' divided by 'h' [points to the formula], *come on, everyone writing that, 'f' prime of 2 will be [she writes it simultaneously] the limit of 'f' at point '2+h' minus 'f' at point 2 divided by 'h', when 'h' tends to zero […] Come on, how much is 'f' in '2+h'?[…] [Anna continues step by step until she gets the final result].*

[…] To understand this [she refers to the concept of derivative at a point], you must practice a lot, until you realize what we do, first, always 'f' in 'a+h', minus 'f' in 'a', divided by 'h'…

-Posing exercises

Exercises are tasks that are assigned to students so that they can put into practice what they have learned in a more autonomous way (Bills et al., 2006). They can be done individually or in groups and are usually corrected in class.

I'm going to propose some exercises for tomorrow [Anna looks at the book], let's see, exercise 10 says, instead of the derivative, compute the slope of the tangent line, which is the same, eh, that is, in number 10, they ask for slopes, okay? … You can also do number 11, and 12, 9… you can do all that, okay? And tomorrow, we will continue with ...

Use of personified instead of alienated language

We consider this one as a motivation routine (Viirman, 2015). Anna uses very frequently first-person plural verb forms instead of impersonal forms (e.g., passive voice), which involves students in the construction of definitions, in the reasoning, or in the computations to solve problems. There are numerous examples of this routine in Anna's discourse. In the protocols below we can see some of them.

Let's begin [...] Let's take [...] We are going to make this 'h' smaller and smaller, and now we are here, and we draw the secant line that passes through here and here, and we would calculate the average rate of change.

Use of paraphrases and synonyms

This type of routine is related to the type of explanation routine that Viirman (2015) named everyday language. Anna uses paraphrases, that is, she says in another way, with different words, what she has just said, although on many occasions, the students do not demand it. She also mentions synonyms of mathematical words, so that students know what they mean in case they appear in different sources of information that they can consult. Two examples of this kind of routine are provided below.

Paraphrase:

First, the abscissa and then the ordinate, the point is 'a', what is the ordinate? How much is the 'y' in 'a' worth?

Synonyms:

There it has a peak, eh, it is named an angular point in mathematics or breakpoint…

Promotion of autonomy in learning

These routines have explanatory and motivational features. Sometimes Anna offers her students tools to check if the calculation is right or wrong, for example, in the form of necessary or sufficient conditions (Gavilán-Izquierdo & Gallego-Sánchez, 2021), as shown for example in the first and second protocols below. Other times, Anna presents her students several valid ways to perform a problem and ask them to choose what it is easier for them.

> *Be careful with the minus that affects everything that goes after, eh, that way you would not get 0/0; always think, when we do not reach an indeterminate form 0/0 it is sure that we have made a mistake...*

> *[...] If they give you a line [Anna draws it] and they ask you to calculate the derivative of 'y' equal to '3x + 1' at point 1, then it will be 3, and calculate the derivative at 0, again it will be 3, because it is a line and the slope of this line is 3, and what I am calculating when I find a derivative is the slope of the tangent line itself, then it is always, always, always 3.*

> *[...] If this is easier for you, do it step by step, each one must choose their way, and do not copy others, if you copy others, you will make it worse…*

Variation

Given a specific example or problem, the teacher can indicate what can vary (to what extent) and what must remain constant in the mathematical objects involved, to obtain another example or problem of the same class.

In the following protocol, Anna makes explicit some of these dimensions (the function and the point), which can make it easier for students to build their own examples or problems once they have learned how they can modify a given one. Also, after solving the problem, Anna invites her students to see the generality of the computation procedure.

> *… And they ask me to find the slope of the tangent line [looking at the student's notebook] If you put 'f' prime already, you must start with the limit, if you do two or three, they are all the same, what varies is the function and the point, of course…*

Outline

This routine complements the explanation routine named "summary and repetition" that appears in the work of Viirman (2015). It usually appears before explaining concepts for the first time, although it is possible that it occurs at the end of the lesson with the purpose of summarizing.

An example of this routine is given by Anna when she makes an outline before starting the topic of the derivative. This schema shows the logical sequencing of the concepts, so that students can differentiate the most general ones, the most particular ones, those which are consequences of others, etc. A related protocol is shown below.

