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Estimation of Counterparty Credit Risk Impact under IFRS Requirements

A modelling proposal under a quantitative market information-based approach

David Delgado-Vaquero

Directors:

Dr. José Morales-Díaz, Professor at Financial Administration and Accounting Department
Universidad Complutense de Madrid

Dr. Constancio Zamora-Ramírez, Professor at Accounting and Financial Economics Department
University of Seville

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Abstract: Counterparty credit risk is one of the main financial risk to be monitored by financial and non-financial institutions, worldwide. It entails a huge impact in areas as diverse as Business, Finance, Risk management, Funding & Liquidity management, Treasury, Trading, Solvency control, Accounting, Reporting, etc.

Concerning valuation and accounting matters, counterparty credit risk is present throughout IFRS rules, with emphasis on a particular way under IFRS 9, IFRS 13 and IFRS 16. Under the IFRS 9, entities must estimate the PD (Probability of Default) for all financial assets (and other elements) not measured at Fair Value through Profit & Loss in turn to compute the Expected Credit Loss for those assets. Also, regarding the potential impact that a modification in a debt instrument terms (i.e., debt restructuring) may have under IFRS 9, the original debt could have to be derecognized and replaced with the present value of the modified debt, which should be computed by discounting its cash-flows with a robust, liquid yield curve according to the company´s credit quality and instrument seniority. Likewise, under IFRS 13 framework, the expected counterparty credit risk should be incorporated to the value of a derivative which is measured at Fair Value. In this case, the derivative credit risk will be determined for both counterparty (CVA – Credit Value Adjustment) and own credit risk (DVA – Debt Value Adjustment). Therefore, the counterparty credit quality (and subsequent PD) and the own PD for the entire life of the instrument should be estimated.

A common problem in this regard is that there is no quoted credit instruments nor credit rating information of a company. For such cases, I propose a regression model that provides a theoretical credit rating for a counterparty as a first, necessary step when estimating the PD or the discounting curve. The model is new in a certain extent in comparison with other recent models in several aspects, such as the size and composition of the database used to calibrate the model variables (financial ratios percentiles within a sector distribution) and the fact that is intended to provide a "*forward-looking*" risk approach. The initial assumption is that financial ratios are a reliable source of information to estimate a rating letter when those are efficiently combined, with no necessity of qualitative nor additional company´s management-related information. I demonstrate that, with a granular sectorial database and by applying optimization in variables via Stepwise AIC process, the model output is reliable and robust to estimate the credit rating for a given company.

On the other hand, under IFRS 16, entities must discount future lease payments to value the leased asset or liability. The discount rate is generally understood as the lessee's IBR (Incremental Borrowing Rate). IFRS 16 states the IBR must consider that the hypothetical loan is collateralized by the leased asset. In this regard, there is a lack of accounting and finance literature focused on analysing how the IBR should be calculated taking into consideration both the counterparty credit risk of the lessee and the quality of the collateral. The starting hypothesis is that this quality is mainly determined by the underlying asset's expected LGD (Loss-Given Default) so that the relationship between the IBR and the LGD could be modelled. In this thesis I propose two quantitative models based on CDS (Credit Default Swap) spreads and liquid bond prices to estimate the IBR given the lessee credit rating and collateral-linked LGD. The results are statistically robust and demonstrates that the relationship between CDS spreads or bonds yield-to-maturity and the LGD implied in their market prices can be translated as a sensitivity measure to estimate the IBR for a lease contract by pivoting from a standard market yield curve.

Keywords: IFRS 9, IFRS 13, IFRS 16, Probability of Default (PD), Credit Rating, Credit Value Adjustment (CVA), Debt Value Adjustment (DVA), Incremental Borrowing Rate (IBR), Recovery Rate (RR), Loss-Given Default (LGD), Yield-to-Maturity (YTM), Credit Default Swap (CDS), Stepwise AIC, Libor Market Model (LMM), Swaptions.

JEL Classification: C13, C33, C52, G33, M41.

TABLE OF CONTENTS

LIST OF ABBREVIATIONS

ABS: Asset-Backed Security

ACF: Autocorrelation Function

AIC: Akaike Information Criterion

AR: Autoregressive model

ARMA: Autoregressive Moving-Average model

ARIMA: Autoregressive Integrated Moving-Average model

ASC: Accounting Standards Codification

ATM: At-the-Money

BCBS: Basel Committee of Banking Supervision

BIC: Bayesian Information Criterion

BIS: Bank for International Settlements

CAPEX: Capital Expenditure

CAPM: Capital Asset Pricing Model

CDS: Credit Default Swap

CFO: Chief Financial Officer

CIR: Cox-Ingersoll-Ross

CVA: Credit Value Adjustment

CRA: Credit Rating Agency

CRR: Capital Requirements Regulation

DPT: Default Point

DTD: Distance-to-Default

DVA: Debt Value Adjustment

EAD: Exposure at Default

ECL: Expected Credit Loss

EBA: European Banking Authority

EBIT: Earnings Before Interest and Taxes

EBITDA: Earnings Before Interest, Taxes, Depreciation and Amortization

ECB: European Central Bank

EMIR: European Market Infrastructure Regulation

EPE: Expected Positive Exposure

EURIBOR: Euro Interbank Offered Rate

ESTR: Euro Short-Term Rate

FCAG: Financial Crisis Advisory Group

FED: Federal Reserve

FRS: Financial Ratios Scoring model

FX: Foreign Exchange

GARCH: Generalized Autorregresive Conditional Heteroskedasticity

GLS: Generalized Least Squares

IAS: International Accounting Standards

IASB: International Accounting Standards Board

IBNR: Incurred But Not Reported

IBR: Incremental Borrowing Rate

ICT: Information and Communications Technology

IFRS: International Financial Reporting Standards

IRD: Interest-Rate Derivative

IRS: Interest-Rate Swap

KMV: Kealhofer, McQuown and Vasicek

LASSO: Least Absolute Shrinkage and Selection Operator

LMM: Libor Market Model

LOOCV: Leave One Out - Cross Validation

MAE: Mean Absolute Error

MC: Monte-Carlo

MtM: Mark-to-Market

NPV: Net Present Value

OLS: Ordinary Least Squares

OTC: Over-the-Counter

PD: Probability of Default

PF: Private Firms

QR: Quick Ratio

RIC: Refinitiv Instrument Code

RMSE: Root Mean Squared Error

ROA: Return on Assets

ROE: Return on Equity

RR: Recovery Rate

US-GAAP: US Generally Accepted Accounting Principles

TIC: Takeuchi Information Criterion

VaR: Value-at-Risk

VBA: Visual Basic for Applications

VIF: Variance-inflation Factor

WACC: Weighted Average Cost of Capital

YTM: Yield-to-Maturity

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1: INTRODUCTION

1.1. General context and motivation of this doctoral thesis

Counterparty credit risk, or in general, credit risk, is the risk of a loss arising from a failure (or default) of a counterparty to meet its contractual obligations (McNeil *et al*., 2015). Credit risk is one of the main financial risks to be monitored by many companies, worldwide. It entails a relevant impact in areas as diverse as Business, Finance, Risk Management, Funding & Liquidity Management, Treasury, Trading, Solvency Control, Accounting, Reporting, etc.

The counterparty credit risk, which is directly translated in potential monetary impact for an institution as a decrease in the value of its assets due to a loss from unpayment (i.e., a "credit loss"), and a source of capital and reserves requirement, is usually understood through two main concepts:

a) ***Probability of Default*** (PD): following most of the definitions given by supranational entities (ECB, EBA, etc.) or the definition of default contained in the CRR, it can be said that the Probability of default is *the term that describes the likelihood of a default of a counterparty over a particular time horizon.* More specifically, the PD *provides an estimate of the likelihood that a borrower will be unable to meet its debt obligations*. Although the definition of "default" can be polysemic, for this research it is not crucial. Hereinafter, we will assume that "default" means "being unable to meet the obligation of payment arising from a debt product (i.e., a loan, a bond, etc.)". Therefore, we will work under the assumption that the PD provides the expected times a default can occur from a borrower in a predefined time horizon.

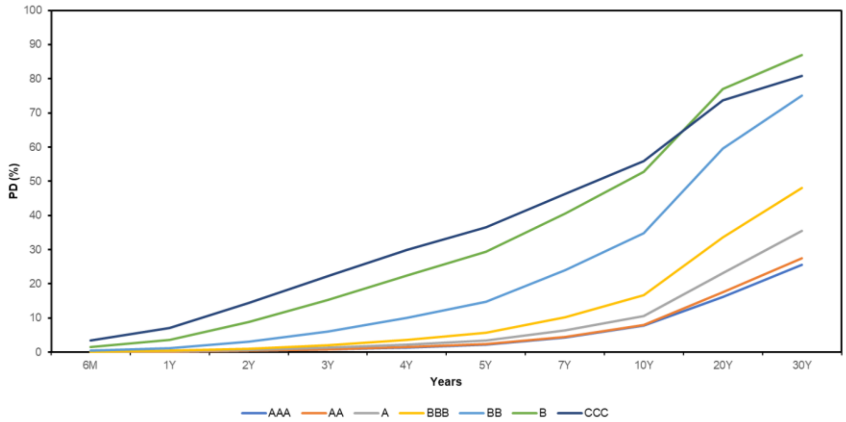
b) ***Loss-Given Default*** (LGD): this concept is inherent to the probability of a default occurrence, and is equally relevant to measure the credit loss, as it represents de amount of losses occurred once the default has taken place. This is, the unrecovered losses once a default has occurred from an obligor. This concept has implicitly attached the concept of *Recovery Rate,* which is the metric that provides the estimated amount recovered once a default occurs. This is, LGD = (1 - *Recovery Rate*). These concepts will be referenced in this thesis in an indistinct way.

