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Local Superderivations on Solvable Lie and Leibniz Superalgebras

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Abstract. Throughout this paper, we show on one hand, that there are nilpotent and solvable Lie superalgebras with infinitely many local superderivations which are not standard superderivations. On the other hand, we show that every local superderivation is a superderivation on the maximal-dimensional solvable Lie superalgebras with model filiform or model nilpotent nilradical. Moreover, we extend the latter result for Leibniz superalgebras by showing that every local superderivation is a superderivation on the maximal-dimensional solvable Leibniz superalgebras with model filiform or model nilpotent non-Lie nilradical.

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1. Introduction

Local derivations were considered for the first time in 1990 by Kadison [22] and also by Larson and Sourour [24]. In particular, Kadison showed that each continuous local derivation from a von Neumann algebra into the dual bimodule is a derivation. Let us note then, that the main problem studied in relation to this research topic is to determine when a local derivation is a derivation, see for instance [9,20]. Additionally, other problem that has been largely studied is to find types of algebras containing local derivations which are not derivations [1]. More recently, in [4,5,13] the authors studied the

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aforementioned problems for Lie algebras, proving in particular that every local derivation of a semi-simple Lie algebra is a derivation and giving examples of solvable Lie algebras with local derivations which are not derivations. Likewise, an analogous study has been developed for Leibniz algebras, see for instance [6] and references therein.

Recently, studying local superderivations on semi-simple Lie superalgebras have drawn a lot of attention [14–16, 27], however none of them tackle local superderivations on solvable Lie superalgebras nor Leibniz superalgebras. Thus, this is the context of our work, studying local superderivations on solvable Lie and Leibniz superalgebras. Note that studying solvable Lie superalgebras presents more difficulties than studying solvable Lie algebras [26]. In particular, Lie's theorem is not verified in general and neither its corollaries. Therefore, for a solvable Lie superalgebra L, $L^2 := [L, L]$ can not be nilpotent, see [25]. Nevertheless, in [11] the authors proved that under the condition of being L^2 nilpotent, any solvable Lie and Leibniz superalgebra over the real or complex field can be obtained by means of outer non-nilpotent superderivations of the nilradical in the same way as occurs for Lie and Leibniz algebras.

In this frame, we investigate local superderivations of solvable Lie and Leibniz superalgebras. First, we prove that there are nilpotent and solvable Lie superalgebras with infinitely many local superderivations which are not ordinary superderivations (see Sects. 3 and 5). Second, we prove on the maximal-dimensional solvable Lie superalgebras with model filiform or model nilpotent nilradical that every local superderivation is a superderivation (see Sect. 4). Finally, we extend this last result for the maximal-dimensional solvable Leibniz superalgebras with model filiform and model nilpotent non-Lie nilradical (see Sect. 6).

2. Preliminary Results

2.1. Preliminary for Lie Superalgebras

A vector space V is said to be $\mathbb{Z}_2$-*graded* if it admits a decomposition into a direct sum, $V = V_{\bar{0}} \oplus V_{\bar{1}}$, where $\bar{0}, \bar{1} \in \mathbb{Z}_2$. An element $X \in V$ is called *homogeneous of degree* $|x|$ if it is an element of $V_{|x|}$, $|x| \in \mathbb{Z}_2$.

In particular, the elements of $V_{\bar{0}}$ (resp. $V_{\bar{1}}$) are also called *even* (resp. *odd*). A *Lie superalgebra* (see [21]) is a $\mathbb{Z}_2$-graded vector space $\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}$, with an even bilinear commutation operation (or "supercommutation") $[\cdot, \cdot]$, which satisfies the conditions

1. $[x, y] = -(-1)^{|x||y|}[y, x]$,
2. $(-1)^{|x||z|}[x, [y, z]] + (-1)^{|x||y|}[y, [z, x]] + (-1)^{|y||z|}[z, [x, y]] = 0$ *(super Jacobi identity)*

for all homogeneous elements $x, y, z \in \mathfrak{g}$.

Thus, $\mathfrak{g}_{\bar{0}}$ is an ordinary Lie algebra, and $\mathfrak{g}_{\bar{1}}$ is a module over $\mathfrak{g}_{\bar{0}}$; the Lie superalgebra structure also contains the symmetric pairing $S^2\mathfrak{g}_{\bar{1}} \longrightarrow \mathfrak{g}_{\bar{0}}$.

Let us note that both the *descending central sequence* and the *derived sequence* of a Lie superalgebra $\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}$ are defined in the same way as for

Lie algebras: $\mathcal{C}^0(\mathfrak{g}) := \mathfrak{g}$, $\mathcal{C}^{k+1}(\mathfrak{g}) := [\mathcal{C}^k(\mathfrak{g}), \mathfrak{g}]$ and $\mathcal{D}^0(\mathfrak{g}) := \mathfrak{g}$, $\mathcal{D}^{k+1}(\mathfrak{g}) := [\mathcal{D}^k(\mathfrak{g}), \mathcal{D}^k(\mathfrak{g})]$ respectively, for all $k \geq 0$. Thus, if $\mathcal{C}^k(\mathfrak{g}) = \{0\}$ (resp. $\mathcal{D}^k(\mathfrak{g}) = \{0\}$) for some k, then the Lie superalgebra is called *nilpotent* (resp. *solvable*). Note that nilpotent Lie superalgebras are in particular solvable. Remark also, that Engel's theorem and its corollaries are still valid for Lie superalgebras. Then, a Lie superalgebra L is nilpotent if and only if $ad_L x$ is nilpotent for every homogeneous element x of L. Additionally, a Lie superalgebra L is solvable if and only if its even part $L_{\overline{0}}$ (a Lie algebra) is solvable. However, Lie's Theorem does not hold for solvable Lie superalgebras.

At the same time, there are also defined two other crucial sequences denoted by $\mathcal{C}^k(\mathfrak{g}_{\overline{0}})$ and $\mathcal{C}^k(\mathfrak{g}_{\overline{1}})$ which will play an important role in our study. They are defined as follows:

$$\mathcal{C}^0(\mathfrak{g}_{\overline{i}}) := \mathfrak{g}_{\overline{i}}, \mathcal{C}^{k+1}(\mathfrak{g}_{\overline{i}}) := [\mathfrak{g}_{\overline{0}}, \mathcal{C}^k(\mathfrak{g}_{\overline{i}})],\ k \geq 0, \overline{i} \in \mathbb{Z}_2.$$

Let us recall now, the definition of superderivations of superalgebras [21]. A superderivation of degree s of a superalgebra L, $s \in \mathbb{Z}_2$, is an endomorphism $D \in End_s L$ with the property

$$D(ab) = D(a)b + (-1)^{s \cdot dega} aD(b)$$

denote $Der_s(L) \subset End_s L$ the space of all superderivations of degree s. Then $Der(L) = Der_{\overline{0}}(L) \oplus Der_{\overline{1}}(L)$ is the Lie superalgebra of superderivations of L, with $Der_{\overline{0}}(L)$ composed by even superderivations and $Der_{\overline{1}}(L)$ by odd ones.

On the other hand, recall also that a homogeneous linear mapping $\Delta : L \longrightarrow L$ of degree s is called a local homogeneous superderivation of degree s if for any element $x \in L$, there exists a superderivation $D_x : L \longrightarrow L$ (depending on x) such that $\Delta(x) = D_x(x)$. Then, the set of all local superderivations can be expressed

$$LocDer(L) = LocDer_{\overline{0}}(L) \oplus LocDer_{\overline{1}}(L)$$

with $LocDer_{\overline{0}}(L)$ (resp. $LocDer_{\overline{1}}(L)$) composed by even (resp. odd) local superderivations. For more details it can be consulted [14].

2.2. Preliminaries for Leibniz Superalgebras

Let us note that many results and definitions of the above sub-section can be extended for Leibniz superalgebras.

Definition 2.1. [2]. A $\mathbb{Z}_2$-graded vector space $L = L_{\overline{0}} \oplus L_{\overline{1}}$ is called a *Leibniz superalgebra* if it is equipped with a product $[\cdot, \cdot]$ which for an arbitrary element x and homogeneous elements y, z satisfies the condition

$$[x, [y, z]] = [[x, y], z] - (-1)^{|y||z|}[[x, z], y] \quad \text{(super Leibniz identity)}.$$

Note that if a Leibniz superalgebra L satisfies the identity $[x, y] = -(-1)^{|x||y|}[y, x]$ for any homogeneous elements $x, y \in L$, then the super Leibniz identity becomes the super Jacobi identity. Consequently, Leibniz superalgebras are a generalization of Lie superalgebras. Also and in the same way as for Lie superalgebras, isomorphisms are assumed to be consistent with the $\mathbb{Z}_2$-graduation.

