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# Centralised resource allocation using Lexicographic Goal Programming. Application to the Spanish public university system

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| Keywords:<br>Centralised DEA<br>Resource allocation<br>Target setting<br>Lexicographic goal programming<br>Spanish public universities | This paper deals with Data Envelopment Analysis (DEA) in centralised settings in which the operating units belong to the same organisation. In such a scenario, a global system-wide perspective may be adopted as regards resource allocation and target setting. In this paper, a new Lexicographic Goal Programming (lexGP) approach is proposed using three priority levels: the aggregated input consumption and output production goals; the input and output goals of the individual operating units; and the technical efficiency of the computed targets. It is assumed that the goals for the overall organisation are established by the Central Decision-Maker (CDM) and that they are consistent with those of the individual operating units. The proposed approach has been applied to the Spanish public university system, comprising 47 institutions. Given the CDM preferences in terms of input and output aggregate goals and relative importance weights, specific technical efficient targets have been computed for each university. The results show that the proposed approach is more suitable than the non-centralised DEA approach and produces targets that are more effective than other centralised resource allocation approaches in the sense that they are much closer to both the aggregate goals of the cDM and the specific goals of each |

## 1. Introduction

Data Envelopment Analysis (DEA) is a non-parametric methodology generally used for the assessment of the relative efficiency of a set of homogeneous operating units (OUs). The OUs are modelled as an inputoutput process that consumes resources to produce products. The key idea in DEA is that the observed OUs enable extrapolation of the socalled Production Possibility Set, which contains all operating points that are deemed feasible. The non-dominated subset of this Production Possibility Set is called the Efficient Frontier and the corresponding operating points are labelled technical efficient. Most DEA models project the observed OUs onto the Efficient Frontier computing targets and also project efficiency scores that measure the distance from the observed OUs to their corresponding targets. This can be carried out using different metrics and orientations (e.g., Refs. [1–3].

Before proceeding further, it should be borne in mind that we use the term OU for the different productive units instead of the more common term Decision-Making Unit (DMU). The reason for this is that the autonomous decision-making capability of individual OUs does not actually apply in the centralised settings considered in this paper, in which the OUs belong to the same organisation and there is a Central Decision-Maker (CDM) that is primarily interested in optimizing the performance of the whole system (e.g., Refs. [4–7]. In such centralised DEA (CDEA) scenarios, resource allocation and target setting are carried out by jointly projecting all the OUs and taking into account their aggregate input consumption and output production. As can be observed in the literature review in the following section, one of the drawbacks of CDEA approaches is that there are often many alternative optima, some of which may lead to targets that are not close to the observed DMUs and to the problems that this entails.

The purpose of this paper is to develop an approach to help the CDM achieve overall efficient performance by computing OU targets that are technical efficient and relatively close to the observed OUs. The proposed approach is based on the Goal Programming (GP) methodology, specifically on a lexicographic GP (lexGP) approach. GP is a well-known operational research/management science methodology that has its roots in the bounded rationality and satisficing concepts as opposed to the conventional optimizing aim. The idea is that, instead of

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determining one or more objective functions to optimize, the decision maker (the CDM in this case) establishes a set of goals. The goals can be of different types  $(=, \leq, \geq)$  and implicitly reflect the preferences of the CDM. The methodology tries to find a solution of the problem that satisfies these goals as much as possible. This is done by minimizing a socalled Achievement Function (AF) which corresponds to the deviations from the different goals. In the lex GP method, which is the one used in this paper, the goals have a hierarchical structure so that goals of a higher priority are considered before goals of lower priority. As explained in Section 3, the CDM needs to establish aggregate input and output goals as well as OU-specific input and output goals. These two sets of goals must be consistent, with the aggregate goals having a higher priority than those of the individual OUs. The established goals act as beacons so that the deviations from them allow the CDM to get a clear picture of the quality of any solution. A third priority level is also included to remove any input and output slacks and guarantee the technical efficiency of the computed targets. Summarising this point, the centralized resource allocation approach proposed in this paper is innovative in its use of lex GP to structure and address the preferences and priorities of the CDM and find a unique solution. To the best of our knowledge this is the first time that this elegant and powerful methodology is considered for this problem. In addition, the structured and parametrized character of the approach grants the CDM a larger degree of control of the resource-allocation and target-setting processes.

Moreover, to illustrate the usefulness of the proposed approach, its application to the Spanish public university system is presented and discussed. Based on the aggregate and OU specific goals and on the importance weights of the different inputs and outputs the proposed approach is able to allocate the available resources among the universities and determine the corresponding output targets which satisfy as close as possible and in some cases exceed the established goals. The proposed approach thus constitutes an effective tool for tactical planning at the organizational level, integrating achieving the overall organizational goals with the goals of the individual OUs and all this while guaranteeing technical efficient targets.

In summary, the proposed approach aims at using the lexGP method, with its hierarchical goal structure, to solve the centralized resource allocation problem. LexGP provides a solid theoretical framework that allows both the CDM and the individual OUs to achieve their respective goals. This harmonious integration of the interests of both sides is missing in existing approaches and should increase the acceptability and usefulness of the proposed approach for the concerned actors. This is confirmed in the application carried out, in which the Ministry of Universities sets input and output goals for the whole system, goals whose achievement is compatible, in a lexicographic sense, with the individual universities also pursuing their own goals. In other words, the proposed approach represents an effective way of taking into account the interests of all parties involved in the centralised resource allocation process.

The structure of the paper is as follows. In Section 2, a review of the relevant CDEA literature is carried out. Section 3 presents and explains the proposed lexGP CDEA approach, which is then applied, in Section 4, to the 47 universities that comprise the Spanish public university system. In the last section, a summary and conclusions are presented.

## 2. Literature review

It should be borne in mind that there are many situations in which a centralized perspective is appropriate, as attested by the large number of CDEA applications spanning many different sectors. Thus, among others, applications can be found in: Education (e.g., Refs. [7–12]; supermarkets and retail stores [13–15]; branches of banks and insurance companies (e.g., Refs. [6,16,17]; regional gas companies (e.g., Ref. [18]; hotel and restaurant chains (e.g., Refs. [19–22]; project selection [23]; local and regional governments and government agencies [24–26]; emission permits and allocation of emission reduction (e.g., Refs. [27–31]; ports and container terminals (e.g., Refs. [32,33]; shipping

companies [34,35]; airports [36,37]; railway stations [38], and hospitals (e.g., Ref. [39].

As regards CDEA models, there are the basic radial and non-radial basic approaches (e.g., Refs. [4,5,7]. These are characterised by the absence of side constraints (more on this topic later), although they can consider non-discretionary variables. Incidentally, most CDEA approaches can accommodate not only non-discretionary variables but also non-allocatable and non-transferable variables. Although the way these variables are handled in CDEA is sometimes analogous to that of non-discretionary variables, they remain conceptually different. Thus, unlike non-discretionary inputs, non-allocatable/non-transferable inputs can be increased or decreased by the DMUs. For these inputs, it remains impossible for reductions in one DMU to be compensated with increases in other DMUs, that is, the released inputs cannot be transferred to other DMUs.

There are more refined CDEA approaches as well as many variants and extensions. Thus, for example [40], proposed a generalised CDEA approach that allows the CDM to select not only those DMUs whose inputs can be reallocated, but also the DMUs that can be used as reference units when computing the targets. [14] proposed a step-by-step improvement path approach where the movement from the operating pints corresponding to the observed OUs to their final targets can be planned in several steps, which facilitates gradually carrying out the required input and output adjustments. More recently [41], have proposed an approach to measure the individual efficiency of OUs in a CDEA setting as an influence index to rank CDEA-efficient OUs.

One category of CDEA approaches involves DEA-based Production Planning models. In this type of study, the aggregate output level (i.e., the product demand) as well as the total input (i.e., the total amount of resources available) are often given and can increase or decrease with respect to the current values. Several approaches (e.g. Refs. [20,21], assume that the inputs and outputs of each OU change by the same proportion and often impose that the changes in the inputs and outputs of all OUs are in the same direction as the change in the corresponding total input or output (e.g., Refs. [18,20,42]. Bounds on the input or output changes of the individual OUs can be imposed (e.g., Refs. [21, 42]. As regards the objective function, it can be the minimisation of costs [43,44], maximisation of the average efficiency [20], maximisation of the production stability [42], or a lexicographic optimisation approach with different objective functions, such the weighted aggregate output increase, the weighted aggregate input reduction, and the sum of the OUs efficiencies (e.g., Refs. [18,21,45].