These two concepts are needed to measure the credit loss for an investment between time *t-1* and time *t*:

$$\textbf{Potential Credit Loss}_{(t-1,\,t)} = \textbf{PD}_{(t-1,\,t)} * \textbf{LGD}_{(t-1,\,t)}$$

A proportional error in either the probability of default or LGD affects potential credit losses identically. Yet, much more resources and efforts are employed in the industry to estimate probability of default. Many different modelling techniques are applied to default probability; from statistical methods based on accounting data to structural models or hybrid approaches. The main reason is that PD are changing over time, and it depends in a wide extent on the rating, sector and geography to which the company belongs. Below is shown a chart of the implied PDs in CDSs[1] per rating letter and maturity, in Europe.

Figure 1. Cumulative Probability of Default curves per Rating letter, 20/04/2022

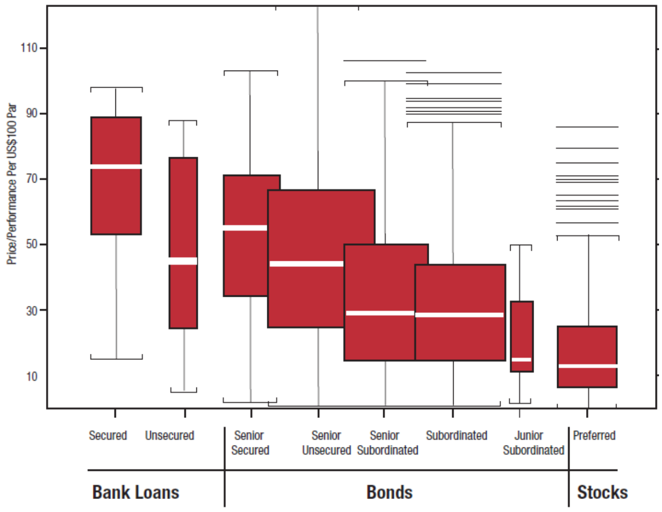


Source: Refinitiv

However, in sharp contrast, LGD is typically estimated by appealing to historical averages, usually segregated by debt type (loans, bonds and preferred stock) and seniority (secured, senior unsecured, subordinated, etc.). Although its levels depend not only on the seniority of the product but on the sector-specific and macroeconomic variables as well, its sensitivity to such factors is not as notable as in the case of PD. Likewise, PD is widely expected to change even between companies of the same sector and rating grade, however historical LGDs, in average, are more static on their seniorities, and even their implicit values in market-traded products, like CDSs and bonds, are assumed to be flat. The main reason for that is that even for same seniority tranches, the LGD could be too much “entity-specific”, and therefore there could be a serious lack of information for a correct modelling. Because of that, LGD uses to be assumed flat and LGD averages per seniority are usually the main input used when estimating potential credit losses.

[1] Credit Default Swap

Figure 2. Loss-Given Default rates on instrument prices, per instrument seniority[2]



Source: Moody`s (2014)

In this context, a correct estimation of the PD and LGD for a given client or counterparty and financial product is therefore a key when measuring its credit risk. It is widely known that financial and non-financial companies face many information-related issues when computing figures for PD and LGD. This is mostly linked to lack of information about the credit quality for a counterparty which usually has no credit-related data available but its own financial statements. This is, there are no credit ratings given by an agency rating, credit-linked instruments with available prices in a financial venue nor historical series of bonds or loans that may provide with a reliable data about the market estimation of the credit quality for such a counterparty. This problem lead companies, auditors, banks and other entities to do a research process to estimate PD and LGD figures for counterparties and clients. The main issue in this case is that the estimation methodology should comply with several accounting criteria and minimum methodological standards that are difficult to reach in an efficient way. In this regard, the motivation of this doctoral thesis is to cover those main modelling gaps found in the academic and industry practice related to the lack of credit quality information and propose new modelling solutions which directly concern the concepts of PD and LGD under the IFRS[3] framework for financial valuation, reporting and accounting.

1.2. IFRS framework and the counterparty credit quality estimation requirement

Over the last ten years, IFRS accounting standards have changed significantly in areas such as *fair value*, *financial instruments*, *lease accounting*, and *revenue recognition*. Generally, the new standards entail a higher use of judgment and estimations, which renders the role of financial

[2] It highlights the variability of recoveries for several seniority classes. The shaded boxes cover the inter-quartile range with the median marked as a white horizontal line. Squared brackets cover the data range except for outliers that are marked as horizontal lines

[3] International Financial Reporting Standards (IFRS) issued by the International Accounting Standards Board (IASB). In Europe, IFRS are applied by quoted entities for the preparation of their consolidated financial statements See http://www.ifrs.org/use-around-the-world/use-of-ifrs-standards-by-jurisdiction/ for a detailed study on the use of IFRS standards by jurisdiction.

analysts and auditors far more difficult. According to Heidhues and Patel (2011), the exercise of accountants' professional judgment has increasingly been recognized as an important and controversial topic.

In this sense, for one purpose or another, several recently issued standards require entities to estimate the credit quality of a third party or their own credit quality. Specifically, IFRS 9 ("Financial Instruments") requires the estimation of an *impairment* from a potential credit loss in the assets (i.e., the Expected Credit Loss or ECL). Likewise, following the implementation of IFRS 13 ("Fair Value Measurement"), when measuring derivatives' fair value, entities must consider the counterparty credit risk adjustment, which generally entails estimating the PD of the derivative's counterparty and the own PD, among other inputs. Furthermore, under IFRS 16 ("Leases"), when a lessee discounts future lease asset cash-flows, if the implicit lease rate is not available, the entity must estimate its own borrowing rate for buying a specific asset with a specific maturity.

In some cases, the inputs required (PD or the bond interest rate/YTM[4]) can be directly estimated from observable market information, such as CDS spread quotes or the issuer bond price quotes[5]. In other cases, however, this information is not available. The counterparty whose credit quality needs to be estimated may not have quoted CDSs nor bonds, nor a credit rating[6] issued by an independent credit rating agency (CRA). In such cases[7], entities need to implement a methodology for internally estimating the credit quality (credit rating) of a company as a basis for obtaining a PD or a YTM/discount rate curve, and also a method to correctly calibrate the adjustments needed on those PD or discount curves due to some particularities of the asset or the counterparty.

Hence, in this thesis I will provide with modelling solutions to tackle the issues arising from the non-existence of indicators of credit quality for a given company nor market information on discounting curves, so that the gaps to comply with the PD and discount rates can be covered under the IFRS 9, 13 and 16 frameworks.

1.3. Objective and starting hypotheses: the necessity of modelling solutions under IFRS 9, 13 & 16 frameworks when there is a lack of counterparty credit quality information

My background and professional experience in the field of financial valuation and risk management have provided me with awareness and expertise on the main problems that companies, both financials and corporates, have when dealing with the compliance of IFRS 9, 13

[4] Yield-To-Maturity.

[5] Or even from internal information such as the yield-to-maturity of a recently obtained, representative banking debt.

[6] Credit ratings are a summary of a firm's expected future creditworthiness. They represent an evaluation of the credit risk of company, i.e., they are related to the probability that a company will default. The higher the rating, the lower the expected credit risk, and the lower the estimated PD. There are independent credit rating agencies that issue public credit ratings for companies/governments or specific bonds issuances. Relevant rating issuers are S&P (Standard & Poors), Moody's, Fitch or DBRS (Dominion Bond Rating Service).

[7] See IFRS 13 *fair value hierarchy* in Chapter 3.

and 16 rules, particularly when the lack of financial and market information about counterparties is relevant. Market models to derivate standard PD values or vanilla YTM curves for companies with credit risk information available are widely known among practitioners. However, when there is no such an information, which is a common issue, new model approaches are needed. In fact, even when there is information on standard credit instruments for a company but no rating nor YTM for longer tenors or non-standard debt seniority tranches, modelling adjustments are duly required. Bearing this in mind, several solutions to cope with these problems are explained, which have been brough together, enhanced and tested alongside my research period, crystallizing in this doctoral thesis.

1.3.1 Credit rating, PD and YTM estimation under IFRS 9 & 13 frameworks

Within the field of finance literature, the interest in counterparty credit risk and credit rating estimation has particularly increased since the 2008 subprime financial crisis. There is a line of research in which authors propose models for obtaining an internal credit rating to challenge the official credit rating issued by CRAs, or to use it in the event that there is no official credit rating available. The first historical work was that by Altman (1968), which used five financial ratios in order to predict bankruptcy. Since then, many authors have also proposed models in which financial variables are used for estimating credit risk. See, for example, Merton (1974); Kaplan and Urwitz (1979); Ohlson (1980); Ederington (1985); Longstaff and Schwartz (1995); Duffee (1999); and Kamstra *et al.* (2001).

More recently, Creal *et al.* (2014) proposed a marked-based rating which makes direct use of the prices on traded assets. The authors use asset pricing data to impute a term structure of risk neutral survival functions or default probabilities. Firms are then clustered into ratings categories based on their survival functions using a functional clustering algorithm. They compare their ratings to S&P and find that, over the period 2005 to 2011, their ratings consistently lead to S&P ones for firms that ultimately default.

Tsay and Zhu (2017) proposed a two-step algorithm involving ARIMA-GARCH modelling and clustering to obtain a market-based credit rating by using easily obtained public information. The algorithm is applied to 3-year CDS spreads of 247 publicly listed firms. The authors compare the ratings obtained with the ratings given by agencies, and show that such market-based credit rating performs reasonably well. Jansen and Fabozzi (2017), assuming a given recovery rate, use the CDS-implied default probabilities to cluster them in rating groups.