> *Well, I'm going to write the outline first, there on the board for you to copy […] this is all I am going to explain about the derivative, okay? … Derivative of a function at a point … Then, calculation of derivatives… graphical representation of a function…*

Location

Sometimes students forget the concepts taught before, so this routine has the purpose of "positioning" the students in the topic, that is, differentiate what they would have to know and what is new for them. Using Shulman's (1986) terminology, we can distinguish between vertical and horizontal location, that is, situate them in the mathematics curriculum (vertical curriculum), or mention the relation with another subject, for instance, physics (horizontal curriculum). In the protocols below Anna indicates which concepts should be known.

> *...We are always going to have indeterminates of the form 0/0, we must solve those indeterminates, we already know how to solve them ...*

> *Well, you may know how to do it like that, but that comes later [Anna refers to the derivation rules, that appeared in physics for simple cases, but Anna introduces them later], now you must take time to do this, because when I tell you, do it applying the definition, you must know how to do it ...*

Linking concepts

This kind of explanation routine points out relationships and analogies between familiar concepts and those just introduced. Thus, this routine stimulates learning since concepts are explicitly related to what is already known, so it is not necessary to build a totally new mental schema; consequently, it also fosters memorization, understanding, and generalization. For example, in the following protocol Anna links the concepts of lateral limits and lateral derivatives. In the second one, Anna links the computation of the instantaneous speed with the derivative (at a point).

> *Thirdly, well ... lateral derivatives, just as in the limits there were lateral limits...*

...therefore, if they asked me to calculate the instantaneous speed at moment 2, you would have to do it like this limit [he points to it], just as we have done with the derivative...

Reference to difficulty

This routine shows that Anna uses pedagogical content knowledge (Carrillo-Yáñez et al., 2018), as she knows the main difficulties of students depending on the content.

When Anna mentions that a problem or concept is difficult, this serves to motivate students since it may lead them to pay more attention and regard them as a challenge. When Anna refers to problems as easy, students may realize that, if on the other hand they seem complicated for them, they may not be performing at the right level because of a lack of understanding of some concepts or a need to study more or pay more attention. This might be an indirect wake-up call to students, sometimes intended to make the students feel less embarrassed than if it were done in an explicit way. Thus, how the teacher knows her students comes into play.

> *Well, let's see it, let's see a simple example of velocity as the derivative of space divided by time...*

> *[...]…x squared and x cubed are very easy to compute by applying the definition of derivative, but x raised to n is a little more complicated...*

DISCUSSION

Some of the routines found can be primarily identified in the secondary-tertiary transition, while others are more common in lower levels. Specifically, the exemplification routines with generic examples serve to introduce a concept (in our case the average rate of change or the derivative of a function at a point) and encourage students not to focus on the specific characteristics of a particular function and to extend the construction process to any function. Hence, deducing general characteristics from examples instead of particular aspects is an objective that we usually find in this transition period (Corriveau & Bednarz, 2017; Rach & Ufer, 2020) rather than in lower educative stages.

Exemplification routines with concrete, worked-out examples, and exercises also appear in the transition stage, as we have just observed in our results, but we can affirm that they are more common in middle school since students at this stage are less capable to infer general characteristics from few examples (Bills et al., 2006).

Sfard (2008) claims that "abstract notions, can be conceived in two fundamentally different ways: Structurally- as objects, and operationally -as processes" (p. 1). Also, she asserts that operational conceptions precede structural conceptions. According to Sfard (2008), the analysis of the discourse can serve us to know about the way that an object is perceived by a student. In the case of processes, the discourse expresses actions and is personalized, whilst in the case of objects, it reveals states and has an impersonal character. In our case, we can see that Anna starts the lesson with procedural definitions (i.e., describing how to construct an object) instead of structural definitions (i.e., describing properties that characterize an object) (Kobiela & Lehrer, 2015). These types of procedural definitions with generic examples justify the use of the remaining types of examples (concrete, worked-out examples, and exercises) to consolidate the use of the procedures or become familiar with the notation. However, concrete examples also appear at the tertiary level, where structural definitions are used, to prove that a certain object that we have defined is not the empty set (Martín-Molina, González-Regaña y Gavilán-Izquierdo, 2018). Therefore, Anna promotes inductive learning, which "implies that the learner is making some generalizations about actions or concepts while working with a range of examples (seeing generality through particulars)" (Bills et al., 2006, pp. 130-131), in contrast with deductive learning, more common in academic mathematics that usually follows the classical definition-theorem-proof model (Viirman, 2021; Weber, 2004).

The use of personified language is a motivation routine since it serves to engage students in action. Also, it is typical of less abstract (objectified) discourses than alienated language (Morgan, 2016; Sfard, 2008). In the protocols provided by Viirman (2015), corresponding to a first-year university course, we can see that both types of language are used, while there is a predominant use of personified language in the case of Anna (upper secondary school).

