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Investigating the relation between heavy ion double charge exchange nuclear reactions and double beta decays

J I Bellone[1,4]**, S Burrello**[1,4]**, M Colonna**[1,4]**, J A Lay**[2,4]**, H Lenske**[3,4]

[1] Laboratori Nazionali del Sud, INFN, I-95123 Catania, Italy
[2] Departamento de FAMN, Universidad de Sevilla, Apartado 1065, E-41080 Sevilla, Spain
[3] Institut für Theoretische Physik, Justus-Liebig-Universität Giessen, D-35392 Giessen, Germany
[4] NUMEN Collaboration

E-mail: bellone@lns.infn.it, burrello@lns.infn.it, colonna@lns.infn.it, lay@us.es, horst.lenske@theo.physik.uni-giessen.de

Abstract. Heavy ion double charge exchange reactions (DCE) are discussed, within DWBA, in synergy with the main goal of the NUMEN project, i. e. to extract information on the $0\nu\beta\beta$ decay nuclear matrix element from DCE cross section measurements. The first step to reach this goal is to assess the analogy between the DCE nuclear matrix element and that describing $0\nu\beta\beta$ decay; then it is necessary to single out a term related to the desired double beta decay nuclear matrix element from the DCE cross-section expression (cross section factorization).

Heavy ion DCE processes, are depicted in terms of sequence of two correlated or uncorrelated Single Charge Exchange (SCE) reactions, resembling in this way $0\nu\beta\beta$ and $2\nu\beta\beta$ decay, respectively. Here, the formalism for describing DCE reactions within the latter scenario is described.

Once provided the formalism leading both to SCE and DCE cross section factorization, together with its limitations, a preliminary comparison with experimental DCE data is shown.

1. Introduction

Charge exchange reactions represent a multipurpose tool to probe different fields of physics, from astrophysics to particle physics. For example, in the 80s a relation between light ion induced single charge exchange (SCE) reaction cross section and single beta decay strength was established [1, 2] and very recentely extended to heavy ion induced SCE nuclear processes [3].

In this paper, we discuss the theoretical studies performed in order to investigate the above relation in the case of heavy ion double charge exchange (DCE) nuclear reactions and double beta decays. The present work is performed in synergy with the NUMEN project, developed at LNS-INFN, in Catania (Italy).

The NUMEN project lays down its basis on several heuristic analogies existing between heavy ion DCE reactions and the theorized $0\nu\beta\beta$ decay [9, 10, 11]. If the latter process were observed, it would allow to discriminate between Dirac and Majorana nature of neutrino mass, thus shedding light on physics beyond the Standard Model. The main goal of this project is to constrain the $0\nu\beta\beta$ decay nuclear matrix element by heavy ion DCE cross section measurements, in order to overcome the big discrepancies (of about a factor 2 - 3) on its value arising from different nuclear structure model calculations. This in turn would allow to determine neutrino effective mass within a significant accuracy from half-life measurements, if $0\nu\beta\beta$ decay were observed. For this purpose, it is necessary to assess the analogy between heavy ion DCE nuclear matrix element and the one describing $0\nu\beta\beta$ decay from an analytical point of view and then to factorize heavy ion DCE cross section expression into the product of a reaction term and a nuclear structure term, with the latter carrying information on the desired weak nuclear matrix element.

In this text, the expressions “SCE” and “DCE processes” refer to those charge changing nuclear reactions proceeding via charged mesons exchange between projectile and target nuclei (collisional or hard processes), which are the processes allowing the analogy with the weak decays. Charge changing reactions can also be induced by the mean field interaction among the nucleons of the the two colliding nuclei (soft processes), like e.g. the multi-nucleon transfer reactions, but this kind of mechanism does not allow to establish the desired analogy mentioned above.

In the following sections, the formalism developed by the authors for heavy ion DCE reactions is described together with the results obtained from the corresponding calculations.

2. Double Charge Exchange formalism

DCE nuclear reactions can be framed within two scenarios:

- sequence of two *uncorrelated* SCE processes ($2\nu\beta\beta$-like mechanism);
- sequence of two *correlated* SCE processes (Majorana-like mechanism).

In the former case, the DCE reaction between heavy ions is characterized by nearly the same analytical and diagrammatical structures as those describing $2\nu\beta\beta$ decay, while in the latter scenario, the heavy ion DCE reaction can show a diagrammatical structure similar to the one illustrating $0\nu\beta\beta$ decay. A proportionality relation between DCE nuclear matrix element (evaluated in closure approximation) and the $0\nu\beta\beta$ decay nuclear matrix element has been provided by some nuclear structure model calculations [12, 13], but the analogy from the analytical point of view is anything but straightforward, due to the different nature of the propagators involved in the two processes and this topic is still a work in progress [14].

