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Modelling inclusive breakup: application to incomplete fusion

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Abstract. We propose an extension of the inclusive breakup model of Ichimura, Austern and Vincent [Phys. Rev. C 32, 431 (1985)] for the evaluation of incomplete fusion (ICF) cross sections in nuclear reactions induced by two-body projectiles. The main idea, adopted in other methods, consists in the separation of the participant-target optical potential into its direct reaction and compound reaction components, the latter being responsible for the ICF contribution of the total nonelastic breakup cross section. Preliminary comparison with experimental data shows encouraging results.

1. Introduction

Fusion of neutron- and proton-rich nuclei is a hot topic with numerous applications, ranging from nuclear astrophysics [1, 2] to the production of superheavy elements [3]. A proper quantitative description of fusion dynamics in reactions involving these exotic species has challenged reaction theory for several decades. Part of the difficulty arises due to the influence of the other reaction channels on the fusion cross sections, such as inelastic excitations of the colliding nuclei, transfer and breakup. There has been also a long-standing debate regarding the interplay and competition between the so-called complete (CF) and incomplete fusion (ICF) mechanisms. Experimentally, they are operationally defined as the fusion processes involving either the total capture (CF) or the partial capture (ICF) of the projectile charge by the target nucleus. There are many experimental evidences indicating that, in reactions induced by light, weakly-bound nuclei, such as ^{9}Be or ^{6,7,8}Li the CF cross sections are suppressed by 20-30% with respect to the prediction of single-barrier penetration models or even coupled-channels calculations accounting for collective excitations of the interacting partners [4–10]. The phenomenon has been attributed to the flux going to either the elastic breakup of the projectile or to ICF. A quantitative assessment of this hypothesis has been hampered by the lack of reliable and predictive models for ICF, a situation that has triggered a significant theoretical and experimental activity in recent years.

In addition to its inherent interest for the understanding fusion dynamics, ICF cross sections are also needed for several practical applications, such as the so-called surrogate method, an indirect method for evaluating compound-nucleus cross sections in reactions for which the direct measurement is difficult or even not possible. An example is the extraction of neutron-induced cross sections of the form (n, χ), where χ is a given decay channel product (γ, fission fragment, etc) from a surrogate reaction giving rise to the same compound nucleus, such as a (d, p) reaction.

In this contribution, we present a practical approach to evaluate ICF cross sections in reactions induced a two-body projectile nucleus. The method is based on the inclusive breakup model developed by Ichimura, Austern and Vincent (IAV) in the middle 1980s [11]. The IAV model provides total inclusive breakup cross section for a generic reaction of the form $A(a, b)B$ where a projectile a, after colliding with target A, dissociates into fragments b and x, but only the former is observed experimentally. The method proposed here tries to isolate the ICF contribution from other breakup contributions provided by the IAV model. This is accomplished by replacing the $x + A$ optical potential by a short-range imaginary potential, which simulates the fusion process. The model is particularly suited for projectiles with a developed two-body structure, such as deuterons, ^{6}Li or ^{7}Li. In this contribution, we consider the ^{7}Li case as an illustrative example and present calculations for the ^{7}Li+^{209}Bi reaction, for which experimental data exist.

The document is organized as follows. In Sec. 2.1 we present a brief remainder of the IAV DWBA formula for nonelastic breakup. Then, in Sec. 2.2 we present the proposed modification of the IAV formula to extract the ICF content of the NEB cross section. In Sec. 3 the model is applied to the reaction ^{7}Li+^{209}Bi and compared with experimental data for this reaction. Finally, in Sec. 4 we summarize the main conclusions of this work and anticipate some future extensions.

