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ARTICLE

The Methods Behind Poincaré's Conventions: Structuralism and Hypothetical-Deductivism

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Abstract
Poincaré's conventionalism has been interpreted in many writings as a philosophical position emerged by reflection on certain scientific problems, such as the applicability of geometry to physical space or the status of certain scientific principles. In this paper I would like to consider conventionalism as a philosophical position that emerged from Poincaré's scientific practice. But not so much from dealing with scientific problems, as from the use of two specific methodologies proper to modern mathematics and the modern natural sciences: methodological structuralism and the hypothetical-deductive method—thus, as a philosophical position which emerged from a way (or rather, two ways) of *doing* science. With this approach, I try to deepen the analysis of connections between Poincaré's scientific practice and his philosophy.

Keywords Poincaré's philosophy · Convention · Geometry · Mechanics

«Let us then watch geometers at work to catch a glimpse of their methods»
«After a study of the working conditions of physicists,
I have thought it necessary to show the latter at work»
Poincaré, 1902.

1 Introduction and Aim of the Paper

The origins of Poincaré's conventionalism have been situated in problems concerning the status of certain scientific principles that had their origin in the development of new scientific theories. Thus, it is first and foremost a philosophical position connected to scientific problems. For example, regarding geometrical conventionalism, it is said that the emergence of non-Euclidean geometries is what prompted the development of a philosophical position that could account for the status of the axioms of geometry without engaging in a discussion about their truth (cf. Giedymin 1982; Zahar 2001). Similarly, conventionalism in physics and mechanics was provoked by work on the evolution of modern branches

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of science—such us electricity, electro-magnetism or thermodynamics—that led to a discussion of the status of their fundamental principles, starting with the Newtonian laws of motion, and to the introduction of ‘convention’ as a distinctive epistemological category (cf. Giedymin 1991; de Paz 2014; Stump 2015).

Conventions have been the object of several contemporary philosophical debates, e. g. concerning their agreement with modern science (for example, with general relativity), their rigidity, flexibility, or constitutivity in scientific theories, their relation to realist or anti-realist positions, and so on (cf. Friedman 1999; Folina 2014; DiSalle 2014). However, the connection of Poincaré’s philosophy to scientific practice has frequently been located in the problems that the different sciences of his time had. In this paper I shall consider this connection from a different perspective and focus more on the methods used by Poincaré to approach scientific problems than on the problems per se. By analyzing his methods, I will strengthen the connection between his philosophical position and his work as a scientist, showing the relation between the use of the notion of ‘convention’ and his scientific approach. Thus, the main purpose of the paper is to throw light on the origins of conventionalism as a philosophical position that originated from two specific methodologies proper to modern mathematics and modern natural sciences: methodological structuralism and the hypothetical-deductive method—hence, as a philosophical position which emerged from a way (or rather, two ways) of *doing* science. I will also show that these two methodologies are connected in both disciplines, geometry and physics. Thus, one of the aims of the paper is to show a possible path that connects geometrical and physical conventionalism by studying the methods underlying these philosophical positions.

The structure of the paper is as follows. After this introduction, the paper is divided into two parts. In the first one, I will present the structuralist method and its relation first to Poincaré’s approach to geometry and to his philosophy of geometry. Then I will consider this method in physics and how it relates to Poincaré’s philosophical views on physical theories. In the second part, the hypothetical-deductive method will be introduced in its relation to geometry and geometrical conventionalism, to be later discussed in connection to physics and physical conventionalism. In the final section I present some conclusions.

2 First Method: Structuralism

To a reader familiar with Poincaré’s philosophy of mathematics, the link with structuralism may seem odd. Structuralism is the idea that mathematics is the science of structures as opposed to the traditional view that mathematics is the science of quantity. It has its roots in abstract axiomatic approaches akin to formalist views. Given Poincaré’s rejection of logicist positions (cf. Goldfarb 1985), this might make his philosophy of arithmetic seem incompatible with structuralism. Regarding the foundation of mathematics, he was convinced that mathematics could not be reduced to logic and he defended the need of intuition even in proofs. However, Poincaré’s philosophy of mathematics has more than one layer, as it is not restricted to arithmetic, but there is also geometry to be considered. Regarding geometry, he rejected the Kantian view of a pure intuition of space in the light of non-Euclidean geometry, he emphasized the role of groups, and he elaborated a sophisticated position known as geometrical conventionalism that I will discuss later. Whether his position in arithmetic is compatible with the one he held regarding geometry or not is beyond the scope of this paper. So, regarding mathematics here I will restrict myself to his views on geometry.

In his works in geometry, Poincaré considered the relevance of studying "the structural relations between Euclidean and non-Euclidean geometry" (Nye 1979, 111). This has led to discussions in the recent literature concerning which of the structuralist positions in the philosophy of mathematics would be compatible with Poincaré's views. Structuralist philosophy of mathematics discusses, among other things, the ontological status of mathematical structures and of the entities instantiating those structures. The philosophical varieties of structuralism have been classified as *in re/ante rem*, eliminative-modal and non-eliminative in the classical works of Resnik (1997), Shapiro (1997) or Hellman (1996). In order to make Poincaré fit within this classification, an attenuated version of *ante rem* structuralism for his views regarding geometry has been defended by Heinzmann (2014, 2017) and Folina (2020). Although my views coincide with theirs in some respects, the approach of this paper is different. Instead of focusing on which philosophical version of structuralism is more appropriate to the ontological or epistemological status that Poincaré confers to mathematical structures, I will approach his structuralism as the orientation of a working mathematician. Firstly, I defend that Poincaré was using a structuralist method—that is, a mathematical method and not a philosophical position—to approach geometry. Secondly, I claim that from the use of this method in connection with the hypothetical-deductive method, which will be discussed in the second part of the paper, his philosophy of geometry emerged. Also, going a step further, I will show his use of the structuralist methodology in physics and the philosophical implications that he extracted from this use. In Poincaré's philosophy of physics, when structural approaches are considered, they usually refer to the structural realism that Worrall (1989a) presented in his famous paper, but structuralism is rarely considered as a method he used for doing physics.