There are also CDEA approaches that consider a network DEA structure. Essentially, the simplest configuration is of two stages in series. To this end [34], propose a non-radial, output-oriented CDEA model in which the OUs are shipping routes of a certain container shipping company and these are modelled as consisting of a Production process and a Service process, linked by intermediate products. They consider that certain inputs (called common inputs) are shared by the OUs and hence can be transferred/reallocated among said OUs, while other inputs (called specific and energy inputs) are not shared. Two Phase II variants (corresponding to a minor and a major adjustment policy) are considered, as are two scenarios, one in which the common inputs are shared only by the Production processes and another in which they are shared by both processes of each route. Hossein Yadolladi and Kazemi Matin (2021) also consider two stages in series (Education and Research) for the modelling of the internal structure of the branches of a private university system in Iran. These authors use a non-radial, non-oriented approach and, like other CDEA models, such as those by Refs. [24,46]; they consider the possibility of OU closures (so-called downsizing).

Finally, there are multiple studies that consider restrictions on the transfer of resources and also consider transfer costs. Thus [47], proposes a CDEA model for the minimisation of the input reallocation costs after solving both the maximum total output increase CDEA model, and a modified CDEA model that maximises the total output increase subject

to either the upper bounds on the input transfers allowed or on the total transfer cost allowed. Assuming that the output unit prices are known, another CDEA model is proposed whose objective function involves maximising the revenue increase (within a certain planning horizon) minus the cost of the resource transfers required. Another common way of restricting the resource transfers is by considering groups of OUs (e.g., based on a geographical or administrative criterion) so that inputs can only be transferred between OUs belonging to the same group. This happens in the local centralised resource-allocation approach of [48]; which considers several groups of OUs, so that resource transfers are possible within each group, and considers an additional group of OUs between which no resource transfer is possible. Groups are also considered in Ref. [16]; which considers two scenarios, one in which all groups of OUs have the same DEA technology and one in which each group has its own DEA technology. Interestingly, even when considering different groups of OUs, they distinguish between inputs that can only be transferred within each group (so-called regional resources) and inputs that can be transferred among OUs of all groups (so-called common resources). [17] assumes the existence of a flow network, with arc capacities and costs, over which the resource transfers would take place. They propose a lexicographic two-phase approach in which the first phase involves the maximisation of the weighted aggregated output increase, with subject bounds on the aggregate and OU input changes, on the OU output changes, and possibly with other types of constraints, including equity constraints on the input allocation. This is followed by a second phase that minimises the cost of resource flows. Another way to strive towards reducing the resource transfers between DMUs is through the application of the production stability concept in Ref. [42]; which, given the aggregate output changes, minimises the weighted changes in inputs and outputs of the different OUs. [22] employ the concept of difficulty coefficient as a weighted measure of the input and output changes for the various OUs. They consider two lexicographic variants (depending on whether aggregate input reduction or aggregate output reduction is given priority), in both of which an upper bound is imposed on the total difficulty of the OUs in reaching the computed targets. A different approach is proposed in Ref. [36]. While many CDEA approaches consider lexicographic optimisation in which the objective is to maintain the targets of the OUs sufficiently close to the observed OUs [36], propose a single-phase approach that uses an objective function that is computed as a ratio of the average absolute relative changes in the inputs and outputs of the OUs and where the denominator is the average increase in the relative aggregate output.

Note that, due to the lack of space, in the above review we have not included the extensive DEA literature on fixed cost and common revenue (e.g., Refs. [49,50]; Lozano 2104; [51], on fixed-sum inputs and fixed-sum outputs (e.g. Refs. [50,52–57], and on structural efficiency (e.g., Ref. [58], which have some of the characteristics of CDEA. However, these DEA problems usually lack a CDM, which makes them conceptually different to CDEA.

As it can be seen from the above literature review, the topic of CDEA and, in particular, centralized resource allocation, has received a great deal of attention in DEA with many approaches having a lexicographic character. However, the different objective functions considered in the existing approaches lack the simplicity and consistency of the AF of the GP methodology. Actually, compared with the GP methodology, existing centralized resource allocation methods look like ingenious, tailored approaches. Hence, it is surprising that, in spite of being a proven and effective methodology, no GP-based approach has been proposed so far. Moreover, the hierarchical character of the lexGP method perfectly suits the centralized resource allocation problem addressed in this paper. Thus, as shown in the next section, the objectives of the individual OUs and the CDM can be harmoniously integrated within the lexGP framework so that any potential conflict between them are avoided. At the same time, the multiobjective character of the problem, with multiples inputs and outputs, is implicitly and effectively handled via the input and output goals established by the CDM.

## 3. Proposed lexicographic Goal Programming CDEA approach

Before presenting the proposed CDEA approach, and in order to appreciate the differences, it may be interesting to first formulate the following basic CDEA model.

| Let,  | Data   |
|---|--|
| i=1,2,,m  | index on inputs                                  |
| k=1,2,,s  | index on outputs                                 |
| j,r=1,2,,n  | indices on observed OUs                          |
| $x_{ij}$  | amount of input <i>i</i> consumed by OU <i>j</i> |
| y <sub>ki</sub>                                     | amount of output k produced by OU                |
| - ,   | i  |
| $\mathbf{x}_{i}^{total} = \sum \mathbf{x}_{ii}$     | current aggregate consumption of                 |
| $i = \sum_{j=1}^{j} i_{j}$                          | input i  |
| $\mathbf{y}_{k}^{total} = \sum \mathbf{y}_{ki}$     | current aggregate production of                  |
| j k <u>j</u>  | output k   |
| Parameters  |  |
| $v_i, w_k$ Importance weights of input <i>i</i> and |  |
| output k, respectively $(\sum v_i + \sum w_k = 1)$  |  |
| $\overline{i}$ $\overline{k}$                       |  |
| Decision variables                                  |  |
| $\widehat{y}_{kr}$                                  | Target of output $k$ of DMU $r$                  |
| $\widehat{x}_{ir}$                                  | Target of input <i>i</i> of DMU <i>r</i>         |
| $(\lambda_{jr})_{i \in \{1,2,\dots,n\}}$            | intensity variables utilised to                  |
| <b>y</b> - <b>c</b> y y y y                         | compute the target of DMU r                      |
| $S_L^{\mathcal{V}}$                                 | increase in aggregate output k                   |
| s <sup>x</sup>                                      | reduction of aggregate input i                   |
| i.  | 00 00 1  |

Non-oriented, weighted additive CDEA model

s.t.

$$Max \quad \sum_{k} w_{k} \frac{s_{k}^{v}}{y_{k}^{total}} + \sum_{i} v_{i} \frac{s_{i}^{i}}{x_{i}^{total}} \tag{1}$$

$$\widehat{x}_{ir} = \sum_{j} \lambda_{jr} x_{ij} \quad \forall i, r$$
<sup>(2)</sup>

$$\widehat{y}_{kr} = \sum_{j} \lambda_{jr} y_{kj} \quad \forall k, r \tag{3}$$

$$\sum_{j} \lambda_{jr} = 1 \quad \forall r \tag{4}$$

$$\sum_{r} \hat{y}_{kr} = y_k^{total} + s_k^y \quad \forall k$$
<sup>(5)</sup>

$$\sum_{r} \widehat{x}_{ir} = x_i^{total} - s_i^x \quad \forall i$$
(6)

$$\lambda_{jr} \ge 0 \quad \forall j, r \quad s_i^x \ge 0 \quad \forall i \quad s_k^y \ge 0 \quad \forall k \quad \widehat{y}_{kr} \ge 0 \quad \forall k, r$$
(7)

The objective function (1) of the above non-oriented CDEA model aims to maximise the sum of the weighted relative increase in aggregate output and the weighted relative reduction of the aggregate inputs. Constraints (2)–(4) compute the input and output targets of each OU as a convex linear combination of the observed OUs. Constraints (5) and (6) compute the aggregate inputs and output improvements. Note that the aggregate inputs cannot increase and the aggregate outputs cannot decrease. Although the model computes targets for the individual OUs, the objective function only takes into account the overall aggregate improvements. Note also that, although the aforementioned model assumes a single Variable Returns to Scale (VRS) DEA technology, other DEA technologies or even multiple (i.e., heterogeneous) technologies (as in Refs. [43,44] can be considered.

