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Towards Prediction of Financial Crashes with a D-Wave Quantum Computer

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Abstract Prediction of financial crashes in a complex financial network is known to be an NP-hard problem, which means that no known algorithm can guarantee to find optimal solutions efficiently. We experimentally explore a novel approach to this problem by using a D-Wave quantum computer, benchmarking its performance for attaining financial equilibrium. To be specific, the equilibrium condition of a nonlinear financial model is embedded into a higher-order unconstrained binary optimization (HUBO) problem, which is then transformed to a spin-1/2 Hamiltonian with at most two-qubit interactions. The problem is thus equivalent to finding the ground state of an interacting spin Hamiltonian, which can be approximated with a quantum annealer. The size of the simulation is mainly constrained by the necessity of a large quantity of physical qubits representing a logical qubit with the correct connectivity. Our experiment paves the way to codify this quantitative macroeconomics problem in quantum computers.

Keywords Quantum Information · Physical Systems · Quantum computation · Financial networks · Adiabatic quantum optimization

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1 Introduction

Economics is a complex science in which the agents' psychology plays an essential role that is often hardly grasped by mathematical models. However, economists do not relinquish to try to predict market behavior employing sophisticate models, leading to the field of quantitative finance. Following this idea, quantitative finance and economics emerged. They were applied to understand the evolution of financial markets and economies, as well as to forecast their possible future. A realistic question in risk management is: would there be a drastic drop in the market values if the prices of assets suffer some small perturbation? The cross-holdings and nonlinear character of financial network dynamics will cause chain reactions, implying that sudden drops of a market value might affect other nodes in the network resulting in a financial crisis. Presently, the prediction of crashes is mainly performed by studying previous cases in history and comparing with the current configuration [1,2,3,4,5,6]. While this empirical approach has been succesful [7], the economic environment is constantly evolving. Hence, we cannot limit ourselves to predicting economic disasters which are qualitatively similar to past events. Therefore, ab initio simulations of financial networks will become essential for avoiding financial crises. This problem was recently shown to be NP-Hard [8]. Therefore, given the current standpoint on complexity theory, this problem is not expected to be efficiently solvable in a classical computer. Indeed, given the global knowledge of a financial network, the time to compute the consequences of a perturbation would by far exceed the age of the universe.

An alternative approach to this problem was presented in Refs. [9,10], where they suggest a possible improvement of the efficiency to tackle this type of problems by using quantum computing. In particular, a mathematically identical problem is simulated and the corresponding result measured [11,12,13,14]. Specifically, it was shown that obtaining the equilibrium configuration of a financial network is equivalent to solving a higher-order unconstrained binary optimization (HUBO) problem, which should be feasible for a quantum annealer allowing for multi-qubit interactions. Unfortunately, this hardware has not been realized yet, as state-of-the-art quantum annealers are restricted to two-qubit interactions [15]. A possible workaround, which comes at the price of introducing ancillary qubits, is to find an effective Hamiltonian with the same low-energy subspace and two-qubit interactions at most. This leaves us with the problem of solving a quadratic unconstrained binary optimization (QUBO) problem whose optimum encodes the equilibrium configuration of a financial network. This problem can be addressed employing a quantum annealer. The D-Wave 2000Q quantum annealer, equipped with a Chimera architecture, requires a large quantity of physical qubits to obtain the desired connectivity and limits the number of institutions and assets considered. An analysis of the changes experienced by the financial network to reach its equilibrium configuration will tell whether a crash has occurred.

In this paper, we experimentally benchmark the study presented in Refs. [9,10]. Specifically, we compute the equilibrium configuration of a financial network after perturbation with a D-Wave 2000Q quantum annealer, and compare the result to alternative methods. Although the D-Wave machine has been successfully used to solve problems in engineering [16], cryptography [17], biology [18], and quantitative finance [19,20] among others, it is the first time that quantum annealing is applied to solve a problem of macroeconomics. This should attract more attention from the finance and economic disciplines towards quantum computing [21,22,23,24,25,26,27], as well as enlarge the amount of feasible problems for quantum annealers.

The contents are organized as follows: in Sec. 2, we introduce the model of financial network that will be considered. Sec. 3 reviews the quantum annealing algorithm to find financial equilibrium. Sec. 4 experimentally proves the validity of the scheme by finding the financial equilibrium of a random network of the largest implementable size with a D-Wave 2000Q quantum annealer; for this network, we also show experimentally how the scheme allows to compute the financial equilibrium. Sec. 5 analyzes the achieved results and discusses further possible improvements. The conclusions drawn from the work are shown in Sec. 6.