Considering also the existing literature, in particular those works mentioned above, the use of the structuralist method has not been presented jointly regarding both physics and geometry. A joint analysis of the use of this method for both disciplines and the philosophical implications for Poincaré will throw some light on the links between his geometrical and physical conventionalism, beyond the common idea that he employed the word 'convention' for both disciplines.

To consider structuralism as a method is not a recent idea. It has been in the game for at least the last twenty years (cf. Reck and Price 2000; Reck 2003) and has been the object of recent studies like the ones contained in Reck and Schiemer (2020). Based on these studies, I want to retain some key points helpful to interpret Poincaré's work as structuralist, or at least as proto-structuralist.[1]

In a famous paper, Saunders Mac Lane defined mathematical structure in general as "a list of mathematical operations and relations and their required properties commonly given as axioms, and often so formulated as to be properties shared by a number of possibly quite different specific mathematical objects" (Mac Lane 1996, 174). This is not only the definition of a notion—structure—but also the description of a way of *doing* mathematics, by using this 'list of mathematical operations' which one can perform on mathematical objects. It is an abstract way of defining something that can have various instantiations: axioms define structures, which are to be taken as relational frameworks for carrying out work in mathematics, and these structures may be instantiated by different objects. Famous examples analyzed by Mac Lane in which this method is displayed are group, metric space, and topological space. Thus, the use of structures in mathematics, or mathematical

[1] The distinction between proto-structuralism and structuralism is presented in Reck and Schiemer (2020, 9–11). I will return to it in order to decide which one of the two fits Poincaré.

structuralism provides a 'general method' (cf. Mac Lane 1996, 176). And as I said above, I would like to focus precisely on method as a way of doing mathematics. It is important to note that methodological structuralism "has primarily to do with mathematical *method*, rather than with semantic and metaphysical issues as the others [i.e., the philosophical variants of structuralism] do." (Reck 2003, 371) This states that the classical philosophical concerns on the ontological status of structures are not the relevant part when using structuralism for doing mathematics, and thus, methodological structuralism is a different variant and should be classified apart. Also "*methodological structuralism* consists then of such a general, largely conceptual approach […]. A mathematician who is a methodological structuralist will not be concerned about the further identity or nature of the objects in the various systems studied" (Reck 2003, 371).

According to this, the structuralist method is a general approach typically connected to axiomatic approaches. That is why the ontology is not relevant, since the aim is to study structures and not objects. I shall take what I have so far presented as minimal landmarks of this structuralist method, according to which the interest is more on structures or concepts expressing properties and relations rather than on particular objects, and the approach is axiomatic. In the following section I discuss the presence of this method in Poincaré's way of doing geometry and how it is related to his philosophical position regarding geometry.

2.1 Methodological Structuralism, Geometry and Conventions

The ideas presented above fit quite well with Poincaré's characterization of what mathematicians do:

> Mathematicians do not study objects, but relations between objects. It is immaterial to them whether objects are replaced by others, as long as the relations do not change. Matter is of no concern to them, only form is of interest (Poincaré 1902/2017, 21).

According to this statement, objects are no longer a major concern for mathematicians, the subject-matter of their work is precisely structures, forms or relations. One can see here how Poincaré is really considering the mathematician in action and the emphasis he places on relations typical for methodological structuralism as described above. Actually, these ideas brought about a shift in nineteenth century mathematics:

> The growing awareness of the fundamental place of structure is characteristic of a general trend that took place during the 19th century in various fields of mathematics, from group theory to projective geometry and number theory (Ben-Menahem 2016, 260).

This trend was clearly in the air before Poincaré (cf. Folina 2020, 275). It consists of a new way of doing mathematics in which the concept of quantity no longer takes central stage. Rather, mathematics becomes a more abstract discipline that will result in the structural approach in the sense of Mac Lane (1996). Regarding geometry, this transformation consists in changing its subject-matter from the study of space or extension, as classically conceived, to the study of structures. This change is accompanied by the introduction of abstract methods in the discipline, such us methods showing the equivalence between different geometries, the reinterpretation of one geometrical theory in terms of another, or the use of model constructions (cf. Schiemer forthcoming).

This transformation is present in Poincaré's approach to geometry when he states: "What we call geometry is nothing but the study of formal properties of a certain continuous group; so we may say, space is a group" (Poincaré 1898, 41). The shift in the subject-matter of geometry can be seen here: it is no longer space as an extended magnitude but the formal (structural) properties of a well-known structural concept.[2] Structural methods—such as the use of the group concept and the introduction of continuous transformations—can be found not only in the description of the subject-matter of geometry but also in Poincaré's way of working, which is important to understand how he thought. His famous dictionary to translate non-Euclidean into Euclidean geometry (Poincaré 1902/2017, 37–38) has as its main aim providing us with a reinterpretation of a geometrical theory in terms of another, which is one of the abstract methods mentioned above.

The nineteenth century tendency to emphasize the role of structures has been characterized as proto-structuralism (Reck and Schiemer 2020, 9). Its key features include: the rejection of the traditional view of mathematics as the science of quantity and number, the adoption of the view that some parts of mathematics are not about particular objects and their properties, or the suggestion that mathematics is about relations. As shown in the quotations above, these are present in Poincaré's approach to geometry, but also are some of those considered as typical of a fully developed structuralist approach, such as basing various parts of mathematics on fundamental concepts (geometry and the concept of group), characterization of various kinds of objects via 'invariants', the substitution of the parallel postulate (cf. Poincaré 1898, 18), or the identification of isomorphic systems such as displacements in a group (cf. Poincaré 1898, 13). Thus, "it is the self-conscious and fruitful use of several of them [the features mentioned] together" that makes a "mathematical methodology structuralist" (Reck and Schiemer 2020, 10). In Poincaré's work in geometry, especially in his work on the concept of group, many of these features are found. He also made fruitful use of these features, in particular regarding his work on automorphic functions (cf. Gray and Walter 1997, 18). So one can assert that the structuralist method is one that he self-consciously used in geometry. In this sense, Poincaré can be very well considered to be a structuralist, rather than a proto-structuralist.