However, there are still present in the financial literature several issues concerning the PD modelling for accounting and reporting purposes, with a relatively global application. Few proposed models for obtaining an internally developed credit rating fulfil all (or most of) the following criteria at the same time:

i. Specifically addressed to accounting purposes (i.e., for complying with accounting requirements) under IFRS, which affect most of companies not reporting under US GAAP.

ii. Specifically focused on complying with IFRS 9 expected loss requirements. The IFRS 9 PD should be based not only on historical information but should also consider *forward-looking* information. By way of example, Altman's and Merton's models do not incorporate *forward-looking* information (related to market quotes).

iii. Able to be applied to non-quoted/non-rated entities. Few models have mainly been developed for non-quoted companies (Beever, 1968; Ohlson, 1980; Campbell *et al*., 2008; Chava and Jarrow, 2004)

iv. Comparable, so that the results can be compared to market or credit rating information.

v. Able to be applied to one specific counterparty/company within a given sector.

vi. Applicable in any jurisdiction.

vii. Able to be implemented by obtaining public information which is readily available, such as the entity's sector; the credit rating issued by official CRAs for other companies in the same sector/country; the entity's financial statements, etc.

viii. The output provided is a credit rating under a scale comparable to the ratings used by CRAs: S&P, Moody's and Fitch. This will make it easier to find companies with similar credit quality and which also have a public credit rating.

ix. Updatable: it provides an updated output based on the current market/sectorial framework.

x. Able to be extrapolated, as the main output could be translated into a Rating Letter, a PD rate, a yield-to-maturity curve, or a credit spread. This fact leads to a solution for lack of counterparty credit information under the IFRS 13 and IFRS 16 frameworks as well.

In this regard, the first objective of this doctoral thesis is to propose a model that provides with a credit rating under the IFRS requirements. Hence, Chapter 5 presents a model that provide a robust output (as a credit rating, a PD or even as a discount rate) to be used as input needed to impairment calculation (ECL) and debt restructuring valuation figures under IFRS 9, as well as CVA and DVA metrics to be estimated under IFRS 13. That model is expected to meet most of above criteria and is intended to provide consistent outputs in this regard.

The model provides the output via a regression scheme which retrieves a theoretical credit rating for a counterparty as a first, necessary step when estimating the PD or the discounting curve. The model is new in a certain extent in comparison with other academic models in several aspects, such as the size and composition of the database used to calibrate the model variables (financial ratios percentiles within a sector distribution for several years in a row) and the fact that is intended to provide a "*forward-looking*" risk approach. The assumption that can be taken as an initial hypothesis is that historical financial ratios are a reliable source of information to estimate a rating letter when those are efficiently combined, with no necessity of qualitative nor additional company´s management-related information. I demonstrate that, with a granular sectorial database and by applying optimization in variables via Stepwise AIC process, the model output is reliable and robust to estimate the credit rating of a given company. Therefore, once the database is accurately treated, the model can be easily implemented and used for different sector and geographies, with a *forward-looking* approach and able to cover the changes in rating criteria throughout time, hence available to be used for accounting and reporting purposes under different audit exercises.

The model has been tested by comparing its output for entities already given with an official credit rating with credit rating agencies (Moody's, Fitch, or Standard & Poor's). Therefore, we obtain a unified framework which incorporates a firm's specific features along with its sectorial and regional factors, and which enables market assessments of credit risk to be incorporated into the book value of financial assets.

1.3.2 Incremental Borrowing Rate estimation for leasing valuation under IFRS 16

On the other hand, the second objective on this thesis is to provide a modeling framework that copes with the necessity of adapting the discounting curve to value leasing contracts with different assets as collateral. It know that entities must discount future lease payments to value the leased asset or liability to comply with IFRS 16 rules. The discounting rate is generally understood as the lessee's IBR (Incremental Borrowing Rate). IFRS 16 states the IBR must consider both the counterparty credit risk of the lessee and the quality of the collateral. Therefore, in this document two quantitative models based on CDS spreads and liquid bond prices are presented, so that the IBR can be estimated given the lessee credit rating and collateral-linked LGD.

This work contributes to the previous literature in three main drawbacks widely found among the industry practice:

- Firstly, the models proposed can be used by researchers when estimating the impact of IFRS 16 on a certain jurisdiction or entity. Studies prior to the issue of IFRS 16 use a unique rate for discounting lease payments (Beattie, *et al*., 1998; Bennett and Bradbury, 2003; Duke *et al*., 2009; Ely, 1995; Imhoff and Lipe, 1997; Singh, 2012; Wong and Joshi, 2015); or discount rates used for pensions and other provisions (Fülbier *et al*., 2008; Pardo *et al*., 2017); or directly use a benchmark rate plus a firm credit spread (Durocher 2008; Fitó *et al*., 2013). Therefore, the method provided for the estimation of lease IBR is in fact more accurate for research purposes because it provides a solution to adapt the IBR at lease-level and contingent LGD, rather than using benchmarks.

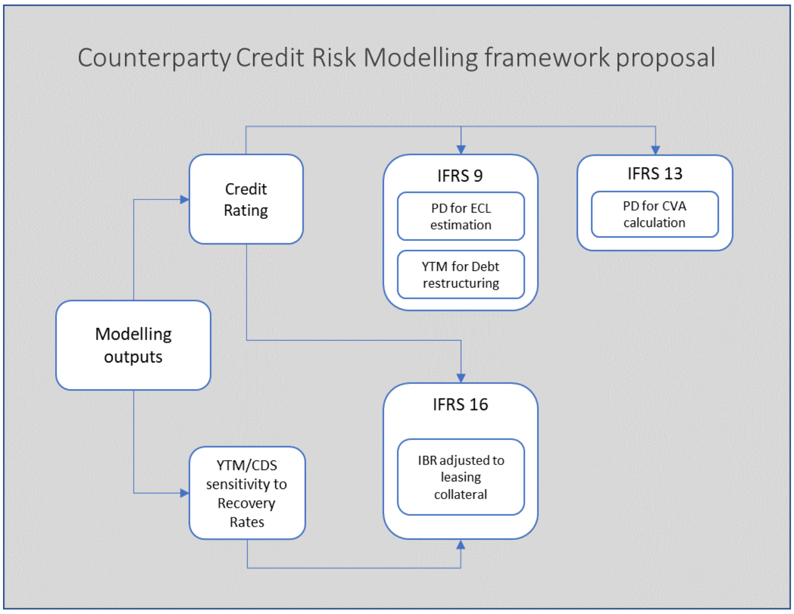
- Secondly, the previous literature related to LGD estimation is not directly applicable to this matter. Although certain authors do present models for estimating the LGD or for analysing the relationship between loan prices and collateral value (with a given sample of loans at a certain date by Akguen and Vanini, 2007; Silagui *et al*., 2020), none of them present a model that explains how a standard yield curve can be adjusted to reflect the sensitivity to a LGD adapted to the collateral quality.

- Thirdly, the models can be used by IFRS 16 practitioners in order to adjust "standard" IBR to the IBR applicable to different lease assets associated with different LGDs. As previously mentioned, there is a gap in the existing literatures in this regard, and entities do not disclose this information in their financial statements. It is worth noting that the model presented is also applicable in many other contexts, such as estimating the fair value of a loan/bond that includes an asset as a collateral (for accounting, trading, valuation, or other purposes). In this case, the model can be used to adjust the discount curve and correctly reflect the higher (or lower) recovery rate expected from the asset. Another potential use would be the calculation of the interest rate of a collateralized loan transaction between a lender and a borrower; in this case the model can be used for adjusting the standard interest rate to the collateral value, calculate additional liquidity margins, etc.

As a summary, it can be said that there is a modelling gap in the accounting and finance literature when analysing how the IBR should be calculated taking into consideration both the counterparty credit risk of the lessee and the quality of the collateral. The starting hypothesis in this regard is that this quality is mainly determined by the underlying asset's expected LGD (Loss-Given Default) so that the relationship between the IBR and the LGD could be modelled. In this research it is demonstrated that the modelling results are statistically robust and demonstrates that the relationship between CDS spreads or bonds yield-to-maturity and the LGD implied in their market prices can be translated as a sensitivity measure to estimate the IBR for a lease contract by pivoting from a standard market yield curve.

Moreover, it is demonstrated that the model functions by using real market data of quoted bonds, i.e., by applying the models to a real sample of quoted bonds and CDS prices, and subsequently analyse whether the model predicts the change in YTM when a change in the recovery rate occurs.

Figure 3. Modelling framework proposal to estimate counterparty credit risk impact under IFRS



Source: Compiled by the author

1.4. Structure of the doctoral thesis

In Chapter 1, the introduction to the problems found in the counterparty credit risk treatment under IFRS framework is presented, including the main objectives of the research and the required fulfilment of the starting hypotheses made. In Chapter 2, I review the main methodology of research steps made and the relevant topics to be outlined within the research period. Chapter 3 covers the global literature review on the IFRS 9, 13 and 16 topics related to the credit risk estimation and its implications under the IFRS space, presenting the main conclusions and gaps found which the models presented aim to fix. Chapter 4 summarizes the main models currently used in the financial industry related to the credit risk estimation, including several approaches. In Chapter 5, I propose a credit rating estimation model named FRS model, that has been developed and improved during my research period, to cover some of the limitations found in the literature. It includes model theory and development, implementation examples, statistical testing and back-testing. Chapter 6 presents the models proposed concerning the IBR estimation under IFRS 16 requirements, also including model theory and development, hypotheses made, implementation and statistical testing. Finally, Chapter 7 includes the doctoral research conclusions, model limitations and future lines of research.

CHAPTER 2: METHODOLOGY OF RESEARCH

2.1. Introduction

The work presented herein focuses on solutions so cope with the main requirements from IFRS rules on counterparty credit risk-related matters, as the relevant drivers motivating this PhD dissertation are linked to issues found during my previous professional and research experience in that regard. Among others, I gained experience in the fields of valuation and risk management with IFRS requirements as one of the most important aspects. Hence, I faced many requirements from clients and projects that were related to that in a wide extent.

The research focuses on the one hand, in the literature review to contextualize the gaps found on the field of counterparty credit risk estimation in the IFRS space, and on the other hand, in the potential solutions that can be developed in terms of modelling by using financial and quantitative data from Refinitiv, Bloomberg, Oxford Economics, Moody´s or S&P databases as main sources. The models proposed throughout this research are entirely based on financial data available on those financial vendors.

Likewise, the mathematical and econometric background developed and discussed along the thesis is then reflected in several model outputs computed via R as the main statistical and computational tool, also with VBA, Excel and Python as auxiliary tools for the rest of computations and formulas application.

