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Robust coalitional model predictive control with negotiation of mutual interactions

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ABSTRACT

This article presents a robust coalitional model predictive control (MPC) approach where neighboring agents negotiate the bounds of their coupling variables. Also, the control variables of each agent are divided into a *private* part that is locally optimized, and a *public* part that is controlled by the corresponding neighbors. Under certain conditions, the agents communicate to update the bounds that determine the constraints on these private and public variables. Moreover, the mutual disturbances induced by coupling are considered using a tube-based approach, guaranteeing recursive feasibility and stability of the closed-loop system. The proposed method is tested on a simulated eight-input coupled tank benchmark to illustrate its benefits.

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1. Introduction

Model predictive control (MPC) is a computer-based control method that uses a model to predict the evolution of a system as a function of the sequence of inputs provided along a given horizon. Then, the minimization of a cost function allows us to determine an optimal sequence to steer the system according to the control designer's goals. To this end, the MPC controller solves an optimization problem at each time step, which may include explicit constraints on the system variables, among other complex issues such as delays, information about expected disturbances, and uncertainties affecting the system. Once a minimizer has been obtained for the optimal control problem, the element of the sequence corresponding to the current time step is implemented and the problem is solved again at the next time instant following a receding horizon approach.

One of the principal drawbacks of MPC is that it cannot be applied to large-scale systems in a centralized way due to the number of decision variables in the optimization problem. Naturally distributed systems such as smart grids and water networks require distributed MPC (DMPC) strategies [1–4]. The main idea is to decompose the overall system into subsystems, which are assigned to local interacting controllers, also known as *agents*. As a consequence, the global problem is partitioned and solved in a distributed manner, preserving the essence of MPC while providing increased scalability and flexibility [5,6]. In the DMPC framework, the communication and cooperation between the set of agents in their distributed computations play a key role to optimize the global performance and provide theoretical guarantees. While complete decentralized strategies minimize communication demands, fully cooperative DMPC algorithms can attain centralized-like behavior. Nonetheless, it may be interesting to restrict the information exchange and minimize the cooperation effort when the latter does not compromise the overall performance. A relevant issue in this regard is that of the coupling that often exists between the subsystems' dynamics. In this regard, authors in [7] apply a distributed tube-based MPC approach where local controllers share information about the size of their currently used constraint space to reduce conservatism by exploiting the varying degree of coupling.

Within the DMPC framework, the so-called *coalitional* control analyzes the dynamic couplings between the different parts of the system so that only strongly coupled elements communicate with each other. In this respect, the communication topology can be chosen based on a function that penalizes the communication cost [8,9] or the coupling strength [10,11]. Consequently, the global control structure is adjusted in real-time, optimizing the existing computational and communication resources. The underlying idea is to dynamically cluster the local agents into cooperative groups, i.e., *coalitions*, whenever this improves the overall system's performance [12], thus leading to time-varying partitions. Accordingly, there may be coalitions of agents exchanging information, while others may be operating in a decentralized fashion. This idea of dynamic clusters of controllers has drawn attention within the context of multi-agent networked systems [13,14]. In particular, coalitional control is presented as an alternative that is halfway between fully cooperative schemes

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and decentralized control. Note that in distributed systems, the lack of coordination may compromise the global performance [5]. Likewise, coordinating all control decisions increases the communication and computation demands [15]. Considering this, coalitional schemes promotes data sharing and coordination under certain conditions. See also [16], which reviews the literature on control by clustering strategies, discussing different criteria for partitioning the system and providing examples of application.

In this article, we present a novel coalitional MPC approach in which input variables are decomposed into private and public versions sharing a common constraint space. The proposed control scheme shares the nature of coalitional MPC strategies introduced in [17–19], which permit partial modes of cooperation, ranging from the fully distributed MPC algorithm, where all agents share information, to agents working in a decentralized manner without any communication. In this way, there may be a group of agents operating cooperatively for the sake of overall performance, while others work independently without exchanging data. Finally, note that this article is closely related to [20], which is used as the starting point for the problem and the algorithm formulation. However, the previously referred work lacks robustness and formal stability guarantees because it is based on a stochastic approach. To remedy this, we employ here a tube-based framework, which leads to significant changes in the control strategy. In particular, we follow a tube-based MPC approach applied in a distributed manner, where disturbances induced by the coupling between subsystems are modeled as bounded disturbances. Finally, we proceed to summarize the main contributions of this paper:

- First, this work strengthens the control algorithm presented in [20] by using a robust approach to guarantee constraint satisfaction, recursive feasibility, and stability. The main idea of the algorithm is to decompose coupling so that agents can *cede a portion* of their inputs to neighboring agents.
- Second, we propose a coalitional scheme where the agents' communication is event-based, and they negotiate the constraint space of the shared inputs by performing an iterative procedure based on dual decomposition. Unlike traditional distributed algorithms, where state or input sequences are communicated, here the agents only broadcast information about the scale factors that bound their inputs' constraint space. In this way, we minimize the amount of data to be shared.
- Third, our approach promotes communication between agents only when it generates a significant benefit in terms of overall performance. In this way, it is possible to reduce the communication and computational burden without significantly compromising the system performance with respect to the centralized behavior.

The outline of the rest of the article is organized as follows. In Section 2, the model of the system is presented, introducing the decomposition of input variables into private and public parts, and defining the setting for coalitional MPC. In Section 3, we describe the application of tube-based MPC to deal with uncertainties. Furthermore, we distribute the problem using the dual decomposition algorithm to negotiate the bounds of coupling variables. Section 4 presents the proposed control scheme. In Section 5, conditions for recursive feasibility and stability are given. Section 6 shows simulation results on an eight input-coupled tank system. Finally, concluding remarks are provided in Section 7.

2. System description

Consider a discrete-time, linear time-invariant system that can be divided into a set $\mathcal{N} = \{1, 2, \dots, N\}$ of input-coupled subsystems modeled as

$$\begin{aligned} x_i^+ &= A_{ii}x_i + B_{ii}u_i + d_i, \\ \text{with } \ d_i &= \sum_{j\in\mathcal{N}_i} B_{ij}u_j, \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^{n_{x_i}}$ and $u \in \mathbb{R}^{n_{u_i}}$ are the state and control input of subsystem $i \in \mathcal{N}$, and x_i^+ is the state at the next time instant. Accordingly, $d_i \in \mathbb{R}^{n_{x_i}}$ represents the input coupling, with the set $\mathcal{N}_i$ of the *neighboring* agents defined as $\mathcal{N}_i \triangleq \{j \in \mathcal{N} : B_{ij} \neq \mathbf{0},\ j \neq i\}$. Moreover, each subsystem $i \in \mathcal{N}$ is subject to local constraints in its state, i.e., $x_i \in \mathbb{X}_i$, and input, i.e., $u_i \in \mathbb{U}_i$, where $\mathbb{X}_i$ and $\mathbb{U}_i$ are convex sets containing the origin.

By aggregating all subsystems' states $x = [x_i]_{i\in\mathcal{N}} \in \mathbb{R}^{n_x}$ and inputs $u = [u_i]_{i\in\mathcal{N}} \in \mathbb{R}^{n_u}$, we can describe the global behavior of the system as

$$x^+ = Ax + Bu, \tag{2}$$

where matrices $A = \text{diag}(A_{ii})_{i\in\mathcal{N}}$ and $B = \left[B_{ij}\right]_{i,j\in\mathcal{N}}$ represent the global model and are defined as the aggregation of (1) for all subsystems. Note that the centralized model (2) does not include uncertainties because interactions are already present in the global matrices.

2.1. Decomposition of coupling variables

In this work, we apply the decomposition method proposed in [20], where input variables are partitioned into public and private parts. Consequently, we can decompose a local variable u_i as

$$u_i = u_i^{\text{pr}} + \sum_{j\in\mathcal{M}_i} u_{ij}^{\text{pu}}, \tag{3}$$

where

(i) u_i^{pr} is the private part of the variable, which is controlled exclusively by the agent that *owns* it, i.e., i, and it must verify $u_i^{\text{pr}} \in \alpha_i\mathbb{U}_i$ with $\alpha_i \in [0, 1]$.

(ii) u_{ij}^{pu} is the public part of u_i controlled by agent j and must verify $u_{ij}^{\text{pu}} \in \alpha_{ij}\mathbb{U}_i$. The set of agents j that can manipulate the public part of u_i are defined as *affected* subsystems $\mathcal{M}_i = \{j \in \mathcal{N} : B_{ji} \neq \mathbf{0},\ j \neq i\}$.

Remark 1. In general, the sets $\mathcal{N}_i$ and $\mathcal{M}_i$ are different and depend on the dynamics of the system, i.e., while $\mathcal{N}_i$ contains the set of neighbors that affect i, $\mathcal{M}_i$ defines the set of agents affected by the input of subsystem i.

Remark 2. Without loss of generality, in this article we consider input-coupled subsystems as defined in (1), i.e., $A_{ij} = 0$ for all $i \neq j$. However, this decomposition could similarly be applied to state-coupled subsystems.

In particular, each agent i will locally control the private part of its input variable, u_i^{pr} and the public part of its neighboring inputs $\{u_{ji}^{\text{pu}}\}_{\forall j\in\mathcal{N}_i}$. Accordingly, since the dynamics of the subsystem i is affected by the inputs u_j of neighboring subsystems, agents $j \in \mathcal{N}_i$ will decide the value of the private input u_j^{pr}. Moreover, the set $j \in \mathcal{M}_i$ of agents affected by subsystem's i input, will decide the public part of u_i, i.e., $\{u_{ij}^{\text{pu}}\}_{\forall j\in\mathcal{N}_i}$. Therefore, variables that cannot be locally controlled such as $\{u_j^{\text{pr}}\}_{\forall j\in\mathcal{N}_i}$ and $\{u_{ij}^{\text{pu}}\}_{\forall j\in\mathcal{M}_i}$ are treated as bounded disturbances from subsystem's i viewpoint. Fig. 1 shows a diagram with three agents illustrating the decomposition and negotiation that occurs in our method.