Let us now denote by R_x the right multiplication operator, i.e., $R_x : L \to L$ given as $R_x(y) := [y, x]$ for $y \in L$, then the super Leibniz identity can be expressed as $R_{[x,y]} = R_y R_x - (-1)^{|x||y|} R_x R_y$.

If we denote by $R(L)$ the set of all right multiplication operators, then $R(L)$ with respect to the following multiplication

$$\langle R_a, R_b\rangle := R_a R_b - (-1)^{\bar{i}\bar{j}} R_b R_a \tag{2.1}$$

for $R_a \in R(L)_{\bar{i}}$, $R_b \in R(L)_{\bar{j}}$, forms a Lie superalgebra. Note that R_a is a derivation. In fact, the condition for being a derivation of a Leibniz superalgebra (for more details see [23]) is $d([x, y]) = (-1)^{|d||y|}[d(x), y] + [x, d(y)]$. Since the degree of R_z as homomorphism between $\mathbb{Z}_2$-graded vector spaces is the same as the degree of the homogeneous element z, that is $|R_z| = |z|$, then the condition for R_z to be a derivation is exactly $R_z([x, y]) = (-1)^{|z||y|}[R_z(x), y]+ [x, R_z(y)]$. This last condition can be rewritten $[[x, y], z] = (-1)^{|z||y|}[[x, z], y]+ [x, [y, z]]$ which is nothing but the super (graded) Leibniz identity. Let us remark that the definition of local superderivation is a natural extension from Lie theory.

Let us note also that the concepts of descending central sequence, nilindex, the variety of Leibniz superalgebras and Engel's theorem are natural extensions from Lie theory.

Let $V = V_{\bar{0}} \oplus V_{\bar{1}}$ be the underlying vector space of L, $L = L_{\bar{0}} \oplus L_{\bar{1}} \in Leib^{n,m}$, being $Leib^{n,m}$ the variety of Leibniz superalgebras, and let $G(V)$ be the group of the invertible linear mappings of the form $f = f_{\bar{0}} + f_{\bar{1}}$, such that $f_{\bar{0}} \in GL(n, \mathbb{C})$ and $f_{\bar{1}} \in GL(m, \mathbb{C})$ (then $G(V) = GL(n, \mathbb{C}) \oplus GL(m, \mathbb{C})$). The action of $G(V)$ on $Leib^{n,m}$ induces an action on the Leibniz superalgebras variety: two laws λ_1, λ_2 are *isomorphic* if there exists a linear mapping $f = f_{\bar{0}} + f_{\bar{1}} \in G(V)$, such that

$$\lambda_2(x, y) = f^{-1}_{\bar{i}+\bar{j}}(\lambda_1(f_{\bar{i}}(x), f_{\bar{j}}(y))), \quad \text{for any } x \in V_{\bar{i}}, y \in V_{\bar{j}}.$$

Furthermore, the description of the variety of any class of algebras or superalgebras is a difficult problem. Different works (for example, [3,7,10,18,19]) are regarding the applications of algebraic groups theory to the description of the variety of Lie and Leibniz algebras.

Definition 2.2. For a Leibniz superalgebra $L = L_{\bar{0}} \oplus L_{\bar{1}}$ we define the *right annihilator of* L as the set $Ann(L) := \{x \in L : [L, x] = 0\}$.

It is easy to see that $Ann(L)$ is a two-sided ideal of L and $[x, x] \in Ann(L)$ for any $x \in L_{\bar{0}}$. This notion is compatible with the right annihilator in Leibniz algebras. If we consider the ideal $I := ideal\langle[x, y]+(-1)^{|x||y|}[y, x]\rangle$, then $I \subset Ann(L)$.

Let $L = L_{\bar{0}} \oplus L_{\bar{1}}$ be a nilpotent Leibniz superalgebra with $\dim L_{\bar{0}} = n$ and $\dim L_{\bar{1}} = m$. From Equation (2.1) we have that $R(L)$ is a Lie superalgebra, and in particular $R(L_{\bar{0}})$ is a Lie algebra. As $L_{\bar{1}}$ has $L_{\bar{0}}$-module structure we can consider $R(L_{\bar{0}})$ as a subset of $GL(V_{\bar{1}})$, where $V_{\bar{1}}$ is the underlying vector space of $L_{\bar{1}}$. So, we have a Lie algebra formed by nilpotent endomorphisms of $V_{\bar{1}}$. Applying Engel's theorem we have the existence of a sequence of subspaces of $V_{\bar{1}}$, $V_0 \subset V_1 \subset V_2 \subset \cdots \subset V_m = V_{\bar{1}}$, with

$R(L_{\bar{0}})(V_{i+1}) \subset V_i$. Then, it can be defined the descending sequences $C^k(L_{\bar{0}})$ and $C^k(L_{\bar{1}})$ and the super-nilindex in the same way as for Lie superalgebras. That is, $\mathcal{C}^0(L_{\bar{i}}) := L_{\bar{i}}, \mathcal{C}^{k+1}(L_{\bar{i}}) := [\mathcal{C}^k(L_{\bar{i}}), L_{\bar{0}}], \quad k \geq 0, \bar{i} \in \mathbb{Z}_2$. If $L = L_{\bar{0}} \oplus L_{\bar{1}}$ is a nilpotent Leibniz superalgebra, then L has *super-nilindex* or *s-nilindex* (p, q) if satisfies

$$\mathcal{C}^{p-1}(L_{\bar{0}}) \neq 0, \qquad \mathcal{C}^{q-1}(L_{\bar{1}}) \neq 0, \qquad \mathcal{C}^p(L_{\bar{0}}) = \mathcal{C}^q(L_{\bar{1}}) = 0.$$

3. Local Superderivations of the Model Filiform Lie Superalgebra

We start our study with one case of nilpotent Lie superalgebra. Among all of them one that has been proved to be very relevant due to its properties is the model filiform Lie superalgebra since all the other filiform Lie superalgebras can be obtained from it by means of infinitesimal deformations [8]. These infinitesimal deformations are given by the even 2-cocycles $Z_0^2(L^{n,m}, L^{n,m})$. We consider then, the model filiform Lie superalgebra $L^{n,m}$, that is, the simplest filiform Lie superalgebra which is defined by the only non-zero bracket products that follow

$$L^{n,m}: \begin{cases} [x_1, x_i] = -[x_i, x_1] = x_{i+1}, & 2 \leq i \leq n-1 \\ [x_1, y_j] = -[y_j, x_1] = y_{j+1}, & 1 \leq j \leq m-1 \end{cases}$$

with a basis $\{x_1, \ldots, x_n\}$ of $(L^{n,m})_{\bar{0}}$ and a basis $\{y_1, \ldots, y_m\}$ of $(L^{n,m})_{\bar{1}}$.

For an even superderivation D of $L^{n,m}$ we have $D(L_{\bar{0}}^{n,m}) \subset L_{\bar{0}}^{n,m}$ and $D(L_{\bar{1}}^{n,m}) \subset L_{\bar{1}}^{n,m}$. Then we set

$$D(x_1) = \sum_{k=1}^{n} a_k x_k, \quad D(x_2) = \sum_{k=1}^{n} b_k x_k, \quad D(y_1) = \sum_{t=1}^{m} c_t y_t.$$

Applying induction and the even superderivation condition for the products $[x_1, x_i]$ we derive

$$D(x_i) = ((i-2)a_1 + b_2)x_i + \sum_{k=i+1}^{n} b_{k-i+2} x_k, \quad 3 \leq i \leq n.$$

Similarly, from the products $[x_1, y_j]$ we get

$$D(y_j) = ((j-1)a_1 + c_1)y_j + \sum_{t=j+1}^{m} c_{t-j+1} y_t, \quad 2 \leq j \leq m.$$

Finally, from the product $[x_2, y_1]$ we obtain $b_1 = 0$. Thus, we conclude

$$Der_{\overline{0}}(L^{n,m})$$

$$= \begin{pmatrix} a_1 & a_2 & a_3 & \dots & a_n & 0 & 0 & \dots & 0 \\ 0 & b_2 & b_3 & \dots & b_n & 0 & 0 & \dots & 0 \\ 0 & 0 & a_1+b_2 & \dots & b_{n-1} & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & (n-2)a_1+b_2 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 & c_1 & c_2 & \dots & c_m \\ 0 & 0 & 0 & \dots & 0 & 0 & a_1+c_1 & \dots & c_{m-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & (m-1)a_1+c_1 \end{pmatrix}.$$

Let now D be an odd superderivation of $L^{n,m}$. Then we have

$$D(x_1) = \sum_{k=1}^{m} a_k y_k, \quad D(x_2) = \sum_{k=1}^{m} b_k y_k, \quad D(y_1) = \sum_{t=1}^{n} c_t x_t.$$

According to the odd superderivation condition on the products of $L^{n,m}$ and induction, similar to even superderivation case, we obtain

$$D(x_i) = \sum_{k=i-1}^{m} b_{k-i+2} y_k, \quad 3 \le i \le n,$$

$$D(y_j) = \sum_{t=j+1}^{n} c_{t-j+1} x_t, \quad 2 \le j \le m.$$

Considering superderivation property for the products $[x_2, y_1]$ and $[x_1, x_n]$, we get c_1 and $b_i = 0$, $1 \le i \le m-n+1$ with $m \ge n$. Thus, we conclude

$$Der_{\overline{1}}(L^{n,m}) = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & a_1 & a_2 & \dots & a_m \\ 0 & 0 & 0 & \dots & 0 & b_1 & b_2 & \dots & b_m \\ 0 & 0 & 0 & \dots & 0 & 0 & b_1 & \dots & b_{m-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & b_1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 0 & c_2 & c_3 & \dots & c_n & 0 & 0 & \dots & 0 \\ 0 & 0 & c_2 & \dots & c_{n-1} & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & c_2 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \end{pmatrix},$$

where $b_i = 0$, $1 \le i \le m-n+1$ with $m \ge n$.