Many philosophically inclined mathematicians began to consider the significance of this structural shift (cf. Reck and Schiemer 2020, 10). As Poincaré is widely recognized as a philosophically inclined mathematician if not a philosopher of mathematics *tout court*, I shall show now that his philosophy of geometry, widely known as geometrical conventionalism, is rooted in his mathematical practice, especially in his structuralist approach to geometry.

Up until around Poincaré's times, i.e. in typical pre-Hilbertian approaches, geometry was considered as the science of space, commonly regarded as three-dimensional extension. So, geometry was the science that provided metric quantification to space. However, the structural shift led Poincaré and others to express their views differently: "Space is only a word that we have believed to be a thing" (Poincaré 1902/2017, XXIX). This assertion, considered in the context of his geometrical practice, involves a structuralist claim, since he is stating that space is not a mathematical (nor a physical) entity, but the word 'space' symbolizes a concept, or better, a conceptual scheme able to have several instantiations. In fact, "Space […] is already constituted as a conceptual scheme, beginning with the primitive group-theoretic conception identified by Poincaré as the basis of our elementary spatial knowledge" (DiSalle 2012, 14). Space is then a conceptual framework characterized by a determined structure, that of a continuous group. And how is this structure defined? I have

[2] Actually, Poincaré combines two structures: the group structure and the structure of continuous transformations.

stated above that structuralism is linked to typical axiomatic approaches. It is the abstract character that axiomatic approaches provide which connects to structuralist methods: "Axioms are about whatever systems fulfill the criteria they jointly stipulate" (Folina 2020, 284). So, structures are defined by axioms. And what are axioms to Poincaré? The answer is well-known: "*Geometrical axioms are then neither synthetic* a priori *judgments, nor experimental facts.* They are *conventions*" (Poincaré 1902/2017, 43). Here one can see the connection between his approach to geometry and the philosophical insights that emerged from it. The important concept is the structure, not the truth of the axioms defining the system. Conventions define the structure that defines a group that characterizes space.

According to the classical pre-Hilbertian views, it made sense to consider which geometry is true, in the sense that geometry was the science of extension and it was applied to physical extension or, for short, physical space. To Poincaré this question no longer makes sense. By using the structuralist method, he changed the subject-matter of geometry, which no longer is the science of space and axioms are no longer propositions defining physical space (whether Euclidean or not). Axioms define structures, conceptual frameworks. And these axioms are conventions.

It is his use of the notion of convention, in particular, for geometry, that made Poincaré a 'conventionalist', a label used by his commentators but never by him. Geometric conventionalism has often been considered to be a position regarding the applicability of geometry to physical space, which is clear in assertions such as "Experience guides us in this choice, which it does not impose upon us. It makes us recognize not which geometry is the truest, but which is the most *useful*" (Poincaré 1902/2017, 56). This is a position defining what Einstein (1921, 212) later called 'the sum geometry + physics', and it is about deciding which geometry should be applied to physical space, connecting it thus with physical laws. Given the flexibility that the notion of convention implies,[3] it has commonly been stated that geometrical conventionalism emerged as a consequence of the plurality of geometries developed in the nineteenth century, with the emergence of non-Euclidean geometries, which provided several possibilities to be applied to physical space (cf. Coffa 1986). One can easily agree that this is the case. But I think that this is only *part* of the case: Geometrical conventionalism is not only a position regarding the applicability of geometry to physical space, but it is also a position considering geometry or geometrical axioms per se. In this sense, one could say that it is also a position about 'pure' geometry. As such, it is not only the emergence of non-Euclidean geometries which prompted Poincaré's philosophical views. Moreover, the structural shift in geometry was not only a consequence of the development of non-Euclidean geometries, but also of the introduction of abstract methods in geometry (transfer principles, reinterpretation, see Schiemer (forthcoming) for more on this). Poincaré's geometrical conventionalism is a philosophical position that emerged from his own introduction of these methods based on the work of Lie on the Riemann-Helmholtz space problem.[4] The use of the axiomatic method leads to a more abstract level of mathematics (the same applies to structures), and that is precisely what Poincaré did when defining geometry as the study of a group. Given the plurality of groups and the abstract insight gained through the axioms, Poincaré introduced a specific

[3] For a study on the notion of convention and its flexibility see de Paz (2018), although not considering geometrical conventions, it provides a useful characterization of the concept.

[4] I say it emerged, as I could say that it is a consequence of it. But I do not want to give the impression that geometrical conventionalism is a necessary consequence and thus, the only possible philosophical position that emerged from the use of structuralism in geometry.

epistemic category to characterize geometrical axioms, and to account for their epistemic status. This provides an additional insight regarding the notion of convention as Poincaré understood it, independently of (or complementary to) the applicability of geometry to physical space.

The use of the notion of convention to describe the epistemic status of geometrical axioms characterises their flexibility, as well as their detachment from truth, since conventions are neither true nor false, but convenient (cf. Poincaré 1902/2017, 99). "Poincaré sees geometry as at least partly detached from a truth-determining subject matter" (Folina 2020, 285), thus if not true as a science, it is conventional. Conventions define the abstract structures that we used to call 'space'. So, his use of structuralist methods and his endorsement of some structuralist views lead him to consider the significance of this new shift in mathematics and develop a philosophical position accordingly. Let us now see how this works for natural science.

