Using constraint programming framework for Semiqualitative Reasoning

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Abstract

A semiqualitative model is the representation of the world by means of qualitative and quantitative knowledge. It is motivated by the lack of quantitative knowledge in determined parameters or variables of the model. We wish to obtain conclusions and generalization derived from these specific models.
In this work, we show as semiqualitative models may be transformed to interval constraint satisfaction problems(ICSP). The inference process in an ICSP is based in combining constraint propagation and *branch and bound* algorithms. The interval domains of the variables are to propagate through the constraints. A defined operator must exist for every qualitative knowledge. It transforms this knowledge to an interval constraint. The application of the constraint programming framework to a ICSP allows the obtaining of conclusions in a automatic way.
Keywords: Qualitative reasoning, Constraint Programming, Consistency techniques.

1 Introduction

Different communities have studied the qualitative representation of the world and they have proposed formalisms to qualitatively reason about these representations. The main motivations are the lack of quantitative knowledge and the excess of them that may obtain conclusions and explanations understandable by humans. Generally the real models are not pure qualitative or pure quantitative but are normally composed of quantitative and qualitative knowledge. It is known as semiqualitative model. All this knowledge must be considered when these models are studied.

In the eighties, the main concepts of the qualitative techniques appeared in the area of Artificial Intelligence(AI), by means of the publication of the systems $ENVISION$ [DeKleer84], $QSIM$[Kuipers86] and QPT[Forbus84]. Different monographs have been published in the 90s [Weld90],[Faltings92],[Kuipers94],[Piera95], [Dague95],[Trave97], that reflect the topics and the different developed techniques in qualitative reasoning(QR). This qualitative knowledge is useful to understand, in a simple way, some of the properties of the models, therefore the simplicity is its principal property. It have been applied in different tasks, such as supervision and diagnosis of chemical industrial process[Penalva91][Bousson93][Chang93], marine motor [Moreno93], the control [Lepetit87], analysis of dynamical systems [Aracil93] [Gasca98], software analysis [Xanthakis94] and conceptual design of structures in engineering [Bozzo98]. In a majority of the approaches to modeling qualitative reasoning in AI, it is assumed explicitly or implicitly that qualitative models are obtained directly from quantitative models, or at least could be refined to a quantitative description of such system.

Different approaches have been proposed to apply qualitative reasoning to real problems. They have significant difficulties, since a lot of importance is given to simple parameters and the relationships among them are generally rejected. The first approach the qualitative reasoning is related to the reasoning about signs, where the qualitative space is +,0,-,?. In most of the research applications of the qualitative calculus, solving a set of qualitative equations is viewed as a constraint propagation problem. Since the possible values for each variable are limited to previous values, we can always use trial-and-error methods.The equations are viewed as a set of constraints that need to be satisfied, and values are propagated through the the equations which have only one variable unknown. If no such equation exist we are obliged to take possibilities and backtrack in case of a dead end. This approach produces sometimes indeterminate conclusions, for example, if A is ***positive*** and B is ***positive***, the sum is easy to infer that its result is ***positive*** but if the subtraction is carried out, the result may be ***negative, zero or negative.*** This fact together with other considerations have given rise to new approximations to formalise structures and tools for QR.

1.1 Order of Magnitude Reasoning

In qualitative reasoning, apart from signs of quantities, also it seems convenient to consider the absolute or relative order of magnitude of the quantities, and of course the approximate numeric values. There are approachs that offers a midway abstraction level between numerical methods and qualitative formalisms is the order magnitude reasoning.

In the absolute orders of magnitude approach is divided the real line into more equivalence classes than the relation ***"equals sign than"***. Every equivalence class have associated a label to distinguish between two magnitudes of the same type. In the bibliography have been proposed ***Qualitative Algebras of Order of Magnitude***[Trave89] [Piera91] and the hybrid algebra [Williams91] that allows to incorporate quantitative information in the models.