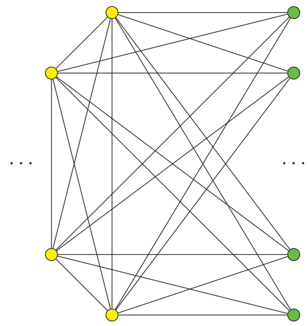

Fig. 1 Example of a financial network: the yellow nodes and green nodes denote institutions and assets, respectively. Links denote ownerships and cross-holdings, described by the ownership matrices $\mathbf{D}$ and $\mathbf{C}$, respectively. Diagonal matrix $\tilde{\mathbf{C}}$ represents the self-ownership of institutions, which would be plotted as self-loops in the graph representation. The equity value V_i of institution i is defined by summing up its ownership of all assets and cross-holdings.

2 Financial network model

A nonlinear network model for financial markets is proposed in Ref. [9]. It is made up of n institutions and m assets, and aims at representing the market values of institutions by mapping it onto a network, as shown in Fig. 1. We codify the prices of the m assets by an $m-$dimensional vector $\vec{p} \in \mathbb{R}^m$, where the element p_k represents the price of asset k. Moreover, an $n \times m$ ownership matrix $\mathbf{D}$ can be defined such that the element $D_{ik} \geq 0$ corresponds to the percentage of asset k owned by institution i. There is also an $n \times n$ ownership matrix $\mathcal{C}$ that describes the cross-holdings and self-ownerships between institutions. The coefficients $\mathcal{C}_{ij}$ denote the percentage of institution j owned by institution i. By considering all self-ownerships (i.e., the diagonal elements) from $\mathcal{C}$ one forms a new diagonal matrix $\tilde{\mathbf{C}}$ which represents the self-ownership only, such that matrix $\mathbf{C} = \mathcal{C} - \tilde{\mathbf{C}}$ codifies all cross-holdings. The equity value V_i of institution i is defined by summing up its ownership of all assets and cross-holdings, $V_i = \Sigma_k D_{ik} p_k + \Sigma_j C_{ij} V_j$. One thus obtains a matrix equation $\vec{V} = \mathbf{D}\vec{p} + \mathbf{C}\vec{V}$, where equity value vector $\vec{V} \in \mathbb{R}^n$ is an $n-$dimensional vector. Accordingly, the market value is the equity value rescaled with its self-ownership, resulting in the $n-$dimensional market value vector $\vec{v} = \tilde{\mathbf{C}}\vec{V}$. The solution to the linear matrix equation thus reads

$$\vec{v} = \tilde{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-1}\mathbf{D}\vec{p}. \tag{1}$$

We introduce the nonlinear effect of *panic* in the model via a Heaviside-theta function Θ; if the market value v_i drops below the critical value v_c^i, *failure* of institution i occurs and its equity value drops by $\beta_i(\vec{p})$ which is governed by the price vector of assets. Once we define the failure vector $\vec{b}(\vec{v}, \vec{p}) = \beta_i(\vec{p})(1 - \Theta(\vec{v} - \vec{v_c}))$, the market value vector with nonlinearity can be written as

$$\vec{v} = \tilde{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-1}(\mathbf{D}\vec{p} - \vec{b}(\vec{v}, \vec{p})). \tag{2}$$

Mathematically, it is the nonlinearity of $\vec{b}(\vec{v}, \vec{p})$ which makes financial networks so hard to be predicted.

This drop may cause an institution's value to *crash*, a behavior which can infect other nodes in the network. Under our definition, a financial crash happens when, after a perturbation in the assets price, the market value of an institution considering the nonlinear term is lower than those pre-perturbation prices calculated with the linear model.

3 Quantum annealing algorithm

As proposed in Ref. [9] finding financial equilibrium can be represented as the minimization of an objective function, which is equivalent to finding the ground state of a classical spin Hamiltonian.

By squaring Eq. (2), we obtain an objective function that meets its minimum value when the market value state is set to be the equilibrium state

(3)

Thus, our task is now to find the $\vec{v}$ that minimizes $Obj(\vec{v})$ for a given financial network.