2.2 Methodological Structuralism, Conventions and Physics

When Poincaré refers to natural science, in general he means mechanics and physics. Although these are different disciplines as becomes clear in well-known ideas regarding the hierarchical structure of science (cf. Friedman 1999, 76), both are considered here as belonging to mathematical physics. And although physics carries more empirical content than mechanics—which in turn carries more empirical content than geometry—, these disciplines share many features. In fact, just like in geometry, basic mechanical principles are conventions,[5] as Poincaré expressed at the end of the third part of *Science and Hypothesis* (1902/2017, 100): "The principles [of mechanics] are conventions and definitions in disguise"; and the same goes for physics: "The principles [of physics], though of experimental origin, are now unassailable by experiment because they have become conventions" (Poincaré 1905, 145). This is stated in his paper on the present and future of mathematical physics presented at the 1904 St. Louis Conference, where the topic was the relevance of some physical principles in the light of Lorentz's theory. So, whether he is talking about mechanics or electromagnetism, the status of the fundamental principles is the same: conventional. But the relevance that Poincaré attaches to principles is not only based on their epistemic status. It also expresses his theoretical approach to mathematical physics, labelled 'the physics of principles' which can be characterized as follows:

> Poincaré favoured a 'physics of principles' in which the compatibility of theories with general principles was more important than the completeness of their physical picture (Darrigol 1995, 1).

The insistence on the relevance of the principles is something very present in Poincaré's technical works on the dynamics of the electron (cf. Poincaré 1906; 1908; 1913) and the new mechanics (cf. Poincaré 1909; 1910), where he made efforts to make compatible Lorentz's electron theory with the principle of relativity. In these works, the physics-of-principles approach is displayed in order to reveal the relativistic dynamics of the electron, as a result from the Lorentz invariance, i.e. the compatibility of Lorentz's theory with the principle of relativity, regardless of the physical details concerning the electron, or even the ether.

[5] Maybe not conventions of the same type, as specified in Stump (2015), but conventions.

This view is particularly clear in Poincaré's lectures on optics, in which he chose to teach his students five different contemporary optic theories in order to show that they share the same system of equations, provided some adjustments (cf. Darrigol 2018, 30–35). He considered this to be an expression of the sameness in structure underlying these theories. That is, in spite of the ontological divergences in luminous ether, waves or particles held by the theories, there was something common—the abstract structure—, and through this structure, different theories represented the same set of phenomena:

> What is essential, that is to say, what must remain common to all theories, is made prominent; all that would only be suitable to a particular theory is nearly always passed over in silence. Thus the reader finds himself in the presence of a form almost devoid of matter, which he is at first tempted to take for a fugitive shadow not to be grasped. But the efforts to which he is thus condemned force him to think and he ends up seeing what was often rather artificial in the theoretic constructs he used to admire (Poincaré 1890, XV–XVI).

But this 'physics of principles' is not only present when dealing with the new electrodynamic optics, it is also embedded in the analytical tradition of mechanics, which expresses a method of approaching nature characterized by the "decline of empirical *and* metaphysical justifications of concepts and the laws combining them. […] The 'first principles' of mechanics become *formal axioms* of science rather than material laws of nature" (Pulte 2009, 81). Both Lagrange's and Jacobi's approaches are placed within this tradition, based as they are on the introduction of abstract principles without searching for underlying metaphysical explanations, and with them mechanics becomes an abstract science. In analytical mechanics, a set of principles imposes a structure on the objects of study. This abstract approach becomes a *method* for doing mathematical physics, just like the structural approach was a method for doing geometry. It is important to remark that the principles situated at the basis of mathematical physics are not themselves considered as based on either a priori or experimental concepts, such as matter or force, but they are principles that organize the structure of science. Poincaré himself expresses this view as a transition from the physics of central forces to the physics of principles:

> The attempt to penetrate into the detail of the structure of the universe, to isolate the pieces of this vast mechanism, to analyze one by one the forces which put them in motion, was abandoned, and we were content to take as guides certain general principles the express object of which is to spare us this minute study (Poincaré 1905, 126).

The traditional method represented by the physics of central forces explained natural phenomena in terms of masses and interacting forces, i.e. with the purpose of finding the ultimate underlying mechanisms of nature. This view tries to disentangle the basic ontology of nature. The physics of principles subsumes phenomena under general principles. It displays a general approach in which those previous significant mechanisms which tried to provide a complete picture are no longer central in providing a physical explanation. Those mechanisms become indifferent hypotheses. Relevance is placed in the principles which provide the necessary framework to explain and predict phenomena. As in the structural approach to geometry, in the new method of physics ontology loses relevance.

The physics of principles is thus an abstract approach to mathematical physics in which theories are formed from several elements, among which principles defining a structure or conceptual framework. A prominent example is Poincaré's interpretation of Newtonian mechanics. Newtonian laws of motion are principles in Poincaré's analysis (cf. 1902/2017, 73ff). If one considers the case of the inertia principle, it posits a privileged motion (uniform and rectilinear) and any deviation from this motion means that a force is acting and can be measured according to the second law or principle. In this sense, these principles define the conceptual framework in which they work: "Within the framework defined by Newton's laws, the investigation of any interacting system can start from the simplest idealized model, and every deviation from the ideal behavior is informative" (de Paz and DiSalle 2014, xiii). A different set of principles—for example a different privileged motion—would have defined a different conceptual framework.

This is a kind of view 'from afar' regarding physics, in which the ontological disputes are avoided, in order to provide general explanations compatible with several theories. These theories might postulate different ontologies, but what matters is that they are compatible with the principles. The latter help in defining a structure, since they are abstract and sophisticated, involving "quite advanced, abstract mathematics" and they "are used to summarize the empirical laws and experimental facts common to two or more theories" (Giedymin 1982, 44). This search for principles expressing the common aspects of different theories through the search for invariants, by introducing a more abstract approach is, if not structuralist per se, at least a proto-structural approach.[6]

But why are these principles considered to be conventions? Or better, what is the connection between the proto-structuralist methodology of the physics of principles and the conventionalist philosophy of physics displayed by Poincaré?[7] This will become clearer in the next part of the paper and once the hypothetical-deductive method for physics is discussed in the context of Poincaré's ideas. But it is important to state that it is connected to the structuralist method by the abstract character held by principles. Of course, this is not a necessary connection, as it was not necessary in the case of geometry. But to Poincaré, the structuralist method of the 'physics of principles' required the introduction of a new epistemic category able to characterize principles that are not fully empirical or a priori. Principles are abstractions that the scientist has elevated to a non-verifiable status. That is what 'conventional' for the principles of mechanics means, something elevated to a status where it is unassailable by experiment. The abstract character expressed in the principles when a theory is compatible with them expresses a mark of the true relations captured by the theory. And of course, expressing relations is important because they are all we can know:

> [T]hat which science captures are not the things themselves, but simply relationships between them. Beyond these relations, there is no knowable reality (Poincaré 1902/2017, 2).