The present work focuses on the study of heavy ion DCE reactions in the two uncorrelated SCE scenario. Thus, according to second order perturbation theory, the corresponding transition matrix element, $\mathcal{M}_{\alpha\beta}$, can be expressed in the following way

$$\mathcal{M}_{\alpha\beta}(\mathbf{k}_\alpha, \mathbf{k}_\beta) = \sum_{\gamma=c,C} \int \frac{d^3k_\gamma}{(2\pi)^3} \mathcal{M}^{(SCE)}_{\beta\gamma}(\mathbf{k}_\beta, \mathbf{k}_\gamma) G_\gamma(\omega_\alpha, \omega_\gamma) \mathcal{M}^{(SCE)}_{\gamma,\alpha}(\mathbf{k}_\gamma, \mathbf{k}_\alpha) \tag{1}$$

i.e. as the convolution of two SCE transition matrix elements and the Green’s function, $G_\gamma = (\omega_\alpha - \omega_\gamma + i\eta)^{-1}$; G_γ is a function of the total energies of the initial and the intermediate reaction channels, ω_α and ω_γ, respectively, in this way accounting for the free propagation of the two nuclei between the first and the second SCE reaction. The pedices α, β, γ denote initial, final and intermediate reaction channels, respectively; $\mathbf{k}_\delta$, $\delta = \alpha, \beta, \gamma$, is the relative momentum of projectile-target system in the channel δ. In the following, the terms referring to projectile are indicated with small letters as pedices, while those describing the target nucleus are denoted with capital letter pedices.

Momentum space representation has been chosen for developing the formalism, because within such representation projectile and target internal degrees of freedom can be treated separately and moreover both SCE and DCE transition matrix elements can be expressed as the momentum integral of a factorized expression:

$$\mathcal{M}_{\alpha\beta}(\mathbf{k}_\alpha, \mathbf{k}_\beta) = \sum_{S,T} \int d^3p\, K^{(S,T)}_{\alpha\beta}(\mathbf{p}) N^D(\mathbf{p}, \mathbf{k}_\alpha, \mathbf{k}_\beta) \tag{2}$$

where the indices S and T identify the spin and isospin transfer characterizing the reaction channel, respectively; $N^D = \int \frac{d^3r}{(2\pi)^3} \chi^*_\beta(\mathbf{r})\chi_\alpha(\mathbf{r})e^{-i\mathbf{p}\cdot\mathbf{r}}$ is the distortion factor, accounting for initial and final state interactions by means of the distorted waves, $\chi_{\alpha,\beta}(\mathbf{r})$.

$K_{\alpha\beta}$ is called reaction kernel and accounts for the whole nuclear structure information on the process, in that it is a combination of the nucleon-nucleon effective local interaction potential, V_{NN}, and of projectile and target transition densities, by means of the form factors $F^{(S,T)}_{ab,AB}$, as shown in eq. (3)

$$\begin{aligned} K^{(ST)}_{\alpha\beta}(\mathbf{p}) &= V^{(C)}_{ST}(p) F^{(ST)\dagger}_{ab}(\mathbf{p}) \cdot F^{(ST)}_{AB}(\mathbf{p}) \\ &+ \delta_{S1}\sqrt{\frac{24\pi}{5}} V^{(T)}_{ST}(p) Y^*_2(\hat{p}) \cdot \left[F^{(ST)\dagger}_{ab}(\mathbf{p}) \otimes F^{(ST)}_{AB}(\mathbf{p})\right]_2 \end{aligned} \tag{3}$$

where the central and tensor components of the effective nucleon-nucleon interaction are denoted by (C) and (T) indices, respectively. This nuclear structure term should carry information on the nuclear matrix element of interest.

Calculations have been performed within Distorted Wave Born Approximation (DWBA) and zero-range approximation.

First of all, we have investigated heavy ion SCE reactions at low momentum transfer and we have found a proportionality relation between SCE reaction kernel square modulus and single beta decay strength [3]. The validity of a factorized expression for heavy ion SCE cross section can be simply assessed by assuming a gaussian reaction kernel; the approximations used make such factorized expression analytically exact for zero momentum transfer, but it can still be applied for momentum transfer values smaller than about 25 MeV [3]. Of course, the same factorized formalism applies in general to a one-step charge exchange process, and thus also to DCE reactions in the two-correlated SCE scenario.