Theorem 3.1. *$Der(L^{n,m}) \subsetneq LocDer(L^{n,m})$.*

Proof. Consider the homogeneous linear mappings $\Delta_t : L^{n,m} \longrightarrow L^{n,m}$, $t \ne 2$ of degree 0 defined on the basis vectors of $L^{n,m}$ by

$$\Delta_t(x_1) = x_1, \ \Delta_t(x_2) = x_2, \ \Delta_t(x_3) = tx_3, \ \Delta_t(x_i) = \Delta_t(y_j) = 0, \ 4 \le i \le n, \ 1 \le j \le m.$$

Clearly, Δ_t is not an even superderivation (because its matrix does not fit with the general matrix of even superderivations).

Consider the following superderivations:

- d_1 is the resultant even superderivation after replacing a_1 by 1 and all the rest of parameters by 0 on the general matrix of $Der_{\overline{0}}(L^{n,m})$,
- d_2 is the resultant even superderivation after replacing b_2 by 1 and all the rest of parameters by 0 on the general matrix of $Der_{\overline{0}}(L^{n,m})$,
- d_t is the even superderivations defined by $d_t := d_1 + (t-1)d_2$.

Clearly,

$$\Delta_t(x_1) = d_1(x_1),\ \Delta_t(x_2) = d_2(x_2),\ \Delta_t(x_3) = d_t(x_3),$$
$$\Delta_t(x_i) = d_0(x_i) = \Delta_t(y_j) = d_0(y_j),\ 4 \le i \le n,\ 1 \le j \le m$$

being d_0 the null superderivation.

For an arbitrary element $e = \alpha_1 x_1 + \cdots + \alpha_n x_n + \beta_1 y_1 + \cdots + \beta_m y_m$ of $L^{n,m}$ we have

$$\Delta_t(e) = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 t x_3.$$

Define superderivation d_e as follows $d_e(x_i) = d_e(y_j) = 0$ with $4 \le i \le n$ and $1 \le j \le m$ and

$$d_e(x_1) = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3, \quad d_e(x_2) = \gamma_2 x_2 + \gamma_3 x_3,$$
$$d_e(x_3) = (\beta_1 + \gamma_2)x_3,$$

where $\beta_1, \beta_2, \beta_3, \gamma_2, \gamma_3$ are some unknowns parameters. From $\Delta_t(e) = d_e(e)$ we derive $\beta_1 = 1$ and the following linear system of equations

$$\begin{pmatrix} \alpha_1 & 0 & \alpha_2 & 0 \\ 0 & \alpha_1 & \alpha_3 & \alpha_2 \end{pmatrix} \begin{pmatrix} \beta_2 \\ \beta_3 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} = \begin{pmatrix} \alpha_2 \\ \alpha_3(t-1) \end{pmatrix},$$

which always has a solution with respect to unknowns $\beta_2, \beta_3, \gamma_2, \gamma_3$. Thus, we obtain the existence a superderivation d_e such that $d_e(e) = \Delta_t(e)$. The proof is complete. □

Remark 3.1. In fact, in the proof of Theorem 3.1 we show the existence of infinitely many local superderivations on the model filiform Lie superalgebra $L^{n,m}$ $(n \ge 3)$ which are not superderivations. Note also, that analogously it can be found infinitely many odd local superderivations which are not odd superderivations.

Along the next sections, we consider non-nilpotent solvable Lie and Leibniz superalgebras with different types of nilradical, starting with abelian nilradical.

4. Local Superderivations of Maximal-Dimensional Solvable Lie Superalgebras with Model Filiform and Model Nilpotent Nilradical

In this section first, we consider the maximal-dimensional solvable Lie superalgebra with model filiform nilradical [12]. This superalgebra is unique for each pair of dimensions (n, m) and can be expressed by the only non-null bracket products that follow:

$$SL^{n,m}:\begin{cases}[x_1,x_i]=-[x_i,x_1]=x_{i+1},\ 2\le i\le n-1, & [t_1,y_j]=-[y_j,t_1]=jy_j,\ 1\le j\le m,\\ [x_1,y_j]=-[y_j,x_1]=y_{j+1},\ 1\le j\le m-1, & [t_2,x_i]=-[x_i,t_2]=x_i,\ 2\le i\le n,\\ [t_1,x_i]=-[x_i,t_1]=ix_i,\ 1\le i\le n, & [t_3,y_j]=-[y_j,t_3]=y_j,\ 1\le j\le m,\end{cases}$$

with $\{x_1,\dots,x_n,t_1,t_2,t_3\}$ a basis of $(SL^{n,m})_{\bar{0}}$ and $\{y_1,\dots,y_m\}$ a basis of $(SL^{n,m})_{\bar{1}}$.

Its superalgebra of superderivations was obtained in [12]. Next, we prove the following result.

Theorem 4.1. *On the maximal-dimensional solvable Lie superalgebra with model filiform nilradical, every local superderivation is a superderivation.*

Proof. First, we are going to express in a more suitable way for our purpose the solvable Lie superalgebra $SL^{n,m}$. Thus, after applying an elementary basis transformation one can express the table of multiplications $SL^{n,m}$ in a new basis $\{e_1,\dots,e_n,e_{n+1},\dots,e_{n+m}\}$ as follows:

$$SL^{n,m}:\begin{cases}[e_1,e_i]=-[e_i,e_1]=e_{i+1}, & 2\le i\le n-1,\\ [e_1,e_{n+j}]=-[e_{n+j},e_1]=e_{n+j+1}, & 1\le j\le m-1,\\ [t_1,e_i]=-[e_i,t_1]=ie_i, & 1\le i\le n+m,\\ [t_2,e_i]=-[e_i,t_2]=e_i, & 2\le i\le n,\\ [t_3,e_{n+j}]=-[e_{n+j},t_3]=e_{n+j}, & 1\le j\le m.\end{cases}$$

In [12] the authors proved every superderivation is exactly the adjoint operator of an element of $SL^{n,m}$. Let us fix an arbitrary element $z=\gamma_1t_1+\gamma_2t_2+\gamma_3t_3+\sum_{p=1}^{n+m}\beta_pe_p$ of the superalgebra $SL^{n,m}$, then for its adjoint operator ad_z we obtain

$$ad_z(t_1)=-\sum_{p=1}^{n+m}p\beta_pe_p,\ ad_z(t_2)=-\sum_{p=2}^{n}\beta_pe_p,\ ad_z(t_3)=-\sum_{p=n+1}^{n+m}\beta_pe_p,$$

$$ad_z(e_1)=\gamma_1e_1-\sum_{p=2}^{n-1}\beta_pe_{p+1}-\sum_{p=n+1}^{n+m-1}\beta_pe_{p+1},$$

$$ad_z(e_i)=(i\gamma_1+\gamma_2)e_i+\beta_1e_{i+1},\ 2\le i\le n,$$

$$ad_z(e_{n+j})=((n+j)\gamma_1+\gamma_3)e_{n+j}+\beta_1e_{n+j+1},\quad 1\le j\le m.$$