Next, we need to deal with the nonlinear terms (modeling failure) of the objective function. The reason is that once the objective function is transformed to a spin-1/2 Hamiltonian, it should ideally be made of polynomial terms only, due to the limitations of quantum annealers. Thus, one expands the failure terms with Heaviside-theta functions in terms of polynomials. This expansion is not unique, and here we choose the Legendre expansion [9],

$$\Theta(x) = \frac{1}{2} + \sum_{l=1}^{\infty} (P_{l-1}(0) + P_{l+1}(0)) P_l(x), \tag{4}$$

in the domain $[-1, 1]$, with $P_l(x)$ to be the l-th Legendre polynomial. By setting $x = (v_i - v_i^c)/v_{max}^i$, Eq. (4) enables us to expand $\Theta(v_i - v_i^c)$ in the range of $v_i \in [0, v_{max}^i]$. Using this expansion as an example, we take the approximation

$$b_i(v_i, \vec{p}) \approx \beta_i(\vec{p}) \left(\frac{1}{2} - \sum_{l=0}^{r} \Gamma_l 2^l \sum_{k=0}^{l} \binom{l}{k} \binom{\frac{l+k-1}{2}}{l} \bar{v}_i^k \right) \tag{5}$$

where $\Gamma_l = \frac{\sqrt{\pi}}{2\Gamma\left(\frac{2-l}{2}\right)\Gamma\left(\frac{3+l}{2}\right)}$ and $\bar{v}_i = \frac{v_i - v_i^c}{v_{\max}^i}$.The polynomial expansion removes the discontinuity while maintaining the strong nonlinearity of the network.

We now encode the continuous variables v_i with classical bits. This will allow rewriting the resulting objective function in digital form. The expansion is straightforward, and reads $v_i = \sum_{\alpha=-\infty}^{\infty} x_{i,\alpha} 2^{\alpha}$. However, due to the limited resources in real-world devices, one must truncate this expansion, i.e., $v_i \approx \sum_{\alpha=-q}^{q} x_{i,\alpha} 2^{\alpha}$, where $x_{i,\alpha}$ are classical bits with binary values 0 or 1. In this way, the market value of institution i is encoded with $2q + 1$ classical bits. The maximal market value v_i^{max} is given by $\sum_{\alpha=-q}^{q} 2^{\alpha}$.

Considering $(v_i - v_i^c)^k = \sum_{h=0}^{k} (-1)^h \binom{k}{h} v_i^{k-h} (v_i^c)^h$ and $v_i^n = \sum_{m_0+\ldots+m_p=n} \binom{n}{m_0,\ldots,m_p} \prod_{0\leq\alpha\leq p}^{m_\alpha\neq 0} 2^{\alpha m_\alpha} x_{i,\alpha}$, the resulting objective function is a polynomial of binary variables $x_{i,\alpha}$ of degree $2r$.

$$\hat{H} = \sum_i \left(\sum_{\alpha=-q}^{q} x_{i,\alpha} 2^{\alpha} - \gamma_i + \sum_j \bar{C}_{ij} b_j(x_{j,\alpha}, \vec{p}) \right)^2 \tag{6}$$

with $\gamma_i = \sum_j \bar{C}_{ij} \sum_k D_{jk} p_k$ and $\bar{C}_{ij} = \tilde{C}_{ii} (\mathbf{I} - C)_{ij}^{-1}$.

To express it as a spin-1/2 Hamiltonian, we replace the binary variables $x_{i,\alpha}$ by qubit operators $\hat{x}_{i,\alpha}$ with eigenvalues 0 and 1, i.e., $\hat{x}_{i,\alpha}|0\rangle = 0$, $\hat{x}_{i,\alpha}|1\rangle = |1\rangle$. The Pauli-$z$ operator satisfies $\hat{x}_{i,\alpha} = (1 + \hat{\sigma}_{i,\alpha}^z)/2$, and therefore the Hamiltonian reads

$$\hat{H} = \mathrm{Poly}_{2r}(\hat{\sigma}_{i,\alpha}^z), \tag{7}$$

which is equivalent to our objective function but written with Pauli matrices. This Hamiltonian includes all types of multi-spin interactions, up to $2r$-body terms.

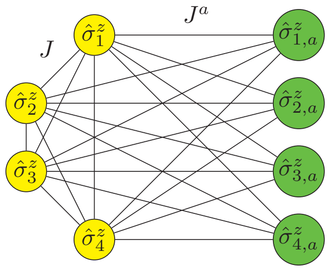

Fig. 2 Recast of quantum Hamiltonian with k-qubit interactions into a modified, effective Hamiltonian with the same low-energy spectrum with two-qubit interactions at most. We illustrate the particular case of a $k = 4$-qubit interactions, which requieres the introduction of 4 ancilla qubits to obtain the effective Hamiltonian.