2. Theoretical considerations

2.1. Reminder of the IAV formula for inclusive breakup

The IAV model has been presented in several recent works [12–15] so we give here only the main formulas. The goal of this model was to provide a practical way of evaluating the singles cross section for a inclusive breakup reaction of the form $A(a, b)B$ (following the same notation used in the introduction) in which on the fragment b is detected and x can undergo any possible interaction with the target nucleus. Thus, the observed b yield includes any possible final state of the $B = x + A$ system. The model separates these contributions into two groups: (i) the elastic breakup part (EBU), in which the fragment x escapes after the collision whereas the target A remains in its ground state, and (ii) the nonelastic breakup part (NEB), which includes any nonelastic interaction of x with A. The ICF is one of the channels contributing to NEB, namely, that in which x is captured (fused) by the target, forming some compound nucleus. Using the Feshbach projection formalism, the model provides compact expressions for the EBU and NEB parts. Since the focus of this contribution is on ICF, we give only the expression for the NEB part, which can be written as follows:

$$\left.\frac{d^2\sigma}{dE_b d\Omega_b}\right|^{\mathrm{IAV}}_{\mathrm{NEB}} = -\frac{2}{\hbar v_i}\rho_b(E_b)\langle\varphi_x^{(+)}|W_x|\varphi_x^{(+)}\rangle, \tag{1}$$

where E_b is the center of mass (c.m.) energy in the $b + B$ channel, Ω_b the c.m. solid angle for b ejectile, v_i is the projectile-target relative velocity in the entrance channel, $\rho_b(E_b)$ is the density of states of particle b, W_x is the imaginary part of the $x + A$ optical potential $\mathcal{U}_x$ and $\varphi_x^{(+)}$ is x-channel wavefunction which, in DWBA post-form representation, is obtained as a solution of the following inhomogeneous equation

$$(E_x - T_x - \mathcal{U}_x)\varphi_x^{(+)}(\vec{k}_b, \vec{r}_x) = \langle\vec{r}_x\chi_b^{(-)}|V_{\mathrm{post}}|\chi_a^{(+)}\phi_a\rangle, \tag{2}$$

where $\chi_a^{(+)}$ and $\chi_b^{(-)}$ are distorted waves describing the relative motion of $a + A$ and $b + B$, respectively, $V_{\mathrm{post}} = V_{bx} + U_{bA} - U_{aA}$ is the post form transition operator, with U_{aA} the distorted

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potential generating the distorted wave $\chi_a^{(+)}$, $\phi_a(\vec{r}_{bx})$ is the initial bound state wavefunction of the $b + x$ system, E_x is the relative energy of the $x + A$ system in the final state and T_x the kinetic energy operator for this system. The relevant coordinates are depicted in Fig. 1. Note that, among these relevant coordinates, are Jacobi coordinates connecting the center of mass of each binary sub-system to the third particle.

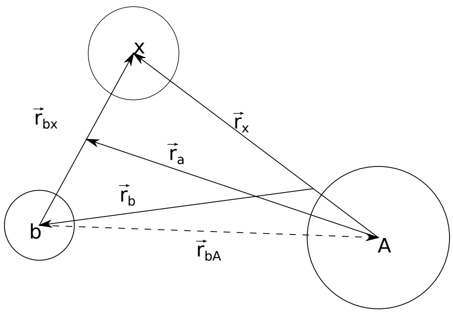

Figure 1. Relevant coordinates for the description of the $A(a, b)B$ reaction.

Note that the energy E_x appearing in (2) is linked to E_b as $E_x = E_a - S_{bx} - E_b$, where S_{bx} is the projectile separation energy into $b + x$ and E_a is the initial c.m. energy. Note also that, above a certain final energy of the b fragment, the energy E_x becomes negative and Eq. (2) corresponds to the transfer of the x fragment to bound states of the $x + A$ system. Therefore, the model accounts simultaneously for both the population of bound and unbound states of the $x + A$ system. For that, it becomes apparent that the $\mathcal{U}_x$ potential must be energy dependent. Ideally, one should use a dispersive potential, as those introduced by Mahoux and Sartor [16]. In practice, dispersive potentials are only available for specific systems so other global, non-dispersive potentials, are commonly adopted in actual calculations.

The IAV model has been recently revisited by several groups [12–14] and its accuracy assessed against experimental data with rather encouraging results [15, 17].