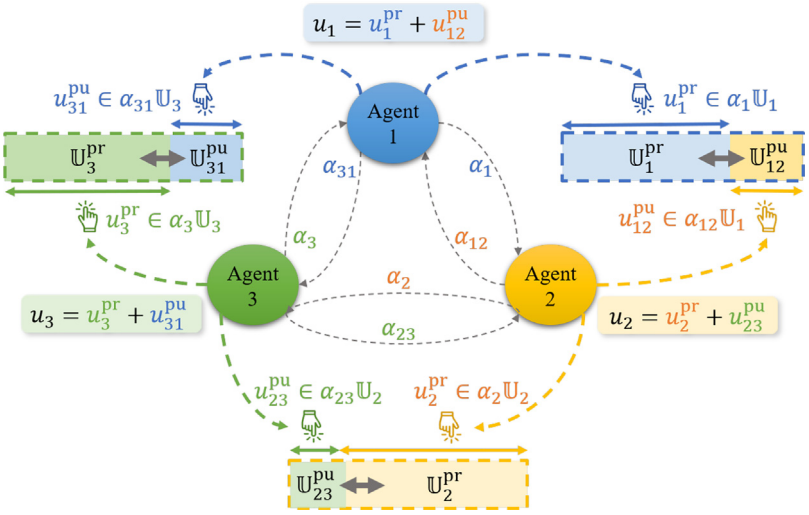


Fig. 1. Scheme of the proposed variable decomposition for three agents. Each agent i manipulates its private variable u_i^{pr} and the public input u_{ji}^{pu} ceded by its neighbor $j \in \mathcal{N}_i$. Agents can negotiate the value of input scale factors α_i, α_{ji}. Focusing on agent 1, we identify $\mathcal{N}_1 = \{3\}$. Therefore, agent 1 can manipulate a public part of u_3. Also, $\mathcal{M}_1 = \{2\}$, meaning that subsystem 2 is affected by u_1 and will manipulate a public part of it.

Hereafter, it is considered that agents should determine the values of u_i^{pr} and u_{ij}^{pu}, $\forall i \in \mathcal{N}$ and $j \in \mathcal{M}_i$, so that the next inequality is satisfied:

$$\alpha_i + \sum_{j \in \mathcal{M}_i} \alpha_{ij} \leq 1, \ \forall i \in \mathcal{N}. \tag{4}$$

Note that if u_i is calculated according to (3) and (4) holds, then the input constraint $u_i \in \mathbb{U}_i$ will be satisfied for all $i \in \mathcal{N}$.

2.2. Control architecture and strategy

Let us assume that a local controller or agent governs each subsystem $i \in \mathcal{N}$. Also, consider that local agents are interconnected through a configurable data network that allows communication among them. We can describe this network using the graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$, being $\mathcal{N}$ the set of nodes that represent the agents, and $\mathcal{L}$ the set of links, with $\mathcal{L} \subseteq \mathcal{L}^{\mathcal{N}} = \{\{i, j\} | i, j \in \mathcal{N}\}$. It is considered that the state of the links can change dynamically to suit the changing requirements of the control scheme, so that the set of agents is *partitioned* into disjoint communication elements or coalitions, i.e., only agents inside a coalition can exchange data. In particular, let $\hat{\alpha}$ be a threshold to enable or disable the agents' communication. Concerning this, the state of the links $\mathcal{L}$ will change depending on the values of α_{ij} with respect $\hat{\alpha}$.

Let $\mathcal{P}(k) = \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_{|\mathcal{P}(k)|}\}$ define the partition of the system at time instant k, where $\mathcal{C}_i \subseteq \mathcal{N}$ describes the ith coalition within the partition, with $\mathcal{C}_i \cap \mathcal{C}_j = \emptyset$ for any $i \neq j$. The size of $\mathcal{C}_i$ can range between the following two extremes:

(i) If all agents work in a decentralized manner, i.e., there is no communication among them, there will be $|\mathcal{N}|$ singletons, i.e., $\mathcal{P}(k) = \{\{1\}, \{2\}, \ldots, \{N\}\}$ and $|\mathcal{C}_i| = 1, \ \forall i$. This happens when $\alpha_{ij} < \hat{\alpha}$, for all $u_{ij}^{pu} \in \alpha_{ij}\mathbb{U}_i$, with $i \in \mathcal{N}$ and $j \in \mathcal{M}_i$.

(ii) If all agents work in a centralized manner, they will form the grand coalition, so there will be a single coalition that will group all agents, i.e. $\mathcal{P}(k) = \{\mathcal{N}\}$. This happens when $\alpha_{ij} \geq \hat{\alpha}$, for all $u_{ij}^{pu} \in \alpha_{ij}\mathbb{U}_i$, with $i \in \mathcal{N}$ and $j \in \mathcal{M}_i$.

As coefficients α_{ij} take values over and below $\hat{\alpha}$ for different combinations of $\{i, j\}$, there will be clusters of agents that communicate, i.e., coalitions, whereas others may be working in a decentralized manner. Therefore, agents operate in a flexible fashion that allows partial modes of cooperation.

The agents' goal is to maximize global performance with a minimum exchange of information. In particular, agents aim to minimize the cost function

$$\sum_{t=0}^{\infty} \sum_{i \in \mathcal{N}} \Big(x_i^{\top}(t+1)\, Q_i\, x_i(t+1) + u_i^{pr\top}(t)\, R_i^{pr}\, u_i^{pr}(t) \Big) + \sum_{j \in \mathcal{N}_i} u_{ji}^{pu\top}(t)\, R_i^{pu}\, u_{ji}^{pu}(t), \tag{5}$$

with Q_i, R_i^{pr}, R_i^{pu} being positive definite weighting matrices. Any agent i will optimize the private part of its input variable u_i^{pr}, the public part of neighboring input variables u_{ji}^{pu}, $\forall j \in \mathcal{N}_i$, and will be able to negotiate scale factors α_i and α_{ji}. In contrast to standard distributed schemes, which negotiate the values of coupling variables, we propose a negotiation of the scale factors that bound these variables. Also, the use of public variables will be penalized with a higher cost to discourage unnecessary cooperation efforts, i.e., $R_i^{pu} \gg R_i^{pr}$.

3. Tube-based MPC for regulation

In contrast to [20], where uncertainties were considered using a scenario-based MPC, we propose a tube-based MPC approach to deal with neighboring uncertainties. In this way, we are able to satisfy the system's constraints regardless of the realization of the disturbance.

3.1. Nominal control problem

Considering the definition of the local state (1), and the decomposition of private and public inputs (3), we can rewrite the dynamics of subsystem i as:

$$x_i^+ = A_{ii}x_i + B_{ii}u_i^{pr} + \sum_{j \in \mathcal{N}_i} B_{ij}u_{ji}^{pu} + w_i, \quad \text{with } w_i \triangleq \sum_{j \in \mathcal{M}_i} B_{ii}u_{ij}^{pu} + \sum_{j \in \mathcal{N}_i} B_{ij}u_j^{pr}. \tag{6}$$

As a result of the interaction between subsystems, the disturbance w_i groups the uncertainties generated by the agents in $\mathcal{M}_i$, which control the public part of u_i, i.e., $\{u_{ij}^{pu}\}_{\forall j \in \mathcal{M}_i}$, and those

Table 1
Notation summary.

u_i	Local variable of subsystem i, decomposed as (3).
$\mathbb{X}_i$, $\mathbb{U}_i$	State and input set of constraints of x_i and u_i.
u_i^{pr}	Private part of u_i controlled exclusively by agent i.
u_{ij}^{pu}	Public part of u_i ceded to agents $j \in \mathcal{M}_i$.
u_j^{pr}	Private part of u_j controlled by agent $j \in \mathcal{N}_i$.
u_{ji}^{pu}	Public part of u_j with $j \in \mathcal{N}_i$, controlled by agent i.
α_i	Tightening factor for set $\mathbb{U}_i$, with $u_i^{\text{pr}} :\in \alpha_i \mathbb{U}_i$.
α_{ji}	Tightening factor for set $\mathbb{U}_j$, with $u_{ji}^{\text{pu}} :\in \alpha_{ji} \mathbb{U}_j$.
$\mathcal{N}_i$	Neighboring subsystems of i: $\{j \in \mathcal{N} : B_{ij} \neq \mathbf{0},\ j \neq i\}$.
$\mathcal{M}_i$	Subsystems affected by u_i: $\{j \in \mathcal{N} : B_{ji} \neq \mathbf{0},\ j \neq i\}$.
$\mathbb{W}_i$	Bound of agent i's disturbances: $w_i \in \mathbb{W}_i$ (7).
$\mathcal{R}_i$	Robust positively invariant set of subsystem i.
$\bar{x}_i$, $\bar{u}_i$	Nominal state and input local variables of i (8).
$\overline{\mathbb{X}}_i$, $\overline{\mathbb{U}}_i$	State and input set of nominal constraints.

in $\mathcal{N}_i$, which control the private part of their input variables, i.e., $\{u_j^{\text{pr}}\}_{\forall j \in \mathcal{N}_i}$. Therefore, uncertainties are bounded by:

$$w_i \in \mathbb{W}_i \triangleq \bigoplus_{j \in \mathcal{M}_i} B_{ii} \mathbb{W}_{ij}^{\text{pu}} \oplus \bigoplus_{j \in \mathcal{N}_i} B_{ij} \mathbb{W}_j^{\text{pr}}, \quad \text{with } \mathbb{W}_{ij}^{\text{pu}} = \alpha_{ij} \mathbb{U}_i, \text{ and } \mathbb{W}_j^{\text{pr}} = \alpha_j \mathbb{U}_j, \tag{7}$$

where $\mathbb{W}_{ij}^{\text{pu}}$ and $\mathbb{W}_j^{\text{pr}}$ are convex closed sets containing the origin, satisfying $u_{ij}^{\text{pu}} \in \mathbb{W}_{ij}^{\text{pu}}$ and $u_j^{\text{pr}} \in \mathbb{W}_j^{\text{pr}}$. To take into account these uncertainties, we will follow a robust approach.

The tube-based MPC approach [21,22], is characterized by solving an MPC problem for the nominal system, that is, without considering disturbances, and by adding an auxiliary control law to keep the evolution of the *real* system within a *tube* around the nominal trajectory. Based on (6), we can derive the nominal model for each subsystem i by *ignoring* interactions w_i, i.e.,

$$\bar{x}_i^+ = A_{ii} \bar{x}_i + B_{ii} \bar{u}_i^{\text{pr}} + \sum_{j \in \mathcal{N}_i} B_{ij} \bar{u}_{ji}^{\text{pu}}. \tag{8}$$

In the nominal control problem, subsystem i ignores private variables of neighboring agents $\mathcal{N}_i$, and the public part of its input variable u_i, which is controlled by agents belonging to $\mathcal{M}_i$. For the sake of clarity, Table 1 has been included to summarize the notation used in this article.