Later on, when needed, from this general expression we will distinguish between even and odd superderivations. Let us consider now an arbitrary local superderivation $\Delta : SL^{n,m} \longrightarrow SL^{n,m}$. Since the value of a local superderivation on any vector coincides with the value on this vector of a superderivation, in particular on the basis vectors we have the following expression:

$$\Delta(t_1) = -\sum_{p=1}^{n+m} p\beta_{1,p}e_p, \quad \Delta(t_2) = -\sum_{p=2}^{n} \beta_{2,p}e_p, \quad \Delta(t_3) = -\sum_{p=n+1}^{n+m} \beta_{3,p}e_p,$$
$$\Delta(e_1) = \gamma_{1,1}e_1 - \sum_{p=2}^{n-1} \delta_{1,p}e_{p+1} - \sum_{p=n+1}^{n+m-1} \delta_{1,p}e_{p+1},$$
$$\Delta(e_i) = (i\gamma_{i,1} + \gamma_{i,2})e_i + \delta_{i,1}e_{i+1}, \qquad 2 \leq i \leq n,$$
$$\Delta(e_{n+j}) = ((n+j)\gamma_{n+j,1} + \gamma_{n+j,3})e_{n+j} + \delta_{n+j,1}e_{n+j+1}, \qquad 1 \leq j \leq m.$$

The goal now is to show that the expressions for ad_z and Δ coincide. Firstly, we will show this coincidence on the generators of the basis vectors, i.e. t_1, t_2, t_3, e_1, e_2 and e_{n+1}. Let us consider $\Delta(st_2 - t_1)$ with a fixed s verifying $2 \leq s \leq n$. Thus as Δ is linear we obtain

$$\Delta(st_2 - t_1) = s\Delta(t_2) - \Delta(t_1) = \sum_{p=1}^{n+m} p\beta_{1,p}e_p - s\sum_{p=2}^{n} \beta_{2,p}e_p$$
$$= \beta_{1,1}e_1 + \sum_{p=2}^{n}(p\beta_{1,p} - s\beta_{2,p})e_p + \sum_{p=n+1}^{n+m} p\beta_{1,p}e_p,$$

on the other hand and by definition the above coincides with the value of a superderivation, named d^e on the vector $st_2 - t_1$, thus

$$\Delta(st_2 - t_1) = d^e(st_2 - t_1) = \beta_1^e e_1 + \sum_{p=2}^{n}(p\beta_p^e - s\beta_p^e)e_p + \sum_{p=n+1}^{n+m} p\beta_p^e e_p,$$

comparing the coefficients of e_s on both expressions it follows that $s(\beta_{1,s} - \beta_{2,s}) = 0$ and $\beta_{1,s} = \beta_{2,s}$. Repeating this process for all the possible s with $2 \leq s \leq n$, leads to $\beta_{1,s} = \beta_{2,s}$, $2 \leq s \leq n$.

Let us consider now $\Delta(st_3 - t_1)$ with a fixed s verifying $n+1 \leq s \leq n+m$. Since Δ is linear map, we obtain

$$s\Delta(t_3) - \Delta(t_1) = \sum_{p=1}^{n+m} p\beta_{1,p}e_p - s\sum_{p=n+1}^{n+m} \beta_{3,p}e_p = \beta_{1,1}e_1 + \sum_{p=2}^{n} p\beta_{1,p}e_p$$
$$+ \sum_{p=n+1}^{n+m} (p\beta_{1,p}e_p - s\beta_{3,p})e_p,$$

on the other hand and by definition the above coincides with the value of a superderivation, named d^e on the vector $st_3 - t_1$, thus

$$\Delta(st_3 - t_1) = d^e(st_3 - t_1) = \beta_1^e e_1 + \sum_{p=2}^{n} p\beta_p^e e_p + \sum_{p=n+1}^{n+m} (p\beta_p^e - s\beta_p^e)e_p,$$

as above, comparing the coefficients of e_s on both expressions it follows that $s(\beta_{1,s} - \beta_{3,s}) = 0$ and $\beta_{1,s} = \beta_{3,s}$. Repeating this process for all the possible s with $n+1 \leq s \leq n+m$, leads to $\beta_{1,s} = \beta_{3,s}$, $n+1 \leq s \leq n+m$. Renaming $\beta_{i,p}$ we have

$$\Delta(t_1) = -\sum_{p=1}^{n+m} p\beta_p e_p, \quad \Delta(t_2) = -\sum_{p=2}^{n} \beta_p e_p, \quad \Delta(t_3) = -\sum_{p=n+1}^{n+m} \beta_p e_p.$$

Let us now consider $\Delta(t_1 - 3t_2 - e_1)$, on one hand we have

$$\begin{aligned}\Delta(t_1) - 3\Delta(t_2) - \Delta(e_1) &= -\sum_{p=1}^{n+m} p\beta_p e_p + 3\sum_{p=2}^{n} \beta_p e_p - \gamma_{1,1}e_1 + \sum_{p=2}^{n-1} \delta_{1,p}e_{p+1} \\ &\quad + \sum_{p=n+1}^{n+m-1} \delta_{1,p}e_{p+1} = (-\beta_1 - \gamma_{1,1})e_1 \\ &\quad + \beta_2 e_2 + \sum_{p=3}^{n} (3\beta_p - p\beta_p + \delta_{1,p-1})e_p \\ &\quad -(n+1)\beta_{n+1}e_{n+1} + \sum_{p=n+2}^{n+m} (\delta_{1,p-1} - p\beta_p)e_p,\end{aligned}$$

on the other hand and by definition the above coincides with the value of a superderivation, named d^e on the vector $t_1 - 3t_2 - e_1$, thus

$$\begin{aligned}&\Delta(t_1 - 3t_2 - e_1) \\ &\quad = d^e(t_1 - 3t_2 - e_1) = -\sum_{p=1}^{n+m} p\beta^e_p e_p + 3\sum_{p=2}^{n} \beta^e_p e_p \\ &\qquad -\gamma^e_1 e_1 + \sum_{p=2}^{n-1} \beta^e_p e_{p+1} + \sum_{p=n+1}^{n+m-1} \beta^e_p e_{p+1} \\ &\quad = (-\beta^e_1 - \gamma^e_1)e_1 + \beta^e_2 e_2 + \sum_{p=3}^{n} (3\beta^e_p - p\beta^e_p + \beta^e_{p-1}) \\ &\qquad \times e_p - (n+1)\beta^e_{n+1}e_{n+1} + \sum_{p=n+2}^{n+m} (\beta^e_{p-1} - p\beta^e_p)e_p,\end{aligned}$$

considering the coefficients of e_2 and e_3 we obtain $\delta_{1,2} = \beta_2$. On account of $\Delta(t_1 - (i+1)t_2 - e_1)$ inductively we get $\delta_{1,i} = \beta_i$ for i verifying $2 \leq i \leq n-1$.

In a similar way from $\Delta(t_1 - (n+2)t_3 - e_1)$ and considering the coefficients of e_{n+1} and e_{n+2} we obtain $\delta_{1,n+1} = \beta_{n+1}$, after and considering the coefficients of e_{n+1}, e_{n+2} and e_{n+3} in $\Delta(t_1 - (n+3)t_3 - e_1)$ we obtain $\delta_{1,n+2} = \beta_{n+2}$. Therefore, by considering $\Delta(t_1 - (n+j+1)t_3 - e_1)$ inductively we get $\delta_{1,n+j} = \beta_{n+j}$ with $1 \leq j \leq m-1$. In summary, we have then

$$\Delta(e_1) = \gamma_1 e_1 - \sum_{p=2}^{n-1} \beta_p e_{p+1} - \sum_{p=n+1}^{n+m-1} \beta_p e_{p+1}.$$

Let us now consider $\Delta(t_1-(i+1)t_2+e_i)$ with a fixed i verifying $2\leq i\leq n-1$. Then

$$\begin{aligned}\Delta(t_1)-(i+1)\Delta(t_2)+\Delta(e_i) &= -\sum_{p=1}^{n+m} p\beta_p e_p+(i+1)\sum_{p=2}^{n}\beta_p e_p\\ &\quad +(i\gamma_{i,1}+\gamma_{i,2})e_i+\delta_{i,1}e_{i+1}\\ &= -\beta_1 e_1+\cdots+\delta_{i,1}e_{i+1}.\end{aligned}$$

On the other hand, we have

$$\begin{aligned}\Delta(t_1-(i+1)t_2+e_i) &= d^e(t_1-(i+1)t_2+e_i)\\ &= -\sum_{p=1}^{n+m} p\beta_p^e e_p+(i+1)\sum_{p=2}^{n}\beta_p^e e_p\\ &\quad +(i\gamma_1^e+\gamma_2^e)e_i+\beta_1^e e_{i+1}=-\beta_1^e e_1+\cdots+\beta_1^e e_{i+1}\end{aligned}$$

which leads to $\delta_{i,1}=\beta_1$. By repeating this process for all i with $2\leq i\leq n-1$, we conclude that $\delta_{i,1}=\beta_1$ for all i, $2\leq i\leq n-1$. Summing up

$$\Delta(e_i)=(i\gamma_{i,1}+\gamma_{i,2})e_i+\beta_1 e_{i+1},\ 2\leq i\leq n.$$

From $\Delta(e_2+e_{n+j})$ we have

$$\begin{aligned}\Delta(e_2)+\Delta(e_{n+j}) &= (2\gamma_{2,1}+\gamma_{2,2})e_2+\beta_1 e_3+((n+j)\gamma_{n+j,1}\\ &\quad +\gamma_{n+j,3})e_{n+j}+\delta_{n+j,1}e_{n+j+1},\end{aligned}$$

and on the other hand

$$\begin{aligned}\Delta(e_2+e_{n+j})=d^e(e_2+e_{n+j}) &= (2\gamma_1^e+\gamma_2^e)e_2+\beta_1^e e_3+((n+j)\gamma_1^e\\ &\quad +\gamma_3^e)e_{n+j}+\beta_1^e e_{n+j+1},\end{aligned}$$

which leads to $\delta_{n+j,1}=\beta_1$, $1\leq j\leq m-1$.