In the relative order of magnitude, the qualitative knowledge is represented by binary ***relations*** expressing orders of magnitude between two quantities (e.g. 'close to', 'negligible', 'distant of'). The first attempt to formalise such reasoning appeared with the formal system FOG[Raiman88] based on three basic relations and 32 rules. Nevertheless, limitations of FOG have been pointed out in the formalism $O(M)$ [Mavrovo90], which prevent it from real use in engineering. In $O(M)$ is considered seven binary primitive relations between positive quantities, with interval semantics. Inference strategies are based on propagation of order of magnitude relations through properties of the relations, solved or unsolved algebraic constraints. In $O(M)$ there is the impossibility, at a formal level, to express a gradual change from one order of magnitude to another, due to the non overlapping nature of the orders of magnitude, then a new formal system $ROM(K)$ [Dague93a] is proposed to introduce a new relation 'distant from'. Determined qualitative labels of $ROM(K)$ correspond to the sets ***Small*** and ***Rough*** defined previously in a set-based general framework [Raiman91], that uses a coarse equilibrium which weighs quantities with a variable level of precision.

In previous techniques, the two most important problems are the difficulty to incorporate quantitative knowledge when available, and the difficulty to control the inference process, in order to obtain valid results in the real world. This last problem can be solved by extending these models with a tolerance calculus that provides a measure of accuracy for the inferred results. Such extensions were proposed first for FOG[Dubois89] and more recently for $ROM(R)$ [Dague93b]. These extensions may cause a spurious deterioration of the results produced in the reasoning process. In order to avoid such problem was proposed a refined definition of the negligibility [Dollinger98]. Also the chance of describing, in a qualitative way, the natural grading of ***negligibility*** allows to obtain a greater precision in particular problems [Sanchez96a].

The first works about the integration qualitative and quantitative knowledge are carried out in $O(M)$ and later in $ROM(R)$, but the obtained results although sound are not in general optimal. Later, a method of reasoning that integrates absolute and relative order of magnitude in qualitative models has been proposed [Sanchez96b]. The last works are aimed at formalizing reasoning when disposes of some real quantitative data and only disposes of the qualitative descriptions of other ones [Sanchez98], [Gasca98].

Also, order of magnitude reasoning is used in model simplification and to find an approximate model that is, in general, tractable and captures the essence of the problem. It is done by identifying and removing insignificant terms and determine the significance of a term by means of the definition of the order of magnitude of a quantity on a logarithmic scale and it uses a set of rules to propagate orders of magnitudes through equations [Nayak92]. Also, model simplification is based in a theory of asymptotic order of magnitude of functions [Yip93].

1.2 Interval Constraint Satisfaction Problem

Reasoning on the ranges of values of variables is another type of reasoning often used in qualitative systems, where there are inaccurate data or partially defined parameters. The first formalisms was Quantity Lattice system [Simmon86], and BOUNDER [Sacks87]. Different problems of interval-based qualitative reasoning has been analyzed in the bibliography [Struss90]

The previous reasoning can be considered as an Interval Constraint Satisfaction Problem ($ICSP$). It is a triple (X, D, C) where X denotes a set of variables, D denotes a set of domains, D_{x_i} denotes the interval containing all aceptable values for x_i and C is the conjunction of constraints that have to be satisfied. A natural way of reasoning on the ranges of values is to propagate the interval domains of the variables through the constraints. Then consistency techniques have been applied to $ICSP$ to detect inconsistent values and delete them. Different techniques have been proposed in the bibliography [Davis87], [Hyvonen92], [Lhomme93], [Lhomme94], [VanHent95a], [SamHaroud95], [Benhamou96], [VanHent97], [Marti97], [Jussien98] and [Granvilliers98]. A lot of techniques have a major drawback since they introduce choice points and they are exponential complexity. Some efficiency aspects of the filtering algorithms based in partial consistency have been mentioned in the bibliography [Lhomme93] [Collavizza99].

These techniques with search algorithms may be included in a constraint programming paradigm to obtain solutions of the $ICSP$. In this paradigm, the semiqualitative models easily can be to express in a declarative way, it provides substantial expressive power. In this paper the semiqualitative models are convert into an interval constraint satisfaction problem by means of rules of transformation. After the inference strategies are based on constraint programming framework. This framework combines constraint propagation and ***"branch and prune"*** algorithms. It efficiently constrains the inferences and obtain results which are suitable for many activities. The automation of the qualitative reasoning is accomplished by means of a program. It express the domains of the qualitative constrained variables and all the constraints of the model where some real quantitative data and qualitative descriptions of other ones are known.