Let us aggregate local input-to-state matrices as $B_i \triangleq \begin{bmatrix} B_{ii} & B_{ij} \end{bmatrix}$ for all $j \in \mathcal{N}_i$. In what follows, we consider the following assumptions:

Assumption 1. For every subsystem i:

- There exists a local feedback gain K_i that guarantees that $A_{K_i} \triangleq (A_{ii} + B_i K_i)$ is stable.
- It is possible to find a robust positively invariant (RPI) set $\mathcal{R}_i$ that satisfies: $A_{K_i} \mathcal{R}_i \oplus \mathbb{W}_i \subseteq \mathcal{R}_i$, $\mathcal{R}_i \subseteq \mathbb{X}_i$, and $K_i \mathcal{R}_i \subseteq \mathbb{U}_i$.
- Nominal state and input constraints are non-empty sets: $\overline{\mathbb{X}}_i \neq \emptyset$ and $\overline{\mathbb{U}}_i \neq \emptyset$, with $\overline{\mathbb{X}}_i \triangleq \mathbb{X}_i \ominus \mathcal{R}_i$ and $\overline{\mathbb{U}}_i \triangleq \mathbb{U}_i \ominus K_i \mathcal{R}_i$.

Remark 3. Assumption 1 is generally considered in the tube-based MPC framework [21,22], as it must be satisfied to have a non-empty solution space for the nominal problem.

For the nominal problem, the goal of each local controller i is to minimize the objective function

$$\mathbf{J}_{\mathbf{c}_i}\left(\bar{x}_i,\ \bar{\mathbf{u}}_i^{\text{pr}},\ \left[\bar{\mathbf{u}}_{ji}^{\text{pu}}\right]_{\forall j \in \mathcal{N}_i},\ \boldsymbol{\alpha}_i\right) = \sum_{t=k}^{k+N_{\text{p}}-1} \ell_i\left(\bar{x}_i(t),\ \bar{u}_i^{\text{pr}}(t),\ \left[\bar{u}_{ji}^{\text{pu}}(t)\right]_{j \in \mathcal{N}_i}\right) + f_i(\boldsymbol{\alpha}_i) + V_i^f\left(\bar{x}_i(k+N_{\text{p}})\right), \tag{9}$$

where t denotes the time step along the prediction horizon N_{p}. Let us describe $\bar{\mathbf{u}}_i^{\text{pr}}$ and $\bar{\mathbf{u}}_{ji}^{\text{pu}}$ as the sequence of inputs $\bar{u}_i^{\text{pr}}(\cdot)$ and $\bar{u}_{ji}^{\text{pu}}(\cdot)$ from $t = k$ to $k + N_{\text{p}} - 1$:

$$\begin{aligned} \bar{\mathbf{u}}_i^{\text{pr}} &= \left[\bar{u}_i^{\text{pr}}(0)^\top,\ \bar{u}_i^{\text{pr}}(1)^\top, \ldots,\ \bar{u}_i^{\text{pr}}(N_{\text{p}}-1)^\top\right]^\top, \\ \bar{\mathbf{u}}_{ji}^{\text{pu}} &= \left[\bar{u}_{ji}^{\text{pu}}(0)^\top,\ \bar{u}_{ji}^{\text{pu}}(1)^\top, \ldots,\ \bar{u}_{ji}^{\text{pu}}(N_{\text{p}}-1)^\top\right]^\top. \end{aligned} \tag{10}$$

Let $\boldsymbol{\alpha}_i$ group the value of the scale factors controlled by i:

$$\boldsymbol{\alpha}_i = \left[\alpha_i,\ \left[\alpha_{ji}\right]_{\forall j \in \mathcal{N}_i}\right]^\top. \tag{11}$$

Note that scale factors α_i and α_{ji} are recalculated every time instant k, but are kept constant over the prediction horizon. Furthermore, the first term of (9) corresponds to the stage cost $\ell_i(\cdot)$, described by the quadratic function:

$$\ell_i\left(\bar{x}_i(t),\ \bar{u}_i^{\text{pr}}(t),\ \left[\bar{u}_{ji}^{\text{pu}}(t)\right]_{j \in \mathcal{N}_i}\right) = \bar{x}_i^{+\top} Q_i\, \bar{x}_i^+ + \bar{u}_i^{\text{pr}\top} R_i^{\text{pr}}\, \bar{u}_i^{\text{pr}} + \sum_{j \in \mathcal{N}_i} \bar{u}_{ji}^{\text{pu}\top} R_i^{\text{pu}}\, \bar{u}_{ji}^{\text{pu}}. \tag{12}$$

Moreover, $f_i(\cdot)$ is introduced as a penalization for the scale factors α_i and α_{ji} for all $j \in \mathcal{N}_i$, i.e.,

$$f_i(\boldsymbol{\alpha}_i) = \rho^{\text{pr}} \alpha_i + \sum_{j \in \mathcal{N}_i} \rho^{\text{pu}} \alpha_{ji}, \tag{13}$$

being ρ^{pr} and ρ^{pu} positive weighting factors. The last term of $\mathbf{J}_{\mathbf{c}_i}$ represents the terminal cost function, with $P_i > 0$:

$$V_i^f\left(\bar{x}_i(k+N_{\text{p}})\right) = \bar{x}_i(k+N_{\text{p}})^\top P_i\, \bar{x}_i(k+N_{\text{p}}). \tag{14}$$

Considering (9), the centralized MPC problem for the nominal system at each time instant k is defined as:

$$\begin{aligned} \min_{\left[\bar{\mathbf{u}}_i^{\text{pr}},\ \left[\bar{\mathbf{u}}_{ji}^{\text{pu}}\right]_{\forall j \in \mathcal{N}_i},\ \boldsymbol{\alpha}_i\right]_{i \in \mathcal{N}}} & \sum_{i \in \mathcal{N}} \mathbf{J}_{\mathbf{c}_i}\left(\bar{x}_i,\ \bar{\mathbf{u}}_i^{\text{pr}},\ \left[\bar{\mathbf{u}}_{ji}^{\text{pu}}\right]_{\forall j \in \mathcal{N}_i},\ \boldsymbol{\alpha}_i\right) \\ \text{s.t.} \\ & \bar{x}_i(k) \in x_i \oplus (-\mathcal{R}_i), \\ & \bar{x}_i(t+1) = A_{ii} \bar{x}_i(t) + B_{ii} \bar{u}_i^{\text{pr}}(t) + \sum_{j \in \mathcal{N}_i} B_{ij} \bar{u}_{ji}^{\text{pu}}(t), \\ & \bar{u}_i^{\text{pr}}(t) \in \alpha_i \overline{\mathbb{U}}_i, \\ & \bar{u}_{ji}^{\text{pu}}(t) \in \alpha_{ji} \overline{\mathbb{U}}_j, \\ & \bar{x}_i(t) \in \overline{\mathbb{X}}_i, \\ & \bar{x}_i(N_{\text{p}}) \in \overline{\mathbb{X}}_i^f = \{0\}, \\ & \forall i \in \mathcal{N},\ \forall j \in \mathcal{N}_i, \\ & \forall t = k, \ldots, k+N_{\text{p}}-1. \end{aligned} \tag{15}$$

where $\overline{\mathbb{X}}_i^f$ represents the terminal region of the nominal model, which has been chosen as the origin. In this way, at the end of the prediction horizon, the state of the plant subject to uncertainties will stay in a neighborhood of the origin given by $\mathcal{R}_i$. Moreover, let $\bar{\mathbf{u}}_i^{\text{pr},*}$ and $\bar{\mathbf{u}}_{ji}^{\text{pu},*}$ be the optimal sequences of private and public inputs, which are solution for subsystem i to problem (15). Note that there will be $|\mathcal{N}_i|$ different sequences $\bar{\mathbf{u}}_{ji}^{\text{pu},*}$, one from each neighboring subsystem.

Since the MPC follows a receding horizon strategy, only the first component of the optimized sequences $\bar{\mathbf{u}}_i^{\text{pr},*}$ and $\bar{\mathbf{u}}_{ji}^{\text{pu},*}$ for all $j \in \mathcal{N}_i$ is applied at time instant k, yielding the nominal control law

$$\bar{\kappa}_i(\bar{x}_i) = \left[\bar{u}_i^{\text{pr},*}(0),\ \left[\bar{u}_{ji}^{\text{pu},*}(0)\right]_{\forall j \in \mathcal{N}_i}\right]^\top. \tag{16}$$

To deal with the mutual disturbances ignored by the nominal model, an auxiliary control law is used so that the trajectory of

the real system follows closely the nominal one, trying to cancel the error between the nominal and real states, i.e., $\bar{x}_i$ and x_i. Consequently, the private and public inputs of each subsystem i are calculated as:

$$\begin{bmatrix} u_i^{\mathrm{pr}} \\ \left[u_{ji}^{\mathrm{pu}}\right]_{\forall j\in\mathcal{N}_i} \end{bmatrix} = \bar{\kappa}_i(\bar{x}_i) + K_i\,(x_i - \bar{x}_i)\,. \tag{17}$$

Remark 4. For all $i \in \mathcal{N}$, the implemented input of subsystem i, expressed according to (1), is the sum of the private and public parts of the manipulated variable u_i defined in (3), whereas (17) is the vector of input variables computed by agent i. In this regard, note that the private part of u_i is computed by agent i, but the public part is decided by agents in $\mathcal{M}_i$.

Solving (15) provides for each subsystem $i \in \mathcal{N}$ the value of the private and public inputs used to obtain the real implemented input (17), i.e., $\bar{u}_i^{\mathrm{pr},*}(0)$ and $\bar{u}_{ji}^{\mathrm{pu},*}(0)$, and the corresponding optimal scale factors $\boldsymbol{\alpha}_i^*$, i.e., $\alpha_i = \alpha_i^*$ and $\alpha_{ji} = \alpha_{ji}^*$, for all neighboring agents $j \in \mathcal{N}_i$. In what follows, we will describe how the centralized problem is solved in a distributed manner among the set of agents by using dual decomposition. Consequently, (15) is not intended to be solved directly.