Consequently, there is no loss of generality in supposing

$$\Delta(t_1)=-\sum_{p=1}^{n+m} p\beta_p e_p,\quad \Delta(t_2)=-\sum_{p=2}^{n}\beta_p e_p,\quad \Delta(t_3)=-\sum_{p=n+1}^{n+m}\beta_p e_p,$$

$$\Delta(e_1)=\gamma_1 e_1-\sum_{p=2}^{n-1}\beta_p e_{p+1}-\sum_{p=n+1}^{n+m-1}\beta_p e_{p+1},\quad \Delta(e_2)=(2\gamma_1+\gamma_2)e_2+\beta_1 e_3,$$

$$\begin{aligned}&\Delta(e_i)=(i\gamma_{i,1}+\gamma_{i,2})e_i+\beta_1 e_{i+1},\ 3\leq i\leq n,\\ &\Delta(e_{n+1})=((n+1)\gamma_1+\gamma_3)e_{n+1}+\beta_1 e_{n+2},\end{aligned}$$

$$\begin{aligned}&\Delta(e_{n+j})=((n+j)\gamma_{n+j,1}+\gamma_{n+j,3})e_{n+j}+\beta_1 e_{n+j+1},\ 2\leq j\leq m-1,\\ &\Delta(e_{n+m})=((n+m)\gamma_{n+m,1}+\gamma_{n+m,3})e_{n+m}.\end{aligned}$$

At this point we are going to distinguish between even and odd local superderivations. Recall that $\{t_1,t_2,t_3,e_1,\dots,e_n\}$ are even basis vectors of $SL^{n,m}$ and $\{e_{n+1},\dots,e_{n+m}\}$ odd ones. Thus, if Δ is an odd local superderivation in particular Δ is a homogeneous linear mapping of degree 1,

$$\Delta:(SL^{n,m})_{\bar 0}\longrightarrow(SL^{n,m})_{\bar 1}\ \text{ and }\ \Delta:(SL^{n,m})_{\bar 1}\longrightarrow(SL^{n,m})_{\bar 0}.$$

Therefore, the only non-null values on the basis vectors for an odd local superderivation are exactly:

$$\Delta(t_1) = -\sum_{p=n+1}^{n+m} p\beta_p e_p, \quad \Delta(t_3) = -\sum_{p=n+1}^{n+m} \beta_p e_p, \quad \Delta(e_1) = -\sum_{p=n+1}^{n+m-1} \beta_p e_{p+1}.$$

Then every odd local superderivation is a standard odd superderivation. Regarding even local superderivations $\Delta : (SL^{n,m})_{\bar{0}} \longrightarrow (SL^{n,m})_{\bar{0}}$ and $\Delta : (SL^{n,m})_{\bar{1}} \longrightarrow (SL^{n,m})_{\bar{1}}$ we have

$$\begin{aligned}
\Delta(t_1) &= -\sum_{p=1}^{n} p\beta_p e_p, \quad \Delta(t_2) = -\sum_{p=2}^{n} \beta_p e_p, \quad \Delta(t_3) = 0, \quad \Delta(e_1) \\
&= \gamma_1 e_1 - \sum_{p=2}^{n-1} \beta_p e_{p+1}, \\
\Delta(e_2) &= (2\gamma_1 + \gamma_2)e_2 + \beta_1 e_3, \quad \Delta(e_i) = (i\gamma_{i,1} + \gamma_{i,2})e_i + \beta_1 e_{i+1}, \ 3 \le i \le n, \\
\Delta(e_{n+1}) &= ((n+1)\gamma_1 + \gamma_3)e_{n+1} + \beta_1 e_{n+2}, \\
\Delta(e_{n+j}) &= ((n+j)\gamma_{n+j,1} + \gamma_{n+j,3})e_{n+j} + \beta_1 e_{n+j+1}, \ 2 \le j \le m.
\end{aligned}$$

Only rest to prove that $\gamma_{i,1} = \gamma_{n+j,1} = \gamma_1$, $\gamma_{i,2} = \gamma_2$ and $\gamma_{n+j,3} = \gamma_3$ in order to have a standard even superderivation. Let us consider $\Delta(t_1 - (j+1)t_2 + e_1 - (j-1)e_2 + \frac{1}{(j-2)!}e_{j+1})$ with a fixed j verifying $2 \le j \le n-1$, on one hand we have

$$\begin{aligned}
&\Delta(t_1) - (j+1)\Delta(t_2) + \Delta(e_1) - (j-1)\Delta(e_2) + \frac{1}{(j-2)!}\Delta(e_{j+1}) \\
&\quad = -\sum_{p=1}^{n} p\beta_p e_p + (j+1)\sum_{p=2}^{n} \beta_p e_p + \gamma_1 e_1 \\
&\qquad -\sum_{p=2}^{n-1} \beta_p e_{p+1} - (j-1)(2\gamma_1 + \gamma_2)e_2 - (j-1)\beta_1 e_3 + \\
&\qquad + \frac{1}{(j-2)!}((j+1)\gamma_{j+1,1} + \gamma_{j+1,2})e_{j+1} + \frac{1}{(j-2)!}\beta_1 e_{j+2} \\
&\quad = (\gamma_1 - \beta_1)e_1 - (j-1)(2\gamma_1 + \gamma_2 - \beta_2)e_2 - [(j-1)\beta_1 \\
&\qquad + \beta_2 - (j-2)\beta_3]e_3 - \sum_{k=4}^{j}[\beta_{k-1} - (j+1-k)\beta_k]e_k - [\beta_j \\
&\qquad - \frac{1}{(j-2)!}((j+1)\gamma_{j+1,1} + \gamma_{j+1,2})]e_{j+1} + \dots
\end{aligned}$$

on the other hand

$$\begin{aligned}
&d^e(t_1 - (j+1)t_2 + e_1 - (j-1)e_2 + \frac{1}{(j-2)!}e_{j+1}) \\
&\quad = (\gamma_1^e - \beta_1^e)e_1 - (j-1)(2\gamma_1^e + \gamma_2^e - \beta_2^e)e_2 - [(j-1)\beta_1^e \\
&\qquad + \beta_2^e - (j-2)\beta_3^e]e_3 - \sum_{k=4}^{j}[\beta_{k-1}^e - (j+1-k)\beta_k^e]e_k - [\beta_j^e
\end{aligned}$$

$$-\frac{1}{(j-2)!}((j+1)\gamma_1^e+\gamma_2^e)]e_{j+1}+\dots$$

On account of the coefficients of $e_1,\dots,e_{j+1}$ we have

$$\begin{array}{ll}
(1) & \gamma_1^e-\beta_1^e=\gamma_1-\beta_1,\\
(2) & 2\gamma_1^e+\gamma_2^e-\beta_2^e=2\gamma_1+\gamma_2-\beta_2,\\
(3) & (j-1)\beta_1^e+\beta_2^e-(j-2)\beta_3^e=(j-1)\beta_1+\beta_2-(j-2)\beta_3,\\
(k)\ 4\le k\le j, & \beta_{k-1}^e-(j+1-k)\beta_k^e=\beta_{k-1}-(j+1-k)\beta_k,\\
(j+1) & \beta_j^e-\frac{1}{(j-2)!}((j+1)\gamma_1^e+\gamma_2^e)=\beta_j-\frac{1}{(j-2)!}((j+1)\gamma_{j+1,1}+\gamma_{j+1,2}).
\end{array}$$