The Hamiltonian obtained in Eq. (7) is appropriate for a quantum annealer that allows many-qubit interactions. However, state-of-the-art quantum annealers only accept inputs with at most two-qubit interactions. Finding the ground state of a spin-1/2 Hamiltonian, as the one obtained in Eq. (7), is equivalent to solving a Quadratic Unconstrained Binary Optimization (QUBO) problem, which is the input of the quantum annealer. Thus, we should recast our quantum Hamiltonian into a modified, effective Hamiltonian with at most two-qubit interactions. Some protocols achieving exactly this are proposed in Refs. [28,29,30,31,32,33,34], in particular we base our protocol in Ref. [34], where k ancilla qubits are introduced to implement an effective k-qubit interaction. Suppose that there is a k-qubit interaction term $\hat{H}_k = J_k \Pi_{i=1}^k \sigma_i^z$ with the same low-energy spectrum of another Hamiltonian term $\tilde{H}_k$ with at most two-qubit interactions. We can express $\tilde{H}_k$ with k logical qubits and k extra ancilla qubits as

$$\begin{aligned}\tilde{H}_k = {} & J\sum_{i=2}^{k}\sum_{j=1}^{i-1}\hat{\sigma}_i^z\hat{\sigma}_j^z + h\sum_{i=1}^{k}\hat{\sigma}_i^z \\ & + J^a\sum_{i=1}^{k}\sum_{j=1}^{k}\hat{\sigma}_i^z\hat{\sigma}_{j,a}^z + \sum_{i=1}^{k}h_i^a\hat{\sigma}_{i,a}^z, \end{aligned} \tag{8}$$

as represented in Fig. 2. This two-qubit Hamiltonian has the same low-energy spectrum than $\hat{H}_k$ when J, J^a, h and h_i^a are set to some appropriate values. As Ref. [34] suggested, this can be achieved once $q_i = (-1)^{k-i+1}J_k + q_0$, $h = -J^a + q_0$, $h_i^a = -J^a(2i-k) + q_i$ and $J = J^a$, with any q_0 that satisfies $|J_k| \lll q_0 < J^a$ and $|J_k| \lll J^a - q_0 < J^a$. These conditions can be relaxed to $|J_k| < q_0 < J^a$ and $|J_k| < J^a - q_0 < J^a$ if one aims at having the same ground state only, instead of the whole low-energy sector.

4 Implementation in a D-Wave 2000Q quantum annealer

Once shown that it is possible to recast the problem of finding financial equilibrium into a language that is amenable to quantum annealers, this section deals with its implementation using a state-of-the-art quantum annealer, namely, the D-Wave 2000Q. This quantum annealer consists of 2048 qubits connected according to the Chimera graph topology (see Fig. 3). It is designed to solve embedded Ising problems or QUBO problems.

Two simulations were produced:

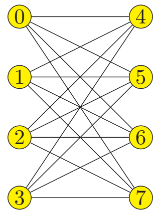

Fig. 3 Chimera graph topology implemented by the D-Wave 2000Q quantum annealer. The 2048 qubits are partitioned into subgraphs of 8 qubits. The connection between subgraphs is sparse, in each of these subgraphs there are two sets of four qubits; each qubit connects to all qubits in the other set but to none in its own, forming a $\mathbf{K_{4,4}}$ bipartite graph.

1. A financial network without failure term, which is simple to solve on a classical computer in order to benchmark the performance of the quantum processor.
2. A financial network with the inherently nonlinear risk of failure. We will perturb the asset price vector in this network to compute the new equilibrium configuration using the quantum annealing algorithm.

We initially generate a financial network with 10 institutions and 15 assets. To demonstrate the algorithm, we randomize the ownership matrix $\mathbf{D}$ with a Dirichlet distribution that satisfies $\sum_{i=1}^{n} D_{ij} = 1$, where D_{ij} are random variables. The cross-holding matrix $\mathcal{C}$ is generated in a similar way but with a constraint that all diagonal elements should be larger than 0.5, ensuring that all institutions can make decisions according to their own wills. Thus, we randomize $\tilde{C}_{ii}$ between 0.5 and 1 and randomize $\sum_{i=1}^{n} C_{ij} = 1 - \tilde{C}_{jj}$ with a rescaled Dirichlet distribution. The price vector $\vec{p}$ is also random, with $p_i \in [10, 40]$. The network configuration is shown in Figs. 4 and 5.

We can calculate the equilibrium state $\vec{v_q}$ and the equity value vector $\vec{V}$ on a classical computer using

$$\vec{v_q} = \tilde{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-1}\mathbf{D}\vec{p}, \tag{9}$$

$$\vec{V} = (\mathbf{I} - \mathbf{C})^{-1}\mathbf{D}\vec{p}. \tag{10}$$

The objective function shown in Eq. (3) was implemented, for benchmarking reasons, both in a quantum annealer and a classical simulator. Variables v_i were encoded, $v_i = \sum_{\alpha=0}^{6} 2^{\alpha} x_{i,\alpha}$, on seven qubits. As such, this constrains the v_i to be integers smaller than 127. A quantum implementation of this algorithm does not require ancilla qubits, as there are no many-qubit interactions.