3.2. Distributed MPC based on dual decomposition

Let us consider the dual decomposition algorithm described in [23], which allows us to compute the solution of (15) in a distributed fashion. In this context, convergence is attained throughout an iterative *negotiation* procedure where Lagrange multipliers λ_i are used to coordinate coupling variables. Consider the problem in (15) and note that if the dynamics of different agents is affected by a shared input u_i, they will need to negotiate the value of the scale factors that define the input constraints, i.e., α_i, $\left[\alpha_{ij}\right]_{j\in\mathcal{N}_i}$. Moreover, the solution needs to satisfy condition (4). The latter will be enforced by the introduction of Lagrange multipliers, which become new parameters of the local objective functions, and thus influence the agents' solutions. These multipliers are assumed to remain constant during the prediction horizon, however, note that they may vary at each time instant k and throughout the dual decomposition iterations. In particular, to comply with constraints (3) and (4), agents that carry out the negotiation are required to satisfy:

$$\lambda_i \left(\alpha_i + \sum_{j\in\mathcal{M}_i} \alpha_{ij} - 1\right) \leq 0 \quad \text{with} \quad \lambda_i \geq 0. \tag{18}$$

Let $\mathcal{S}_i$ be the set containing subsystem i and its neighboring agents, i.e., $\mathcal{S}_i \triangleq \{\{i\} \cup \mathcal{N}_i\}$. Accordingly, there will be $|\mathcal{S}_i|$ Lagrange multipliers in the local objective function of subsystem i. In other words, there will be as many Lagrange multipliers as private and public input local variables. These auxiliary variables are introduced in the local objective function of each subsystem $i \in \mathcal{N}$ as

$$\Lambda_i\left(\boldsymbol{\alpha}_i, [\lambda_m]_{m\in\mathcal{S}_i}\right) = \lambda_i\alpha_i + \sum_{j\in\mathcal{N}_i} \lambda_j\alpha_{ji}. \tag{19}$$

Remark 5. Note that (18) needs to be fulfilled for every agent $i \in \mathcal{N}$. Consequently, (19) is the result of keeping all the terms of the $|\mathcal{N}|$ expressions of (18) that multiply the variables controlled by agent i.

As a result, to distribute the global problem (15), we can rewrite (9) taking into account the agents' negotiation. The local objective function is formulated as the sum of the stage cost $\ell_i(\cdot)$ (12), the penalization of scale factors $f_i(\cdot)$ (13), the term introducing the Lagrange multipliers for the dual decomposition algorithm $\Lambda_i(\cdot)$ (19), and the terminal cost $V_i^f(\cdot)$ (14):

$$\begin{aligned} &\mathbf{J}_i\left(\bar{x}_i, \bar{\mathbf{u}}_i^{\mathrm{pr}}, \left[\bar{\mathbf{u}}_{ji}^{\mathrm{pu}}\right]_{\forall j\in\mathcal{N}_i}, \boldsymbol{\alpha}_i, \left[\lambda_m^p\right]_{m\in\mathcal{S}_i}\right) = \\ &\sum_{t=k}^{k+N_{\mathrm{p}}-1} \ell_i\left(\bar{x}_i(t), \bar{u}_i^{\mathrm{pr}}(t), \left[\bar{u}_{ji}^{\mathrm{pu}}(t)\right]_{j\in\mathcal{N}_i}\right) \\ &+ f_i\left(\boldsymbol{\alpha}_i\right) + \Lambda_i\left(\boldsymbol{\alpha}_i, \left[\lambda_m^p\right]_{m\in\mathcal{S}_i}\right) + V_i^f\left(\bar{x}_i(k+N_{\mathrm{p}})\right). \end{aligned} \tag{20}$$

Consequently, at each time instant k and iteration step p, each agent $i \in \mathcal{N}$ solves:

$$\begin{aligned} \min_{\left[\bar{\mathbf{u}}_i^{\mathrm{pr}}, \left[\bar{\mathbf{u}}_{ji}^{\mathrm{pu}}\right]_{\forall j\in\mathcal{N}_i}, \boldsymbol{\alpha}_i\right]_{i\in\mathcal{N}}} \quad & \mathbf{J}_i\left(\bar{x}_i, \bar{\mathbf{u}}_i^{\mathrm{pr}}, \left[\bar{\mathbf{u}}_{ji}^{\mathrm{pu}}\right]_{\forall j\in\mathcal{N}_i}, \boldsymbol{\alpha}_i, \left[\lambda_m^p\right]_{m\in\mathcal{S}_i}\right) \\ \text{s.t.} \quad & \\ & \bar{x}_i(k) \in x_i \oplus (-\mathcal{R}_i), \\ & \bar{x}_i(t+1) = A_{ii}\bar{x}_i(t) + B_{ii}\bar{u}_i^{\mathrm{pr}}(t) + \sum_{j\in\mathcal{N}_i} B_{ij}\bar{u}_{ji}^{\mathrm{pu}}(t), \\ & \bar{u}_i^{\mathrm{pr}}(t) \in \alpha_i^p\overline{\mathbb{U}}_i, \;\; \bar{u}_{ji}^{\mathrm{pu}}(t) \in \alpha_{ji}^p\overline{\mathbb{U}}_j, \;\; \forall j \in \mathcal{N}_i, \\ & \bar{x}_i(t) \in \overline{\mathbb{X}}_i, \\ & \bar{x}_i(N_{\mathrm{p}}) \in \overline{\mathbb{X}}_i^f = \{0\}, \\ & \lambda_m^p \geq 0, \;\; \forall m \in \mathcal{S}_i, \\ & \forall t = k, \ldots, k+N_{\mathrm{p}}-1. \end{aligned} \tag{21}$$

where $[\lambda_m^p]_{m\in\mathcal{S}_i}$ are introduced as parameters.

The iterative negotiation takes the values of the variables involved in (19) obtained at any iteration p and compares them with those resulting from the previous iteration $p-1$. Practical convergence is attained when $\Delta \triangleq |\alpha^p - \alpha^{p-1}|$ is below a threshold $\epsilon > 0$. In this way, while $\Delta > \epsilon$ agents will negotiate the bounds on shared variables. On the other hand, when $\Delta \leq \epsilon$, it is considered that consecutive values are similar enough to finish the negotiation for time step k. Meanwhile, Lagrange multipliers are updated in each step of the negotiation according to:

$$\lambda_i^{p+1} = \lambda_i^p + \gamma\left(\alpha_i^p + \sum_{j\in\mathcal{M}_i} \alpha_{ij}^p - 1\right), \tag{22}$$

where $\gamma > 0$ is the step size. Finally, the scale factors α_i and α_{ij} involved in (18) come from the solution to (21) at time instant k and iteration step p.

4. Control scheme

The proposed coalitional control approach aims to reduce communication among agents and is summarized in Algorithm 1. For convenience, in what follows we use superscript $^-$ to refer to a variable at the previous time instant, e.g., x^- denotes x at time instant $k-1$.

We consider event-based coordination, meaning that communication links will be enabled or disabled according to the condition introduced in Step 9 of Algorithm 1. The communication link between two agents i and j will be disabled if the public variable u_{ji}^{pu} is small enough during two consecutive time steps. It is considered that there is no need for coordination depending on the value of α_{ji} because it scales the constraint space of the public variable. When α_{ji} takes small values, the region of the public variable is negligible compared to the values that the private variable can take. Nevertheless, communication will be restored if at a single time step k agent i requires greater use of the public variable u_{ji}^{pu}, being the value of its scale factor α_{ji} above the threshold $\hat{\alpha}$.

Agents that are communicating follow an iterative procedure under the distributed dual decomposition algorithm. They share the values of the scale factors α_i and α_{ji} until the negotiation converges, updating the value of the Lagrange multipliers at every iteration step p. It is worth mentioning that, unlike common distributed approaches, agents share the variables that bound input constraints and not input variables directly. In addition, agents will keep calculating the value of their public variables even when they are not communicating since the negotiation can be suddenly resumed.

Algorithm 1 Control Scheme

Initialization: At the first time instant $k = 0$ and iteration $p = 0$, the values of the scaling factors are set to a positive non-zero value: $\alpha_i^{p=0}(0) = \alpha_{\text{aux}}$, $\alpha_{ji}^{p=0}(0) = \alpha_{\text{aux}}$ $\forall i, j \in \mathcal{N}$, with $\alpha_{\text{aux}} > 0$. If $k > 0$, we will set the value of the scale factor at $p = 0$ as the one in the previous time instant, i.e., $\alpha_i^{p=0}(k) = \alpha_i(k-1)$, $\alpha_{ji}^{p=0}(k) = \alpha_{ji}(k-1)$ $\forall i, j \in \mathcal{N}$. Moreover, set $[\lambda_m]_{m\in\mathcal{S}_i} = 0$ at $p = 0$ $\forall k$.

At each sample time k, each agent $i \in \mathcal{N}$ proceeds as follows:

1: **while** $\Delta > \epsilon$ **do**
2: Update the disturbance set $\mathbb{W}_i^p$ (7) according to the scale values from the previous iteration, i.e., $\left[\alpha_{ij}^{p-1}\right]_{\forall j\in\mathcal{M}_i}$ and $\left[\alpha_j^{p-1}\right]_{\forall j\in\mathcal{N}_i}$.
3: Compute the corresponding invariant set $\mathcal{R}_i^p$.
4: **if** $x_i - \bar{x}_i \notin \mathcal{R}_i^p$ **then**
5: Set $\mathcal{R}_i^p = \mathcal{R}_i^-$.
6: **end if**
7: Solve (21) to obtain the optimal nominal sequences $\bar{\mathbf{u}}_i^{\text{pr},*}$, $\bar{\mathbf{u}}_{ji}^{\text{pu},*}$ and the optimal scale factors $\boldsymbol{\alpha}_i^*$.
8: Calculate the real inputs u_i^{pr} and u_{ji}^{pu} $\forall j \in \mathcal{N}_i$ according to equation (17).
9: **if** $\alpha_{ji}^p < \hat{\alpha}$ and $\alpha_{ji}^- < \hat{\alpha}$ **then**
10: Disable the communication between agents j and i.
11: Ignore the value of the public variable: $u_{ji}^{\text{pu}} = 0$.
12: The associated Lagrange multiplier to α_{ji} will remain constant in the next iteration: $\lambda_j^{p+1} = \lambda_j^p$.
13: Set $\Delta_j = 0$.
14: **else**
15: Enable the communication between agents j and i.
16: Update the neighboring Lagrange multiplier $\lambda_j^{p:+1}$ according to (22).
17: Compute $\Delta_j = \max\{|\alpha_j^p - \alpha_j^{p-1}|, |\alpha_{ji}^p - \alpha_{ji}^{p-1}|\}$.
18: **end if**
19: Set $p \leftarrow p + 1$ and compute $\Delta = \max\{\Delta_j\}_{\forall j\in\mathcal{N}_i}$.
20: **end while**

The set $\mathcal{R}_i^p$ has to be recalculated for each iteration p and for every time instant k. Note that, in the literature, there are methods such as [24,25] that allow us to perform this calculation. In particular, the invariant sets for Algorithm 1 are designed according to [26], where a one-step procedure is applied to compute a polytopic minimal robust positively invariant (mRPI) set. In particular, this method solves a single LP to compute the so-called (P, r)-mRPI, where r denotes the number of inequalities defining the set and P is a predefined matrix. Therefore, the update of $\mathcal{R}_i^p$ entails a light computational load.