2.2. Incomplete fusion within the IAV model

As discussed in the previous section, the IAV model is intended to provide the total inclusive cross section corresponding to the detection of the b fragment in reactions of the form $A(a, b)B$. This results from the fact that the imaginary part appearing in the expectation value of Eq. (1) accounts in principle for *all* processes in which the participant fragment x interacts non-elastically with the target nucleus. This will include the ICF cross section, but also other NEB processes not associated with the formation of a compound nucleus of the $x + A$ system, such as target excitation or nucleons exchange between x and A. Keeping with the fact that compound nucleus formation occurs at small inter-cluster separations, when the nuclei have overcome their mutual barrier, it is plausible to associate the ICF with the absorption due to a short range $x + A$ imaginary potential. This idea has been exploited by other authors in alternative approaches [18–21] with reasonable success. With this simple prescription, the ICF cross section is evaluated using the IAV expression, Eq. (1), in which the imaginary part of $\mathcal{U}_x$ is replaced by a short-range potential. Note that, for a meaningful and predictive application of this idea, the calculated cross section should be independent of the choice of this potential, provided the latter is sufficiently short-ranged and deep, so that it can simulate the usual ingoing boundary condition for fusion.

3. Applications

To illustrate the method outlined in the previous section, we consider the reaction ^{7}Li+^{209}Bi, for which complete and incomplete fusion data have been measured at the ANU facility at Canberra [6, 7, 22].

The following potentials were used in the IAV calculations: for the entrance channel (^{7}Li+^{209}Bi) we used the ^{7}Li+^{208}Pb potential from Ref. [23]. For potentials for α+^{209}Bi and α+^{211}Po* were taken from [24] and the triton optical potentials from [25]. The ^{7}Li bound state wavefuntion was obtained with the potential model of Buck and Merchant [26].

In the left panel of Fig. 2 we compare the measured ICF data for this reaction with the present calculations. As explained in [6,7], ICF was identified from the decay of the At and Po products, presumably stemming from the partial fusion of the projectile. For the calculations we show the individual contributions to the ICF cross section, namely, α-ICF (i.e., α absorbed) and t-ICF (t-absorbed) as well as their sum. To compute the α-ICF (t-ICF), the imaginary part of the α+^{209}Bi (t+^{209}Bi) system was replaced by a short-range imaginary potential of Woods-Saxon form and parameters $W_0 = 50$ MeV, $R_i = 1.0 \times 209^{1/3}$ fm, $a_i = 0.2$ fm.

The calculated ICF agrees well with the data for incident energies up to ∼35 MeV, but overestimate them at higher energies. Interestingly, the results are very similar to those reported in Ref. [20], in which the authors employed a very different approach based on the CDCC method.

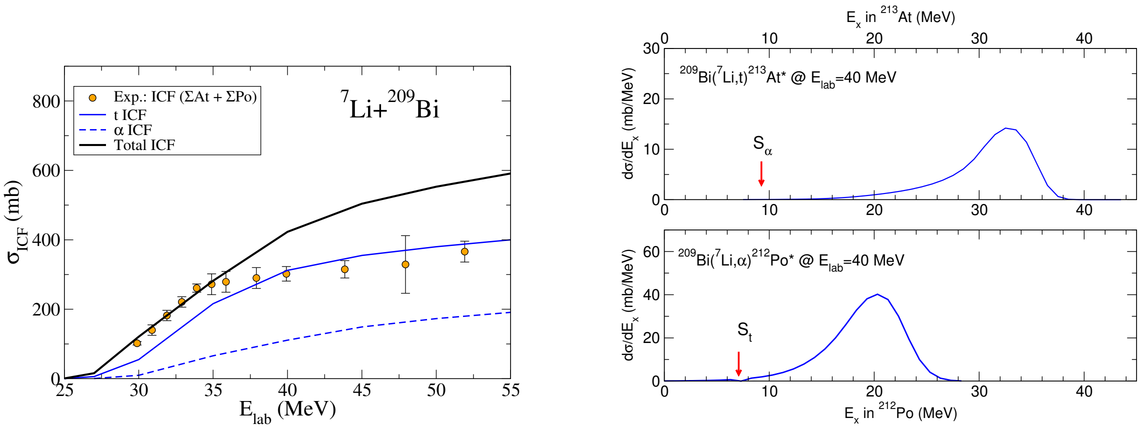

Figure 2. Left: ICF cross sections for ^{7}Li+^{209}Bi as a function of the incident energy. Experimental data from [6, 7] are compared with the calculations based on the IAV model. Right: Excitation energy distributions for the α-ICF (top) and t-ICF (bottom) processes at an incident energy of 40 MeV. The arrows indicate the α and triton separation energies in the respective residual nuclei.