Although in this document both the methodological section, modelling framework and outputs have been partially extended, the basis of the research has been developed throughout previous research articles on which I have been working during my doctoral period. These articles have been already published in several financial and accounting journals or are expected to be published soon, thanks to the critical work done by Prof. Constancio Zamora-Ramírez and Prof. José Morales-Díaz as co-authors, who continuously motivated and endorsed me to join their work in turn to start this doctoral program.

2.2. Questions to initial hypotheses and research methodology

As previously discussed, counterparty credit risk is present in a particular way under IFRS 9, IFRS 13 and IFRS 16. As explained above, under IFRS 9 and 13, the counterparty credit quality for the entire life of the instrument should be estimated. When there is no quoted credit instruments nor credit rating information of a company, a model should be developed to estimate such a credit quality, which should be ultimately transposed to a robust PD for a given timeframe.

The main question in this regard is that if a model can be robust enough to be used in those cases to estimate the PD with a sufficient degree of confidence, covering the main aspects required by IFRS and also able to be used by most of practitioners in a relevant extent. The question also requires that the main gaps found in the literature can be also bridged.

Also, under IFRS 16, entities must discount future lease payments to value the leased asset or liability. IFRS 16 states the Incremental Borrowing Rate to be used for discounting purposes must consider that the hypothetical loan is collateralized by the leased asset. There is a relevant lack of literature researching on how the IBR should be estimated taking into consideration both the counterparty credit risk of the lessee and the quality of the collateral. The question in here is how to construct a modeling framework to estimate an IBR for a given lessee and collateral considering that hypothetical relationship and proof that this relationship is consistent and demonstrable.

Therefore, in turn to provide answers to the abovementioned questions and initial hypotheses, there should be a research plan that follows a methodology to reach the final goals with an enough degree of confidence. The application of the methodology of research took into consideration the objectives of the dissertation as well as the potential sources of information, and can be hence summarized as follows:

1) Awareness of the main issues and casuistries to be researched
2) Assessment on the potential issues to be found in the academic literature and industry modelling fields
3) Assessment on the potential solutions to cover the gaps found
4) Assessment on the potential data sources that can be used to build the models and challenge the outputs and the initial hypotheses
5) Estimation of the time length to develop the research and establish a research plan accordingly
6) Carry out the research in an organized way, periodically checking the progress with the PhD program directors so as to check if appropriate methodologies, models, variables and model uses are correctly described and tested, to achieve the objectives and demonstrate the hypotheses initially made.

2.3. Sample and data input collection for modelling purposes

The databases used to calibrate the models and compute the outputs from those models are built upon financial and quantitative information from public companies for which Refinitiv, Bloomberg, Moody´s and S&P provide financial statements data, credit rating letters, PD and LGD from both historical and current views, etc. Also, bond prices, YTM curves and CDS spreads are directly taken from Refinitiv and Bloomberg quoted information.

Most of this information is available upon subscription to those data providers. Refinitiv, Bloomberg and Oxford Economics are vendors with annual payment subscription to which I have access through my current employer. Most of Moody´s and S&P data used in this research are public upon subscription to their websites, but some other reports and databases are available upon fee paid upfront.

2.5. Final considerations

The application of the previous explained research methodology, including the treatment and usage of research done in previous articles, crystallizes in the output of this dissertation, which can be concluded as satisfactory in terms of the starting hypotheses testing and robustness of the results.

For the first issue found in the academic literature and among practical experience concerning IFRS 9 PD estimation, this thesis proposes a regression model that provides a theoretical credit rating for a given company. The assumption that the financial ratios are a reliable source of information to estimate a rating letter when those are efficiently combined, is demonstrated in a relevant way. Likewise, with regards to the IFRS 16 - IBR space, the results of the models proposed are statistically robust and demonstrates that the relationship between CDS spreads or bonds yield-to-maturity and the LGD implied in their market prices can be translated as a sensitivity measure to estimate the IBR for a lease contract by pivoting from a standard market yield curve.

The models developed in this research project, as well as the input sources and outputs are transparent enough to be used by other researchers and practitioners that want to contribute to the increase of the academic literature of methodologies and models related to the counterparty credit risk and their application to the financial world.

CHAPTER 3: LITERATURE REVIEW

Since 2007, the world economy has gone through a critical period. A crisis in terms of both debt and financial confidence arose rapidly and spread through many countries, particularly affecting the United States and Europe. Financial markets suffered significant credit uncertainty, which in turn affected almost every counterparty involved in a transaction.

An increasing tension of weak debts, both on the micro and macro scale, emphasized the threat existing to the financial stability of not only specific entities, but also the market as a whole and even countries. The credit reliability of counterparties and clients became the main point of interest for a growing number of market participants, while leaving the market (price) risk and trading itself out of the main scope. Those investing in credit instruments started to consider them to be of even greater risk than other types of investments.

Against this background, the regulatory framework in many relevant jurisdictions focused on supervising credit and counterparty risk of financial markets and their participants, ensuring that the actual credit risk was reflected in both a bank's trading and banking book, as well as in the financial statements of any company involved in relevant financial transactions (particularly derivatives). From primary markets to OTC[8] derivatives (and with significant effects on retail clients), the change in principal credit risk factors has stimulated the research into more effective methods of credit and counterparty risk management.

Literature review. Dealing with the necessity for counterparty credit risk estimation and the academic contributions that can help cope with IFRS requirements.

As previously explained in Chapter 3, under IFRS accounting standards there are many scenarios in which a credit quality estimation is called for in turn to obtain a PD or a YTM. In this sense, entities from many different sectors and sizes are currently facing a variety of situations which require them to estimate the credit quality of a third party (or their own credit quality), and that information may not be observable in the market. In the below paragraphs I make a summary of the relevant IFRS requirements in the field of credit risk and bring together the main contributions found from academics and practitioners which could be helpful to cover, in some extent, the IFRS 9, 13 and 16 rules in this regard.

[8] Over-the-counter.

3.1. IFRS 9: Financial Assets Expected Loss Provision and Liabilities restructuring to Fair Value

IFRS 9 is the financial instruments accounting standard that has replaced IAS 39 for annual reporting periods commencing on or after 1st January 2018. One of the areas in which IFRS 9 will have a higher impact is the new impairment model (applicable to financial assets not measured at fair value through profit and loss, lease receivables, contract assets and financial guarantee contract - see IFRS 9.5.5.1).

IAS 39 followed an incurred loss model: an impairment loss could not be recognized until it was incurred. Additionally, in terms of the "generic" provision, only what was known as Incurred But Not Reported (IBNR) losses could be recognized: losses related to debtors for which, at the date of the financial statements, the credit event has occurred but has not yet been revealed/reported.

Conversely under IFRS 9, as soon as the debt instrument is recognized, at least part of the expected losses should be recognized. Loans are classified in three steps: *step 1*, *step 2* and *step 3*. In step 1, 12-month expected credit losses are recognized, while lifetime expected credit losses are recognized in steps 2 and 3.

Broadly speaking, the expected credit losses are calculated as $EAD_t \cdot PD_t \cdot LGD_t$, where EAD_t represents the Exposure at Default (expected instrument exposure) at time t; PD_t represents the Probability of Default at time t; LGD_t represents the Loss Given Default at time *t*. LGD_t represents at the same time the following: (1 – *Recovery Rate*)

Therefore, one of the necessary inputs for calculating the expected credit losses is the PD of the borrower.

However, IFRS 9 has introduced several changes with respect to IAS 39. For example, the categories for financial assets are different to those of IAS 39 (classification criteria is also different), and changes have been made to hedge accounting rules.

One aspect significantly affected by the IFRS 9 changes is loan loss provisioning (impairment rules). For many entities, this has proved to be the most important change (that with the highest impact). It is not only banks that have been impacted by the new impairment rules; in fact all kinds of entities are making changes to their provisioning criteria (EY, 2018; EY, 2016; Novotny-Farkas, 2016; Beerbaum, 2015; Hronsky, 2010).

The IAS 39 impairment model was based on “incurred losses”. Several regulators and authorities argued that this model led to procyclical effects, and asked standard issuers to develop a new model that entailed a more forward-looking provisioning (e.g. BCBS, 2009; FCAG, 2009; G20, 2009). The new IFRS 9 model is based on “expected losses” instead of “incurred losses”; however, it is not a full expected loss model.

With certain exceptions[9], under IFRS 9 all financial assets[10] not measured at fair value through profit or loss should be classified in three different "stages". For financial assets included in stage 1, 1-year expected loss should be estimated and recognized. For financial assets included in stages 2 and 3, expected loss until maturity should be estimated and recognized. In other words, for all financial assets (and other elements) subject to IFRS 9 impairment rules, the entity should estimate a PD for 1 year or maturity. The measure of the loan loss allowance will require the use of data not previously considered under IAS 39 (Holt & McCarroll, 2015).

3.1.1 Expected Credit Loss

As previously stated, IFRS 9 impairment rules are based on an expected loss model (in contrast with the IAS 39 incurred loss model). All financial assets subject to IFRS 9 impairment rules (with certain exceptions), are classified in three different stages. Depending on the stage involved, the impairment calculation is based on 1 year expected loss ("12-month expected credit losses"), or on expected loss until maturity ("lifetime expected credit losses"). In theory, all financial assets are included in stage 1. They progress to stage 2 when "credit risk on that financial instrument has increased significantly since initial recognition" (IFRS 9 paragraph 5.5.3). Finally, they are classified as stage 3 when the loss is incurred. The general formulas for estimating impairment (ECL) according to the stage to which the instrument belongs are as follows:

$$Stage\ 1\colon\ ECL_{12m} = EAD_{12m} \cdot PD_{12m} \cdot LGD_{12m} \cdot DF(0,1) \tag{1}$$

$$Stage\ 2\colon\ ECL_{Lifetime} = \sum_{t=1}^{n} EAD_t \cdot PD_t \cdot LGD_t \cdot DF(0,t) \tag{2}$$

$$Stage\ 3\colon\ ECL_{Lifetime} = EAD_{Mat} \cdot LGD_{Mat} \cdot DF(0,Mat) \tag{3}$$

where EAD_t represents the Exposure at Default (expected instrument exposure) at time t; EAD_{12m} is the Exposure at Default at 12 months; EAD_{Mat} represents the Exposure at Default at maturity; PD_t represents the Probability of Default at time t; LGD_t represents the Loss Given Default at time t. *LGD* is calculated as (1 – *Recovery Rate*); $DF(0,t)$ represents the discount factor from the calculation date to *t*. *t* is 1 year in stage 1 (or less than 1 year if the instruments mature in less than 1 year) and the time in years to maturity in stage 2 and stage 3. In stages 2 and 3, *t* can be divided into sub-periods (always considering all periods to maturity of the instruments).