4.1. Verify whether $x_i - \bar{x}_i \in \mathcal{R}_i^p$

Since input, state, and uncertainty constraint sets vary over time, the RPI set must be recomputed at every instant k to suit the new constraints. Following [7], a *checking step* is introduced to verify if the difference between the predicted and nominal states in $k+1$ belongs to the new RPI calculated at that time instant in order to maintain constraint satisfaction and recursive feasibility.

According to [22], if the state belongs to the set $\bar{x}_i \oplus \mathcal{R}_i$ for a certain time instant k, we can guarantee that for the successive time instant $k+1$ the state will remain within the set $\bar{x}_i^+ \oplus \mathcal{R}_i$. At the beginning of the algorithm, $k = 0$, $\mathcal{R}_i^0$ is calculated, and we guarantee that $x_i^0 - \bar{x}_i^0 \in \mathcal{R}_i^0$, since the initial nominal state is optimized according to $\bar{x}_i(0) \in x_i \oplus (-\mathcal{R}_i)$. The problem arises when we have to recalculate for each time step k the RPI set because the constraints that affect $\mathcal{R}_i$ may have changed with respect to the previous instant. According to $x_i \in \bar{x}_i \oplus \mathcal{R}_i$, at time instant k, successor states x_i^+ and $\bar{x}_i^+$ are calculated using the invariant set $\mathcal{R}_i$. As condition $x_i^+ \in \bar{x}_i^+ \oplus \mathcal{R}_i$ holds, we have to guarantee that the new RPI calculated at $k+1$ contains the difference between the predicted real and nominal current states:

$$x_i^+ - \bar{x}_i^+ \in \mathcal{R}_i^+. \tag{23}$$

To pursue this goal, Step 3 of Algorithm 1 is introduced with the same procedure as the checking step introduced in [7] (Subsection 4.1). If (23) is met, the following is satisfied:

$$A_{K_i}\left(x_i^+ - \bar{x}_i^+\right) \oplus \mathbb{W}_i^+ \subseteq \mathcal{R}_i^+. \tag{24}$$

Consequently, the predicted state and input trajectory will verify the original constraints $\mathbb{X}_i$ and $\mathbb{U}_i$. Conversely, if $x_i^+ - \bar{x}_i^+ \notin \mathcal{R}_i^+$ we will discard the new RPI set and use the previous one, i.e., $\mathcal{R}_i := \mathcal{R}_i^-$, since $x_i^+ \in \bar{x}_i^+ \oplus \mathcal{R}_i$ is always fulfilled.

5. Recursive feasibility and stability

In what follows, we proceed as in [7] to develop the conditions that ensure recursive feasibility and stability since the applied schemes are somewhat similar except for the novel distribution of inputs among agents, and the exchanges of communication that are necessary to adjust the bounds on shared variables.

5.1. Decreasing trend of the disturbance set $\mathbb{W}$

Hereafter, we will introduce several assumptions that allow us to preserve the guarantees of recursive feasibility and stability when using the previous RPI if the new set does not satisfy condition (23).

Assumption 2. The set of disturbances are non-increasing from one time instant k to another $k+1$, i.e., $\mathbb{W}_i^+ \subseteq \mathbb{W}_i$.

Remark 6. Since we are working with a regulation control problem, the state will approach the origin as time goes by, and hence the inputs will also tend to zero. In this regard, any non-increasing evolution of factors α_j ($j \in \mathcal{N}_i$) and α_{ij} ($j \in \mathcal{M}_i$) will lead to coupling disturbances sets satisfying Assumption 2 (recall (6) and (7)).

In this way, the bounds of the uncertainties evolve into smaller sets over time. This implies that if the disturbance set changes in two consecutive instants, the successor set will be a subset of the previous one. As the size of the RPI set depends on the disturbance set, a smaller disturbance set entails a smaller RPI set, leading to the following assumption:

Assumption 3. The invariant sets are non-increasing from one time instant k to another $k+1$, i.e., $\mathcal{R}_i^+ \subseteq \mathcal{R}_i$.

Let us consider that at time instant k, subsystem i satisfies: $A_{K_i}\mathcal{R}_i \oplus \mathbb{W}_i \subseteq \mathcal{R}_i$. Considering the above, for the successor instant, it holds that $\mathbb{W}_i^+ \subseteq \mathbb{W}_i$, and $\mathcal{R}_i^+ \subseteq \mathcal{R}_i$. Hence, for $k+1$ is satisfied: $A_{K_i}\mathcal{R}_i^+ \oplus \mathbb{W}_i^+ \subseteq \mathcal{R}_i^+$. Let us define $\mathbf{W}_i$ as any uncertainty set contained in $\mathbb{W}_i$, i.e., $\mathbf{W}_i \subseteq \mathbb{W}_i$. As disturbances decrease over time, $A_{K_i}\mathcal{R}_i \oplus \mathbf{W}_i \subseteq \mathcal{R}_i$ is fulfilled for any uncertainty set contained in $\mathbb{W}_i$. A fail-safe option for $k+1$ if $x_i^+ - \bar{x}_i^+ \notin \mathcal{R}_i^+$, is to replace the new RPI with the preceding one, i.e., set $\mathcal{R}_i^+ = \mathcal{R}_i$. This is why Step 4 of the control algorithm is proposed as an alternative when the necessary conditions are not met, as we are being more conservative with a larger RPI set that satisfies the problem's constraints.

5.2. Conditions for recursive feasibility

Theorem 1 (*Recursive Feasibility*). *Consider that Assumptions 2 and 3 hold. Then, if at the initial time instant $k = 0$ it is possible to find a feasible solution of problem (21) for all agents $i \in \mathcal{N}$, it is guaranteed that it exists a feasible solution for any instant $k \in \{0, 1, 2, \dots\}$.* □

Proof. Let $\bar{\mathbf{x}}_i^* = \left[\bar{x}_i^*(0)^\top,\ \bar{x}_i^*(1)^\top, \dots,\ \bar{x}_i^*(N_\mathrm{p})^\top\right]^\top$ and

$$\bar{\mathbf{u}}_i^{\mathrm{pr},*}(k) = \begin{bmatrix} \bar{u}_i^{\mathrm{pr},*}(0) \\ \bar{u}_i^{\mathrm{pr},*}(1) \\ \vdots \\ \bar{u}_i^{\mathrm{pr},*}(N_\mathrm{p}-1) \end{bmatrix}, \bar{\mathbf{u}}_{ji}^{\mathrm{pu},*} = \begin{bmatrix} \bar{u}_{ji}^{\mathrm{pu},*}(0) \\ \bar{u}_{ji}^{\mathrm{pu},*}(1) \\ \vdots \\ \bar{u}_{ji}^{\mathrm{pu},*}(N_\mathrm{p}-1) \end{bmatrix} \forall j \in \mathcal{N}_i$$

be the optimal state and input sequences derived from the solution of (21) at instant k. Additionally, note that Assumptions 2 and 3 imply that $\mathbb{W}_i^+ \subseteq \mathbb{W}_i$ and $\mathcal{R}_i^+ \subseteq \mathcal{R}_i$, with $\mathbb{W}_i^+$ and $\mathcal{R}_i^+$ being respectively the disturbances and RPI sets at instant $k+1$, and $\mathbb{W}_i$ and $\mathcal{R}_i$ those at k. Finally, let us build up the following candidate solution for instant $k+1$:

$$\tilde{\mathbf{x}}_i^+ = \begin{bmatrix} \tilde{x}_i^+(0) \\ \tilde{x}_i^+(1) \\ \vdots \\ \tilde{x}_i^+(N_\mathrm{p}-1) \\ \tilde{x}_i^+(N_\mathrm{p}) \end{bmatrix} = \begin{bmatrix} \bar{x}_i^*(1) \\ \bar{x}_i^*(2) \\ \vdots \\ \bar{x}_i^*(N_\mathrm{p}) \\ 0 \end{bmatrix}, \tag{25}$$

$$\tilde{\mathbf{u}}_i^{\mathrm{pr},+} = \begin{bmatrix} \bar{u}_i^{\mathrm{pr},*}(1) \\ \bar{u}_i^{\mathrm{pr},*}(2) \\ \vdots \\ \bar{u}_i^{\mathrm{pr},*}(N_\mathrm{p}-1) \\ 0 \end{bmatrix}, \tilde{\mathbf{u}}_{ji}^{\mathrm{pu},+} = \begin{bmatrix} \bar{u}_{ji}^{\mathrm{pu},*}(1) \\ \bar{u}_{ji}^{\mathrm{pu},*}(2) \\ \vdots \\ \bar{u}_{ji}^{\mathrm{pu},*}(N_\mathrm{p}-1) \\ 0 \end{bmatrix} \forall j \in \mathcal{N}_i. \tag{26}$$

From the constraints of (21), we have that $x_i - \bar{x}_i^*(0) \in \mathcal{R}_i$ and $\bar{x}_i^*(N_\mathrm{p}) = \{0\}$. Additionally, if we apply control law (17), it holds that $x_i^+ - \bar{x}_i^*(1) \in \mathcal{R}_i$. However, as the RPI is recalculated at each time step, condition $x_i^+ - \bar{x}_i^{*+}(1) \in \mathcal{R}_i^+$ must be satisfied at $k+1$. Since $\tilde{x}_i^+(0) = \bar{x}_i^*(1)$, then

$$x_i^+ - \tilde{x}_i^+(0) \in \mathcal{R}_i.$$

Moreover, according to the checking step described in Section 4.1, if $x_i^+ - \tilde{x}_i^+(0) \notin \mathcal{R}_i^+$, then we set $\mathcal{R}_i^+ := \mathcal{R}_i$ as a fail-safe option, i.e., $\bar{x}_i^*(1)$ will always satisfy the constraint on the initial nominal state of problem (21).