Poincaré's efforts in showing the compatibility of different theories with the same principles and in showing what is invariant in the theories must be considered a proto-structural methodology in natural science. The abstract character of the principles able to subsume several phenomena and experimental laws connects to the notion of convention as

[6] I am here more cautious in the characterization of Poincaré as a proto-structuralist instead of a structuralist, since there are several structural aspects—as the emphasis on relations suggests—, but one cannot fully assert that he developed for physics a fully structural approach as he did for geometry. In particular, this is so, because he kept in mind the relevance of learning about the models (e.g., ether models) which helped us in visualizing the structure, something that in geometry is not so prominent.

[7] Although Ivanova (2015) connects Poincaré's conventionalism with his structuralism, she focuses on the emphasis on relations and on the possibility that these relations capture the outer structure of the world (structural realism) and not on the methodological proposal I am considering here, in which a particular scientific practice prompts a particular philosophical position.

something beyond empirical verification. Principles are not directly based on experiment nor are they valid a priori. They are abstractions introduced by the scientist in the constitution of science. They cannot be overturned by experiment, and this is what happens with conventions: not empirical, not a priori and not able to be demoted by experimental results. One can get rid of them once they are no longer useful, but this is not the same as being refuted. Moreover, the flexibility involved in the notion of convention explains the non-unique character of the principles and the possible existence of alternatives, as in the case of the inertia principle.

3 Second Method: Hypothetical-Deductivism

Now I turn to the second method 'behind Poincaré's conventions'. Unlike structuralism, hypothetical-deductivism is a well-known method in science, and it is also popular amongst philosophers of science at least since Popper. A common way of characterizing it is the following: it consists of an approach in which a supposition is made to explain a set of phenomena, and deductive logic is applied to extract consequences from that hypothesis. Then the consequences are tested, and if proved correct, the hypothesis is considered correct, at least, until something contradicts it. The use of hypotheses in Poincaré's philosophy is known to the reader familiar with his work, and there have been studies on the different kinds of hypotheses that Poincaré uses (e.g., Heinzmann and Stump 2017). However, to the best of my knowledge, there is no study analyzing the use of this method in connection to Poincaré's philosophical ideas, and the point there might be in common when it is applied to geometry and to physics.

A good example of the use of this method are Riemann's methodological remarks in a paper written around 1866, where he presents a process of research explained in three phases[8]:

1. The search for a hypothesis which is sufficient to account for what the organ accomplishes.
2. An investigation of the extent to which such a hypothesis is necessary.
3. Comparison with experience in order to confirm or to correct it (Riemann 1876, 317).

The origin of this text is a paper on "The Mechanism of the Ear", thus the first step refers to a hypothesis that is sufficient to explain the phenomena related to this organ. This is a clear description of the hypothetical-deductive methodology as explained above, from the formulation of a hypothesis to its testing in experience. I have chosen the example of Riemann because it enlightens Poincaré's procedure and I will show that there are some common points in the methods employed by both mathematicians.

In what follows I will proceed as in the previous part. I will consider the display of this method in Poincaré's geometrical work and then in his works on physics. Of course, in the case of pure geometry only the 'conceptual' aspect is taken into account, and the key ingredient of experimental testing is lacking. In any event, around 1900 it was usual to emphasize the 'hypothetical-deductive' method in geometry (an outstanding example is Mario

[8] Cf. de Paz and Ferreirós (2020) for further discussion of Riemann's methodology.

Pieri, an Italian mathematician).[9] Later, philosophers have tended to reserve that name for the method as employed in empirical sciences.

3.1 Hypothetical-Deductivism and Geometry

The notion of hypothesis is central to Poincaré's work, it is not for no reason that it forms part of the title of what has been named 'his first philosophical book', *Science and Hypothesis*. It is a key notion in his approach to science, but it can be found in his writings even before the publication of that book. In 1887 Poincaré published a paper entitled "On the fundamental hypotheses of geometry" (Sur les hypothèses fondamentales de la Géométrie). It is a technical paper, not often quoted in philosophical analysis of Poincaré's work, because of the complex mathematical approach it contains.

According to the title and the content of the text, geometrical axioms are considered hypotheses, in the Greek sense of the word, i.e. as something placed 'under' or at the basis of research. As I will show, the aim of the paper is to expose the hypotheses at the basis of a determined geometry. To any historian of geometry, the title of that paper sounds familiar, usually not associated to Poincaré, but to Riemann's Habilitationsvortrag "Über die Hypothesen, welche der Geometrie zu Grunde liegen" (On the Hypotheses which lie at the basis of geometry). In fact, Poincaré's choice of title is certainly inspired by Riemann's ideas, and Riemann's work is mentioned in the article. But I will try to show that not only the title, but part of the approach in the text is guided by Riemann's ideas and method. In this sense, one can say that Poincaré and Riemann are using the same scientific method, although they differ in the mathematical details they use in their respective presentations. Let us now see how Poincaré proceeds in that paper in which he copies the title of Riemann's work.