Then, we have focused on heavy ion DCE cross section within the scenario of two-uncorrelated SCE processes

$$\frac{d^2\sigma}{dEd\Omega} = \frac{E_\alpha E_\beta}{4\pi(\hbar c)^2} \frac{1}{(2J_A+1)(2J_a+1)} \frac{k_\beta}{k_\alpha} \sum_{\substack{m_a,m_b \\ m_A,m_B}} |\sum_{\substack{\tau=C,T_n, \\ so}} \mathcal{M}^{(\tau)}_{\alpha\beta}(\mathbf{k}_\alpha, \mathbf{k}_\beta)|^2 \tag{4}$$

In eq. (4), k_α (k_β) is the relative momentum and $E_\alpha = \sqrt{\mu_\alpha^2 + k_\alpha^2}$ ($E_\beta = \sqrt{\mu_\beta^2 + k_\beta^2}$) is the reduced energy of the initial (final) reaction channel, with $\mu_{\alpha,\beta}$ indicating the reduced mass of the given reaction channel; J_A and J_a represent the total angular momenta and m_A, m_a (m_B, m_b) denote the corresponding third-component projections of target and projectile (target-like fragment and ejectile), respectilvely; $\mathcal{M}_{\alpha\beta}$ given by eq. (1), with the index τ identifying the central (C), tensor (T_n) and spin-orbit (so) components of the effective nucleon-nucleon interaction.

Within the two-uncorrelated SCE scenario, it is quite straightforward to recover the analogy between DCE and $2\nu\beta\beta$ nuclear matrix elements, by assuming that only one nuclear state dominates in the intermediate channel, i. e. within the Single State Dominance (SSD) approximation. In order to reach a factorized expression of DCE cross section, we have also assumed that only the pole of the intermediate propagator gives a significant contribution to the integral in $\mathcal{M}_{\alpha\beta}$ (pole approximation), thus obtaining the following formula for DCE cross section in the two-uncorrelated SCE scenario, in non-relativistic energy regime, within SSD,

$$\begin{aligned} \frac{d^2\sigma}{dEd\Omega} &= \frac{E_\alpha E_\beta}{4(\hbar c)^4}\frac{k_\beta}{k_\alpha}\frac{1}{(2J_A+1)}\frac{1}{(2J_a+1)}\frac{\mu_\delta^2 k_\delta^2}{(2\pi)^6} \\ &\sum_{\substack{m_a,m_A\\ m_b,m_B}} |\sum_\tau \int_0^{2\pi} d\varphi_\delta \int_0^\pi d\theta_\delta \, \sin\theta_\delta \mathcal{M}_{\delta\beta}^{(\tau,SCE)}(k_\delta,k_\beta,\theta_{\delta\beta})\,\tilde{\mathcal{M}}_{\alpha\delta}^{(\tau,SCE)}(k_\alpha,k_\delta,\theta_{\alpha\delta})|^2 \end{aligned} \tag{5}$$

where $\tilde{\mathcal{M}}$ represent the transition matrix element that depends on the distorted waves originating from an optical potential with a positive imaginary part (source term), instead of a negative one (absorption term), due to the non-hermiticity of the Hamiltonian describing the projectile-target relative dynamics. The scattering angle of the second SCE step has been properly referred to the same reference frame of the first SCE reaction, by means of the following relation

$$\theta_{\delta\beta} = \arccos(\sin\theta_{\alpha\beta}\sin\theta_\delta\cos\varphi_\delta + \cos\theta_{\alpha\beta}\cos\theta_\delta) \tag{6}$$

In the above formulas, the δ subscript refers to a fixed intermediate reaction channel (while a generic one is indicated with γ in the text below).

Hence, eq. (5) leads to a factorized cross section only in the Plane Wave Born Approximation (PWBA) limit or assuming plane waves in the intermediate channel, but not in full DWBA case; moreover, in the above three cases the DCE reaction kernel cannot simply be expressed as the product of two SCE ones, because of the integral over the off-shell relative momentum, k_γ.

To check the effect of the absorption in the intermediate channel, we compare the results obtained by eq. (5) with FRESCO calculations [15]. In this way, we found that the order of magnitude of the DWBA DCE cross section can be recovered by simply assuming plane waves in the intermediate channel, while the diffraction pattern is very similar to the one obtained by simply performing the product of two SCE cross sections; such result implies that the assorption effects do not act in the intermediate channel.

Because we practically deal with heavy ions systems, we expect that a lot of intermediate states could give a significant contribution to the DCE cross section, thus making it possible to factorize the cross section. Hence, we performed calculations beyond SSD, summing over all the intermediate nuclear states, mantaining the pole approximation, obtaining the following formula for the DCE transition matrix element

$$\begin{aligned} \mathcal{M}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) &= -i\pi N^D(\mathbf{k}_\alpha,\mathbf{k}_\beta)\sum_{J_1,J_2}\sum_{\substack{S_1,S_2\\ m_{S_1},m_{S_2}}}\sum_{\substack{L_1,L_2\\ m_{L_1},m_{L_2}}}(S_1 m_{S_1} S_2 m_{S_2}|S M_S) \\ &(L_1 m_{L_1} L_2 m_{L_2}|L M_L)\,(L M_L S M_S|J M)\int\frac{d^3k_\gamma}{(2\pi)^3}\frac{K_{\beta\gamma}^{(SCE)}(\mathbf{k}_\beta,\mathbf{k}_\gamma)K_{\gamma\alpha}^{(SCE)}(\mathbf{k}_\gamma,\mathbf{k}_\alpha)}{E_{\max}^{(\gamma)}} \end{aligned} \tag{7}$$

where $E_{\max}^{(\gamma)}$ is the maximum total excitation energy considered in the intermediate channel.