[9] For example, purchased or originated credit-impaired financial assets (IFRS 9 paragraphs 5.5.13 and 5.5.14), or trade receivables, contract assets and lease receivables to which the simplified model is applied (IFRS 9 paragraphs 5.5.15 and 5.5.16).

[10] IFRS 9 impairment rules do not only apply to financial assets; they also apply to lease receivables (under IFRS 16); to contract assets (under IFRS 15); and in many cases to loan commitment and financial guarantee contracts (IFRS 9 paragraphs 2.1, 4.2.1(c), 4.2.1(d) and 5.5.1).

In Chapter 1 we saw that LGD value depends on several factors and is not even a certain value for a counterparty but depends on the loan's seniority and the value of any specific guarantee at a given time period. In practice, if no information is available, LGD is assumed to be 60% (the recovery rate being 40%)[11].

In the following table the average corporate debt recovery rates measured by trading prices from 1983 to 2017 is shown. This type values can be used as a robust source to estimate the LGD for most of seniorities:

Table 1: Average corporate debt recovery rates measured by trading prices

Class	Average recovery rate
1st Lien Bank Loan	63.74%
2nd Lien Bank Loan	27.73%
Sr. Unsecured Bank Loan	40.21%
1st Lien Bond	53.80%
2nd Lien Bond	43.63%
Sr. Unsecured Bond	33.48%
Sr. Subordinated Bond	26.34%
Subordinated Bond	27.55%
Jr. Subordinated Bond	13.97%

Source*:* Moody's 2018.

Also, it should be noted that IFRS 9 establishes that the estimated PD must include not only past due information, but also *forward-looking* information (in relation to expected changes in default rates). In this sense, observed past default rates should be adapted to changes in macroeconomic variables and market expectations.

With the above context clear, it can be said that, generally, there are several methods for obtaining a PD depending on the availability of market and financial reliable sources:

i. If market information of quoted inputs is available, the PD can be directly calibrated from quoted CDS spreads, quoted bonds yields or by using official credit rating and peer information. In theory, it is assumed that this market information already incorporates *forward-looking* adjustments.

ii. A PD can also be obtained by using internal historical default data adjusted by *forward-looking* estimations. This data is generally held by large corporate and banking companies.

iii. Finally, if no market or internal historical information is available, an internal model can be used for estimating the PD based on other companies' default rates, or on information from the company's financial statements or from other sources. The models can be split into two groups:

[11] See Ou *et al*. (2016) and Koulafetis (2017) for an empirical study of average recovery rates according to collateral.

- Structural models: based on Merton (1974) and on Black and Scholes (1973) option pricing models.

- Non-structural (analytical) models (as Altman *et al*. 1977).

With regard to the abovementioned third method (which is the focus of this research), several authors have proposed internal models for estimating a company's probability of default. Altman (1968) proposed an initial analytical model in which he used financial metrics (accounting ratios) for predicting an entity's default. Other authors have proposed structural and analytical models for estimating credit risk or default probability, such as Merton (1974); Kaplan and Urwitz (1979); Ederington (1985); Longstaff and Schwartz (1995); Duffee (1999); and Kamstra *et al*. (2001).

In this regard, there are also lines of research by other authors proposing a model whereby they obtain their own internal credit rating for a counterparty (also known as an "unofficial" or "shadow" rating). They compare this rating with the official credit rating (in order to challenge the official credit rating). The most recent papers in this area are those by Creal *et al*. (2014), Tsay and Zhu (2017), and Jiang (2018).

Nonetheless, there is a lack of studies focused on non-quoted/non-rated entities. According to Duan *et al.* (2018), the relative paucity of academic attention is partly due to the lack of publicly available data on privately held firms. Even if accounting data for private firms is available, the lack of market data such as stock prices entails an additional obstacle to studying their defaults, since recent advancements in the credit risk model typically require some form of market information.

Duan *et al*. (2018) propose a model for such cases. They obtain the distance-to-default (DTD) for quoted companies, and then identify macro and firm-specific factors related to the DTDs. Subsequently they locate macro and firm-specific values for private firms, and utilize the coefficients estimated from public firms to obtain the public-firm equivalent DTDs for the private firms. In addition, they improve the efficiency of estimating the default probabilities by adopting the newly developed doubly stochastic Poisson forward intensity model suggested in Duan *et al*. (2012).

Cappon *et al*. (2018) propose an alternative model which they apply to Brazilian banks. They develop a regression model to estimate the "synthetic rating" of Brazilian banks from financial variables. They achieve an R^2 higher than 80% to explain the ratings. However, they do not disclose the main internal aspects of the model.

Ivanovic *et al*. (2015) also propose a model for obtaining a "shadow rating", but it is focused on countries (and not on entities).

3.1.2 The Effective Interest Rate for Liabilities restructuring

Although the main objective in this dissertation is the credit rating modelling with regards to the Expected Credit Loss estimation, in should be noted that IFRS 9 also includes the abovementioned rule on recognition of changes in a liability measured at amortized cost. In July 2017 the IASB confirmed the accounting for modifications of financial liabilities under IFRS 9. That is, when a financial liability measured at amortized cost is modified without this resulting in derecognition, a gain or loss should be recognized in profit or loss. The gain or loss is calculated as the difference between the original contractual cash flows and the modified cash flows discounted by the original effective interest rate. (IFRS 9, paragraph B5.4.6) This is consistent with the tentative agenda decision of the IFRS Interpretations Committee ('IC'). However, the IC decided not to finalize this decision on the grounds that an agenda decision was not an appropriate mechanism to address the issue.

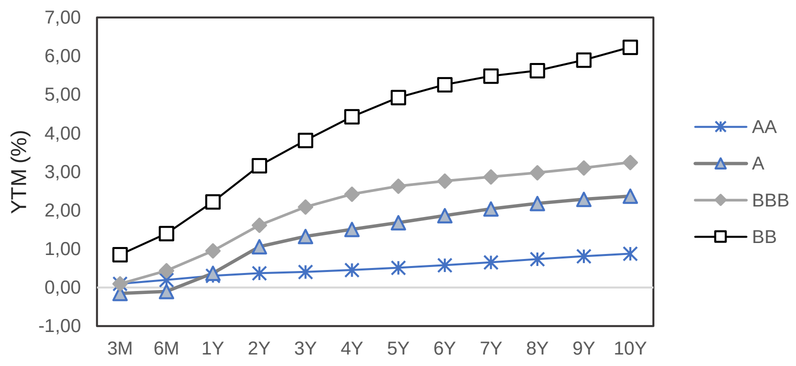
The Board has decided instead to amend the Basis for Conclusions to IFRS 9 to highlight that the accounting under IFRS 9 is clear and that no changes to the standard are required. This will impact all preparers, particularly those applying a different policy for recognizing gains and losses today. Under *IAS 39, Financial instruments: Recognition and measurement*, many preparers did not recognize a gain or loss at the date of modification of a financial liability. Instead, the difference between the original and modified cash flows was amortized over the remaining term of the modified liability by re-calculating the effective interest rate. This will need to change on transition to IFRS 9 because the accounting will change. Whilst it is not expected that entities will be required to change their existing accounting policy under IAS 39, the impact on transition to IFRS 9 should be considered. IFRS 9 is required to be applied retrospectively, therefore modification on gains and losses arising from financial liabilities that are still recognized at the date of initial application (e.g., 1st January 2018 for calendar year end companies) would need to be calculated and adjusted through opening retained earnings on transition.

Although changes in debt terms are common in today's environment and at first glance, it may appear that IFRS 9 does not change the accounting for financial liabilities as it retains almost all the existing guidance under IAS 39. However, IFRS 9 has introduced new guidance on how to account for changes in debt terms and this new requirement is expected to result in a significant change in practice for many companies.

Modifications to debt can occur when the borrower and lender agree on changes to the contractual terms of the liability, e.g., changing the interest coupon or extending the expiration date. Under IAS 39, a change that is considered "substantial" would be assumed to be extinguishing, which means that the initial liability is derecognized, implying a gain or loss to be recorded in profit & loss, and subsequently that a new financial liability will be recorded based on the new terms. If the change is not considered "substantial", then the original liability remains on the books and no profit and loss impact will be recorded. Nonetheless, under IFRS 9, a gain or loss at the date of the modification will be recognized notwithstanding if the change in contractual terms is substantial or not. This entails that the original liability will have to be derecognized and replaced with the present value of the modified liability. Also, if there were any costs or fees incurred to change the terms, they would be adjusted to the carrying amount of the modified debt and amortized over the remaining term of the modified debt. This means that the modified debt should be measured at fair value. This leads to the problem of looking for a reliable, adapted YTM curve. This curve should reflect the credit risk inherent to the debtor, so that the yield curve used should be in line with its credit rating. Hence, the model to estimate credit rating which is to be presented in next chapters not only covers the ECL calculation itself, but provides with the credit rating that can be used to look for a YTM curve to discount cash-flows of a modified debt.

To provide a hint about the relevance of being accurate when estimating the credit rating for a given company, see the below chart which provides a view on the difference between YTM curves per rating notch, for the same currency and sector (in this example, Automobile sector):

Figure 4. Automobile & auto parts sector, EUR-denominated YTM curves (%) per Rating notch, 18/04/22



Source: Refinitiv

3.2. The Counterparty Risk in Derivatives trades: IFRS 13 and CVA

From 2008, new, innovative financial regulation was implemented and was increasingly focusing on counterparty risk and OTC derivatives. The US Dodd–Frank Reform and Consumer Protection Act 2009 (Dodd–Frank) and European Market Infrastructure Regulation (EMIR) were designed to enhance the stability of OTC derivative markets. Basel III rules were introduced to strengthen bank capital bases and introduce new requirements on liquidity and leverage.