The main issue with these types of basic CDEA approaches is that they generally have multiple alternative optima. The targets computed for different OUs can be arbitrarily permutated and, since the objective function only involves the aggregated inputs and outputs, then its value would not change. Furthermore, it can be shown that these basic CDEA models are equivalent to a projection of the average OU (i.e., a virtual OU whose input consumption is the average of the input consumption of the observed OUs and whose outputs are the average of the outputs of the observed OUs) using a conventional DEA model. Moreover [5], have shown how alternative optimal disaggregate targets can easily be generated once the conventional DEA projection of the average OU has been carried out.

The existence of alternative optima poses no problem in itself. The problem arises when the specific optimal solution computed sets targets that are far from the current operation point of several of the OUs. As indicated in the introduction, the further the target of an OU is, the harder the task becomes to make the corresponding input and output adjustments. In certain cases, even OUs that are Technically Efficient may be asked to modify their operating points, which may be considered unreasonable (e.g., Ref. [5]. In practice, this problem can be mitigated if there are non-discretionary/quasi-fixed inputs or non-transferable/non-reallocatable inputs as occurs in Ref. [59]; Wu et al. [9,16,30,31]; and [60]; among others. Moreover, a simple way of precluding the targets from being sufficiently close to the observed OUs is by imposing bounds on the changes in the inputs and outputs of the OUs as carried out, for example, in Refs. [21,61,62]; and [63]; among others. Another way of precluding the targets from being not too far from the observed OUs is by means of solving a Phase II model that minimises the changes in the inputs of the OUs. This occurs in, for example, [32,34,38]; and [26]; among others. There are also variants of this strategy; in Ref. [42]; the total output increases are given and the minimisation of the input and output changes (called production stability in the paper) is not the Phase II objective function but the main objective function. Alternatively, instead of two phases [36], propose a single-phase slack-based approach for centralised resource reallocation. Another way of both increasing the acceptability of the targets by each OU and anchoring the OUs around their current operating points so that their computed targets sufficiently close to them involves imposing constraints that guarantee that the current output level of each DMU is maintained (e.g., Refs. [12,27,33,37,43,44,64].

In this paper, an alternative way to allocate the resources and set the targets of the OUs in a CDEA setting is presented. It is based on GP so that goals on the input and output changes are established by the CDM, and the AF consists of minimizing the deviations from such goals. Note, however, that although the CDM sets aggregated input and output goals for the whole system, specific goals for each individual OU are also established. These goals have a hierarchical relationship whereby the aggregate goals have precedence over the goals of the individual OUs. This leads to a lexicographic GP approach that considers two basic priority levels. In the first level, the weighted sum of the deviations from the aggregated input and output goals is minimised, while in the second level, the same is carried out but instead for the weighted sum of the deviations from the OU specific input and output goals. Finally, an additional priority level is considered in order to guarantee the efficiency of the computed targets.

Additional parameters

- $\Delta y_k$  Desired minimum increase in the total amount of output *k*. This implies that the aggregate goal for output *k* is  $y_k^{total} + \Delta y_k$
- $\Delta x_i$  Desired maximum increase in the total amount of input *i*. This implies that the aggregate goal for input *i* is  $x_i^{iotal} + \Delta x_i$
- $\delta y_{kr}$  Desired minimum increase in the amount of output *k* produced by OU *r*. This implies that the goal for output *k* of OU *r* is  $y_{kr} + \delta y_{kr}$
- $\begin{aligned} \delta x_{ir} & \text{Desired maximum increase in the amount of input } i \text{ consumed by OU } r. \text{ This implies that the goal for input } i \text{ of OU } r \text{ is } x_{ir} + \delta x_{ir} \end{aligned}$
- $\omega_r$  Importance weight of OU r ( $\sum_r \omega_r = 1$ )

Note that, in order to achieve consistency between the aggregated and the OU specific input and output goals, the corresponding parameters should be consistent, that is,

$$\sum_{r} \delta y_{kr} \approx \Delta y_k \quad \forall k \tag{8}$$

$$\sum_{r} \delta x_{ir} \approx \Delta x_i \quad \forall i \tag{9}$$

Indeed, without loss of generality and in order to reduce the number of parameters, henceforth it will be assumed that

$$\delta y_{kr} \approx \omega_r \Delta y_k \quad \forall k, r \tag{10}$$

$$\delta x_{ir} \approx \omega_r \Delta x_i \quad \forall i, r \tag{11}$$

that is, the desired minimum output increase and the desired maximum input increase of a OU are assumed to be proportional to the importance weight of that OU.

Similarly, without loss of generality and in order to reduce the number of parameters, henceforth, it will be assumed that

$$\nu_r \approx \sum_i v_i \frac{x_{ir}}{x_i^{loral}} + \sum_k w_k \frac{y_{kr}}{y_k^{loral}} \quad \forall r$$
(12)

that is, the importance weight of each OU is assumed to be proportional to its weighted relative output level.

Therefore, the number of independent parameters of the proposed approach can be reduced to a minimum, just the importance of each input and output ( $\nu_i$  and  $w_k$ , respectively) and the desired total increase for each input and output ( $\Delta x_i$  and  $\Delta y_k$ , respectively). Note that sometimes we are interested in resource reallocation, in which case  $\Delta x_i = 0$   $\forall i$ . Alternatively, one may assume a growth scenario in which the CDM desires all the total outputs and all the total inputs to increase, that is,  $\Delta y_k \ge 0$   $\forall k$  and  $\Delta x_i \ge 0$   $\forall i$ . Of course, other scenarios in which some or all the outputs or some or all the inputs should be reduced can also be considered.

Additional decision variables

- $D_k^-$  Negative deviation from the desired minimum increase on the total amount of output k
- $D_i^+$  Positive deviation from the desired maximum increase on the total amount of input i
- *d<sub>kr</sub>* Negative deviation from the desired minimum increase on the amount of output *k* produced by OU *r*
- $d_{ir}^+$  Positive deviation from the desired maximum increase on the amount of input *i* consumed by OU *r*

Proposed lexGP CDEA model

$$Min \quad P_1\left(\sum_{i} v_i \frac{D_i^+}{x_i^{total} + \Delta x_i} + \sum_{k} w_k \frac{D_k^-}{y_k^{total} + \Delta y_k}\right) + P_2\left(\sum_{r} \omega_r \left[\sum_{i} v_i \frac{d_{ir}^+}{x_{ir} + \delta x_{ir}} + \sum_{k} w_k \frac{d_{kr}^-}{y_{kr} + \delta y_{kr}}\right]\right) + P_3\left(\sum_{r} \omega_r \left[\sum_{i} \frac{\widehat{x}_{ir}}{x_{ir} + \delta x_{ir}} - \sum_{k} w_k \frac{\widehat{y}_{kr}}{y_{kr} + \delta y_{kr}}\right]\right)$$

$$(13)$$

s.t.

$$\widehat{x}_{ir} = \sum_{j} \lambda_{jr} x_{ij} \quad \forall i, r$$
(14)

$$\widehat{y}_{kr} = \sum_{j} \lambda_{jr} y_{kj} \quad \forall k, r$$
(15)

$$\sum_{i} \lambda_{jr} = 1 \quad \forall r \tag{16}$$

 $\widehat{y}_{kr} + d_{kr}^{-} \ge y_{kr} + \delta y_{kr} \quad \forall k, r$ (17)

$$\sum_{r} \hat{y}_{kr} + D_k^- \ge y_k^{total} + \Delta y_k \quad \forall k$$
(18)

$$\widehat{x}_{ir} - d_{ir}^+ \le x_{ir} + \delta x_{ir} \quad \forall i, r$$
(19)

$$\sum_{r} \hat{x}_{ir} - D_{i}^{+} \leq x_{i}^{total} + \Delta x_{i} \quad \forall i$$

$$\lambda_{jr} \geq 0 \quad \forall j, r; \quad d_{kr}^{-} \geq 0 \quad \forall k, r; \quad d_{ir}^{+} \geq 0 \quad \forall i, r; \quad D_{k}^{-} \geq 0 \quad \forall k; \quad D_{i}^{+} \geq 0 \quad \forall i$$
(21)