The QUBO for this linear problem is a 70×70 matrix, with 210 couplers which cannot be solved directly due to the topology structure of the quantum annealer. D-Wave provides a software named *qbsolv* that allows to combine the classical computer with its quantum annealer by splitting the QUBO matrix into partition matrices that can be embedded in the quantum annealer. As a decomposing solver, it finds a minimum value of a large QUBO problem by splitting it into pieces solved either via a D-Wave system or a classical tabu solver (both approaches were considered here for comparison purposes). Since the D-Wave 2000Q processor is a quantum annealer, 20 results would be obtained from a *qbsolv* process with a default setting; these results should be handled by a correction process, e.g., majority voting, to help us identify the most plausible answer. The result of this QUBO problem is shown in Fig. 6, where the exact solution via solving a linear matrix equation, *qbsolv* solution with classical tabu solver, and *qbsolv* solution with D-Wave quantum annealer, are compared. It is straightforward to observe that a quantum annealer provides a similar solution to the exact solution than the classical tabu solver.

While the failure-free model only has linear and quadratic terms in v_i, the nonlinear model has powers of v_i up to order $2r$, as shown in Eq. (7). For large r, this can be extremely resource-consuming in terms of

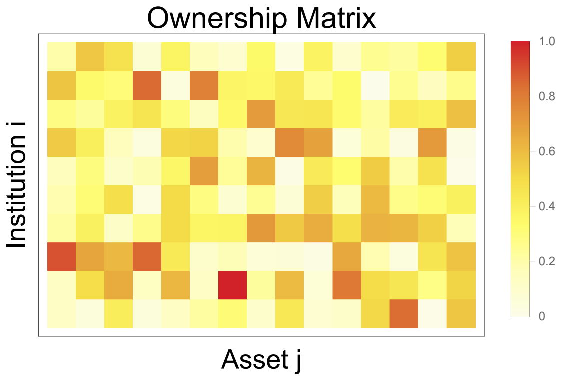

Fig. 4 Ownership matrix $\mathbf{D}$ for the linear model. The element $D_{ik} \geq 0$ corresponds to the percentage of asset k owned by institution i. We randomize the ownership matrix $\mathbf{D}$ with a Dirichlet distribution that satisfies $\sum_{i=1}^{n} D_{ij} = 1$.

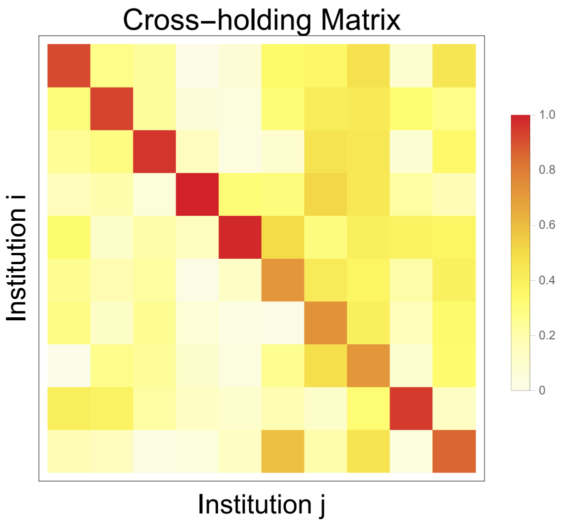

Fig. 5 Cross-holding matrix $\mathcal{C}$ for the linear model that describes the cross-holdings and self-ownerships between institutions. Cross-holding matrix is generated in a similar way to ownership matrix but with a constraint that all diagonal elements should be larger than 0.5, ensuring that all institutions can make decisions according to their own wills.

ancillary qubits due to the requested connectivity. An estimation of the number of qubits can be made by counting the number of interaction terms; Eq. (7) indicates that $\hat{H}$ can have up to $\sum_{\alpha=0}^{2r} \binom{n(2q+1)}{\alpha}$ terms, where $n(2q+1)$ denotes the logical qubits that are required. In each term, 3-to-$2r$ new ancilla qubits are needed, depending on the number of logical qubits in this term. Therefore, the number of necessary qubits grows rapidly with the degree of the polynomial expansion r. Note that the aforementioned QUBO problem is NP-hard for any $n \geq 2$. In practice, this is an upper bound to the required resources, calculated assuming that $\hat{H}$ has all possible terms up to order $O(2r)$.