The following linear combination of the above equations

$$(j-1)(1)+(2)+(3)+\sum_{k=4}^{j+1}\frac{(j-2)!}{(j+1-k)!}(k)$$

leads to $(j+1)\gamma_{j+1,1}+\gamma_{j+1,2}=(j+1)\gamma_1+\gamma_2$. Repeating this process for all j with $2\le j\le n-1$ allow us to assume

$$\Delta(e_i)=(i\gamma_1+\gamma_2)e_i+\beta_1e_{i+1},\quad 3\le i\le n.$$

Finally, let us consider $\Delta(t_1+e_1-e_{n+1}+e_{n+j})$ for a fixed j verifying $2\le j\le m$. Then on one hand, we have

$$\begin{aligned}
&\Delta(t_1)+\Delta(e_1)+\Delta(e_{n+1})+\Delta(e_{n+j})\\
&\quad=-\sum_{p=1}^{n}p\beta_pe_p+\gamma_1e_1-\sum_{p=2}^{n-1}\beta_pe_{p+1}+((n+1)\gamma_1+\gamma_3)e_{n+1}+\beta_1e_{n+2}\\
&\qquad+((n+j)\gamma_{n+j,1}+\gamma_{n+j,3})e_{n+j}+\beta_1e_{n+j+1}\\
&\quad=(\gamma_1-\beta_1)e_1+\dots+((n+1)\gamma_1+\gamma_3)e_{n+1}+\beta_1e_{n+2}+((n+j)\gamma_{n+j,1}\\
&\qquad+\gamma_{n+j,3})e_{n+j}+\beta_1e_{n+j+1}.
\end{aligned}$$

On the other hand, we get

$$\begin{aligned}
&d^e(t_1)+d^e(e_1)+d^e(e_{n+1})+d^e(e_{n+j})\\
&\quad=(\gamma_1^e-\beta_1^e)e_1+\dots+((n+1)\gamma_1^e+\gamma_3^e)e_{n+1}+\beta_1^ee_{n+2}+((n+j)\gamma_1^e\\
&\qquad+\gamma_3^e)e_{n+j}+\beta_1^ee_{n+j+1}
\end{aligned}$$

which leads to $((n+j)\gamma_{n+j,1}+\gamma_{n+j,3})=((n+j)\gamma_1+\gamma_3)$. Hence, we obtain

$$\Delta(e_{n+j})=((n+j)\gamma_1+\gamma_3)e_{n+j}+\beta_1e_{n+j+1},\ 2\le j\le m,$$

which completes the proof of the theorem. □

Let us consider now the maximal-dimensional solvable Lie superalgebra with model nilpotent nilradical [12]. We denote this superalgebra by $SN(n_1,\dots,n_k,1|m_1,\dots,m_p)$ and it can expressed by the following products:

$$\begin{cases} [x_1,x_j]=-[x_j,x_1]=x_{j+1}, & 2\le j\le n_1,\\ [x_1,x_{n_1+\cdots+n_j+i}]=-[x_{n_1+\cdots+n_j+i},x_1]=x_{n_1+\cdots+n_j+i+1}, & 1\le j\le k-1,\ 2\le i\le n_{j+1},\\ [x_1,y_j]=-[y_j,x_1]=y_{j+1}, & 1\le j\le m_1-1,\\ [x_1,y_{m_1+\cdots+m_j+i}]=-[y_{m_1+\cdots+m_j+i},x_1]=y_{m_1+\cdots+m_j+i+1}, & 1\le j\le p-1,\ 1\le i\le m_{j+1}-1,\\ [t_1,x_i]=-[x_i,t_1]=ix_i, & 1\le i\le n_1+\cdots+n_k+1,\\ [t_1,y_j]=-[y_j,t_1]=jy_j, & 1\le j\le m_1+\cdots+m_p,\\ [t_2,x_i]=-[x_i,t_2]=x_i, & 2\le i\le n_1+1,\\ [t_{j+2},x_{n_1+\cdots+n_j+i}]=-[x_{n_1+\cdots+n_j+i},t_{j+2}]=x_{n_1+\cdots+n_j+i}, & 1\le j\le k-1,\ 2\le i\le n_{j+1}+1,\\ [t'_1,y_i]=-[y_i,t'_1]=y_i, & 1\le i\le m_1,\\ [t'_{j+1},y_{m_1+\cdots+m_j+i}]=-[y_{m_1+\cdots+m_j+i},t'_{j+1}]=y_{m_1+\cdots+m_j+i}, & 1\le j\le p-1,\ 1\le i\le m_{j+1}, \end{cases}$$

with $\{x_1,\dots,x_{n_1+\cdots n_k+1},\ t_1,\dots,t_{k+1},\ t'_1,\dots,t'_p\}$ even basis vectors and $\{y_1,\ \dots,\ y_{m_1+\cdots+m_p}\}$ odd basis vectors.

In [12] it is proved that all the superderivations are inner, then and following the spirit of the proof of the theorem for model filiform nilradical we have the next result. We omit the computations because they are rather cumbersome and do not contain any new idea.

Theorem 4.2. *On the maximal-dimensional solvable Lie superalgebra with model nilpotent nilradical, $SN(n_1,\dots,n_k,1|\ m_1,\dots,m_p)$ every local superder -ivation is a superderivation.*

5. Local Superderivations of Solvable Lie Superalgebras with Non-model Nilradical

Along this section, we use an example of solvable Lie superalgebra whose nilradical is a non-model one, in particular the nilradical is the only one Lie superalgebra of maximal nilindex $K^{2,m}$ (for more details regarding $K^{2,m}$ see Theorem 4.17 of [17]). We build over this solvable Lie superalgebra infinitely many local superderivations which are not superderivations.

Thus, consider for any m odd positive integer $m\ge 3$, the $(m+3)$-dimensional solvable Lie superalgebra L^{m+3} (named $L^{m+3}_{1,\frac{2-m}{2},1,0,\dots,0}$ in [11]). For that Lie superalgebra there exists a basis, namely $\{z,x_1,x_2,y_1,\dots,y_m\}$ with $\{z,x_1,x_2\}$ even basis vectors and $\{y_1,\dots,y_m\}$ odd basis vectors, in which L^{m+3} can be expressed by the only non-null bracket products that follow:

$$\begin{cases} [x_1,y_i]=-[y_i,x_1]=y_{i+1}, & 1\le i\le m-1,\\ (y_i,y_{m+1-i})=(y_{m+1-i},y_i)=(-1)^{i+1}x_2, & 1\le i\le \frac{1}{2}(m+1),\\ [z,x_1]=-[x_1,z]=x_1+x_2 & \\ [z,x_2]=-[x_2,z]=x_2 & \\ [z,y_1]=-[y_1,z]=\frac{2-m}{2}y_1, & \\ [z,y_i]=-[y_i,z]=((i-1)+\frac{2-m}{2})y_i, & 2\le i\le m, \end{cases}$$

being its nilradical:

$$K^{2,m}: \begin{cases} [x_1, y_i] = -[y_i, x_1] = y_{i+1}, & 1 \le i \le m-1, \\ (y_i, y_{m+1-i}) = (y_{m+1-i}, y_i) = (-1)^{i+1} x_2, & 1 \le i \le \frac{1}{2}(m+1). \end{cases}$$

Note that $[\cdot, \cdot]$ is the standard skew-symmetric bracket product whereas $(\cdot, \cdot)$ denotes the symmetrical ones, recall that a Lie superalgebra structure $\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}$ contains in particular the symmetric pairing $S^2\mathfrak{g}_{\bar{1}} \longrightarrow \mathfrak{g}_{\bar{0}}$. Moreover, because of these symmetric products $K^{2,m}$ is not a model nilpotent Lie superalgebra.

Consider now the even superderivations on L^{m+3}, that is $d \in Der_{\bar{0}}(L^{m+3})$ with

$$\begin{aligned} &d(z) = \alpha_0 z + \alpha_1 x_1 + \alpha_2 x_2, \\ &d(x_i) = \alpha_{i0} z + \alpha_{i1} x_1 + \alpha_{i2} x_2, \quad 1 \le i \le 2, \\ &d(y_j) = \beta_{j1} y_1 + \beta_{j2} y_2 + \cdots + \beta_{jm} y_m, \quad 1 \le j \le m. \end{aligned}$$

A straightforward computation leads to the following general matrix of any even superderivation on L^{m+3} :

$$Der_{\bar{0}}(L^{m+3}) = \begin{pmatrix} 0 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & \dots & 0 \\ 0 & \alpha_{11} & \alpha_{12} & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & \alpha_{11} & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \beta_{11} & -\alpha_1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \beta_{22} & -\alpha_1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & -\alpha_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & \beta_{mm} \end{pmatrix}$$

with $\beta_{ii} = ((i-1) + \frac{2-m}{2})\alpha_{11}$ for i, $1 \le i \le m$. Note also, from the matrix that $\beta_{i,i+1} = -\alpha_1$ and $\beta_{ij} = 0$ for the remaining possibilities.