Note that this model always has a feasible solution and that the first three sets of constraints are the same as in the basic CDEA approach. Thus, (14)–(16) compute the input and output targets of each OU as a convex linear combination of the observed OUs. Apart from the AF (13), the main differences lie in the goal constraints (17)-(20) and associated deviation variables. Neither goals nor variables that measure the deviation from them appear in the basic CDEA approach (nor in any non-GP centralized resource allocation approach for that matter). Conversely, they play an essential role in GP approaches like this. In our case, the output goals (both aggregate and OU specific) are of  $\geq$  type and hence they have associated negative deviation variables. The opposite occurs for the input goals (both aggregate and OU specific), which are of  $\leq$  type and therefore have associated positive deviations. These deviations appear in the AF (13). Thus, the model first tries to satisfy the aggregate input and output goals as much as possible. Once this first-priority objective  $(P_1)$  is attained, the model re-computes the targets of the OUs in an effort to satisfy all the OU-specific goals  $(P_2)$  as much as possible. Both the aggregate and OU-specific output goals represent minimum increases (maximum increases in the case of aggregate and OU specific input goals), which means that, once these minimum (respectively, maximum) amounts are satisfied, further improvements in input and outputs are not sought by the first two priority levels of the AF (13). Hence, in order to guarantee that the computed targets are technically efficient, it is necessary to consider an additional priority level  $(P_3)$  that exhausts any possible remaining input and output slacks in the computed targets. This third priority level can be viewed as minimizing the weighted sum of the deviations from an implicit utopian operation point with zero input consumption and infinite output goals. Note that, although the proposed approach above uses lexicographic GP, it can easily be modified to an extended lexicographic GP approach [65].

Finally, in the formulation presented above it has been assumed that all the inputs and outputs are discretionary. The proposed approach can also take into account non-discretionary variables. Thus, denoting the sets of discretionary and non-discretionary inputs as  $I^D$  and  $I^{ND}$ , respectively, and the sets of discretionary and non-discretionary outputs as  $O^D$  and  $O^{ND}$ , respectively, only the following adaptations need to be made to the above formulation:

$$D_i^+ = \Delta x_i = 0 \quad \forall i \in I^{ND}; \ d_{ir}^+ = \delta x_{ir} = 0 \quad \forall r, i \in I^{ND}$$

$$(22)$$

$$D_k^- = \Delta y_k = 0 \quad \forall k \in O^{ND}; \ d_{kr}^- = \delta y_{kr} = 0 \quad \forall r, k \in O^{ND}$$
(23)

$$Min \quad P_1\left(\sum_{i\in I^D} v_i \frac{D_i^+}{x_i^{total} + \Delta x_i} + \sum_{k\in O^D} w_k \frac{D_k^-}{y_k^{total} + \Delta y_k}\right) + P_2\left(\sum_r \omega_r \left[\sum_{i\in I^D} v_i \frac{d_{ir}^+}{x_{ir} + \delta x_{ir}} + \sum_{k\in O^D} w_k \frac{d_{kr}^-}{y_{kr} + \delta y_{kr}}\right]\right) + P_3\left(\sum_r \omega_r \left[\sum_{i\in I^D} \frac{\widehat{x}_{ir}}{x_{ir} + \delta x_{ir}} - \sum_{k\in O^D} w_k \frac{\widehat{y}_{kr}}{y_{kr} + \delta y_{kr}}\right]\right)$$

$$(24)$$

Expression (22) means that for non-discretionary inputs both the aggregate goal and the OU specific goals coincide with the observed values. Also, by fixing the corresponding deviations to zero, we are preventing those inputs to exceed the observed values. The interpretation of (23) is similar, i.e. for non-discretionary outputs both the aggregate goal and the OU specific goals coincide with the observed values. Also, by fixing the corresponding deviations to zero, we are preventing those output to fall below the observed values. As regards (24), it takes into account that the deviations for the non-discretionary inputs and outputs are fixed to zero and that, due to its non-

discretionary character, the model should not seek reducing them (in the case of inputs) or increase them (in the case of outputs).

## 4. Application

As indicated in the literature review, one of the fields where CDEA has been applied is that of Education, especially Higher Education Institutions. [8]; for instance, consider 11 branches of the Islamic Azad University in Iran. Their study focuses on the outputs of these branches and considers four outputs: assessment score, scientific publications, external research funding obtained, and the number of students. They employ a bi-objective DEA-based production planning approach using a multiplier formulation with estimated OU allocation ratios based on the current efficiency levels of the OUs, which are assumed to be maintained. [10] studies 64 Chinese universities (34 of which are funded by the 985 Project) using five inputs (the number of R&D staff, the number of faculty members, the number of postgraduates, the amount of research grant from the government, and the amount of research grant from the industry) and three inputs (the number of papers published, the number of domestic and international patent applications submitted, and the revenue gained from knowledge transfer to the private sector). The study focuses on the re-allocation of government funding. An envelopment CDEA model is utilised to maximise the weighted increase in aggregate outputs subject to bounds on the input and output changes for the overall system as well as for the individual OUs. The latter are assumed to maintain their current efficiency levels. Hossein Yadolladi and Kazemi Matin (2021) consider 20 campus branches of the Islamic Azad University, and model them using a two-stage network DEA system. Stage 1 (Education Department) consumes three inputs (number of employees, number of students, and number of faculty members) and produces two intermediate products (number of graduates and tuition fees), which are in turn consumed by Stage 2 (Research Department) to produce two final outputs (total university income and number of research products). Their study considers three scenarios. In the first two, a non-radial, non-oriented optimisation of aggregate inputs and outputs is considered with a flexible and a fixed downsizing criterion, respectively. In the third scenario, in Phase I, the number of OUs to keep open is minimised subject to constraints on the minimum aggregate outputs and on the maximum aggregate inputs, while in Phase II, for the minimum number of OUs computed, the non-radial, non-oriented optimisation of aggregate inputs and outputs is carried out. In Ref. [12]; the optimal structure of the Spanish public universities is analysed, whereby possible splits and mergers are considered. As a final step, a non-oriented, slack-based resource reallocation CDEA model is used with bounds on the target outputs of the OUs to guarantee that no decrease in these outputs occurs in any case. Of the three inputs considered, two (labour cost and other costs) are discretionary and the third input (number of students) is non-discretionary. The three outputs (number of student credits passed, number of publications, and R&D income) are all discretionary. It should be borne in mind that the bargaining approach proposed in Ref. [66] has not been listed among the existing CDEA studies: their approach leads to mathematical models that have similarities to CDEA models but where the OU targets are computed bottom-up instead of top-down, that is, they are computed by the OUs voluntarily reaching a type of satisfactory, decentralised agreement on the resource allocation (instead of the resource allocation being decided by a CDM).

The application of the proposed approach to the Spanish public university system is presented below. The Spanish university system comprises a total of 84 universities, including 50 public universities and 34 private institutions. In this study, only the public institutions have been considered, since our interest resides in the reallocation of public resources among these institutions. On-line and special universities have also been excluded to ensure the homogeneity of OUs. Thus, the dataset employed herein involves 47 OUs.

The inputs and outputs considered are the same as in Ref. [66] but

the most recent data available from the year 2019 is used. Thus, the first input considered in the analysis measures the number of students enrolled in each university in 2019 (labelled as Students). This input has traditionally been considered to calibrate the size of the institutions and is included as a non-discretionary variable. The second input considered corresponds to the public funds received in 2019 (Public funding). The largest source of income of Spanish public universities are the public funds received from central and regional governments. Hence, this input is a surrogate of the financial resources of each OU. The third input considered is a measure of the labour costs of each university (including both academic and administrative staff) in 2019 (Labour Cost). All this data has been drawn from the official website of the Spanish Ministry of Universities [67].