Here, we implement an enhanced model with failure terms on the basis of the linear model previously simulated. We perturb the vector of asset prices, leaving the ownership matrix $\mathbf{D}$ and cross-holding matrix $\mathcal{C}$ invariant, and recompute the equilibrium state. Specifically, we set the price of some random assets to zero (to

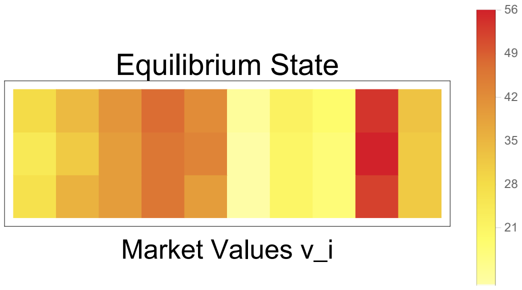

Fig. 6 Linear-model result. The first row shows the result if the matrix equation is solved exactly, the second row if *qbsolv* with tabu classical solver is used, and the third row if *qbsolv* with D-Wave 2000Q solver is employed. We observe that a quantum annealer provides a similar solution to the exact solution than the classical tabu solver.

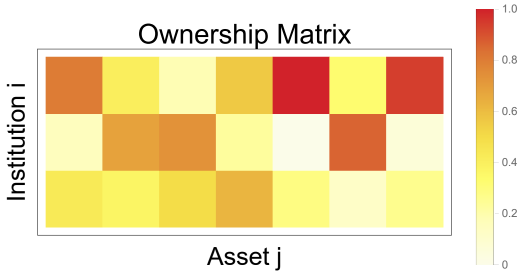

Fig. 7 Ownership matrix $\mathbf{D}$ for the implemented network with failure terms. The element $D_{ik} \geq 0$ corresponds to the percentage of asset k owned by institution i. We randomize the ownership matrix $\mathbf{D}$ with a Dirichlet distribution that satisfies $\sum_{i=1}^{n} D_{ij} = 1$.

simulate, e.g: the assets' destruction). In this study, we will use an expansion of $\hat{H}$ to third order, which still characterizes the phenomenon of sudden drop near the critical value. Moreover, this approach provides strong nonlinearity while saving plenty of qubit resources. As a result, 70 logical qubits and 872,690 ancilla qubits are required with 4,446,575 couplers in the problem. This results in the requirement of about 6TB memory, since each element has an accuracy of double float in *qbsolv*. Due to the limitations of state-of-the-art techniques, the network is reduced to three institutions and each market value v_i is encoded by five qubits, bounding the maximum market value to be 31. New 3×7 ownership matrix $\mathbf{D}$ and 3×3 cross-holding matrix $\mathcal{C}$ are generated while the price vector $\vec{p}$ before perturbation is $\vec{p} = \{8.43,\ 14.47,\ 6.75,\ 8.09,\ 19.11\ ,11.32,\ 7.19\}^T$. The network configuration is shown in Figs. 7 and 8. The equilibrium state before perturbation without nonlinearity is given as $\vec{v_q} = \{21.18\ \ 23.33,\ 30.83\}^T$, and the critical value vector is still set to be 80% of the original equilibrium state, while the failure strength $\vec{\beta}$ is considered to be 30% of the original equity value. The corresponding perturbed price vector is given as $\vec{p} = \{8.43,\ 14.47,\ 0,\ 8.09,\ 0\ ,11.32,\ 7.19\}^T$. Before calculating the new equilibrum state with nonlinearity and perturbation, some parameters, like J^a and q_0, must be set. For the minor embedding of a submatrix in the D-Wave quantum annealer, this is done by introducing a penalty function between qubits in the Chimera graph requiring $J^m \geq J^a$, which means that the J^a for mapping multi-qubit interactions to two-qubit interactions should be in the proper scale. Meanwhile, as we mentioned in the theory part, we need to sample out the thermal fluctuation by assuming that $|\hat{H}_k|$ is much smaller than J^a, or the protocol will break down because those ancilla qubits will not be in the corresponding ground state anymore. Thus, in the implementation we took $J^a = 20J_k$ and $q_0 = 10J_k$, such that this could ensure that either q_0 or $J^a - q_0$ would be at least 10 times larger than J_k.