The use of paraphrases and synonyms is more frequent at the upper secondary school level than at the university level because the secondary education discourse is more student-centered, as evidenced by the role of these routines. In comparison, academic language is precise and univocal, as well as content-centered rather than student-centered (Clark & Lovric, 2008; Rach & Heinze, 2017). In fact, the language of advanced mathematics is mostly symbolic (Corriveau & Berdnarz, 2017), and several difficulties have been detected when students approach it for the first time (Gueudet, 2008).

Routines promoting autonomy throughout the learning process are essential to ease the transition to university (Gueudet, 2008; Rach y Heinze, 2017). For this reason, they are more frequent as the students progress through the different educational stages, where they are urged to be more flexible and autonomous (Clark & Lovric, 2008), which is also an incentive for their motivation. Thus, teachers are not the only knowledge transmitters and substantiators (Sfard, 2008) since students turn to other sources such as books or the internet. Indeed, Rach and Heinze (2017) affirm that the most important factor to have success at university is the ability to develop appropriate strategies of learning.

The theory of variation (Marton & Booth, 1997) states that learning can also be defined as achieving awareness of one or more dimensions of variation that an example may have. Variation routines are clearly linked with this theory, and they can contribute also to enhance autonomy in learning. Indeed, these routines provide students with tools that, besides their usefulness to distinguish examples and problems of the same type, can be used to construct their own examples and problems.

Concerning outline routines, we have not found any evidence in the literature of its frequency in middle school, upper secondary school or first years at university. Outlines usually appear in textbooks as indexes or on the first pages of each lesson, so teachers often do not elaborate outlines, but they may comment on them during class.

Location routines are normally more frequent in middle school and upper secondary school than at university because many university teachers have a distorted idea of the average student (De Guzmán et al., 1998; Gueudet, 2008). Indeed, they tend to think that most of their students have successfully completed a quality secondary education, as well as they understand and remember all contents that they have been taught, and so these teachers do not believe that these routines are necessary. However, the reality is just the opposite since the level of preparation of students coming to university is increasingly lower, and there is a reduction in the time that they are able or willing to devote to study (Clark & Lovric, 2009). Again, the lower frequency of occurrence of these routines at tertiary level is because they are more student-centered than content-centered.

The routines of linking concepts are more typical of the transition stage than of previous courses and constitute a useful resource for students, since, as they progress through the educational stages, they are given fewer routinary or algorithmic exercises and more exercises to interrelate concepts (Gueudet, 2008; Lithner, 2000; Sierpinska, 2000). According to Gueudet (2008), the organization of contents might be seen as a necessary condition for developing the modes of representation and reasoning that are required at the tertiary level. Furthermore, Rach and Ufer (2020) claim that school mathematical knowledge and the connections between their concepts are key factors of success in the exams of a first-year university course.

Finally, routines referring to the difficulty are useful to eliminate students' prejudices about their own mathematical skills and motivate them to complete those tasks. Indeed, Gafoor and Kurukkan (2015) and Harun et al. (2021) ensure that students who perceive mathematics as very difficult tend to abandon the tasks having made less effort than those who consider it easy. We have not found in the literature any data about the use of this routine, but we are convinced that they are more common in the preceding courses of transition, again because of their focus on students instead of content.

CONCLUSION

We have analyzed the pedagogical discourse of an upper secondary school teacher when introducing the concept of derivative. We have found examples of the types of pedagogical routines proposed by Viirman (2015): Explanation, motivation, and question posing routines. Furthermore, we have obtained other subtypes of explanation and motivation routines that complement those by this author, i.e., exemplification routines, with four subtypes (use of generic, concrete, and worked-out examples, and exercises), use of personified language, use of paraphrases and synonyms, promotion of autonomy in learning, variation, outline, location, linking concepts and reference to difficulty. We suggest that mathematics teachers can use this classification to be more aware of the routines they use and the consequences they have on the teaching-learning processes, for instance, generic examples could be a valuable tool to facilitate the transition from procedural to structural definitions, being the latter more typical of university mathematics.

To conclude, we would like to comment on some limitations of our work. The first one comes from our methodology: We have conducted exploratory research using a case study. This allowed us to identify and classify some examples of routines that complement those in the work of Viirman (2015). Nevertheless, these results must be complemented by studying the discourse of more teachers in different phases of the transition. Another line of future research could be analyzing the effect of these routines in students' learning.

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