The paper starts by questioning the name that the fundamental premises of geometry should receive—axioms, hypotheses, postulates—and what their status might be—experimental facts, analytical judgments, synthetic a priori judgments. So the philosophical problem of the status of geometric propositions is posed right at the beginning, in a similar way as Riemann posed it at the beginning of his Habilitationsvortrag. But the central problem for Poincaré would be, before reflecting on the status of geometrical propositions, to consider what these propositions are, that is, to make explicit the propositions required to perform geometrical demonstrations. Thus, the aim of the paper is "to formulate every necessary hypothesis" to constitute a geometry, "and only those" (Poincaré 1887, 204). In this paper, he used the group-theoretic approach to geometry to characterize and restrict the number of possible geometries he discusses. But what is relevant to my point is the way he presents his procedure:

> We can formulate the necessary and sufficient hypotheses to serve as premises for plane geometry:
> A. The plane has two dimensions.
> B. The position of a plane figure in the plane is determined by three conditions [parameters] […]

[9] In 2002 Avellone, Brigaglia & Zappulla offered this evaluation of his contribution to geometry: "Pieri's work was very influential. B. Russell and L. Couturat rightly regarded him as the founder of mathematics as a hypothetical-deductive science." (418) One might also think about the work of Riemann, who in his Habilitation lecture used precisely this method, before Pieri, of course.

> These two first hypotheses allow us to choose among the several quadratic geometries [2-dimensional geometries of constant curvature] […]
> These two geometries are excluded if we also make the following hypotheses:
> C. When a figure does not abandon its plane and two of its points remain immobile, the whole figure stays immobile (Poincaré 1887, 213–214).

The statement ‘formulating the necessary and sufficient hypotheses’ sounds clearly Riemannian.[10] Poincaré starts by defining the hypotheses for plane geometry and then he continues formulating hypotheses (D., E., F…) which restrict the number of possibilities until he reaches Euclidean geometry. Thus he has formulated all the necessary hypotheses required to construct this geometry, i.e. the hypotheses which lie at the basis of (Euclidean) geometry. As in the Riemannian methodology quoted above, he has searched for hypotheses and investigated the extent to which they were necessary. Actually, Riemann also introduced “more and more restrictive hypotheses (postulates in our terminology), leading from pure topology to the concretion of Euclidean space” (Ferreirós 2006, 88), which is a procedure extremely similar to Poincaré’s.[11]

In the second section of the paper Poincaré applies Lie’s group theory to unearth the common hypotheses to what he calls “quadratic geometries”. From the hypotheses the possible operations or movements are defined, and these form a group. The different possible groups constitute different possible geometries. As we know, groups are algebraic structures defined by the formal properties and possible operations expressed by the group axioms. These axioms are called hypotheses by Poincaré in his 1887 paper. Depending on the axioms or hypotheses one introduces and the consequences one derives from them, one obtains one geometry or another. So, the hypothetical-deductive approach to geometry is connected to the structuralist approach, as the use of the group concept in Poincaré’s paper shows. It is also interesting to notice that this kind of approach was quite influential in the formulation of later structuralist ideas, such as Heinzmann has shown for the case of Bourbaki (Heinzmann 2017), in which he names Bourbaki’s approach precisely as ‘Hypothetico-Deductive Structuralism’.

The naming of geometrical propositions as hypotheses cannot be considered to be a casual choice by Poincaré, since right at the beginning of the paper he discusses how to name those premises, and in the whole paper he uses the word ‘hypothesis’. This choice is influenced by how strongly the concept of axiom was laden with the connotation of being ‘necessarily true’ and ‘self-evident’, given its traditional classical meaning.[12] ‘Hypothesis’ offers a more flexible alternative, since it is a proposition not true or false and in any case not self-evident. In a sense, the hypothetical-deductive approach comes to substitute the classic axiomatic-deductive approach. This becomes even clearer when, at the end of the

[10] Compare: “After this investigation into the determination of the metric relations between n-dimensional entities, it is now possible to indicate a set of necessary and sufficient conditions for the determination of these relations in space. We make the hypotheses that the lengths of lines are independent of position, and that the length of the infinitesimal line element is expressible as the square root of a quadratic differential expression, so that flatness in the smallest parts is assumed” (Riemann 1876, 265).

[11] The difference lies essentially in the mathematics they used. Riemann approaches the subject using the methods of differential geometry and Poincaré that of Lie’s continuous groups. But they are methodologically and philosophically very close.

[12] The concept of axiom would change in 1899 with Hilbert’s work, but in Riemann and Poincaré the concept is the traditional one.

paper, Poincaré goes back to the other problem posed at the beginning of the text, i.e. the status of geometrical premises:

> Now we can ask what these hypotheses are. Are they experimental facts, analytical or synthetic a priori judgments? We have to answer no to these three questions. If these hypotheses were experimental facts, geometry would be submitted to a constant revision, it would not be an exact science. If they were synthetic a priori judgments, or even more, analytical judgments, it would be impossible to found anything upon their negation.
> We can show that analysis is based on a certain number of synthetic a priori judgments; but it is not the same for geometry (Poincaré 1887, 214–215).

After this paragraph a reader used to Poincaré's ideas would expect the introduction of the notion of convention, since clearly he is in search for a distinctive epistemic category to characterize the hypotheses which lie at the basis of geometry, and he rejects the Kantian categories that were available until then. We know this new category is 'convention', but he does not use that word in the text, although the 'conventional mood'—possibility of choice, rejection of the other categories—is undoubtedly present. Only in 1891 would he repeat some of the arguments present in this 1887 paper, and for the first time introduce the notion of convention applied to the fundamental premises of geometry. In any case, it is the consideration of geometrical premises as hypotheses and the use of the hypothetical-deductive method in the best Riemannian sense that led Poincaré to introduce a new epistemic category and thus to develop geometric conventionalism.[13]

3.2 Hypothetical-Deductivism and Physics

When it comes to physics and natural sciences, the hypothetical-deductive method might be best qualified as experimental or empirical hypothetical-deductivism, since the consequences of hypotheses are supposed to be tested against natural phenomena.[14] This method was popular among physicist of the time. Two examples of this use with which Poincaré was certainly familiar are Fresnel's works on the diffraction of light and Weber's works on electricity. Fresnel took as a fundamental hypothesis that light is a wave in motion in a medium:

> I am now going to show that one can give a satisfactory explanation and a general theory [of diffraction] within the system of waves without the aid of any secondary hypothesis, by depending only on the Huygens principle and that of interferences, which are both consequences of the fundamental hypothesis (Fresnel 1819, 282–283).