Although not specifically driven by the effects from the 2008 crisis, IFRS 13 accounting rules were introduced from 2013 to replace IAS 39. IFRS 13 rules provide a single framework around fair value measurement for financial instruments and started to create convergence in practices around CVA. In particular, IFRS 13 uses the concepts of *fair value* and *exit price*, which entails the usage of market-implied quantitative information as much as possible. This is particularly relevant in default risk estimation, as market credit spreads must be used instead of historical default probabilities (somehow following the principle of *forward-looking* estimation approach). “Exit price” also introduces the notion of “own credit risk” and leads to DVA as the CVA charged by a replacement counterparty when exiting a transaction.

The IFRS 13 standard was issued in 2011 and came into effect for annual reporting periods commencing on or after 1st January 2013. This standard represents a general fair value framework. If another IFRS requires or permits the use of fair value as a measurement basis, generally the entity should follow IFRS 13 for measuring the fair value (with the exceptions included in paragraphs 6 and 7 of IFRS 13).

3.2.1 CVA definition

Prior to IFRS 13, and as a general rule, in order to measure the fair value of a financial derivative, future cash flows were estimated using different techniques, and these cash-flows were subsequently discounted using a "risk free" curve (based on interbank rates, such as the EURIBOR 6M swap curve or OIS curves).

In this regard, it was assumed that the potential credit risk adjustment that could arise was not material, or that the credit risk assigned to both counterparties was netted. An adjustment for credit risk was only carried out in those scenarios where incurred losses had to be provisioned. In these cases, the positive value of the derivative was priced downwards to reflect an estimated recoverable amount.

IFRS 13 clarified that when measuring the fair value of derivatives, credit risk must always be considered (see paragraphs 3, 42 - 44 and 69 of IFRS 13). This includes both the risk that the derivative may end with a positive value and the counterparty does not meet its obligations (which means, inherently, the inclusion of the Credit Value Adjustment or commonly, CVA)[12], as well as the risk that the derivative may end with a negative value and the company itself does not meet its obligations (Debt Value Adjustment or DVA, which was not considered prior to IFRS 13).

Credit Value Adjustment (hereinafter, CVA) measurement is similar to the one shown for the case of ECL under IFRS 9, although there is a critical difference concerning the Exposure at default amount. In the case of IFRS 9 impairment the EAD amount could be assumed as the amortized cost of the asset, constant for the remaining lifetime of the contract. However, the exposure to be taken into account for CVA should be understood as a double way exposure, i.e., bilateral. A derivative can take positive or negative values for both counterparties throughout its life span. Hence, it is necessary to model not only the default risk of each counterparty, but also the potential exposure values the derivative might have until maturity. This is understood as the potential exposure amount, which determines the amount of CVA for each counterparty to be subtracted to the current derivative Mark-to-Market. Therefore, the Fair value (i.e., the exit price) for a derivative at time *t* under IFRS 13 would be:

$$Derivative\ Fair\ Value_t = MtM_t - CVA_t + DVA_t \tag{4}$$

where MtM_t the derivative mark-to-market at time *t*, CVA_t the Credit-Value Adjustment and DVA_t the Debt-Value Adjustment at valuation time *t*.

The CVA metric is relevant for OTC derivatives that have no full collateralization, i.e., there is no full hedge of counterparty risk, meaning that there is no full collateral posted daily for both counterparties to hedge the current CVA value. For those derivatives that need the CVA included in its fair value calculation, the future exposure estimation is critical. The CVA/DVA would be calculated as follows[13]:

[12] CVA is not specifically mentioned in IFRS 13. Nevertheless, the standard states that an entity should measure the fair value of an asset or a liability using the assumptions that market participants would use when pricing the asset or liability, assuming that market participants act in their economic best interest. CVA is considered by market participants when pricing the derivative.

[13] See Kenyon and Stamm (2012), Morales (2015) or Gregory (2015) among others, for further details on CVA/DVA estimation

$$CVA/DVA = (1 - R)\int_{0}^{T} E_t^{\mathbb{Q}}\left[(DF(0,t)\cdot V(t)^{+}\right]\cdot PD_c(t)dt \quad (5)$$

where $E_t^{\mathbb{Q}}[(DF(0,t)\cdot V(t)^{+}]$ is the expected discounted value of the derivative's positive exposure $V(t)^{+}$ under a probability measure $\mathbb{Q}$; $PD_c(t)$ is the conditional PD at t; and R is the estimated Recovery Rate.

Therefore, it can be seen that one of the necessary inputs for CVA estimation is the conditional PD of the counterparty between $t = 0$ and $t = T$ while in the case of DVA estimation, one of the necessary inputs is the own conditional PD in the same context. For this, the model proposed in this dissertation for credit risk estimation is fully useful for those counterparties that have no rating nor credit instruments with liquid prices. However, the estimation of the future exposure is something critical to be modelled. In the next chapter, some current market modelling solutions to cope with this issue are presented, for interest rate vanilla and non-vanilla derivatives.

3.2.2 IFRS 13: Fair value hierarchy and the relevancy of credit risk data

The way in which a company should consider the corresponding credit quality in the situations described in section 3.1.1., and the way in which the inputs are developed should be consistent with the fair value hierarchy included in IFRS 13.

Fair value hierarchy refers to the inputs used in order to measure fair value. IFRS 13 prioritises observable inputs over those that are not observable (i.e., that are internally developed by an entity). There are three levels within IFRS 13 fair value hierarchy (IFRS 13 Appendix A):

- *Level 1 inputs*: quoted prices in active markets for identical assets or liabilities that the entity can access at the time of measurement.
- *Level 2 inputs*: inputs other than quoted prices included within Level 1 that are observable for the asset or liability, either directly or indirectly.
- *Level 3 inputs*: unobservable or difficult-to-obtain model inputs for the asset or liability.

IFRS 13 focuses on prioritizing the inputs used in the valuation techniques and not the techniques themselves (see IFRS 13.74), (however, the availability of inputs could affect the valuation technique used).

Therefore, as stated above, when obtaining a PD or a YTM within this context, it is important to consider fair value hierarchy. For example, to obtain a PD for a specific counterparty and maturity:

1. The best input would be the PD calibrated with CDS spreads (on bonds issued by the same counterparty with the same maturity), quoted in a liquid market.
2. Should that information not be available, other potential sources in order to estimate the PD are:

- The quoted YTM of bonds issued by the same counterparty with the same maturity in an active market.
- The quoted CDSs spread (over bonds issued by the same counterparty with the same maturity) in a non-active market.
- The quoted YTM of bonds issued by the same counterparty with the same maturity in a non-active market.
- The quoted CDSs spread (over bonds issued by the same counterparty with similar maturity) in a non-active market. The spread should be adjusted for the difference in maturity.
- The quoted YTM of bonds issued by the same counterparty with similar maturity in an active or non-active market. The PD is adjusted for the difference in maturity.

3. Should the specific counterparty not have quoted CDSs or bonds, nor a public credit rating, it could be internally estimated a credit rating for the specific counterparty in order to obtain the PD from quoted CDSs or bonds of companies with the same rating and characteristics (sector, country, size, etc.). In both cases, as much market information as possible should be used.

The model proposed in Chapter 5 to estimate credit rating and PD would only be used in the case of this last scenario.

3.3. Lease accounting and valuation under IFRS 16

IFRS 16 is the new lease accounting standard that will replace the current IAS 17 for annual reporting periods commencing on or after 1st January 2019.

The implementation of IFRS 16 will specifically affect contracts in which the entity is the lessee. In the majority of these contracts, the entity will have to apply the so-called "capitalization model" which the new standard introduces.

In the capitalization model, the lease asset (right-of-use) and the lease liability are initially measured by discounting future lease payments. Subsequently, the asset is depreciated (in most cases on a straight-line basis), and the liability is accounted for as a debt in which the financial expense is accrued based on the discount rate used.

In addition, in case of subsequent modification of the lease payments (due to changes in variable payments, changes in the lease term, etc.), the lease liability should be recalculated; that is, future cash-flows should be discounted once again (using the original interest rate in some cases and a new interest rate in others).

IFRS 16 establishes the following in relation to the interest rate to be used by a lessee when discounting future lease payments (IFRS 16.26, 41 and 45):

- 1) In principle, the so-called "*implicit interest rate in the lease*" should be used. This is the rate that the lessor obtains from the financing transaction implied by the lease.

- 2) The IASB recognizes that in many cases, the lessee will not be able to obtain the interest rate implicit in the lease because he/she does not possess information on aspects such as the initial costs incurred by the lessor or the residual value of the asset at the end of the lease period (IFRS 16. BC161). In these cases, IFRS 16 allows for the use of the "*lessee's incremental borrowing rate*". This is the rate that the lessee would have to pay on a debt in order to buy the leased asset while taking into consideration the following aspects (IFRS 16.BC161):

- Moment in time.
- The maturity of the lease.
- The economic environment in which the transaction occurs.
- The credit quality of the lessee.
- The nature and quality of the collateral.

Generally speaking, it is expected that many entities will use the incremental borrowing rate instead of the lease implicit rate (see Morales and Zamora, 2017). Therefore, an estimation of the lessee's credit quality is required in order to obtain the borrowing rate.

3.3.1 Introduction to IFRS 16

Under IFRS 16 (as well as under ASC[14] Topic 842), a lessee must apply the capitalization model for the accounting of all lease transactions (except if two voluntary exceptions are applied) (Morales-Díaz and Zamora-Ramírez, 2018a). The capitalization model entails recognizing an asset ("right-of-use") and a liability ("lease liability") in the statement of financial position. Both elements are initially measured as the present value of future lease payments for the duration of the lease term. In order to discount future lease payments (and calculate the present value), IFRS 16 (along with ASC Topic 842) offers the lessee two options (IFRS 16, paragraph 26/ASC Topic 842-20-30-3):

A) "*Interest rate implicit in the lease*" which is defined as "*the rate of interest that causes the present value of (a) the lease payments and (b) the unguaranteed residual value to equal the sum of (i) the fair value of the underlying asset and (ii) any initial direct costs of the lessor*" (IFRS 16 Appendix A; the ASC Topic 842 definition is similar).