2 Semiqualitative Models

2.1 Elements of representation

A semiqualitative model can be considered as a constraint network where the elements may be:

- Basic Operators, represent the set of unary and binary arithmetic operators. A example of these operators may be: $\{+, -, *, /, ...\}$
- Order of Magnitude Operators (OM operators),
  - Absolute OM operators allow represents the orden relation the set of equivalence classes of every qualitative magnitude of the problem. For example $U \equiv$ ***large, small, medium, negative, short, acid, high,...***
  - Relative OM operators allow represents the set of equivalence classes of the relative order of magnitude

between magnitudes . For example $B \equiv$ *much smaller than, moderately smaller than, slightly smaller than, exactly equal to, much larger than, negligible, distant from....*

- Functions and Envelope Functions, let $\mathbb{R}$ be the set of reals numbers, then F represents a set of functions $f : \mathbb{R} \rightarrow \mathbb{R}$. It is defined by $f(x) \equiv < e(x), I_1, I_2 >$, where I_1 and I_2 are of domain and range of f respectively. The envelope function F represents a family of functions between two functions of one variable f_1 and f_2,

$$F(x) \equiv < f_1(x), f_2(x), I_1, I_2 >$$

$$\text{so that} \quad \forall x \in I_1 : f_1(x) < f_2(x) \qquad (1)$$

where I_1 and I_2 stand for the domain and range of F. Also we permit to express *"piecewise functions"* and those ones that are not continuous in certain points. They are defined by

$$f(x) \equiv < e^1(x), I_1^1, I_2^1 >, ..., < e^n, I_1^n, I_2^n >$$

$$\text{so that} \quad I_1^i \bigcap_{i \neq j} I_1^j = \emptyset \qquad (2)$$

In the same way this concept may be used to envelope functions.

- Predicates, P where every p_i is a unary predicate $u_i(e)$ of the set of operators U that is used to stand for the qualitative knowledge of the expression e, or a binary predicate $b_i(e_1, e_2)$, where $b_i \in B$. It stands for a qualitative *relation* between the values of e_1 and e_2.
- Constraints, C where every c_i is a predicate about the variables of the model that must satisfy all values of the model, and represents the knowledge of the problem.
- Single Queries, Q where every q_i is a query. It may be a unary query, such as $e?$, that indicates the qualitative value of the expression e and a binary query such as $e_1?e_2$, which indicates the relative order of magnitude *relation* between e_1 and e_2.
- Compound Queries are boolean expressions of queries.

2.2 Transformation to an ICSP

The rules of transformation of the previous elements transform an initial model into a normalized one. If r always denotes a new variable, the transformations applied to the initial model are the following:

- Renamed of constants that are intervals: Every interval constant of model is substituted by a variable and a constraint. $C(..., I, ..) \equiv C(..., r, ...), r \in I$
- Absolute OM operators: The following transformation is carried out

$$u(e) \equiv e - r = 0, \; r \in I_u \qquad (3)$$

where I_u is the associated interval to the unary operator u. This transformation is carried out to express the qualitative knowledge that somebody has about the expression. In the bibliography there are different spaces of qualitative description, one of them uses two landmarks, denoted as α and β [Trave89] and other uses more landmarks [Agell98]. It depends on every magnitude of the reasoning problem and the level of precision to denote a quantity. This association between operators and intervals is carried out according to the knowledge of the expert. The absolute order of magnitude scale for every quantity of the model must be coherent with the corresponding relative order of magnitude scale.

- Relative OM operators: Binary predicates are related to the division and they have the following semantics

$$b(e_1, e_2) \equiv e_1 - e_2 * r = 0, \; r \in I_b \qquad (4)$$

I_b is the interval corresponding to symbol b. In the bibliography there are different spaces of relative order of magnitude description, one of them uses one tolerance parameter [Mavrovo90] and other uses two parameters [Dague93b]. This may express a gradual change from one order of magnitude to another and the first may not express it. The consistency between both scales to every magnitude dependent of the choosing of the values of the parameters.