As regards the outputs, the first one is the total number of credits passed by graduate and postgraduate students in year 2019 (Student Credits), as a measure of the teaching outputs of each university (sourced from Ref. [67]. The second and third output aims to gauge the research outputs of the institutions. To this end, the number of publications in journals indexed on the Science and Social Sciences Citation Indices in year 2019 (Publications) has been considered together with the total income obtained from R&D and consulting projects in year 2018 (R&D Projects). The source of this data is the IUNE Observatory [68]. Table 1 shows the summary of the statistics of the variables considered in this study.

First of all, we solve a conventional non-oriented, weighted additive non-centralised DEA model to determine the OUs technical inefficiency scores. A weight  $v_i = 0.25$  has been assigned to each of the two discretionary variables and, for the outputs, a weight of 0.1, 0.2, and 0.2 have been assigned to Students Credits, Publications, and R&D Projects, respectively. This weighting structure is balanced between inputs and outputs and gives greater weight to the research outputs than to the teaching output. Although Variable Returns to Scale (VRS) have been assumed, the corresponding Constant Returns to Scale (CRS) model has also been solved in order to determine both the scale inefficiency (which can be approximately gauged by the difference between the CRS and VRS inefficiency scores) and the returns to scale of each OU. Fig. 1 shows all this information. As can be observed there are only seven OUs with the Most Productive Scale Size (MPSS). These technically efficient and scale-efficient universities include UAB, UAM, UC3M, UPF, UPO, UPV, and URJC. Six other universities (namely UNICAN, UPCT, UNIRIOJA, UB, UV, and UCM) are technically efficient but not scale efficient. Of these universities, UNICAN, UPCT and UNIRIOJA exhibit Decreasing Returns to Scale (DRS) while UB, UV, and UCM exhibit Increasing Returns to Scale (IRS). The other 34 universities are inefficient and most exhibit DRS, except UMA, US, UGR, UM, and EHU, which exhibit IRS.

Fig. 2 shows the normalised input and output slacks of the three outputs and the two discretionary inputs. In order to be consistent with (1), these slacks are normalised by the total observed inputs and outputs. The ordering of the universities is the same as that used in Fig. 1, that is, in increasing order of their VRS inefficiency scores. Thus, the slacks for the first 13 technical efficient universities are all zero. For the remaining universities, the grey level indicates the potential improvement in the corresponding variables. Several universities, such as UGR, UM, UMA, and US, have slacks in certain variables but not in others although most inefficient university is EHU, with the largest normalised slacks in three of the variables. But even that most inefficient university has a zero slack

| Table 1 |        |            |            |        |        |          |
|---------|--------|------------|------------|--------|--------|----------|
| Summary | of the | statistics | of input a | and ou | tput v | ariables |

for the R&D Project output. The usefulness of Fig. 2 is that the information can be assessed row-wise or column-wise. Row-wise it shows, for each university, the dimensions in which improvements are possible and the magnitude of these potential improvements. Therefore, for example, the US has margin for improvement in Labour costs, Publications, and R&D Projects, in that order. Column-wise it is apparent which universities have more room for improvement in a specific dimension. Thus, for example, as regards Labour costs, the universities with the most excessive input are EHU, UPC, UGR, US, and UMA, followed by USC, UNIOVI, UVA, and UCLM.

The left panel of Fig. 3 shows the aggregated target values computed by the non-centralised DEA model, the centralised DEA model (1)-(7), and by the three priority levels (labelled as P1, P2, and P3) of the proposed lexGP approach. For comparative purposes, in the right panel, we also show the results from the approaches proposed in Refs. [10,26]. In both cases, we have adapted the models for the case study presented here, maintaining the preference structure for the relative importance of inputs and outputs, and including the goals for each disposal variable as constraints of the models. That is to say, the increase (respectively, decrease) of the aggregated outputs (respectively, inputs) should achieve at least the values fixed for the goals in our proposal. For the model proposed in Ref. [10]; the author includes bounds for the permissible variations of each variable of each OU. We have considered that the increase (respectively, decrease) of each output (respectively, input) can be up to two times the value used for the OU specific goals. For the model proposed in Ref. [26]; the reallocation of the aggregate input and output changes, proposed as a second phase, has been done considering, for each OU, its current share of each output variable in the corresponding aggregate output.

In all the cases, in Fig. 3, a value of 1.0 represents the total observed inputs and outputs. Hence, a value of 0.95 for an input dimension indicates that the aggregated target corresponds to 95% of the total observed value, that is, a 5% reduction in total input consumption. Similarly, a value of 1.2 for an output dimension corresponds to a 20% increase in the corresponding total output (with respect to the observed value). In order to interpret Fig. 3, let us first look at the results of the non-centralised approach. They lead to significant reductions (of 10%–15%) in the total consumption of the two discretionary inputs as well as significant increases in Publications and R&D Projects (30%–35%) and a smaller increase (of just 5%) in the Student Credits output. The centralised approach, model (1)–(7), leads to smaller reduction (less than 10%) in the discretionary inputs, a similar increase (5%) in the Student Credits output and larger increases (40%–50%) in Publications and R&D Projects.

As regards the proposed lexGP approach, the CDM has first to establish the goals on the total inputs and outputs. As indicated in Fig. 3, these goals are reductions of 10% and 5% in Public Funding and Labour Costs, respectively, an increase of 5% in Student Credits, and increases of 25% and 20% in Publications and R&D Projects, respectively. Note that the three priority levels considered in the proposed approach do not depend on the application considered. Thus, the first priority level corresponds to the aggregate level and minimises the deviations from the aggregate input and output goals. The second priority level, on the hand, corresponds to the individual OUs level and minimises the sum of the deviations from the input and output goals of each OU. The third priority level, finally, aims at minimizing the inputs consumed and the outputs produced by each OU and, hence, guarantees that the computed

|      | Students  | Public funding (10 <sup>3</sup> euros) | Labour costs (10 <sup>3</sup> euros) | Student Credits (10 <sup>3</sup> ECTS) | Publications | R&D Projects (10 <sup>3</sup> euros) |
|------|-----------|--|--------------------------------------|--|--------------|--------------------------------------|
| Min  | 3526.0    | 36,402.88                              | 34,618.23                            | 145.90                                 | 345.0        | 2211.00                              |
| Max  | 57,360.0  | 366,679.11                             | 389,793.52                           | 2855.91                                | 6074.0       | 81,475.00                            |
| Mean | 20,286.5  | 131,047.58                             | 140,453.97                           | 941.10                                 | 1685.0       | 24,910.47                            |
| Sum  | 953,463.0 | 6,159,236.38                           | 6,601,336.44                         | 44,231.61                              | 79,195.0     | 1,170,792.00                         |



Fig. 1. Non-centralised inefficiency scores and returns to scale

Note: VRS=Variable Returns to Scale, CRS=Constant Returns to Scale, IRS=Increasing Returns to Scale, DRS = Decreasing Returns to Scale, MPSS = Most Productive Scale Size.

targets are technical efficient, i.e. after getting as close as possible to the aggregate and OU specific input and output goals any remaining input and output slacks are exhausted.

In the present application, the targets computed by the first level of priority (P1) are such that there are no deviations from the

corresponding aggregate goals (zero value of the P1 achievement function). For the second priority level, the input and output goals of each university must be established. Although in principle these should be agreed by the CDM (i.e., the Ministry of Universities) with each university, we have derived these goals using (10)-(12). With these OU