Additionally, given that $\mathcal{R}_i^+ \subseteq \mathcal{R}_i$, nominal constraints sets $\overline{\mathbb{U}}_i$ and $\overline{\mathbb{X}}_i$ can only enlarge in time, i.e., $\overline{\mathbb{U}}_i^+ \supseteq \overline{\mathbb{U}}_i$, $\overline{\mathbb{X}}_i^+ \supseteq \overline{\mathbb{X}}_i$. Therefore, states $\bar{x}_i^*(n)$ and inputs $\bar{u}_i^{\mathrm{pr},*}(n)$, $\bar{u}_{ji}^{\mathrm{pu},*}(n)$ $\forall j \in \mathcal{N}_i$ in (25) and (26) are admissible for all $n = 1, \dots, N_\mathrm{p}$. Finally, since $\bar{x}_i^*(N_\mathrm{p}) = \{0\}$, we can always apply zero as the terminal input to stay at the origin. Therefore, (25) and (26) constitutes a feasible solution of problem (21) for time step $k+1$. By induction, the theorem is proven. ■

5.3. Conditions for stability

Theorem 2 (*Stability*). *For all agents $i \in \mathcal{N}$, the origin is asymptotically stable.* □

Proof. To establish stability, we have to prove that the nominal cost function $\mathbf{J}_i$ (20) decreases during the evolution of the system. Let the optimal sequences in 1 $\bar{\mathbf{x}}_i^*$, $\bar{\mathbf{u}}_i^{\mathrm{pr},*}$ and $\bar{\mathbf{u}}_{ji}^{\mathrm{pu},*}$ at k lead to the optimal cost $\mathbf{J}_i^*$. Additionally, the candidate state and input sequence for $k+1$, i.e., $\tilde{\mathbf{x}}_i^+$, $\tilde{\mathbf{u}}_i^{\mathrm{pr},+}$ and $\tilde{\mathbf{u}}_{ji}^{\mathrm{pu},+}$, lead to the cost $\tilde{\mathbf{J}}_i^+$, which provides a value for the upper bound of the optimal cost: $\tilde{\mathbf{J}}_i^+ \geq \mathbf{J}_i^{*+}$. Moreover, we can calculate the difference $\tilde{\mathbf{J}}_i^+ - \mathbf{J}_i^*$ as:

$$\begin{aligned} &\ell_i\left(\tilde{x}_i^+(N_\mathrm{p}-1), \tilde{u}_i^{\mathrm{pr},+}(N_\mathrm{p}-1), \left[\tilde{u}_{ji}^{\mathrm{pu},+}(N_\mathrm{p}-1)\right]_{\forall j\in\mathcal{N}_i}\right) \\ &- \ell_i\left(\bar{x}_i^*(0), \bar{u}_i^{\mathrm{pr},*}(0), \left[\bar{u}_{ji}^{\mathrm{pu},*}(0)\right]_{\forall j\in\mathcal{N}_i}\right) \\ &+ \tilde{f}_i^+(\boldsymbol{\alpha}_i) - f_i^*(\boldsymbol{\alpha}_i) \\ &+ \tilde{\Lambda}_i^+\left(\boldsymbol{\alpha}_i, [\lambda_m]_{m\in\mathcal{S}}\right) - \Lambda_i^*\left(\boldsymbol{\alpha}_i, [\lambda_m]_{m\in\mathcal{S}}\right) \\ &+ V_i^f\left(\tilde{x}_i^+(N_\mathrm{p})\right) - V_i^f\left(\bar{x}_i^*(N_\mathrm{p})\right). \end{aligned} \tag{27}$$

Unlike private and public inputs $\bar{u}_i^{\mathrm{pr}}$ and $\bar{u}_{ji}^{\mathrm{pu}}$, optimal scale factors in $\boldsymbol{\alpha}_i^*$ remain constant along the prediction horizon. Moreover, the values of the Lagrange multipliers for the candidate solution do not vary along N_p, so $\tilde{\lambda}_m^+ = \lambda_m^*$. We have assumed that the values of optimization variables that remain constant over the prediction horizon are also kept constant from the optimal solution in k to the candidate solution in $k+1$. Therefore, $\tilde{f}_i^+ = f_i^*$ and $\tilde{\Lambda}_i^+ = \Lambda_i^*$, so they cancel themselves in (27). As a result, in the subtraction $\tilde{\mathbf{J}}_i^+ - \mathbf{J}_i^*$ there are only left terms referring to the stage and terminal costs. Furthermore, the terminal cost is a continuous Lyapunov function at the origin, meaning that $V_i^f(k+1) - V_i^f(k) \leq 0$, which entails:

$$\begin{aligned} &\ell_i\left(\tilde{x}_i^+(N_\mathrm{p}-1), \tilde{u}_i^{\mathrm{pr},+}(N_\mathrm{p}-1), \left[\tilde{u}_{ji}^{\mathrm{pu},+}(N_\mathrm{p}-1)\right]_{\forall j\in\mathcal{N}_i}\right) \\ &+ V_i^f\left(\tilde{x}_i^+(N_\mathrm{p})\right) - V_i^f\left(\bar{x}_i^*(N_\mathrm{p})\right). \end{aligned} \tag{28}$$

Therefore, we can rewrite (27) as:

$$\tilde{\mathbf{J}}_i^+ - \mathbf{J}_i^* \leq -\ell_i\left(\bar{x}_i^*(0), \bar{u}_i^{\mathrm{pr},*}(0), \left[\bar{u}_{ji}^{\mathrm{pu},*}(0)\right]_{\forall j\in\mathcal{N}_i}\right), \tag{29}$$

which also implies

$$\mathbf{J}_i^{*+} - \mathbf{J}_i^* \leq -\ell_i\left(\bar{x}_i^*(0), \bar{u}_i^{\mathrm{pr},*}(0), \left[\bar{u}_{ji}^{\mathrm{pu},*}(0)\right]_{\forall j\in\mathcal{N}_i}\right). \tag{30}$$

We can extend the (30) for all subsystems $i \in \mathcal{N}$:

$$\mathbf{J} = \sum_{\forall i\in\mathcal{N}} \mathbf{J}_i \rightarrow \mathbf{J}^{*+} - \mathbf{J}^* \leq -\ell\left(\bar{x}(0), \bar{u}(0)\right). \tag{31}$$

As the stage cost $\ell(\cdot)$ is strictly positive, it is proven that the cost function $\mathbf{J}_i$ defined in (20) decreases over time, ensuring the stability of the system. ■

6. Simulation results

In this section, we apply the proposed coalitional control algorithm to the academic example shown in Fig. 2. The eight input-coupled tanks plant consists of four top tanks (5, 6, 7, 8) that discharge flow into four bottom tanks (1, 2, 3, 4), and these, in turn, discharge into a shared storage tank. Four pumps (Q_a, Q_b, Q_c, Q_d) are used to fill the tanks, carrying water from the storage tank to the tanks indicated in Fig. 2. As can be seen, flow regulation is done through three-way valves, which divide the pumped flow into two ways to fill the tanks.

The global system can be divided into $N = 4$ subsystems that consist of a top and bottom tank. Thus, the first subsystem

is formed by tanks #1 and #5; tanks #2 and #6 describe the second one; the third subsystem is composed of tanks #3 and #7; and the fourth one is formed by tanks #4 and #8. As shown in Fig. 4, subsystems are physically coupled through the colored pipes that connect their tanks. On the other hand, we refer to the data connections between their corresponding local controllers as communication *links*. In particular, link (i, j) represents the bidirectional data connection between agents i and j. Whenever a given link is enabled, the agents connected through it will form a coalition and will be able to share information. In this regard, we consider the following links: (4, 1), (2, 3), (1, 2), and (3, 4).

The target is to regulate the lower tanks towards their operating point in terms of water level, i.e., h_i°. To this end, the state of each subsystem is defined as the water level measured from the operating point. Taking the fourth subsystem as an example, its state is given by: $x_4 = \left[h_4 - h_4^\circ,\ h_8 - h_8^\circ\right]^T$. Additionally, the inputs are given by the difference of the pump flow and its value at the operating point $u_i = Q_k - Q_k^\circ$, with $i = 1, \dots, N$ and $k \in \{a, b, c, d\}$. Let us describe the operating point of the plant as:

$$\begin{aligned} &h_1^\circ = 0.6487,\ h_2^\circ = 0.6639,\ h_3^\circ = 0.6534,\ h_4^\circ = 0.6521,\\ &h_5^\circ = 0.6498,\ h_6^\circ = 0.6592,\ h_7^\circ = 0.6594,\ h_8^\circ = 0.6587,\\ &Q_a^\circ = 1.63,\ Q_b^\circ = 2,\ Q_c^\circ = 1.8,\ Q_d^\circ = 2, \end{aligned} \tag{32}$$

where water level is measured in meters and flows in cubic meters per hour. Moreover, we can characterize each subsystem with the following matrices:

$$A_{11} = \begin{bmatrix} 0.8257 & 0.1178 \\ 0 & 0.8703 \end{bmatrix}, B_{11} = \begin{bmatrix} 0.0379 \\ 0 \end{bmatrix}, B_{14} = \begin{bmatrix} 0.0056 \\ 0.0843 \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} 0.8163 & 0.1023 \\ 0 & 0.8867 \end{bmatrix}, B_{22} = \begin{bmatrix} 0.0503 \\ 0 \end{bmatrix}, B_{21} = \begin{bmatrix} 0.0053 \\ 0.0916 \end{bmatrix},$$

$$A_{33} = \begin{bmatrix} 0.8232 & 0.1077 \\ 0 & 0.8813 \end{bmatrix}, B_{33} = \begin{bmatrix} 0.0442 \\ 0 \end{bmatrix}, B_{32} = \begin{bmatrix} 0.0047 \\ 0.0783 \end{bmatrix},$$

$$A_{44} = \begin{bmatrix} 0.8194 & 0.1050 \\ 0 & 0.8840 \end{bmatrix}, B_{44} = \begin{bmatrix} 0.0441 \\ 0 \end{bmatrix}, B_{43} = \begin{bmatrix} 0.0050 \\ 0.0849 \end{bmatrix}.$$