Eq. (7) is very interesting, in that it mimics the Closure Approximation (CA) and for this reason we refer this formalism as "simil Closure Approximation" (sCA). sCA in turn resembles the 0^{th} order term of the perturbative expansion of $0\nu\beta\beta$ decay nuclear matrix element, i. e. the term obtained by assuming a constant propagator. Hence, in this way, the expression in (7) represents a first naïve step towards an analogy between Majorana-like DCE and $0\nu\beta\beta$ decay nuclear matrix elements, even if it is of crucial importance to determine an analogy including the effect of the intermediate propagator.

3. Results

To check both SSD and sCA formalisms illustrated above, we compared our results with NUMEN data for the pilot DCE reaction ^{40}Ca+^{18}O $\to^{40}$Ar $+^{18}$Ne, at 270 MeV [9]. This comparison shows that SSD result underestimates the data by about 2 orders of magnitude, while the result from sCA allows to reproduce the order of magnitude of the data, but not the full trend. Both results have been obtained adopting the same double-folding optical potentials [16, 17] allowing to reproduce NUMEN SCE data (not yet published). These DCE results are preliminary, in that we need to check the effects of different nuclear structure models on the magnitude and above all on the diffraction pattern of the angular distribution.

Calculations within sCA have also been performed for the nuclear reaction ^{116}Cd $+^{20}$Ne $\to^{116}$Sn $+^{20}$O at 306 MeV, that is one of the DCE reactions of main interest within the NUMEN collaboration and it is currently under study. A preliminary check on the optical potential to employ has been performed. As a starting point, for this check we used double folding optical potentials with two different parameterizations, e.g. Franey-Love [17] and Akyuz-Winther [18], thus finding that they lead to similar elastic cross sections in $\theta_{CM} \in [0°, 24°]$ angular range, but very different DCE angular distributions. The ambiguity of the optical potential paramenterization in describing forward elastic angular distribution is a well known problem [19, 20] and it is due to the fact that elastic cross section at forward angles is due to peripheral scatterings and hence it is not sensible to the absorption effects, accounted for by the imaginary part of the optcal potential, while both SCE [3] and DCE cross sections are strongly affected by the absorption effects, thus justifying the differences found in magnitude and diffraction pattern.

Table 1 shows the differences in the integrated cross section values obtained by using the 2 different optical potential parameterizations mentioned above.

Table 1. DCE cross section for the reaction ^{116}Cd $+^{20}$Ne $\to^{116}$Sn $+^{20}$O, at 306 MeV, integrated in $\theta_{lab} \in [3°, 13°]$, calculated by using a double folding optical potential with two different parameterizations leading to almost the same elastic cross section. Cross sections are expressed in nb.

Franey Love [17]	Akyuz Winther [18]
0.28	12

Hence, to disentangle this optical potential ambiguity, we need to fit elastic angular distributions up to backward scattering angles or alternatively to fix optical potential parameters by trying to simultaneously fit both DCE [21] and SCE cross sections.

4. Summary and Conclusions

Summarizing, heavy ion SCE cross section factorization has been derived by simply assuming gaussian reaction kernels. This simple formalism applies also to a generic one-step charge exchange reaction, e.g. to a DCE reaction within the two-correlated SCE scenario, and it is valid in the low momentum transfer range and for small scattering angles. Instead, a factorized expression is anything but straightforward to be reached for heavy ion DCE reaction within the two-step process that we investigate here, because of the intermediate relative momentum integral and the sum over all the nuclear states involved in the intermediate channel, appearing in the DCE transition matrix element expression. To simplify such expression we used the pole approximation and assumed first SSD and then sCA. A preliminary comparison with data leads to the conclusion that a lot of intermediate nuclear states give significant contribution to heavy ion DCE cross section, in that sCA allows to recover the order of magnitude of angular distribution data. However, the results here presented need further checks, because it is necessary to test the effects of different nuclear structure models and above all it is necessary to solve the ambiguity in the optical potential parameterization exploited to perform DCE calculations. To accomplish the latter task, a more extensive comparison to experimental data (i.e. SCE, DCE, transfer, elastic channels), is in order.

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