|          | Public  | Labor costs | Students | Bublications | R&D      |     |
|----------|---------|-------------|----------|--------------|----------|-----|
|          | funding | Labor costs | Credits  | Fublications | Projects |     |
| UAB      |         |             |          |              |          |     |
| UAM      |         |             |          |              |          |     |
| UC3M     |         |             |          |              |          |     |
| UPF      |         |             |          |              |          |     |
| UPO      |         |             |          |              |          |     |
| UPV      |         |             |          |              |          |     |
| URJC     |         |             |          |              |          |     |
| UB       |         |             |          |              |          |     |
| UV       |         |             |          |              |          |     |
| UCM      |         |             |          |              |          |     |
| UNICAN   |         |             |          |              |          | Mir |
| UPCT     |         |             |          |              |          |     |
| UCO      |         |             |          |              |          |     |
| UNIRIOJA |         |             |          |              |          | Ma  |
| имн      |         |             |          |              |          |     |
| UMA      |         |             |          |              |          |     |
| UDL      |         |             |          |              |          |     |
| UBU      |         |             |          |              |          |     |
| UDG      |         |             |          |              |          |     |
| URV      |         |             |          |              |          |     |
| UNILEON  |         |             |          |              |          |     |
| UAL      |         |             |          |              |          |     |
| UIB      |         |             |          |              |          |     |
| USC      |         |             |          |              |          |     |
| ILU      |         |             |          |              |          |     |
| UNAVARRA |         |             |          |              |          |     |
| US       |         |             |          |              |          |     |
| UHU      |         |             |          |              |          |     |
| UJAEN    |         |             |          |              |          |     |
| UDC      |         |             |          |              |          |     |
| ULPGC    |         |             |          |              |          |     |
| UNEX     |         |             |          |              |          |     |
| UVIGO    |         | 1           |          |              |          |     |
| UAH      | _       |             |          |              |          |     |
| UGR      |         |             |          |              |          |     |
| USAL     |         |             |          |              |          |     |
| UPM      |         |             |          |              |          |     |
| ULL      |         |             |          |              |          |     |
| UNIOVI   |         |             |          |              |          |     |
| UCA      |         |             |          |              |          |     |
| UVA      |         |             |          |              |          |     |
| UNIZAR   |         |             |          |              |          |     |
| UA       |         |             |          |              |          |     |
| UM       |         |             |          |              |          |     |
| UPC      |         |             |          |              |          |     |
| UCLM     |         |             |          |              |          |     |
| EHU      |         |             |          |              |          |     |

Fig. 2. Normalised input and output slacks Note: Slacks normalised by total observed inputs and outputs.

specific goals, the P2 priority level recomputes the targets so that, as indicated in Fig. 3, the total input reductions and output improvements are similar to those of P1. From this point of view, this may not be seen as a major step. However, the opposite is true. In fact, although the total input and output targets hardly change from P1 to P2, the deviations of the OU targets from their corresponding goals have been minimised (to a P2 achievement function of 0.0518444). Since the P1 priority level has many widely differing alternative optima, this second lexicographic step is essential for the computation of reasonable targets that are sufficiently close to the OU specific goals. Finally, since the P1 and P2 levels penalise positive deviation in the input consumption goals and negative deviation in the output production goals, input and output slacks may remain in certain OUs, that is, there may be universities that could still reduce their inputs below their stated goals and/or increase their outputs above their stated goals. From a purely satisficing point of view, the P3 priority level is unnecessary since both the aggregate and the OU specific goals have already been taken into account and achieved as much as possible. However, in the DEA, achieving technically efficient targets is also an implicit goal. Hence, the P3 priority level strives to exhaust all remaining input and output slacks from the targets thus leading to minor additional reductions (of approximately 2%) in the inputs, no increase in

the Student Credits output, and a significant additional increase (of approximately 10%) in Publications and R&D Projects. Note that although, in the end, the total inputs and outputs computed by the proposed lexGP approach have a level similar to those of the noncentralised approach, the OU specific targets are completely different. Thus, the non-centralised approach only considers targets that dominate the observed DMU and, by its own nature, fails to consider and control the total input and output improvements that can be achieved. In contrast, the proposed lexGP approach adopts a centralised, systemwide perspective that allows the CDM to set not only aggregate but also specific OU goals that involve increasing or decreasing each input and output by a specific amount. The proposed projection process is more controlled and more satisficing than both the non-centralised DEA approach and the basic centralised DEA approach that does not use a Phase II to choose from among the large number of alternative optima. In the case of the [10] results, the aggregate inputs and outputs practically coincide with the aggregate goals but do not improve upon them. The [26] aggregate results, on the other hand, achieved and improved the established aggregate goals. Actually, except as regards Labour costs, the improvements over the established goals are larger than those of the proposed approach. Note in this regard that the rationale behind GP is one of satisficing the goals. Exceeding them is permitted but not incentivized.

Fig. 4 shows three cases that can illustrate the variety of situations that can occur as regards the targets computed for each OU. Consider, for example, EHU. Note that in this figure, the reference value 1.0 corresponds to the observed inputs and outputs. The non-centralised, the centralised, and the P1 priority level of the proposed approach would reduce the two discretionary inputs (by approximately 40%) and increase the Publications output (by as much as 80%). The P2 priority level, however, considers goals of a more modest nature although they are goals that represent improvements in all the input and output dimensions and, as a result, lead to a solution that exceeds all those goals and remains technically efficient and cannot be further improved by the P3 priority level. Now consider the UMA case. The non-centralised approach only improves (by approximately 25%) Labour Costs and Publications. The centralised approach significantly increases both the inputs and the outputs, (by approximately 200% in the case of Publications). The P1 priority level of the proposed approach computes target inputs and outputs equal to the observed levels. The P2 priority level is able to satisfy the specific targets for this university as regards Labour Costs and Publications although there are deviations from the goals in the other three dimensions. The P3 priority level does not seem to improve any of the variables, which means the P2 targets were already technically efficient. Finally, let us consider the case of URJC. This university is technically efficient and that is why the non-centralised approach fails to produce any improvement in any dimension. The centralised approach assigns targets to this university that increase the inputs and two of the outputs significantly. Such targets are unrealistic and unlikely to be achieved in the medium term. In the P1 priority level, the proposed approach leaves the inputs and outputs equal to their current values. Such values do not satisfy the specific goals of this university, which require a small reduction in the inputs and a small increase in the outputs. In the end, the target computed by P2 (and P3) priority levels is able to satisfy the Publications goal and almost satisfies the other two output goals but at the cost of increasing the two inputs, and hence incurs a deviation from the goals in these two dimensions.

Finally, Fig. 5 shows the boxplots of the ratio of the OU input and output targets over their respective goals for the different universities. The figure shows the results of the proposed lexGP approach as well as those of [10,26]. In all cases, the value 1.0 along the vertical axis corresponds to the OU specific goals. Values below 1.0 for the inputs and above 1.0 for the outputs imply that the goals are satisfied (i.e., there are no deviations). Conversely, values above 1.0 for the inputs and below 1.0 for the outputs imply deviations from the goals. Although these deviations are minimised in the P2 priority level (i.e., after the P1



**Fig. 3.** Aggregated target values of proposed and existing approaches Note: Value 1.0 corresponds to total observed inputs and outputs.

priority level has been solved), many universities deviate from their OU specific goals. This has already been shown in Fig. 4. Note, however, that the average ratio is always slightly less than unity for the inputs, slightly above unity for the Student Credits output, and significantly above unity for Publications and R&D Projects. The latter is in fact the variable in which the OU specific goals are more generally exceeded. Note that the horizontal line within each boxplot represents the median. Given that the median is very close to unity for all variables, this means that the cases of targets exceeding the goals are almost as frequent as the opposite cases, that is, cases with targets that fail to reach the corresponding goals. Finally, note that the results obtained of [10] present the smallest range of variation around the OU specific goals, followed the proposed lexGP approach and, lastly, by Ref. [26]. The large range of variation of the latter means in some cases greatly exceeding the OU specific goals but also, in other cases, falling short by a significant amount.

## 5. Conclusions

In this paper, a parametrised approach for centralised resource allocation and target setting is proposed. It uses lexicographic GP and considers three priority levels. The P1 priority level refers to the goals on the total input consumption and output production. This reflects the centralised character of the problem and the system-wide performance criterion adopted. It is not an optimisation approach as are the basic radial and non-radial CDEA approaches. Indeed, it is based on a satisficing criterion, which means that satisficing the established goals is the main objective. The goals are set by the CDM according to their preferences and objectives and, in particular, can involve increases or decreases of total inputs and simultaneous increases or decreases of total outputs. Once these aggregate input and output goals are maximally achieved, it is the turn of the P2 priority level, which is based on OU specific goals. From a mathematical modelling point of view, these goals anchor the targets of the OUs which help to choose from among the many alternative optima of the first priority level in an effort to deviate minimally from the OU specific targets. From a management perspective, the specific OU goals allow the CDM to consider the specific circumstances (e.g., local competition, management quality, and local returns to scale), adverse or otherwise, of each OU, thus adjusting its targets to these circumstances. The third priority level P3 aims to remove any input and output slacks that may exist and reflects the implicit goal that the final targets be technically efficient. This last step



offers the OUs incentives to reduce their input consumption below the levels of the corresponding goals and to increase the outputs above the levels of the corresponding goals. In other words, this step (but subject to the two previous higher-priority levels) aims not only to satisfice the goals but also to exceed them, whenever possible.