- Rules of transformation of functions and envelope functions: In accordance with the definition of these functions the following transformation is applied

$$r = f(x) \equiv \begin{cases} f(x) - r = 0 \\ x \in I_1, \; r \in I_2 \end{cases}$$

$$r = g(x) \equiv \begin{cases} g(x) - r = 0 \\ g(x) = \underline{g}(x) + (1 - r_1)\overline{g}(x) \\ r_1 \in [0, 1], \; x \in I_1, \; r \in I_2 \end{cases}$$

The envelope functions express qualitative aspects, and represent a family of functions enveloped in an upper function $\overline{g} : \mathbb{R} \rightarrow \mathbb{R}$ and lower one $\underline{g} : \mathbb{R} \rightarrow \mathbb{R}$. Then, if $\alpha = 0 \Rightarrow g(x) = \overline{g}(x)$ and if $\alpha = 1 \Rightarrow g(x) = \underline{g}(x)$ and any other value of α belonging to the interval [0,1] represents a set of values between $\underline{g}(x)$ and $\overline{g}(x)$.

The transformation that we carry out in the *piecewise function* is:

$$r = f(x) \equiv \begin{cases} e^1(x) - r = 0 \\ x \in I_1^1, \; r \in I_2^1 \\ ... \\ e^n(x) - r = 0 \\ x \in I_1^n, \; r \in I_2^n \end{cases}$$

- Rules of transformation of single query: Depending on the type of query, the following transformation is carried out

$$e_1? \equiv \begin{cases} e_1 - r = 0 \\ r? \end{cases}$$

$$e_1?e_2 \equiv \begin{cases} e_1 - e_2 * r = 0 \\ r? \end{cases}$$

- Rules of transformation of compounds query: For all the single queries the previous transformation is carried out and the boolean operators are not transformed.

3 Semiqualitative Reasoning

3.1 Interval methods

Interval methods are based on interval analysis techniques, seem to be the only methods which are capable of infallibly solving the problem of Global Solution[Kolev98]. However, all interval methods known to date suffer from a serious drawback which severely limits their applicability, numerical complexity grows too rapidly with the dimension n of the system. The main problem is the function evaluations in interval form to locate the solutions. Global search algorithms have been widely used in the constraint programming framework to solve constraint systems over continuous domains.

3.2 Constraint Programming Framework for Semiqualitative Reasoning

In this work the field of constraint network is focused on producing a library of efficient procedures to solve general constraints over qualitative data types(transformed in interval data). These procedures are defined and implemented in a constraint programming paradigm. In this paradigm different bases are proposed with respect to other programming paradigm. These ones provides the computer with fully deterministic procedure for carrying out a computation. Constraint languages, in contrast, implement a form of declarative programming in which only the relations between objects are specified by the programmer while leaving the procedural details of how enforce these relations up to the constraint-solving system. As a result, constraint languages require significantly more intelligence in their interpreter, whose operation is thus harder to understand.

In constraint programming paradigm, the representation of the problem is the input and a problem solver would produce a solution. It is known that these problems are NP-hard. In other words, a general algorithm uses can grow exponentially but only in the worst case In this problem solver is possible that the user may adapt the algorithm in order to achieve better running time. It is accomplished by means of different heuristics.

3.3 Heuristics

We can make the search itself more efficient by exploiting knowledge about the problem. The algorithms for constraint satisfaction do not specify the order in which variables and values are selected. It is well known that these orderings have a dramatic effect on the algorithm's efficiency [Dechter94]. That kind of information is known as strategic knowledge since it deals with the way the problem should be solved. Heuristics can be grouped into two categories:

- Static Heuristic, that establish an ordering before the search starts and that maintains this ordering throughout all the search.
- Dynamic heuristic make selections dynam-

ically during search, the decisions about variable and value orderings is decided at each search node.

A well-known static heuristic is to consider first the most constrained variables because they are likely to be more difficult to assign. Inconsistencies are expected to be found at early tree levels, where recovering from mistakes is less costly. This has been often used in the bibliography.

It is strongly believed that dynamic variable orderings are more effective than static ones. The more popular variable ordering heuristic selects the variable with the minimum number of values in its current domain. A general criterion which can be followed in the interactive framework tends to minimize knowledge acquisitions.

In a work about numeric constraint satisfaction problem is proposed [Hyvonen92] to select a cutset variable x by some criterion (e.g. select the variable with the largest width and split x exhaustively into intervals by some criterion(e.g. bisect X). In another work [VanHent97] is used a round-robin heuristic to split the domains of the variables.