B) In those cases where the implicit rate "*cannot be readily determined*", a lessee may use what IFRS 16 names as the "*lessee's incremental borrowing rate*" (IBR), defined as "*the rate of interest that a lessee would have to pay to borrow over a similar term, and with a similar security, the funds necessary to obtain an asset of a similar value to the right-of-use asset in a similar economic environment*" (IFRS 16 Appendix A; the ASC Topic 842 definition is similar5).

[14] Accounting Standards Codification

In principle, the use of the interest rate implicit in the lease is the preferred option, while the IBR is only utilized if the implicit rate "*cannot be readily determined*". Nevetheless, it can in fact easily be demonstrated that, in practice, almost all entities use the IBR (while the interest rate implicit in the lease is not widely used). This is because lessees, generally speaking, do not possess sufficient information to be able to obtain the interest rate implicit in the lease. They do not have information regarding the initial fair value of the leased asset; the direct costs of the lessor; nor the residual value of the leased asset. This is recognized by the IASB in paragraph BC161 of the standard Basis for Conclusions, and also indicated by Deloitte (2018, p.6) and KPMG (2017, p.11). In the case of Spanish IBEX 35 companies, all of them use the IBR as the discount rate (for all or certain specific leases).

One of the greatest technical difficulties of IFRS 16, along with having higher levels of diversity as regards its application, is the estimation of the discount rate (generally the IBR is used, as stated above). In relation to other IFRS standards which also require the estimation of a discount rate, several authors have shown that the calculation and application of discount rates across firms is both inconsistent and arbitrary (Michelon *et al*., 2020; Blum and Thérond, 2019; Schneider *et al*., 2017). Yet, no similar studies have focused on the IFRS 16 discount rate (it being a new standard).

Therefore, entities need to develop their own IBR for discounting lease operation cash flows. In general terms (while following IFRS 16/ASC Topic 842 principles and considering the availability of market information), we can divide the process for estimating the IBR into two steps:

1. **Step 1**. This consists of estimating the yield of a hypothetical loan (to be received by the lessee) with the same maturity as the lease operation (IFRS, 2019, p.6). This initial yield (which we can call the "standard yield") may be easily obtained if the company has quoted bonds or has recently obtained a loan with a maturity similar to the lease contract term. In other cases, it could be obtained from the yield of bonds issued by peer companies (companies with same rating (credit quality) and same sector, currency, geography, etc.).

2. **Step 2.** Depending on the characteristics of the yield obtained in Step 1 (in relation to guarantees and collaterals) and the characteristics of the leased asset, that yield should be adjusted in order to reflect the recovery rate associated with the underlying leased asset. ASC Topic 842 defines the IBR as the rate of a hypothetical loan "*on a collateralized basis*". According to the IFRS Interpretations Committee, "*in determining its incremental borrowing rate, the Board explained in paragraph BC162 that, depending on the nature of the underlying asset and the terms and conditions of the lease, a lessee may be able to refer to a rate that is readily observable as a starting point. A lessee would then adjust such an observable rate as is needed to determine its incremental borrowing rate as defined in IFRS 16*" (IFRIC7, 2019). In other words, once the standard yield is obtained (Step 1), Step 2 would consist of adjusting the initial yield in order to consider the applicable Loss-Given Default (LGD).

Currently, a gap exists in the accounting and finance literature in relation to the models and methodologies that may be applied – so as to adjust the initial yield in order to consider the applicable LGD – given that all of the following characteristics need to be included:

a) The model is able to estimate how the initial standard yield (obtained in Step 1) can be modified in order to reflect a specific LGD related to the leased asset.

b) The model is simple, and able to be used by all kinds of companies currently implementing IFRS 16 (companies from different types of sectors and size).

c) The model uses updated and easily accessible market data (and not historical data) as the principal input. As IFRS 13 explains, observable market data should be used if it is available.

d) Its results can be validated with real market data.

3.3.2 The role of collateral

Lease contracts are collateralized. If the lessee (the “borrower”) fails to make the corresponding payments, the lessor (“lender”) will repossess the asset. Therefore, the lessor would recover the remaining nominal amount of the theoretical loan and will only lose the unmade payments. This also depends on aspects like the physical state of the property, the kind of asset, the possibility of using the assets or leasing it again, etc.

According to Laurentis and Mattei (2009), there is clear evidence that lessors are ex-ante able to balance the probability of default and the loss given default case-by-case, using proper contract structures, as well as carefully managing recovery procedures and strategies according to each operation’s characteristics.

The previous literature extensively covers the LGD (the role of the collateral in all kind of lending agreements) within several contexts including loan pricing; measuring loan loss provisioning; and in estimating the CVA/DVA adjustment for derivatives valuation.

Credit Default Swaps play an important role in loan rates (which is an important basis of one of the models presented in Chapter 6. Previous works have examined how CDS affect the bank loan market, not only with regard to prices but also loan monitoring (Hu and Black, 2008; Shan *et al*., 2019); the availability and cost of credit (Subrahmanyam *et al*., 2014; Saretto and Tookes, 2013; Hirtle, 2009; Shan *et al*., 2016); and the structure of debt contracts (Shan *et al*., 2019). Norden and Wagner (2008) analyze the relationship between the CDS market and banks’ pricing of syndicated loans to US corporates. They find that CDS prices are are strongly linked to the spreads on new syndicated loans. They have also become the dominant factor in explaining these spreads. This suggests that CDS influence loan rates because they represent the opportunity costs of taking on risk, for example, or because they represent the new pricing benchmark. Akguen and

Vanini (2007) propose a model for loan valuation in which they decompose a secured loan into a linear combination of an unsecured loan and a credit derivative (a CDS).

The fact that the value of a derivative is affected by the value of collateral is also covered by Silaghi *et al*. (2020) and Drago *et al*. (2019). Many authors have analyzed the direct influence of the recovery rate (value of the collateral) on loan pricing. In one of the most recent studies, Bellucci *et al.* (2021) use a variety of estimation methods to explore the empirical relationship between interest rate and collateral requirements in bank loan contracts. They conclude that there is a strong relationship between the loan interest rate and collateral, and that the higher the value of the collateral, the lower the interest rate will be with a relevant degree of confidence. This is another basis for the models proposed, from both a fixed income product and a CDS pricing approach.

Luck and Santos (2021) analyze the valuation of collateral by comparing spreads on loans by the same bank to the same borrower at the same original date, but backed by different types of collateral. Their data source is the FED FR Y-14Q database (US). They find that pledging collateral reduces borrowing costs by on average 23 basis points. This effect varies according to the type of collateral, with marketable securities being the most valuable, and with real estate, accounts receivables and inventory being more valuable than fixed assets and a blanket lien. They also find that collateral proves most valuable for riskier firms as proxied by leverage, interest coverage and size. Conversely, collateral is of little or no value for large and publicly listed firms, which tend to be safer and less informationally opaque.

Benmelech and Bergman (2009) investigate how loan pricing in the airline industry varies according to the redeployability of collateral (aircrafts). They find that debt tranches that are secured by more redeployable collateral carry lower credit spreads, higher credit ratings, and higher loan-to-value ratios, thereby confirming that pledging collateral is valuable. Cerquerio *et al*. (2016) use a sample from the loans given by a Swedish bank. They take into consideration a change in the law in Sweden which restricted banks' claims on certain types of their borrowers' collateral. In the case of loans where the collateral was affected, loan pricing increased more in comparison to loans with unaffected collateral. Other studies along these lines are those by Duo and Meder (2020); Lara-Rubio *et al*. (2016); Matias and Dias (2015); Benmelech and Bergman (2011); and Bo (2010).

The conclusions reached by the previous literature (namely that the higher the value of the collateral, the lower the interest rate) are true, since the collateral reduces the lender's loss if the borrower defaults on the loan (Blazy and Weill, 2013; Gonas *et al*., 2004). These findings play an important role in the proposal for the models. Other works like that of Han (2017) demonstrate the correlation between the PD and the LGD in a relevant way as well. Nonetheless, the modelling question remains. There is no practical, implementable model in the literature that is based on current market information and tested against debt prices, that can be adapted to the expected collateral LGD, at least with predefined formulae. That is something I try to cover in the presented models in Chapter 6.

3.3.3 LGD for lease operations

Given the objective of this research, it is important to analyze whether any previous works quantify (using statistical or, in general, quantitative data) the LGD for lease operations depending on the nature of the leased asset. The data obtained in such papers may be used as input for the models presented in Chapter 6. If this data is not available, then the model will not be able to be practically applied, since a percentage change in the LGD due to a change in the leased asset is required.

In general (and not specifically for lease operations), market information concerning recovery rates can easily be obtained for the main sectors, including historical data on LGDs. This information is usually provided by the main rating agencies. An example is the information provided on the recovery rates (LGD) by Moody's (2018) for corporate debt measured by trading prices, split into priority position (from 1st lien bank loans to junior, unsecured and subordinated securities). Using Moody's Ultimate Recovery Database, Khieu *et al*. (2012) estimate a model for bank loan recoveries using variables reflecting loan and borrower characteristics, industry, and macroeconomic conditions, along with several recovery process variables.

Shifting the focus to lease operations, Hartmann-Wendels *et al*. (2014) estimate the recovery rate in lease operations according to the type of leased asset. They used a dataset from three German leasing companies with 14,322 defaulted leasing contracts in order to analyze different approaches to estimating the LGD. They differentiate between the following leased assets: vehicles, machinery, information and communications technology (ICT), equipment and others. As a global average, the LGD for vehicles is approximately 39.5%; for machinery 49%; for ICT 88.2%; for equipment 66%; and 46% for others.