The lexicographic GP framework represents a solid theoretical support for the proposed approach and suits perfectly the hierarchical nature of the resource allocation process, allowing a harmonious and effective integration of the organizational goals with those of the individual OUs. This is facilitated by the fact that the GP methodology is based on the satisficing concept which is a more flexible and accommodating approach than conventional optimisation. All these features are missing in existing centralized resource allocation methods.

In order to illustrate the working and the usefulness of the proposed approach, it has been applied to the Spanish public university system, comprising 47 institutions. The data used corresponds to that most recent available (i.e., data from year 2019) and considers Number of Students (non-discretionary), Public Funding, and Labour Costs as inputs, and Student Credits passed, Publications, and Income from R&D Projects as outputs. A non-centralised efficiency assessment is first carried out to identify global and technically efficient universities and to estimate potential input and output improvements for each university and for the system as a whole. A basic CDEA approach aimed at maximising the weighted improvements in total inputs and outputs is subsequently applied. The problem with this type of basic CDEA approach is that they have many alternative optima and hence require a Phase II to choose from among them using a reasonable criterion. As an alternative, the proposed lexicographic GP approach is used after having set appropriate goals for the total consumption of the two discretionary inputs and for the total production of the three outputs. After maximally satisficing these aggregate goals, specific input and output goals for each university are considered. Although a type of proportional method has been assumed to consistently translate the aggregate system goals into specific input and output goals for each university, the method allows the CDM to establish specific goals tailored to the characteristics and strategies of each university. The targets thus obtained minimise the deviations from these specific university goals but may be technically inefficient. This is corrected by the third priority level, which finally exhausts any input and output slacks that might remain. The results reported show the validity of the proposed approach and the higher level of control that it provides for the CDM to allocate the resources among the universities and to set their targets. The lexicographic character of



Fig. 4. Target values of non-centralised, centralised, and proposed lexGP models for several OUs. Note: Value 1.0 corresponds to the observed inputs and outputs of the corresponding OU.



**Fig. 5.** Boxplots of the ratio of OU input and output targets over their respective goals for proposed and existing approaches Note: Value 1.0 corresponds to input and output goals of the corresponding OUs.

the proposed approach suits the hierarchy of global and local objectives associated with the Ministry and the universities, respectively, and its orderly and transparent character facilitates the explanation of the process and communication of its results.

It is interesting to point out that the case study developed here corresponds to the base scenario that have been studied for the Spanish university system for the coming years. Specifically, this scenario, which anticipates a looming economic and public finance crisis, considers a small reduction of the resources consumed by the system compatible with a moderate increase of the outputs. Note that these targets, together with the weighing scheme assigned to the variables, represent implicitly a preference structure fixed by the CDM. The consideration of the lexicographic GP methodology provides degree of control and flexibility. Once the models have been solved, the given preference structure can be reconsidered by the policymakers in the light of the outcomes, so that more emphasis on some variables at the aggregate or at the OU specific level may be given.

As possible continuations of this research, on the practical level, it would be very useful to build a Decision Support System based on the proposed approach to facilitate the task of selecting input and output weights and aggregate and specific OU goals. The idea is that the results of the method depend on these parameters and hence a tool to assist the CDM to set those parameters by using certain metacriteria could be of practical use. On the theoretical level, an extension of the proposed approach to network DEA systems as well as to scenarios with integer or fuzzy data is also desirable. It is also desirable to apply the proposed approach to other centralised settings not only in education (e.g., primary and secondary schools) but also in transportation (e.g., bus services, airlines), energy (e.g., power plants), health care (e.g., medical clinics, hospitals), and tourism (e.g., restaurant and hotel chains).

## Author statement

Both authors have contributed to the final version of the paper. In particular, Sebastian Lozano developed the theoretical formalism. Ignacio Contreras and Sebastian Lozano performed the analytic calculations and performed the numerical simulations.

At the final stage, both authors contributed to the final version of the manuscript.

Sebastian Lozano supervised the project.

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#### References

- Banker RD, Charnes A, Cooper WW. Some models for estimating technical and scale inefficiencies in data envelopment analysis. Manag Sci 1984;30(9):1078–92.
- [2] Tone K. A slacks-based measure of efficiency in data envelopment analysis. Eur J Oper Res 2001;130:498–509.
- [3] Fukuyama H, Weber WL. A directional slacks-based measure of technical inefficiency. Soc Econ Plann Sci 2009;43:274–87.
- [4] Lozano S, Villa G. Centralized resource allocation using data envelopment analysis. J Prod Anal 2004;22:143–61.
- [5] Asmild M, Paradi JC, Pastor JT. Centralized resource allocation BCC Models. Omega 2009;37:40–9.
- [6] Hosseinzadeh Lofti F, Noora AA, Jahanshahloo GR, Gerami J, Mozaffari MR. Centralized resource allocation for enhanced Russell models. J Comput Appl Math 2010;235:1–10.
- [7] Mar-Molinero C, Prior D, Segovia MM, Portillo F. On centralized resource utilization and its reallocation by using DEA. Ann Oper Res 2014;221:273–83.