In general a constraint programming framework allows the user the introduction of different splitting strategies. The user can choose static orderings or dynamic orderings, and if the user said nothing then all the variables of the constraint network are split and the ordering is based on to consider first the most constrained variables. The different orderings run the version of the program noticeably faster or slower.

3.4 Generation of redundant constraints

The problem solver in constraint programming languages as PROLOG IV [Colmerauer96], CLP(BNR) [Older93], NEWTON [VanHent97] and systems addressing numerical constraints are based on local applications of operators reducing the domains of possible values for some variables, followed by a search phase recursively applying the operators to selected sub-domains. A consequence of the local application of these operators is that the computacional efficiency can be drastically improved by adding redundancies to the constraint network. The generation of redundant constraints is based on the method of Gröbner basis [Buchberger85], that were used in non-linear constraint solving [Benhamou97b]. The basic idea is to transform a set of polynomials into a certain standard form. Given a system of multivariate polynomials equations, its Gröbner basis is an equivalent system, that is, a system that has the same solutions with the same multiplicities. The Gröbner basis are computed by Buchberger's algorithm, that is an algorithm that generalizes both Gaussian elimination for linear multivariate equations and the Euclidean algorithm for univariate polynomial equations. Using Gröbner basis has the following advantages:

- A Gröbner basis has better computational properties than the original system. In particular, it is very easy to determine whether the system is solvable.
- The over-constrained problems with redundant equations, a Gröbner basis eliminates the redundant ones.
- In over-constrained and inconsistent network is obtained the constraint 1=0, that it is obviously inconsistent
- If the under-constrained problems, the new network give useful information in order to resolve the problem.

These redundant constraints in the constraint programming framework improve the performance of the search algorithms. Other experience proposed [Cheng99] the idea of increasing constraint propagation by redundant modeling.

3.5 Constraint Solver in Continuous domains

In the constraint programming paradigm to continuous domains, the Constraint Solver includes the procedures of propagation of interval values and application of heuristics. For it is necessary to have ***narrowing operators***, whose aim is to help the constraint solving to obtain optimal results. There are single narrowing operators to arithmetic interval extension and mathematical functions, that are based

in previous works [Cleary87] [Hyvonen92] and complex narrowing operators are based upon partial consistency filtering techniques. Informally speaking, a ICSP satisfies a partial consistency property if a relaxation of it is consistent. In the bibliography have been proposed ***hull consistency*** [Lee93] [Lhomme93] ***box-consistency*** [Benhamou94] [VanHent95a], ***3B-consistency*** [Lhomme93], ***Bound-Consistency*** [VanHent97] [Puget98] and combination of the box-consistency and hull consistency [Granvilliers98]. A analysis of these partial consistency have been investigated and a set of properties have been obtained[Collavizza99].

In this work we proposed to have a library of these different partial consistency. They are goals in the constraint programming paradigm. The propagation of interval values of the variables is accomplished by means of the application of the different narrowing operators to the constraint network until no pruning takes place. The goal of the function **ConstSolver** is to obtain the list of boxes that are possible solutions of the $ICSP$. It eliminates most of the spurious solutions. The function has four parameters, where the first one represents the queries, the second one the selected search algorithm, the third one represents the type of partial consistency used by means of narrowing operators during the search and the last one the heuristic of selection of variables. The rules of transformation applied to a semiqualitative model generates a program in the constraint paradigm framework. This program may be:

Program
  Parameters
    $\epsilon = 10^{-1}, \sigma = 10^{-3}$
  Variables
    x, y, z, w
  Domains
    D_x, D_y, D_z, D_w
  Constraints
    C_1, C_2, C_3
  Solving
    $ConstSolver(Q, BB, HullC, H_1)$
endProgram

The execution of this program determines the boxes that are solutions of the queries. They are interpreted later to obtain qualitative labels of the semiqualitative model. The propagation and branching of boxes takes place until there is not pruning possible. Also we allows users to specify the accuracy. The key idea of this algorithm is to split the set of boxes solutions by a determined face only if it is necessary. If the result of the application of narrowing operators obtains neighbor boxes then we carries out the intelligent join of these to improve the efficiency of the algorithm. This program depends on two parameters ϵ and σ that determine respectively the size of *canonical* interval and the quantity to consider two boxes which are neighbor.