Theorem 5.1. *$Der(L^{m+3}) \subsetneq LocDer(L^{m+3})$.*

Proof. Consider the homogeneous linear mappings $\Delta_t : L^{m+3} \longrightarrow L^{m+3}$ $(t \neq 1)$ of degree 0 defined on the basis vectors of L^{m+3} by

$$\Delta(z) = 0,\ \Delta(x_1) = x_1,\ \Delta(x_2) = t x_2,\ \Delta(y_j) = 0,\ 1 \le j \le m.$$

It can be easily checked that Δ is not an even superderivation because its matrix does not fit with the general matrix of even superderivations. Nevertheless, for any t the map Δ_t is an even local superderivation. Indeed, we have

$$\begin{aligned} &\Delta_t(z) = d_0(z),\ \Delta_t(x_1) = d_1(x_1),\ \Delta_t(x_2) = t d_1(x_2), \\ &\Delta_t(y_j) = d_0(y_j),\ 1 \le j \le m, \end{aligned}$$

where d_1 is the resultant even superderivation after replacing α_{11} by 1 and all the rest of parameters by 0 on the general matrix of $Der_{\bar{0}}(L^{m+3})$ and d_0 is the null superderivation.

Analogously to proof of Theorem 3.1 for a fixed element $e = \alpha_0 z + \alpha_1 x_1 + \alpha_2 x_2 + \beta_1 y_1 + \cdots + \beta_m y_m$ we have $\Delta_t(e) = \alpha_1 x_1 + \alpha_2 t x_2$.

Now our goal is to prove the existence of a derivation d_e such that $\Delta_t(e) = d_e(e)$.

Let $d_e(z) = d_e(y_j) = 0$ be with $1 \leq i \leq m$ and

$$d_e(x_1) = a_1 x_1 + a_2 x_2, \qquad d_e(x_2) = a_3 x_2$$

where a_1, a_2, a_3 are unknowns. From the constraint $\Delta_t(e) = d_e(e)$ we have that $a_1 = 1$ and the following equation $\alpha_1 a_2 + \alpha_2(a_3 - t) = 0$. This equation always has solution with respect to unknowns a_2, a_3. Replacing one of theses solutions of d_e we get that Δ_t is a local superderivation.

□

Remark 5.1. In fact, in the proof of Theorem 5.1 we show the existence of infinitely many local superderivations on the $(m+3)$-dimensional solvable Lie superalgebra L^{m+3} which are not superderivations. Note also, that analogously it can be found infinitely many odd local superderivations which are not odd superderivations.

6. Local Superderivations of Maximal-Dimensional Solvable Leibniz Superalgebras with Model Filiform and Model Nilpotent Non-Lie Nilradical

We consider the maximal-dimensional solvable Leibniz superalgebra with filiform nilradical [12]. This superalgebra is unique for each pair of dimensions (n, m) and can be expressed by:

$$SLP^{n,m}: \begin{cases} [x_i, x_1] = x_{i+1}, & 2 \leq i \leq n-1; \\ [y_j, x_1] = y_{j+1}, & 1 \leq j \leq m-1; \\ [t_1, x_1] = -x_1, & \\ [x_1, t_1] = x_1, & \\ [x_i, t_1] = (i-2)x_i, & 3 \leq i \leq n; \\ [y_j, t_1] = (j-1)y_j, & 2 \leq j \leq m; \\ [x_i, t_2] = x_i, & 2 \leq i \leq n; \\ [y_j, t_3] = y_j, & 1 \leq j \leq m; \end{cases}$$

with $\{x_1, x_2, \ldots, x_n, t_1, t_2, t_3\}$ a basis of $(SLP^{n,m})_{\bar{0}}$ and $\{y_1, y_2, \ldots, y_m\}$ a basis of $(SLP^{n,m})_{\bar{1}}$. Its superalgebra of superderivations was obtained in [12]. Next, we prove the following result.

Theorem 6.1. *On the maximal-dimensional solvable Leibniz superalgebra with model filiform nilradical, every local superderivation is a superderivation.*

Proof. We rewrite the table of multiplications of the superalgebra $SLP^{n,m}$ as follows:

$$SLP^{n,m}:\begin{cases}[e_i,e_1]=e_{i+1}, & 2\le i\le n-1;\\ [e_{n+j},e_1]=e_{n+j+1}, & 1\le j\le m-1;\\ [t_1,e_1]=-e_1,\\ [e_1,t_1]=e_1,\\ [e_i,t_1]=(i-2)e_i, & 3\le i\le n;\\ [e_{n+j},t_1]=(j-1)e_{n+j}, & 2\le j\le m;\\ [e_i,t_2]=e_i, & 2\le i\le n;\\ [e_{n+j},t_3]=e_{n+j}, & 1\le j\le m;\end{cases}$$

The superderivations of $SLP^{n,m}$ are inner [12]. Let us fix an arbitrary element

$$z=\gamma_1 t_1+\gamma_2 t_2+\gamma_3 t_3+\sum_{p=1}^{n+m}\beta_p e_p$$

of $SLP^{n,m}$. Then for its right operator R_z we obtain

$$\begin{aligned}R_z(t_1)&=-\beta_1 e_1\quad R_z(t_2)=R_z(t_3)=0,\\ R_z(e_1)&=\gamma_1 e_1\\ R_z(e_i)&=((i-2)\gamma_1+\gamma_2)e_i+\beta_1 e_{i+1},\ \ 2\le i\le n-1,\\ R_z(e_n)&=((n-2)\gamma_1+\gamma_2)e_n,\\ R_z(e_{n+j})&=((j-1)\gamma_1+\gamma_3)e_{n+j}+\beta_1 e_{n+j+1},\ 1\le j\le m-1,\\ R_z(e_{n+m})&=((m-1)\gamma_1+\gamma_3)e_{n+m}.\end{aligned}$$

In [12], we prove that all superderivations are even. Let us consider an arbitrary local superderivation $\Delta: SLP^{n,m}\longrightarrow SLP^{n,m}$. Similar to Lie superalgebra we have that:

$$\begin{array}{ll}\Delta(t_1)=-\beta_{1,1}e_1 & \Delta(t_2)=\Delta(t_3)=0,\\ \Delta(e_1)=\gamma_{1,1}e_1 & \\ \Delta(e_i)=((i-2)\gamma_{i,1}+\gamma_{i,2})e_i+\beta_{i,1}e_{i+1}, & 2\le i\le n-1,\\ \Delta(e_n)=((n-2)\gamma_{n,1}+\gamma_{n,2})e_n, & \\ \Delta(e_{n+j})=((j-1)\gamma_{n+j,1}+\gamma_{n+j,3})e_{n+j}+\beta_{n+j,1}e_{n+j+1}, & 1\le j\le m-1,\\ \Delta(e_{n+m})=((m-1)\gamma_{n+m,1}+\gamma_{n+m,3})e_{n+m}. & \end{array}$$

The goal now is to show that the expressions for R_z and Δ coincide. Firstly, we will show this coincidence on the generators of the basis vectors, i.e. t_1,t_2,t_3,e_1,e_2 and e_{n+1}. Let us consider $\Delta(t_1+e_2)$. Thus as Δ is linear we obtain

$$\Delta(t_1+e_2)=\Delta(t_1)+\Delta(e_2)=-\beta_{1,1}e_1+\gamma_{2,1}e_2+\beta_{2,1}e_3,$$

on the other hand and by definition the above coincides with the value of a superderivation, named R^e on the vector t_1+e_2, thus

$$\Delta(t_1+e_2)=R^e(t_1+e_2)=-\beta_1^e e_1+\gamma_2^e e_2+\beta_1^e e_3,$$

comparing the coefficients of e_1 and e_3 leads to $\beta_{2,1}=\beta_{1,1}$. Let us consider $\Delta(t_1+e_j)=-\beta_{1,1}e_1+((j-2)\gamma_{j,1}+\gamma_{j,2})e_j+\beta_{j,1}e_{j+1}$ for $3\le j\le n-1$

and on the other hand and by the definition of local derivation we get that $R^e(t_1+e_j) = -\beta_1^e e_1 + ((j-2)\gamma_1^e + \gamma_2^e)e_j + \beta_1^e e_{j+1}$. Comparing the coefficients of e_1 and e_{j+1} we obtain $\beta_{j,1} = \beta_{1,1}$ for $3 \leq j \leq n-1$.

We consider now $\Delta(t_1 + e_{n+1})$ and similar as the above we have that $\beta_{n+1,1} = \beta_{1,1}$. Analogously, if we take $\Delta(t_1 + e_{n+j})$ with $2 \leq j \leq m-1$ we have that $\beta_{n+j,1} = \beta_{1,1}$ with $2 \leq j \leq m-1$.