Accordingly, the disturbance vectors (recall (7)) for the simulation example are such that:

$$\begin{aligned} &w_1 \in \mathbb{W}_1 \triangleq B_{11}\mathbb{W}_{12}^{\text{pu}} \oplus B_{14}\mathbb{W}_4^{\text{pr}},\\ &\text{with } \mathbb{W}_{12}^{\text{pu}} = \alpha_{12}\mathbb{U}_1, \text{ and } \mathbb{W}_4^{\text{pr}} = \alpha_4\mathbb{U}_4,\\ &w_2 \in \mathbb{W}_2 \triangleq B_{22}\mathbb{W}_{23}^{\text{pu}} \oplus B_{21}\mathbb{W}_1^{\text{pr}},\\ &\text{with } \mathbb{W}_{23}^{\text{pu}} = \alpha_{23}\mathbb{U}_2, \text{ and } \mathbb{W}_1^{\text{pr}} = \alpha_1\mathbb{U}_1,\\ &w_3 \in \mathbb{W}_3 \triangleq B_{33}\mathbb{W}_{34}^{\text{pu}} \oplus B_{32}\mathbb{W}_2^{\text{pr}},\\ &\text{with } \mathbb{W}_{34}^{\text{pu}} = \alpha_{34}\mathbb{U}_3, \text{ and } \mathbb{W}_2^{\text{pr}} = \alpha_2\mathbb{U}_2,\\ &w_4 \in \mathbb{W}_4 \triangleq B_{44}\mathbb{W}_{14}^{\text{pu}} \oplus B_{43}\mathbb{W}_3^{\text{pr}},\\ &\text{with } \mathbb{W}_{14}^{\text{pu}} = \alpha_{14}\mathbb{U}_4, \text{ and } \mathbb{W}_3^{\text{pr}} = \alpha_3\mathbb{U}_3. \end{aligned}$$

Lastly, the system is subject to the following constraints: $0.2 \leq h_n \leq 1.3,\ \forall n = 1, \dots, 8$, $0 \leq Q_a \leq 3.26$, $0 \leq Q_b \leq 4$, $0 \leq Q_c \leq 3.6$, and $0 \leq Q_d \leq 4$. Note that these conditions must be adapted to the state and input constraints by subtracting the operating point (32) from the previous limit values. The simulation has been done using as weighting matrices: $Q_i = \mathbf{I}_2$, $R_i^{\text{pr}} = 0.20$, $R_i^{\text{pu}} = 2R_i^{\text{pr}}$, and $\rho^{\text{pr}} = R^{\text{pr}}$, $\rho^{\text{pu}} = R^{\text{pu}}$ penalize scale factors in the objective function (13), calculated by trial and error. The prediction horizon has been set to $N_{\text{p}} = 10$, and the simulation lasts 25 s. Finally, the threshold $\hat{\alpha}$ is a tunning parameter, and its value has been set as $\hat{\alpha} = 0.015$ by trial and error.

In what follows, the proposed coalitional method is compared with a decentralized MPC, where agents optimize their local control objectives regardless of the control goals of the rest of the system, and with a centralized MPC, representing a fully cooperative scheme. Note that the centralized MPC solution can also be found in a distributed fashion, e.g., using dual decomposition DMPC after convergence is attained. To that end, the agents need to share information regarding their input sequences at every time instant, whereas communication in the proposed coalitional method is less demanding and event-based (recall Algorithm 1).

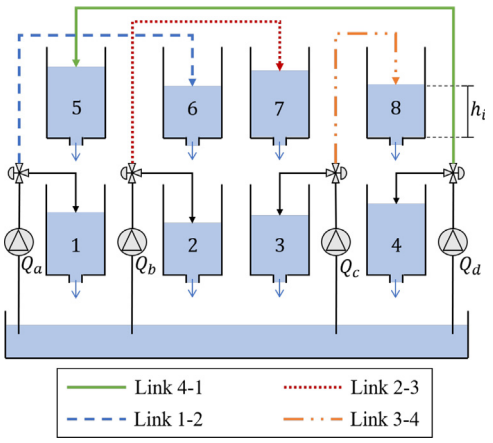


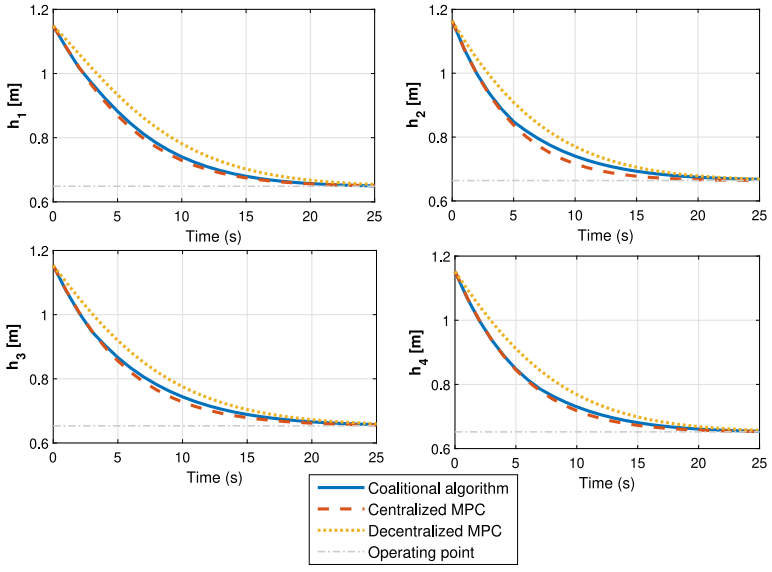
Fig. 2. Diagram of the eight tanks plant, where the coupling relations are represented by the colored pipes.

Fig. 3(a) represents the evolution of the water level of lower tanks towards their equilibrium point. As they all start with a water level that is above their operating point, they must be slightly emptied and, therefore, the flow rate of the four pumps must decrease. This is shown in Fig. 3(b), which illustrates the performance of Algorithm 1 in terms of the evolution of flow rates. As can be seen, when using the coalitional scheme with $\hat{\alpha} = 0.015$, water level and pump flow trajectories stay close to the centralized solution. This fact highlights the benefits provided by the proposed method, as a very similar behavior to the centralized one is achieved without permanent communication between the agents. On the other hand, the water level evolution towards their operating point h_i° is slower with the decentralized MPC due to the lack of coordination, which makes the agents act independently of each other's control objectives. As a consequence, the pump operation differs substantially from that implemented with the centralized and coalitional approaches.

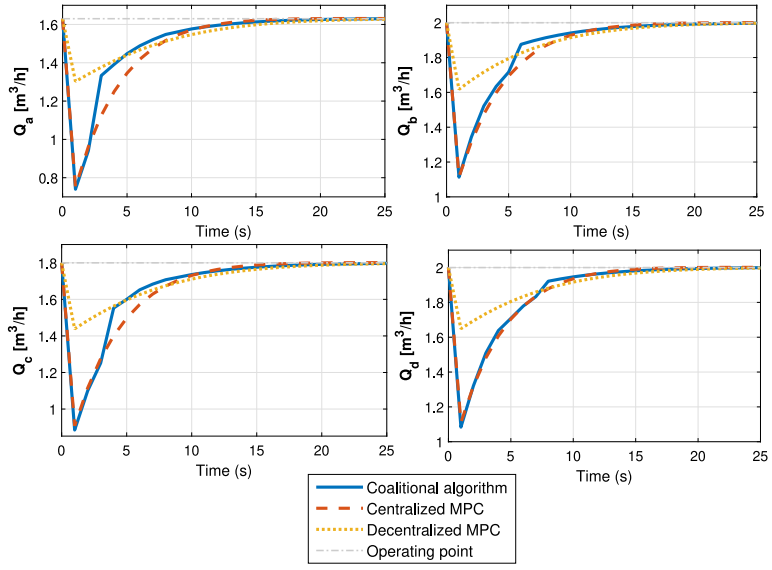
Let us define $\mathbf{P}$ as the index that evaluates the system's performance:

$$\mathbf{P} = \sum_{k=1}^{T} \sum_{i \in \mathcal{N}} u_i^\top(k) R_i^{\text{pr}} u_i(k) + x_i^\top(k+1) Q_i x_i(k+1), \tag{33}$$

being T the simulation length. Note that the definition of the performance index allows one to compare the three assessed methods. In the coalitional approach, the input variable $u_i(k)$ in (33) is defined as the sum of the private and public parts of the variable according to (3). In this regard, Table 2 provides a comparison of the performance index when using centralized, coalitional, and decentralized MPC strategies. Applying the proposed coalitional approach results in a 1.28% decrease in the overall performance regarding the centralized solution. This was expected since coalitional control leads to a slight loss of performance in exchange of savings in cooperation and communication efforts. In turn, the decentralized MPC strategy implies a performance loss of a 17.06% compared to the centralized solution, as there is no communication between the different agents. Thus, it can be seen that, in terms of performance, it is beneficial to have some coordination between agents, as there is a significant



(a) Water level of lower tanks



(b) Flow rates of the four pumps

Fig. 3. Evolution of the water level of the lower tanks and flow rate of the four pumps. Solids lines represent the result using the proposed coalitional algorithm using $\hat{\alpha} = 0.015$, while dashed and dotted lines represent the evolution using a centralized and decentralized MPC respectively. The fine dashed–dotted line represents the operating point of the plant (32).

Table 2
Comparison of the overall performance for different control approaches.

	Performance index **P**
Centralized MPC	4.8189
Coalitional algorithm with $\hat{\alpha} = 0.015$	4.8805
Decentralized MPC	5.6411

loss in the performance when agents work independently in the decentralized scheme.

Our coalitional approach aims to reduce communication between agents while minimizing performance losses with respect to the centralized approach. Therefore, performance is slightly decreased in order to save on cooperation efforts. To check this, we have calculated a value for the communication cost as the total number of links activated along the simulation. While in

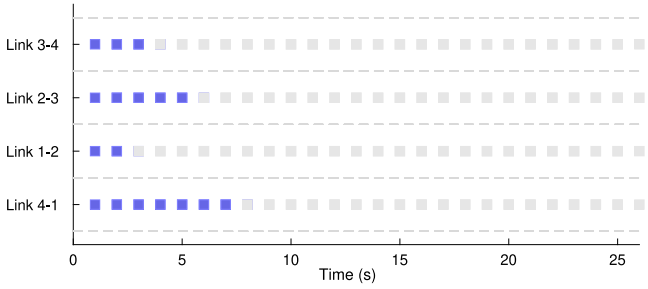


Fig. 4. Communication topology by means of the state of the communication links, i.e., enabled/disabled, using $\hat{\alpha} = 0.015$.