3.6 Examples

The main utility of qualitative reasoning in practical applications is most important when the studied models are more complex. But we only show a simple example, whose description and steps can the reader easily understand. In the bibliography [Mavrovo90] has been studied qualitatively the comparison of the order of magnitude behavior of a continuos tank reactor(CSTR) and a plug-flow reactor(PFR), for the irreversible first-order reaction:

$$A \to B \tag{5}$$

In these systems whose rate r is given by $r = k[A]$ where $[A]$ stands for the concentration of A. In both reactors we define the residence time as $T = V/F$ where V stands for the reactor volume and F is the flowrate through the reactor. If for isothermal operation the reaction's time constant is $t = 1/k$, then we can obtain constraint involving the concentration of A in the feed(C_1) and the concentration of A in the effluent of the reactor from the mass-balance for the reactor. It depends on the reactor type. For a PFR reactor the constraint between is:

$$ln(C_1/C_2) = T/t \tag{6}$$

while for a CSTR reactor is:

$$C_1t - C_2t - C_2T = 0 \tag{7}$$

The queries in these systems are C_2/C_1? for different relative order of magnitude relation T/t. Then the generated program by means of the rules of transformation and generation of redundant constraint is:

Program
  Parameters
    $\epsilon = 10^{-1}, \sigma = 10^{-3}$
  Variables
    $C1, C2, T, t, r_1, r_2$

Domains

$[0,\infty],[0,\infty],[0,\infty],[0,\infty],[0,\infty],[0,\infty]$

Constraints

$(C_1t - C_2t - C2T = 0, T - t * r_1 = 0,$
$C_2 - C_1 * r_2 = 0, -1 + r_2 + r_1r_2 = 0,$
$C_1 - C_2 - C_2r_1 = 0); (r_1 = 1);$
$(r_1 \geq 0, r_1 \leq 0.1); (r_1 > 0.1, r_1 \leq 0.9)$
$(r_1 > 0.9, r_1 < 1.0); (r_1 > 1, r_1 \leq 1.1)$
$(r_1 > 1, r_1 \leq 10); (r_1 > 10)$

Solving

$ConstSolver(r_2, BB, BoxC, SH_1)$

endProgram

where the disjunction of constraints is represented by means of ";" and the conjunction is represented by means of ",". In the **Const-Solver** function has been chosen as parameters, the variable r_2 that stands for the relation C_2/C_1, the search algorithm *branch and bound*, the box-consistency techniques and a static heuristic to the selection of variables.

The execution of this program produce the following results The results is similar to the re-

	Semiqualualitative Answers	
r_1	PFR	CSTR
$T \ll t$	$C_2 \sim< C_1$	$C_2 \sim< C_1$
$T- < ... >$	$C_2- < C_1$	$C_2- < C_1$
$T > -t$	$C_2 \ll ..- < C_1$	$C_2- < C_1$
$T \gg t$	$C_2 \ll C_1$	$C_2 \ll C_1$

Tabla 1: Table of results of the questions of the models of PFR and CSTR

sults obtained in the bibliography [Mavrovo90] and show the qualitative behavior of the different reactor. The introduction of quantitative in the previous model is easy. It corresponds to interval whose bounds are the quantitative value.

4 Conclusions

This paper has provided a way to carry out automatically the semiqualitative reasoning by means of the transformation of a semiqualitative model to a interval constraint satisfaction problem. Qualitative knowledge is transformed to interval labels by means of the defined rules of transformation. The knowledge qualitative may be order of magnitude knowledge and different functions.

The generation of a program allow the application of constraint programming framework. The results obtained are valid in the real world and they are obtained with an adjusted efficiency. The main improvements of our work are the automation of the semiqualitative reasoning and the possibility of the user to adapt the process of search of solutions in accordance different heuristics. In the future, we are going to apply the previous techniques to more complex real problems. We would like to enrich the expressivity of the qualitative knowledge, with purely qualitative functions and to apply our techniques to semiqualitative analysis of dynamic system where it is possible obtain the stability and bifurcations regions. Another possible field of applications of our methodology is the semiqualitative simulation of dynamical systems which must hold a set of constraints.

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