Consequently, there is no loss of generality in supposing

$$\begin{array}{ll}
\Delta(t_1) = -\beta_1 e_1, & \Delta(t_2) = \Delta(t_3) = 0, \\
\Delta(e_1) = \gamma_1 e_1 & \Delta(e_2) = \gamma_2 e_2 + \beta_1 e_3, \\
\Delta(e_i) = ((i-2)\gamma_{i,1} + \gamma_{i,2})e_i + \beta_1 e_{i+1},\ 3 \leq i \leq n-1, & \Delta(e_n) = ((n-2)\gamma_{n,1} + \gamma_{n,2})e_n, \\
\Delta(e_{n+1}) = \gamma_3 e_{n+1} + \beta_1 e_{n+2}, & \\
\Delta(e_{n+j}) = ((j-1)\gamma_{n+j,1} + \gamma_{n+j,3})e_{n+j} + \beta_1 e_{n+j+1},\ 2 \leq j \leq m-1, & \\
\Delta(e_{n+m}) = ((m-1)\gamma_{n+m,1} + \gamma_{n+m,3})e_{n+m}. &
\end{array}$$

Let us consider $\Delta(t_1 + e_1 + e_2 + e_3) = (\gamma_1 - \beta_1)e_1 + \gamma_2 e_2 + (\beta_1 + \gamma_{3,1} + \gamma_{3,2})e_3 + \beta_1 e_4$ on the other hand and by definition the above coincides with the value of a superderivation, named R^e on the vector $t_1 + e_1 + e_2 + e_3$, thus

$$\Delta(t_1 + e_1 + e_2 + e_3) = R^e(t_1 + e_1 + e_2 + e_3) = (\gamma_1^e - \beta_1^e)e_1 + \gamma_2^e e_2 + (\beta_1^e + \gamma_1^e + \gamma_2^e)e_3 + \beta_1^e e_4.$$

On account of the coefficients of $e_1, \ldots, e_4$ we have

(1) $\gamma_1 - \beta_1 = \gamma_1^e - \beta_1^e$,
(2) $\gamma_2 = \gamma_2^e$,
(3) $\gamma_{3,1} + \gamma_{3,2} + \beta_1 = \gamma_1^e + \gamma_2^e + \beta_1^e$,
(4) $\beta_1 = \beta_1^e$.

From (1) + (2) + (4) we have that $\gamma_1 + \gamma_2 = \gamma_1^e + \gamma_2^e$ and from (3) – (4) we get $\gamma_{3,1} + \gamma_{3,2} = \gamma_1^e + \gamma_2^e$. Then, $\gamma_{3,1} + \gamma_{3,2} = \gamma_1 + \gamma_2$.

Now, we study $\Delta(t_1 + e_1 + e_2 + e_j)$ for $4 \leq j \leq n$ and we obtain the following equations:

(1) $\gamma_1 - \beta_1 = \gamma_1^e - \beta_1^e$,
(2) $\gamma_2 = \gamma_2^e$,
(3) $\beta_1 = \beta_1^e$,
(4) $(j-2)\gamma_{j,1} + \gamma_{j,2} = (j-2)\gamma_1^e + \gamma_2^e$,

From $(j-2)\times(1)+(2)+(j-2)\times(3)$ we get $(j-2)\gamma_1^e+\gamma_2^e = (j-2)\gamma_1+\gamma_2$ and from (4) we leads to $(j-2)\gamma_{j,1} + \gamma_{j,2} = (j-2)\gamma_1 + \gamma_2$.

We now consider the vectors $t_1 + e_1 + e_{n+1} + e_{n+j}$ for a fixed j verifying $2 \leq j \leq m$. For $j = 2$, we have that

$$\Delta(t_1) + \Delta(e_1) + \Delta(e_{n+1}) + \Delta(e_{n+2}) = (\gamma_1 - \beta_1)e_1 + \gamma_3 e_{n+1} + (\gamma_{n+2,1} + \gamma_{n+2,3} + \beta_1)e_{n+2} + \beta_1 e_{n+3}$$

on the other hand, we get

$$R^e(t_1) + R^e(e_1) + R^e(e_{n+1}) + R^e(e_{n+2}) = (\gamma_1^e - \beta_1^e)e_1 + \gamma_3^e e_{n+1} + (\gamma_1^e + \gamma_3^e + \beta_1^e)e_{n+2} + \beta_1^e e_{n+3},$$

which leads to

$$\begin{aligned}&(1)\ \gamma_1-\beta_1=\gamma_1^e-\beta_1^e,\\&(2)\ \gamma_3=\gamma_3^e,\\&(3)\ \gamma_{n+2,1}+\gamma_{n+2,2}+\beta_1=\gamma_1^e+\gamma_2^e+\beta_1^e,\\&(4)\ \beta_1=\beta_1^e.\end{aligned}$$

From $(1)+(2)+(3)$ we obtain $\gamma_1+\gamma_3=\gamma_1^e+\gamma_3^e$ and from $(3)-(4)$ we get $\gamma_{n+2,1}+\gamma_{n+2,3}=\gamma_1+\gamma_3$.

We repeat the calculations for the vectors $t_1+e_1+e_{n+1}+e_{n+j}$ with $3\le j\le m$ and we derive the following equations:

$$\begin{aligned}&(1)\ \gamma_1-\beta_1=\gamma_1^e-\beta_1^e,\\&(2)\ \gamma_3=\gamma_3^e,\\&(3)\ \beta_1=\beta_1^e,\\&(4)\ (j-1)\gamma_{n+j,1}+\gamma_{n+j,3}=(j-1)\gamma_1^e+\gamma_3^e,\end{aligned}$$

From $(j-1)\times(1)+(2)+(j-1)\times(3)$ we get $(j-1)\gamma_1^e+\gamma_3^e=(j-1)\gamma_1+\gamma_3$ and from (4) we derive $(j-1)\gamma_{n+j,1}+\gamma_{n+j,3}=(j-1)\gamma_1+\gamma_3$ with $3\le j\le m$ which completes the proof of the theorem. □

Let us consider now the complex maximal-dimensional solvable Leibniz superalgebra with model nilpotent non-Lie nilradical [12]. We denote this superalgebra by $SNP(n_1,\cdots,n_k,1|m_1,\ldots,m_p)$ given by:

$$\begin{cases}[x_j,x_1]=x_{j+1}, & 2\le j\le n_1;\\ [x_{n_1+\cdots+n_j+i},x_1]=x_{n_1+\cdots+n_j+i+1}, & 1\le j\le k-1,\ 2\le i\le n_{j+1};\\ [y_j,x_1]=y_{j+1}, & 1\le j\le m_1-1;\\ [y_{m_1+\cdots+m_j+i},x_1]=y_{m_1+\cdots+m_j+i+1}, & 1\le j\le p-1,\ 1\le i\le m_{j+1}-1;\\ [t_1,x_1]=-x_1, & \\ [x_1,t_1]=x_1, & \\ [x_i,t_1]=(i-2)x_i, & 3\le i\le n_1+1;\\ [x_{n_1+\cdots+n_j+i},t_1]=(i-2)x_{n_1+\cdots+n_j+i}, & 1\le j\le k-1,\ 3\le i\le n_{j+1}+1;\\ [y_j,t_1]=(i-1)y_j, & 2\le j\le m_1;\\ [y_{m_1+\cdots+m_j+i},t_1]=(i-1)y_{m_1+\cdots+m_j+i}, & 1\le j\le p-1,\ 2\le i\le m_{j+1},\\ [x_i,t_2]=x_i, & 2\le i\le n_1+1;\\ [x_{n_1+\cdots+n_j+i},t_{j+2}]=x_{n_1+\cdots+n_j+i}, & 1\le j\le k-1,\ 2\le i\le n_{j+1}+1;\\ [y_j,t_1']=y_j, & 1\le j\le m_1;\\ [y_{m_1+\cdots+m_j+i},t_{j+1}']=y_{m_1+\cdots+m_j+i}, & 1\le j\le p-1,\ 1\le i\le m_{j+1};\end{cases}$$

with $\{x_1,\ldots,x_{n_1+\cdots n_k+1},t_1,\ldots,t_{k+1},t_1',\ldots,t_p'\}$ even basis vectors and $\{y_1,\ldots,y_{m_1+\cdots+m_p}\}$ odd basis vectors.

In [12] it is proved that all superderivation are inner. Analogously to Theorem 6.1, we have the following result.

Theorem 6.2. *On the maximal-dimensional solvable Leibniz superalgebra with model nilpotent non-Lie nilradical, $SNP(n_1,\ldots,n_k,1|m_1,\ldots,m_p)$ every local superderivation is a superderivation.*

Proof. The proof is carrying out by arguments that used in the previous theorem. We omit the proof of this theorem because the computations are rather cumbersome and do not contain any new idea. □

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