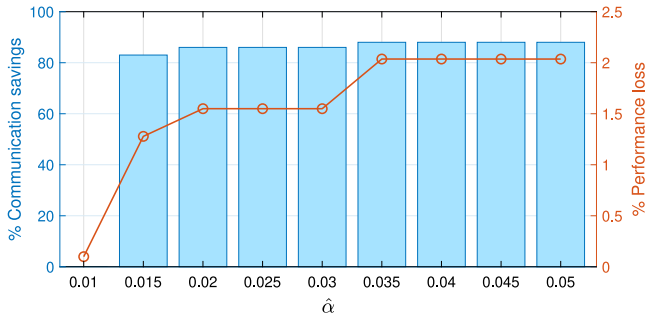


Fig. 5. Evolution of the public variables u_{ji}^{pu} and scale factors α_{ji} controlled by each agent i.

the centralized solution agents are constantly coordinating, and, therefore, all communication links are always active, the state of the links in the coalitional scheme depends on Step 9 of Algorithm 1 (see Fig. 4, which represents the state of the links during the simulation of the coalitional scheme). In this regard, the communication cost for the coalitional scheme (using the threshold value $\hat{\alpha} = 0.0015$) decreases by a 83% concerning the centralized approach. Therefore, we can conclude that our proposal saves communication efforts while preserving system performance.

In order to study the effect of the threshold that enables and disables communication, Fig. 5 compares the performance loss and the communication savings of the coalitional approach in comparison with the centralized control when using different values of $\hat{\alpha}$. Note that the value of this threshold for the given simulation results ($\hat{\alpha} = 0.015$) provides a tradeoff between performance and savings in coordination efforts.

Finally, Fig. 6 shows the evolution of the public variables u_{ji}^{pu} and scale factors α_{ji}, for each agent $i \in \mathcal{N}$ with $j \in \mathcal{N}_i$. Each column of the graph represents the variables controlled by agent i. The first row represents in solid lines the trajectory of the public part of the neighboring input u_j which has been ceded to agent i. The second row shows the evolution of the scale factors α_{ji}, that bound the constraint of the public variables u_{ji}^{pu}. Moreover, the value of the selected threshold $\hat{\alpha} = 0.015$ is represented in a continuous red dotted line in the bottom plots. In this way, along with the evolution of u_{ji}^{pu} in the upper row, the product $\alpha_{ji}\mathbb{U}_j$ has been represented in the graph with a gray dashed line to illustrate the values that the public variable can take. To allow a better comparison between agents, we have represented public inputs and scale factors using the same range. As it can be seen, when the value of a given scale factor α_{ji} is below the threshold $\hat{\alpha}$ at two consecutive time-steps, the corresponding communication link is disabled according to Step 9 of Algorithm 1, and the value of u_{ji}^{pu} is set to zero accordingly. Fig. 4 also illustrates this, as we can see that, for example, at the time instant $k = 6$ s, the link $(2, 3)$ is disabled. This happens because at $k = \{5, 6\}$ s the scale factor α_{23} that bounds the public part that agent 2 cedes to agent 3 (u_{23}^{pu}) satisfies $\alpha_{23} < \hat{\alpha}$.

6.1. The case of milder coupling

The previous case study does not allow the comparison with conventional tube-based MPC because the minimal invariant set $\mathcal{R}_i$ for every subsystem exceeds the considered constraint set as can be seen in Fig. 7(a). This highlights the advantages of the method we present, for it dynamically scales the tube size based on the requirements of the agents.

To further highlight the benefits of our method, an additional simulation in which we have reduced the terms of the coupling matrices B_{ij} by a scale factor of 0.35 is included. In this way, we can compute a mRPI set for every subsystem, which satisfies $\mathcal{R}_i \subseteq \mathbb{X}_i$, as shown in Fig. 7(b). A comparison in terms of the evolution of the pump flows is given in Fig. 8, where the coalitional approach is compared with a classic tube-based MPC (TMPC), as well as with the centralized and decentralized MPC schemes. As expected, if we decrease the coupling terms B_{ij} there is less difference with the decentralized approach. Nonetheless, our coalitional algorithm provides an intermediate solution with a loss of a 0.86% in the overall performance **P** regarding the centralized approach, while the decentralized and the classic TMPC approach resulted in a 5.97% and 5.59%, respectively.

7. Conclusion

This paper presents a novel coalitional strategy, where coupling variables are decomposed and distributed among neighboring agents into multiple versions that share a common constraint space. Agents willing to communicate will be coordinated using the dual decomposition scheme, negotiating the bounds of shared variables. Instead of communicating input or state sequences, the distributed algorithm will only broadcast information about the scale factors of the constraints. In this framework, we propose a tube-based MPC approach to robustify agents against disturbances induced by coupling. When agents communicate, they will share and negotiate the values that bound the input constraints, and the tube size will depend on these variables. In this way, we

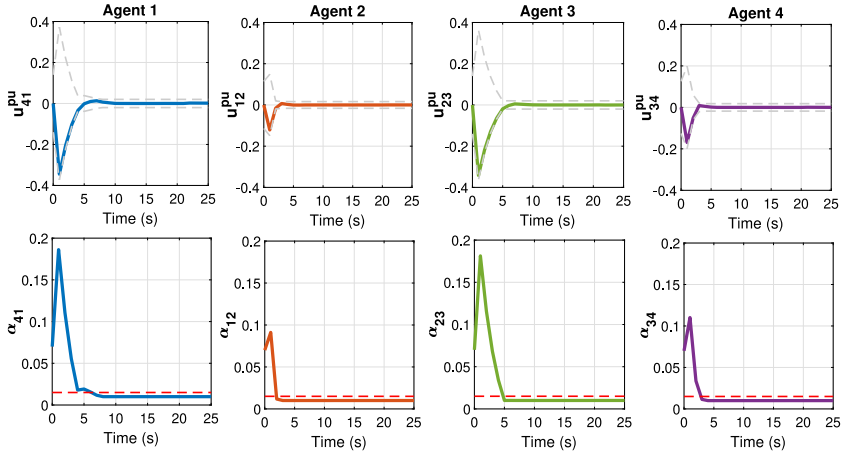
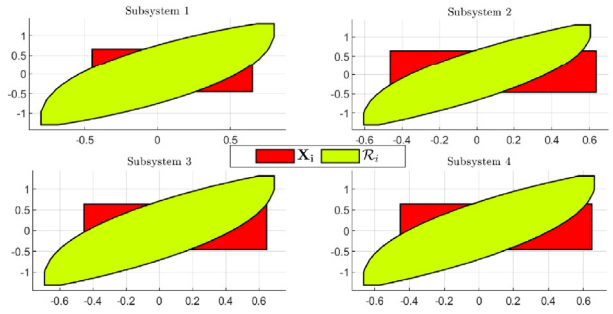
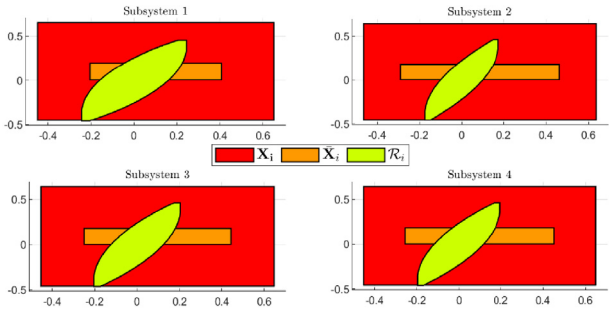


Fig. 6. Evolution of the public variables u_{ji}^{pu} and scale factors α_{ji} controlled by each agent i when using $\hat{\alpha} = 0.015$.



(a) Original system



(b) New system with $0.35B_{ij}$

Fig. 7. Representation of the state constraint set $\mathbb{X}_i$ (red), mRPI set R_i (yellow) and (on the bottom plot) nominal constraint set $\bar{\mathbb{X}}_i$ (orange) for every subsystem. The mRPI has been calculated using the methods proposed in [21,26], both leading to the same set.

demonstrate conditions for recursive feasibility and stability of the algorithm.

Simulation results on an 8-tanks system show that the proposed coalitional approach provides an overall performance similar to the one obtained with the centralized MPC algorithm. Event-based communication reduces the communication burden since agents will only coordinate when deemed necessary for the sake of better fulfillment. Further research will consider applying this idea to larger systems, such as solar plants, irrigation canals or electrical networks, to fully exploit the algorithm's scalability. In addition, future lines of work will consider the application of learning methods, such as in [27], to optimize the selection of $\hat{\alpha}$.

CRediT authorship contribution statement

A. Sánchez-Amores: Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing – original draft, Writing – review & editing, Visualization. **P. Chanfreut:** Methodology, Validation, Formal analysis, Investigation, Data curation, Writing – original draft, Writing – review & editing, Visualization, Supervision, Funding acquisition. **J.M. Maestre:** Conceptualization, Methodology, Formal analysis, Writing – review & editing, Supervision, Funding acquisition. **E.F. Camacho:** Conceptualization, Methodology, Formal analysis, Validation, Supervision, Project administration, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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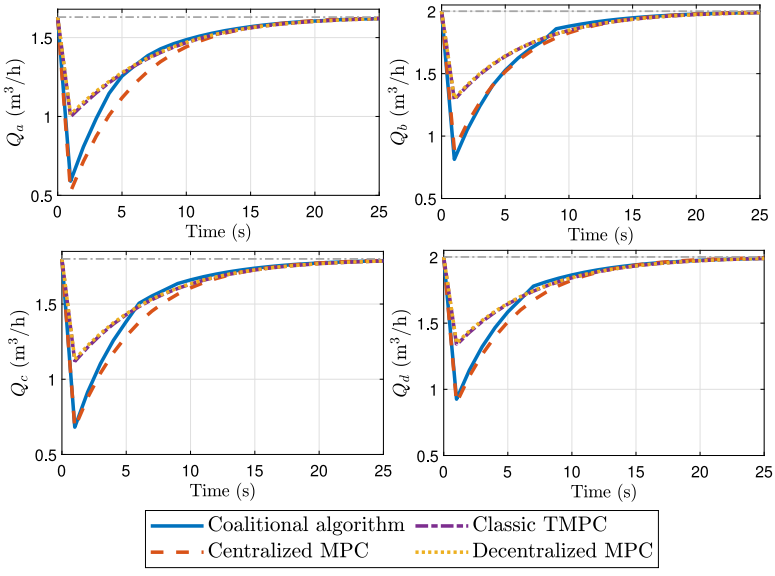

Fig. 8. Evolution of the flow rates of the four pumps for the system with reduced coupling terms B_{ij}.

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