Although there are problems in the way Fresnel extracts the consequences of his fundamental hypothesis (cf. Worrall 1989b), what matters to us is the method he uses, starting from an assumption and then inferring consequences from it. This is also, when talking about Weber's physics, what Dedekind called 'truly scientific research':

[13] Important is also what Coffa (1986) stresses: the existence of alternative geometries and their relation to physical space. But he introduced this category first to deal with pure geometry.
[14] This is also clear in Riemann's text on "The Mechanism of the Ear" quoted above.

> The strict separation between the fundamental facts, discovered by means of the simplest experiments, and the hypotheses linked with them by the thinking human mind, afforded an unmatchable model of the truly scientific research (Dedekind's comments on lectures of circa 1850; quoted in Ferreirós 2007, 25).

As his lectures on optics and electricity prove, Poincaré was well acquainted with the work of both, Fresnel and Weber. It is very likely that in his lessons he decided to show the variety of optical theories in order to stress not only the structure common to them, but also their hypothetical character, given the assumptions that their proponents established to develop those theories. In fact, Poincaré acknowledged this procedure as the usual one in mathematical physics:

> All our great scientists, from Laplace to Cauchy, proceeded in the same manner. Starting from clearly stated hypotheses, they deduced all their results with mathematical rigor before comparing them to the results of experiment (Poincaré 1890, V–VI).

This was also the way in which he proceeded in his courses in order to extract the ideas common to the different optical theories he expounded. He started by supposing an elastic medium formed of molecules (he called this the first hypothesis, cf. Poincaré 1889, 3). Then, he supposes that the molecules in the medium are in equilibrium, so the attraction forces between them can be studied as functions of their motion (second hypothesis, cf. 1889, 3). Based on this, he deduced the linear equations to study motion and energy conservation, given the equilibrium in the medium. Subsequently he introduced a third hypothesis neglecting the interaction among particles when they are very distant (cf. 1889, 13). This allowed him to simplify the functions of the interacting forces. From these initial hypotheses he deduced the common structure of the optical theories he would later develop:

> In fact, were we to look at the situation more closely, we would see that only two things are borrowed from the molecular hypotheses: the principle of the conservation of energy and the linear form of the equations that is the general law of small motions, as it is of all small variations (1889, III).

This is what happens when Fresnel's theory is substituted by Maxwell's: wave theory is replaced by an electromagnetic theory that explains phenomena in terms of variations of the electromagnetic field, but Fresnel's results are preserved, thus what changes is the physical interpretation of the formulae. It does not matter if there are one or two electric fluids, or variations in a field, those are hypothetical parts of the theories, and when one acknowledges their hypothetical character, one is able to acknowledge the hidden structure common to them. This also reminds one of the second point in Riemann's hypothetical-deductive methodology: "An investigation of the extent to which such a hypothesis is necessary" (Riemann 1876, 317). One can now see the link between the proto-structural approach displayed in his 'physics of principles' and the hypothetical-deductive method: hypotheses are introduced to deduce some consequences from them that help us in finding the structure of the theories, in establishing the true relations that are preserved when the theories change:

> The equations express relations and if these equations remain true, it is because the relations preserve their reality. Now as before, they teach us that there is a particular relationship between something and something else. We formerly called this something *motion* and we now call it *electric current*, but these labels were only images standing in for the real objects that nature forever hides from us. The true relations

> between these real objects are the only reality we can reach, the only condition being that the relations between the objects are the same as those between the images standing in for the objects. It does not matter whether we find it useful to replace one image by another, as long as these relations are known to us (Poincaré 1902/2017, 115).

This connects also with the structuralist topic of the relevance of relations in contrast with the irrelevance of objects: it does not matter what is called electric current, what matters is that relations are preserved and known. Wave motion or electric current are outer forms of representation that help us in understanding different theories. The use of one or the other defines a different framework for testing the phenomena. In this sense theories work as conceptual constructions defining their range of applicability, but some of the elements in the theory are in the end neither true nor false, but remain hypothetical and dependent on certain choices. As we know, Poincaré named these choices conventions and established a link between hypotheses and conventions:

> We shall also see that there are many kinds of hypotheses; […] that others are hypotheses in appearance only, and amount to definitions or to conventions in disguise. The latter are found mostly in mathematics and its related sciences (Poincaré 1902/2017, 1–2)

By emphasizing the physics of principles as a new method that substitutes the physics of central forces, Poincaré highlighted the role of structures in the natural sciences and the use of the hypothetical-deductive method. This led him to introduce a new epistemic category that captures the flexible character of some elements present in the theories that enable us to discover the hidden relations among things and that connect our mathematical structures to experience.

Moreover, when he analyses the science of mechanics, he states the conditions under which this science works (e.g., absolute space, absolute time, equality of time intervals, Euclidean geometry, cf. Poincaré 1902/2017, 71). These define a framework in which the principles of mechanics are applied. They are not preexistent to mechanics, they can be derived from the use of the principles, but they are not necessary. They are conventions that are also dependent on other conventions, such as the principles of mechanics. Let us take the principle of inertia as an example. Poincaré says that it is neither an experimental fact nor an a priori truth, hence we know it is a convention. But, before applying that epistemic category, he analyzes it in what he calls its generalized form: "The acceleration of a body depends only on its position and velocity and those of neighboring bodies" (1902/2017, 73) and proposes hypothetical alternatives to the principle that may lead to different conclusions. If one puts together the fact that there are possible alternatives to the principle, with the idea that it is not a priori or empirical, and with the fact that its validity cannot be proved since "[w]hen a body is not subject to any force, rather than suppose that its speed does not change, we could suppose that its position or even its acceleration should not change" (1902/2017, 73), then it becomes a hypothetical assumption, or, in Poincaré's words, a convention.[15] The fact that Poincaré treats the principle of inertia as a supposition or hypothesis will not be surprising if one takes into account that there is a whole tradition

[15] Of course, there are different kinds of hypotheses and conventions, but how to classify them and how they relate to each other is beyond the scope of this paper. Works that discuss Poincaré's ideas concerning hypotheses are Ly (2008), de Paz (2015) or Heinzmann and Stump (2017).

questioning its status as an axiom.[16] A prominent example within this tradition is precisely Riemann:

> The distinction that Newton makes between laws of motion, or axioms, and hypotheses, does not seem tenable to me. The law of inertia is the [following] hypothesis […] (Riemann 1876, 525).