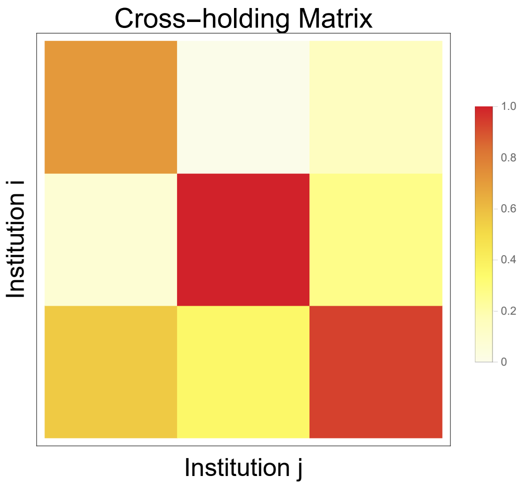

Fig. 8 Cross-holding matrix $\mathcal{C}$ for the implemented network with failure terms that describes the cross-holdings and self-ownerships between institutions. Cross-holding matrix is generated in a similar way to ownership matrix but with a constraint that all diagonal elements should be larger than 0.5, ensuring that all institutions can make decisions according to their own wills.

For this problem, the QUBO matrix had the size of 8280×8280, with 15 logical qubits, 8265 ancilla qubits and 38,790 couplers. Remark that the available quantum annealer structure is not optimized for this problem and, also, that the translation is not efficient because of sparse connectivity of the quantum processor. Therefore, the only benefit here is to explore the possibility of having certain sub-exponential speedup in the implementation of the problem in a quantum annealer [35], by optimizing submatrices generated by the partition algorithm in *qbsolv*. Finally, we compare our results from the quantum annealer with the integer equilibrium solution calculated with a straightforward method by trying 32^3 times in Fig. 9, which shows a good agreement and the accuracy of the proposed method. Comparing the results after the perturbation with the pre-perturbation values, we can conclude that we have detected the financial crash.

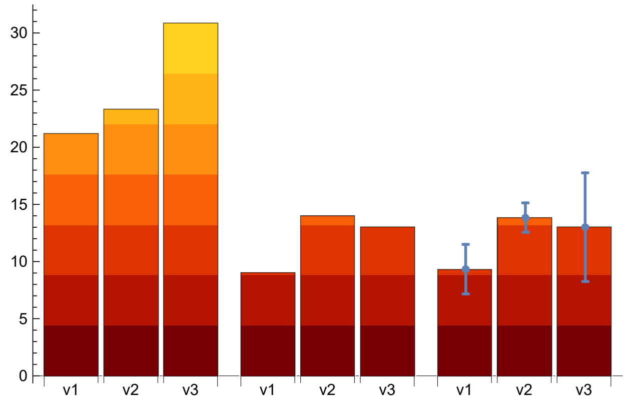

Fig. 9 The first group (left) is the equilibrium without failure term before perturbation. The second group (center) depicts the integer result of the implemented network with polynomial expansion calculated with a straightforward method by trying 32^3 times. The third group (right) shows the outcome from *qbsolv* software in D-Wave 2000Q. The error bar characterizes a 95% confidence interval. The agreement between integer and annealer solutions confirms the feasibility and accuracy of the method. Additionally, by comparing both with the pre-perturbation values, we can conclude that we have detected the financial crash.

5 Results and discussion

D-Wave is a quantum annealer designed to deal with QUBO problems, e.g., Ising model. However, the problem faced in this paper, namely, financial crisis prediction with nonlinearity associated to panic, is not QUBO but HUBO instead, thus requiring multi-qubit interactions. In order to approximate this HUBO problem with two-qubit interactions, at the current stage of hardware and software we were limited to simulate a small financial network, made up of three institutions and cross-holdings.

An effective two-qubit quantum Hamiltonian could still not be read directly in D-Wave system which requires QUBO type input or Ising type input. Although some open-source software like *pyqubo* can generate it, the input size must be very small in order to avoid a stack overflow associated with recursion errors. A possible solution is to produce a Mathematica script that reads each term, write it as a string of coefficients and qubits in an input file for the D-Wave system. Once we generate the input for this problem, this is still too large to be embedded in the D-Wave 2000Q quantum annealer because of the graph structure. Thus, *qbsolv* is an inevitable option for us, which works by separating the large matrix to submatrices and solve them by a classical tabu solver or D-Wave solver. This kind of hybrid computation provides the possibility to solve the complicated problem but brings some new constraints, namely: (i) *Local hardware.* Once the QUBO matrix is provided, *qbsolv* allocates dynamic memory before separating it to submatrices with elements of double precision floats, by requiring a size of $8n \times n$ bytes of memory. However, the bottleneck is not the memory size but the performance of CPU since a large QUBO matrix will consume exhaustive CPU time if one needs high accuracy of the optimized result; (ii) *Algorithm.* Instead of a real quantum annealing process for the whole matrix, *qbsolv* provides a tabu algorithm or D-Wave 2000Q quantum annealer for submatrices. The partition strategy for generating submatrices may get stuck in a local minimum instead of the global minimum that quantum annealing guarantees with high probability under ideal conditions, i.e. in absence of decoherence and in the adiabatic limit. Considering that the logical qubits only encode less than 1% in the QUBO matrix, the risk of getting stuck is still high, even if we sample over the thermal distribution or give a huge repeat limitation in the main loop to improve its accuracy. We would have to customize a random seed for the separation, and check the final result manually, to see whether the result is near from the equilibrium. Another option is that one may send the QUBO matrix to the solver many times and average the result to obtain the best solution. (iii) *Quantum annealer.* The submatrices will be sent to D-Wave 2000Q quantum annealing device for optimization after they are generated by Glover's algorithm [36]. In the quantum annealing process, magnetic fields are applied to the processors and the strength should be accurate because J^k, J^a in the QUBO matrix and J^m for the embedding belong to different magnitudes. Any imprecision in the system preparation will cause significant deviation from the correct result.