- [8] Amirteimoori A, Daneshian B, Kordrostami S, Shahroodi K. Production planning in data envelopment analysis without explicit inputs. Oper Res 2013;47:273–84.
- [9] López-Torres L, Prior D. Centralized allocation of human resources. An application to public schools. Comput Oper Res 2016;73:104–14.
- [10] Wang DD. Performance-based resource allocation for higher education institutions in China. Soc Econ Plann Sci 2019;65:66–75.
- [11] Hossein Yadolladi A, Kazemi Matin R. Centralized resource allocation with the possibility of downsizing in two-stage network production systems. Oper Res 2021; 55:2583–98.
- [12] Contreras I, Lozano S. Size efficiency, splits and merger gains, and centralized resource reallocation of Spanish public universities. Soc Econ Plann Sci 2022;81: 101190.
- [13] Korhonen P, Syrjänen M. Resource allocation based on efficiency analysis. Manag Sci 2004;50:1134–44.
- [14] Fang L. Centralized resource allocation based on efficiency analysis for step-by-step improvement paths. Omega 2015;51:24–8.
- [15] Yang FC. Application of centralized DEA in an automobile parts retail network in Taiwan. Int J Retail Distrib Manag 2020;48(7):667–86.
- [16] Shao Y, Bi G, Yang F, Xia Q. Resource allocation for branch network system with considering heterogeneity based on DEA method. Cent Eur J Oper Res 2018;26: 1005–25.
- [17] Tao X, Xiong B, An Q. DEA-based centralized resource allocation with network flows. Int Trans Oper Res 2021;28:926–58.
- [18] Amirteimoori A, Kordrostami S. Production planning: a DEA-based approach. Int J Adv Manuf Technol 2011;56:369–76.
- [19] Giménez-García VM, Martínez-Parra JL, Buffa FP. Improving resource utilization in multi-unit networked organizations: the case of a Spanish restaurant chain. Tourism Manag 2007;28:262–70.
- [20] Du J, Liang L, Chen Y, Gon-bing B. DEA-based production planning. Omega 2010; 38:105–12.
- [21] Zhang Y, Zhang H, Zhang R, Zeng Z, Wang Z. DEA-based production planning considering influencing factors. J Oper Res Soc 2015;66:1878–86.
- [22] Yang T, Wang P, Li F. Centralized resource allocation and target setting based on data envelopment analysis model. Math Probl Eng 2018:3826096. 2018.
- [23] Villa G, Lozano S, Redondo S. Data envelopment analysis approach to energysaving projects selection in an energy service company. Mathematics 2021;9:200.
- [24] Narasimhan R, Talluri S, Sarkis J, Ross A. Efficient service location design in government services A decision support system framework. J Oper Manag 2005;23: 163–78.
- [25] Liang C, Wang B, Ding T, Ma Y. DEA based production planning considering technology heterogeneity with undesirable outputs. Oper Res 2020;54:325–39.
- [26] Arocena P, Cabasés F, Pascual P. A centralized directional distance model for efficient and horizontally equitable grants allocation to local governments. Soc Econ Plann Sci 2022;81:100947.
- [27] Lozano S, Villa G, Brännlund R. Centralized reallocation of emission permits using DEA. Eur J Oper Res 2009;193:752–60.
- [28] Sun J, Wu J, Liang L, Zhong RY, Huang GQ. Allocation of emission permits using DEA: centralized and individual points of view. Int J Prod Res 2014;52:419–35.
- [29] Feng C, Chu F, Ding J, Bi G, Liang L. Carbon Emissions Abatement (CEA) allocation and compensation schemes based on DEA. Omega 2015;53:78–89.
- [30] Wu J, Chu JF, Liang L. Target setting and allocation of carbon emissions abatement based on DEA and closest target: an application to 20 APEC economies. Nat Hazards 2016;84:S279–96.
- [31] Wu J, Zhu Q, Liang L. CO2 emissions and energy intensity reduction allocation over provincial industrial sectors in China. Appl Energy 2016;166:282–91.
- [32] Lozano S, Villa G, Canca D. Application of centralized DEA approach to capital budgeting in Spanish ports. Comput Ind Eng 2011;60:455–65.
  [33] Chang SM, Wang JS, Yu MM, Shang KC, Lin SH, Hsiao B. An application of
- [33] Chang SM, Wang JS, Yu MM, Shang KC, Lin SH, Hsiao B. An application of centralized data envelopment analysis in resource allocation in container terminal operations. Marit Pol Manag 2015;42:776–88.
- [34] Yu MM, Chen LH. Centralized resource allocation with emission resistance in a two-stage production system: evidence from a Taiwan's container shipping company. Transport Res Part A 2016;94:650–71.
- [35] Chen MC, Yu MM, Ho YT. Using network centralized data envelopment analysis for shipping line resource allocation. Int J Environ Sci Technol 2018;15:1777–92.
- [36] Yu MM, Hsiao B. Single-phase slack-based centralized DEA for resource reallocation. Int Trans Oper Res 2018;25:737–51.
- [37] Ripoll-Zarraga AE, Lozano S. A centralized DEA approach to resource reallocation in Spanish airports. Ann Oper Res 2020;288:701–32.
- [38] Hsiao B, Shu LC, Yu MM, Shen LK, Wang DJ. Performance evaluation of the Taiwan railway administration. Ann Oper Res 2017;259:119–56.
- [39] Yang CC. Measuring health indicators and allocating health resources: a DEA-based approach. Health Care Manag Sci 2017;20:365–78.
- [40] Fang L. A generalized DEA model for centralized resource allocation. Eur J Oper Res 2013;228:405–12.
- [41] Davtalab-Olyaie M, Mahmudi-Baram H, Asgharian M. Measuring individual efficiency and unit influence in centrally managed systems. Ann Oper Res 2022. https://doi.org/10.1007/s10479-022-04676-6.
- [42] Wu H, Yang J, Chen Y, Liang L, Chen Y. DEA based production planning considering production stability. INFOR Inf Syst Oper Res 2019;57:477–94.
- [43] Lozano S. Company-wide production planning using a multiple technology DEA approach. J Oper Res Soc 2014;65:723–34.
- [44] Lozano S. Nonradial approach to allocating fixed-costs and common revenue using centralized DEA. Int J Inf Technol Decis Making 2014;13:29–46.
- [45] Amirteimoori A, Kordrostami S. Production planning in data envelopment analysis. Int J Prod Econ 2012;140:212–8.

#### S. Lozano and I. Contreras

- [46] Lozano S, Villa G. Centralized DEA models with the possibility of downsizing. J Oper Res Soc 2005;56:357–64.
- [47] Pachkova EV. Restricted reallocation of resources. Eur J Oper Res 2009;196: 1049–57.
- [48] Yang CC. Local resource reallocation considering practical feasibility: a centralized data envelopment analysis approach. Appl Econ 2017;49:5709–21.
- [49] Li Y, Yang M, Chen Y, Dai Q, Liang L. Allocating a fixed cost based on data envelopment analysis and satisfaction degree. Omega 2013;41:55–60.
- [50] Du J, Cook WD, Liang L, Zhu J. Fixed cost and resource allocation based on DEA cross-efficiency. Eur J Oper Res 2014;235:206–14.
- [51] Li F, Yan Z, Zhu Q, Yin M, Kou G. Allocating a fixed cost across decision making units with explicitly considering efficiency rankings. J Oper Res Soc 2021;72: 1432–46.
- [52] Yang M, Li YJ, Liang L. A generalized equilibrium efficient frontier data envelopment analysis approach for evaluating DMUs with fixed-sum outputs. Eur J Oper Res 2015;246:209–17.
- [53] Fang L. A new approach for achievement of the equilibrium efficient frontier with fixed-sum outputs. J Oper Res Soc 2016;67:412–20.
- [54] Amirteimoori A, Masrouri S, Yang F, Kordrostami S. Context-based competition strategy and performance analysis with fixed-sum outputs: an application to banking sector. J Oper Res Soc 2017;68:1461–9.
- [55] Amirteimoori H, Amirteimoori A, Karbasian M. Performance measurement of gas companies with fixed-sum inputs: a DEA-based model. J Econ Stud 2020;47: 1591–603.
- [56] Zhu Q, Wu J, Song M, Liang L. A unique equilibrium efficient frontier with fixedsum outputs in data envelopment analysis. J Oper Res Soc 2017;68:1483–90.
- [57] Zhu Q, Song M, Wu J. Extended secondary goal approach for common equilibrium efficient frontier selection in DEA with fixed-sum outputs. Comput Ind Eng 2020; 144:106483.
- [58] Cesaroni G. Technically and cost-efficient centralized allocations in data envelopment analysis. Soc Econ Plann Sci 2020;70:100734.
- [59] Fang L, Li H. Centralized resource allocation based on the cost-revenue analysis. Comput Ind Eng 2015:85:395–401.
- [60] Sadeghi J, Dehnokhalaji A. A comprehensive method for the centralized resource allocation in DEA. Comput Ind Eng 2019;127:344–52.
- [61] Nasrabadi N, Dehnokhalaji A, Kiani NA, Korhonen PJ, Wallenius J. Resource allocation for performance improvement. Ann Oper Res 2012;196:459–68.
- [62] Wu J, Chu J, An Q, Sun J, Yin P. Resource reallocation and target setting for improving environmental performance of DMUs: an application to regional highway transportation systems in China. Transport Res Part D 2018;61:204–16.

- [63] Fang L, Li H, Wang Z. Centralized resource allocation based on the bargaining approach. Asia Pac J Oper Res 2021;38:2050049.
- [64] Wu J, An Q, Ali S, Liang L. DEA based resource allocation considering environmental factors. Math Comput Model 2013;58:1128–37.
- [65] Romero C. Extended lexicographic goal programming: a unifying approach. Omega 2001;29:63–71.
- [66] Contreras I, Lozano S. Allocating additional resources to public universities. A DEA bargaining approach. Soc Econ Plann Sci 2020;71:100752.
- [67] Ministerio de Universidades. https://www.universidades.gob.es/portal/site/universidades/; 2022.
- [68] IUNE Observatory. http://www.iune.es; 2022.

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Sebastian Lozano: My first publications were related with Production Management, field in which I did my Ph.D. My main first research line in those days was cellular manufacturing and flexible manufacturing systems and the research tools I used most frequently were artificial neural networks and metaheuristics (genetic algorithms, tabu search, GRASP, etc.). Later on, although I have kept doing research on Production Management (for example, scheduling, disassembly, bullwhip effect, reconfigurable manufacturing systems, etc.) my main research filed move progressively to efficiency and productivity assessment using Data Envelopment Analysis (DEA). This line of research includes theoretical works (e.g. on integer DEA, fuzzy DEA or network DEA) as well as applications in different sectors (transportation, finance, sports, environmental impact, telecommunications, etc.) and overlaps in a synergic way with other research lines I am interested in, such as Cooperative Game Theory and Complex Network Analysis. Another line research line that I keep active is Logistics and Transportation, field in which I have worked on reverse logistics and coulaborative logistics, efficiency, environmental impacts, etc.