So, the use of hypotheses in physics is introduced as a method to explain the phenomena, as a supposition lying at the basis of research from which by deduction one can extract consequences and contrast them. From this use it emerges also as an epistemic tenet that questions the axiomatic status (in the old sense of true and evident) of classical mechanics just as the use of hypotheses in geometry questions the axiomatic status (also in that sense) of Euclidean geometry.

4 Conclusions

The previous sections discussed Poincaré's claim that geometry is the study of the formal properties of certain continuous groups and argued that this is typically a structuralist claim. I have shown that geometrical conventionalism may be understood as a consequence of Poincaré's way of doing and understanding geometry. Poincaré's defense of a 'physics of principles' where the conventional status of the principles is made explicit can also be linked to a movement of abstraction in nineteenth century physics that is similar to the conceptual approach in mathematics, characteristic of the emergence of the structuralist position. The structuralist method in geometry is connected to the hypothetical-deductive method when the status of the axioms is transformed into hypotheses (in the way Riemann did), and the hypothetical-deductive methodology substitutes the axiomatic-deductive one. This is shown in the Riemannian path that Poincaré follows. Poincaré qualifies the axioms of geometry as hypotheses and conventions and connects these two notions. Similarly, hypothetical-deductivism is a method of the natural sciences, and Poincaré applies it to mathematical physics. This method also challenges the traditional understanding of natural sciences as axiomatic-deductive, as it appears in traditional works of early Modernity such as Newton's *Principia*. By challenging this conception, the status of the fundamental principles becomes susceptible to revision, that is hypothetical, and in the eyes of Poincaré this requires also the introduction of a new epistemic category: convention.

The notions of convention and hypothesis are deeply interconnected. The title of Poincaré's book *Science and Hypothesis*, where he displays his 'conventionalist ideas' shows this connection, as well as the introduction of the book where he stresses the role of hypotheses in science.[17] Both notions share their uncertain and flexible character. These propositions are not true or false, but assumed as suppositions guiding scientific research. They express the existence of a range of possibilities or alternatives among which one has

[16] Regarding this tradition, see the works of Helmut Pulte, esp. (2005; 2009; 2012).

[17] As Heinzmann (2009, 169) states, the title of the book was probably a suggestion by Gustave Le Bon (there is a letter from Ernst Flammarion to Poincaré indicating that much: http://henripoincarepapers.univ-lorraine.fr/chp/image/flammarion-e-1902-04-18-1.jpg). Poincaré did not modify the title (although he could have done so). Thus, he probably considered the title adequate, and in any case, the relevance of the notion of hypothesis in the book is explicitly stated in the introduction which supports our argument. I want to thank the reviewer who called my attention to Flammarion's letter.

to choose to proceed in subsequent scientific stages. Poincaré himself interpreted conventions as some kind of hypotheses, as hypothetical in appearance, and stressed that those were precisely those present in mathematics and related sciences (cf. Poincaré 1902/2017, 1–2). The words 'convention' and 'hypothesis' have to do with methodological flexibility, that is, with the idea of having different conceptual possibilities.[18]

Given that the questioning of the traditional epistemic status of both—the fundamental premises of geometry and the principles of mathematical physics—is connected to the use of modernist scientific methods, one can say that the introduction of certain methodological elements led to Poincaré's philosophical conception regarding geometry and physics. Of course, the thread that guided him to introduce the category of convention is not a necessary path, in the sense that he could have thought of different possibilities, but "his structuralist views are not tangential to the rest of his philosophy" (Folina 2020, 283), in particular if one considers his philosophy as emerged from his scientific practice, and his structuralist views as a method of doing science as I have done and not as a philosophical position defining the ontological status of structures.

The use of parallel methods in geometry and physics connects his philosophical ideas in both disciplines, as does the use of the notion of convention. One can see these relations not only regarding the hypothetical status of the fundamental principles of both disciplines, but in the reduced interest in the ontological details consequent upon the introduction of the structuralist methods. However, this does not mean that geometry and mathematical physics share exactly the same epistemic status. Geometrical conventions and the conventions present in mathematical physics are different, because Poincaré's philosophy is 'regional' (cf. Schmid 2001, 12), meaning that it is adapted to different sciences. In fact, Poincaré himself states the difference in status of geometry and physics:

> It is tempting then to say that either mechanics must be regarded as an experimental science, and then the same must be true of geometry, or rather on the contrary, that geometry is a deductive science and then the same can be said of mechanics.
> Such a conclusion would be unjustified (Poincaré 1902/2017, 99).

The fact that he uses similar methods does not make the philosophies of physics and geometry equivalent, but connects them. Both sciences employ experience in different ways[19] and while geometry is an 'exact science'—a science that leads to precise and definitive conclusions, a mathematical science—, mechanics and mathematical physics in general are empirical, and as empirical sciences, their conclusions can be overturned. However, it can be asserted that Poincaré saw a practical and philosophical connection in these sciences: the crisis in the foundations of geometry led to the introduction of new methods and to the questioning of traditional epistemic categories. The same happened with the crisis in mathematical physics. By working scientifically on both crises, Poincaré developed his scientific philosophy.

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[18] This is not the case when I use the notion of 'convention' in a trivial linguistic sense: e.g. the name of the thing called 'table' is 'table' by convention. In this sense, it does not display different conceptual possibilities, but it is clear that Poincaré is not dealing with this trivial semantic sense.

[19] Experience is at the basis of the formation of geometry, but there are no proper geometrical experiences (cf. Poincaré 1902/2017, 98).

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