In this implementation, the accuracy is not especially high, since we are not optimizing the objective function rigourously because the market values are integers $v_i \in [0, v_{max}]$ constrained by the qubits we take to encode them. The computation time is also long, considering that there is a straightforward but equivalent classical algorithm by testing the value of the objective function 32^3 times by brute force, corresponding to all de possible combinations. Although mapping it to a QUBO problem and optimizing it with a general quantum annealer is not efficient enough for current technology, we believe it is a valuable example of how one can solve an NP-hard problem via quantum computation. With quantum annealers designed for solving HUBO problems that allow an efficient mapping multi-qubit interactions to a QUBO problem, we may obtain a speed up factor in forecasting the behavior of complex financial networks over the use of general purpose annealers. We expect this kind of quantum solver may be available in the near future. Meanwhile, D-Wave has recently announce its next generation of quantum annealers called the Advantage system [37]. It would consist of more than 5000 qubits connected with each other according to the Pegasus topology. In this manner, one could improve the number of qubits and the connectivity by a factor of 2.5.

Considering that a specialized quantum annealer for HUBO problems would not be available to the public anytime soon, we now analyze the possible ways to enhance the performance of D-Wave 2000Q quantum annealer on this problem. After compromising on the maximum two-qubit interactions in hardware, the subsequent strategy will be reducing the number of ancilla qubits. With fewer ancilla qubits, the size and accuracy of a solvable network can be improved. As proposed in Ref. [34], the multi-to-two mapping is a general method, but for three-to-two, for example, a more efficient mapping can be constructed with only one ancilla qubit. Suppose there is a sub-Hamiltonian of three-qubit interactions

$$\hat{H}_3 = J_3 \hat{\sigma}_1^z \hat{\sigma}_2^z \hat{\sigma}_3^z. \tag{11}$$

A subgraph with full connectivity of three logical qubits and one ancilla qubit is shown in Fig. 10, where the equivalent Hamiltonian is given as

$$\tilde{H}_3 = J \sum_{i=2}^{3} \sum_{j=1}^{i-1} \hat{\sigma}_i^z \hat{\sigma}_j^z + h \sum_{i=1}^{3} \hat{\sigma}_i^z + J^a \sum_{i=1}^{3} \hat{\sigma}_i^z \hat{\sigma}_a^z + h^a \hat{\sigma}_a^z. \tag{12}$$

At variance with the previous protocol, $J^a = 2J > h$ and $h^a = 2h = 2J_3$. Also, for sampling out the thermal fluctuation, we take $J^a \geq J_3$, to prevent the protocol to fail for the same reason. The ancilla qubits can be reduced to about 7000 with this method. Meanwhile, the partition method in *qbsolv* may cause the system to get stuck in local minima which requires a better algorithm in the main loop.

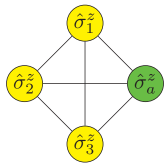

Fig. 10 An efficient encoding of three qubits, making use of only one ancilla qubit. The multi-to-two interaction Hamiltonian mapping is a general method, but for three-to-two, a more efficient mapping can be constructed via a subgraph with full connectivity of three logical qubits and one ancilla.

6 Conclusion

We have implemented in a D-Wave quantum computer the algorithm proposed in Ref. [9], to solve the equilibrium state of a complex financial network that predicts financial crashes. Although the size of the studied financial network is limited, this proof of principle is in agreement with the result of an exhaustive search. Moreover, this work is a convincing evidence that quantum computation can be used to study quantitative finance and help institutions anticipate risks. This result may be improved with the design of a customized "financial quantum annealer": a quantum processor with suitable connectivity for efficient embedding of this kind of problems. Such coherent quantum annealers can be built with current technology [38, 39, 40], providing convenient multi-qubit couplings.

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