A virtue of the present method is the possibility of computing, in addition to angle and energy integrated cross sections, more detailed observables, such as angular and/or energy distributions of the detected fragments.

As an example, we show in the right panel of Fig. 2 the excitation energy distribution of the residual nucleus formed after the capture of an α particle (upper panel) or a triton (lower panel) for a projectile incident energy of 40 MeV. The arrows indicate the respective α and triton separation energies of the residual nuclei, namely, ^{213}At and ^{212}Po, respectively. It can be seen that most of the α-ICF (t-ICF) corresponds to the α (triton) capture to very high excited states of the target, forming a highly excited state of the ^{213}At (^{212}Po) nucleus that will eventually decay mostly by neutron emission.

The calculations provide also the angular momentum distribution of the populated states. This is shown in Fig. 3 for the t-ICF and α-ICF processes. In these calculations, the intrinsic spins are ignored so the angular momentum of the compound nucleus stems from the relative orbital angular momentum of the captured fragment and the target. It is seen that states with very high angular momentum are populated (ℓ ∼15) for both the t and α capture. We note that

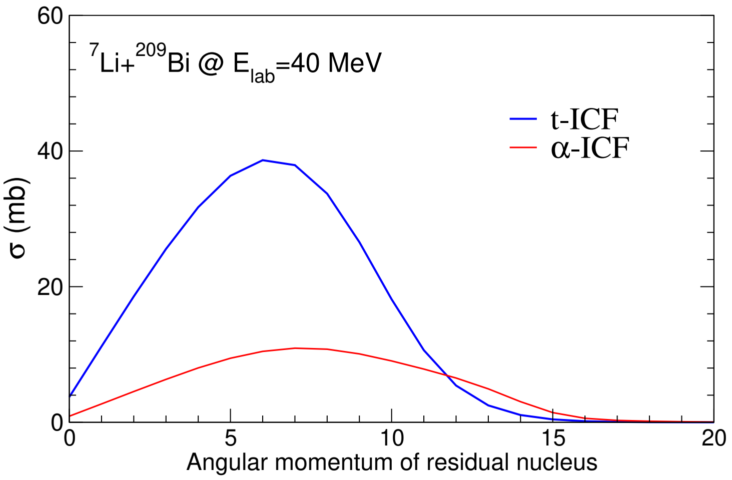

Figure 3. Angular momentum distribution of populated states in the t and α ICF for the reaction ^{7}Li+^{209}Bi at 40 MeV. Note that the intrinsic spins of t and ^{209}Bi are ignored.

the combined excitation energy and angular momentum distributions could be used as input of Hauser-Feshbach calculations to study the subsequent decay of the residual nucleus, a possibility that would be worth exploring in future works. It is also worth noting that a more realistic spin distribution could be obtained incorporating the intrinsic spins of the colliding partners and adding, when appropriate, some spin-dependent terms between the fragments. For example, the inclusion of a spin-orbit interaction in the $x - A$ system would produce the usual $j = \ell \pm 1/2$ splitting of the cross section. We believe however that the adopted model without intrinsic spins is realistic enough for the present illustrative purposes.

A more detailed analysis of this and similar reactions, including comparison with available data, will serve to better evaluate the validity and predictability of these calculations.

4. Summary and conclusions

We have presented a model to evaluate incomplete-fusion cross sections in the collision of a two-body composite nucleus with a target. The model uses a modified version of the IAV model for inclusive breakup in which, to extract the ICF component of the NEB cross section, the potential of the captured fragment with the target is modeled by a short-range absorptive potential.

The model has been compared with data for the ^{7}Li+^{209}Bi reaction with encouraging results. Comparison with data from other reactions will be necessary to assess the validity of this prescription. For example, the same model could be applied to reactions induced by ^{6}Li, described as $\alpha + d$. Work in this direction is in progress and will be presented elsewhere.

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