Kaposty *et al*. (2020) analyze the models for predicting loss given default in lease operations. Using a proprietary data set of 1,184 defaulted corporate leases in Germany, the authors explore different parametric, semi-parametric and non-parametric approaches that attempt to predict the LGD. Miller and Töws (2018) introduce a multi-step approach to estimate the overall LGD of leases, based on their economic composition.

The findings of these studies may be used by companies to generate an estimation of the recovery rates of the different leased assets, and subsequently apply the models proposed in Chapter 6.

Other works such as those by Qi *et al*. (2011), Kim and Kim (2006), and Frontczak and Rostek (2015) propose or describe modelling methods for creating LGD estimations, or alternatively estimate the recovery rates for other operations, including mortgage loans for example (Huang and Ozdemir, 2020).

3.3.4 IFRS discount rates

This dissertation is partially concerned with discount rates in IFRS. The model scheme proposed focuses on improving the IFRS 16 discount rate estimation so as to reflect the correct recovery rate related to the leased asset. A line of research into IFRS discount rates does exist, but there is no research specifically dedicated to IFRS 16 (or, previously, to IAS 17); rather the research focuses on other standards under which the discount rate is also important. Husmann and Schmidt (2008) provide guidance in relation to the discount factor for impairment calculation under IAS 36 ("Impairment of Asset"). Under this standard, the entity is able to choose between three alternative starting points (the cost of capital according to the Capital Asset Pricing Model (CAPM); the entity's incremental borrowing rate; and other market borrowing rates). The authors demonstrate that the Weighted Average Cost of Capital (WACC) is the only suitable starting point for all scenarios. Within the IAS 36 context, Kvaal (2010) analyzes the standard's option to use 'the entity's incremental borrowing rate'. According to the author, the incremental borrowing rate may be a useful approximation to the cost of capital within a CAPM framework.

The case presented in this dissertation, however, is different. WACC is not considered in the model developed because the WACC includes the cost of capital, and the cost of capital is different to the cost of debt. According to IFRS 16, the IBR is assimilated to the cost of debt and not to the cost of capital.

Carlin and Finch (2009, 2010) focus on the discount rate in goodwill impairment testing under IAS 36. It is clear that for the authors, decisions related to discount rate selection are of paramount importance in influencing the outcomes of impairment-testing processes implemented pursuant to IFRS. Using empirical data, they also claim that if bias in the selection of discount rates exists, fundamental questions must be asked about the quality of reported earnings, the validity of valuations ascribed to goodwill, and about the status to be accorded to financial statements produced in conformity with the IFRS regime.

Schauten *et al*. (2010) attempt to determine the discount rate for the valuation of intangible assets within the context of USGAAP. The authors use a sample of US Standard and Poor's 500 index, and show that for all the identified sectors, the required return on intangible assets is higher than the WACC. They also show that the return is higher than the levered or unlevered cost of equity of the company as a whole. In six of the eight sectors analyzed, the levered cost of equity appears to be the best proxy for the required return on intangible assets.

Meanwhile, Michelon *et al.* (2020) concentrate on the discount rates used in accounting for decommissioning costs, clean-up costs, and other related environmental liabilities within the context of IAS 37 ("*Provisions, Contingent Liabilities and Contingent Assets*"). They conduct their research using a multi-method approach (including interviews). They find that there is significant diversity in discount rate choices and related disclosures across both industry sectors and countries. In fact, Eckel *et al.* (2003) propose that the standard setters should issue a completely new standard dedicated solely to the discount rate.

Blum and Thérond (2019) find that the discount rate is used in many standards across the different IFRS. They do not adhere to one consistent definition, and likewise not all of them are consistent with finance theory. They also conduct a survey of 30 European accountants, CFOs, auditors and executives with financial functions, covering a variety of topics in relation to discount rates in IFRS. The results show how the entities and the auditors are faced with many challenges in their efforts to comply with IFRS discount rates, and how there is a wide diversity in practice. IFRS 16 is mentioned, but there are no references as to how entities comply with IFRS 16.

CHAPTER 4: INDUSTRY MODELLING REVIEW

In this chapter, we will see a review on some of the main models used in the financial industry related to counterparty credit risk and probability of default estimation. It is a summary of the methodologies used by rating agencies and practitioners at a high-level approach. This means that these methodologies may provide a robust, initial grounding to model developers, but additional modelling aspects are surely considered to adapt them to particular industries and data sources. In fact, the contribution to the credit rating and PD estimation field in this document is partially based on the Moody´s credit rating models presented in next sections.

Likewise, in this chapter we will see some modelling solutions used in the financial industry which cope with the Exposure at Default (EAD) estimation for derivatives concerning IFRS 13. Although the models developed during my doctoral research period have been mainly focused on the credit rating estimation and the subsequent modelling of the IBR, it is important to highlight the relevancy of a robust estimation of the exposure concerning CVA calculation under IFRS 13, as outlined in section **3.2.1**.

4.1. Rating agencies and Credit Rating letter models

Credit rating agencies, like Moody’s, publish research and methodology for a wide spectrum of casuistries related to credit analysis, rating methodologies, credit markets, among others.

In this section, the basic, public Moody’s rating methodology will be shown[15] for the telecommunication sector. It describes the key qualitative and quantitative considerations that are usually most important for assessing credit risk in a given sector. These considerations can be understood as a set of guidance to assigning ratings backed by conceptual background. Also, it is explained through a step-by-step basis:

4.1.1 Qualitative and Quantitative Scorecard/Grid

The main tool that the rating assignment process leverages is a scorecard or grid, which is a reference that can be used to approximate credit profiles within this sector in most cases and to explain, in summary, the factors that are generally most important in assigning ratings to companies in each industry. The scorecard is a summary that does not include every rating consideration, and other quantitative or qualitative considerations. The weights shown for each

[15] Visit www.moodys.com for further information and public papers on methodology for rating estimation for different sectors.

factor in the scorecard represent an approximation of their importance for rating decisions, but actual importance may vary substantially between companies and sectors.

Although the telecommunication sector to be explained is an example of the different sectors for which variable rating methodologies can be applied, the below factors use to be common when using the grid:

1) Company's Scale
2) Business Profile
3) Profitability and Efficiency
4) Leverage and Coverage
5) Company's Financial Policy

4.1.2 *Scorecard Factors and Weighting*

The below table summarizes the Telecommunications sector scorecard published by Moody's in their rating assignment methodology summaries (Moody's, 2018). This scorecard assigns a weight to a given rating concept or metric, so that the weighted average of all the scores will represent the credit rating.

Table 2: Scorecard Factors and relative weights, Telecommunications sector

Rating Factors	Factor weight	Subfactors	Subfactor Weight
Company's Scale	12.5%	Revenue	12.5%
Business Profile	27.5%	Business Model, Competitive Environment and Technical Positioning	12.5%
		Regulatory Environment	7.5%
		Market Share	7.5%
Profitability and Efficiency	10%	Revenue Trend and Margin Sustainability	10%
Leverage and Coverage	35%	Debt/EBITDA	15%
		Retained Cash/Debt	10%
		(EBITDA-CAPEX)/Interest Expense	10%
Financial Policy	15%	Financial Policy	15%
Total	**100%**	**Total**	**100%**

Source: Moody's (2018).

In the development of the credit rating and PD model proposed in this dissertation, the potential correlation between the quantitative variables used by rating agencies (the scorecard factors above) and the ones used as explanatory variables within the model will be checked. As a matter of fact, and as previously outlined in Chapters 1 and 2, one of the initial hypotheses on the model is that the variables used in its construction should be somehow aligned to the ones used by rating agencies.

4.1.3 Mapping Scorecard Factors to a Numerical Score

After estimating or calculating each sub-factor, the outcomes for each of the sub-factors are mapped to a broad Moody's rating category (Aaa, Aa, A, Baa, Ba, B, Caa, or Ca, also called alpha categories) and to a numerical score.

Qualitative factors are scored based on the description by broad rating category in the scorecard. The numeric value of each alpha score is based upon the scale below.

Table 3: Possible numerical outputs for qualitative metrics and related rating, Telecommunications sector.

Rating	**Aaa**	**Aa**	**A**	**Baa**	**Ba**	**B**	**Caa**	**Ca**
Numeric value	1	3	6	9	12	15	18	20

Source: Moody's.

Quantitative factors are scored on a linear continuum. For each metric, the scorecard shows the range by alpha category. The scale below is used to convert the metric, based on its placement within the scorecard range, to a numeric score, which may be a fraction.

Table 4: Possible numerical outputs for quantitative metrics and related rating, Telecommunications sector.

Rating	**Aaa**	**Aa**	**A**	**Baa**	**Ba**	**B**	**Caa**	**Ca**
Numeric value	0.5-1.5	1.5-4.5	4.5-7.5	7.5-10.5	10.5-13.5	13.5-16.5	16.5-19.5	19.5-20.5

Source: Moody's.

4.1.4 Determining the Overall Scorecard - Indicated Outcome

The numeric score for each sub-factor (or each factor, when the factor has no sub-factors) is multiplied by the weight for that sub-factor (or factor), with the results then added to produce an aggregate numeric score. The aggregate numeric score is then mapped back to an alphanumeric scorecard indicated outcome based on the ranges in the table below.

Table 5: Aggregated numeric score and mapping to alphanumeric scorecard

Scorecard indicated outcome (Rating)	Aggregate Weighted Factor Score				
Aaa	0	<	x	≤	1.5
Aa1	1.5	<	x	≤	2.5
Aa2	2.5	<	x	≤	3.5
Aa3	3.5	<	x	≤	4.5
A1	4.5	<	x	≤	5.5
A2	5.5	<	x	≤	6.5
A3	6.5	<	x	≤	7.5
Baa1	7.5	<	x	≤	8.5
Baa2	8.5	<	x	≤	9.5
Baa3	9.5	<	x	≤	10.5
Ba1	10.5	<	x	≤	11.5
Ba2	11.5	<	x	≤	12.5
Ba3	12.5	<	x	≤	13.5
B1	13.5	<	x	≤	14.5
B2	14.5	<	x	≤	15.5
B3	15.5	<	x	≤	16.5
Caa1	16.5	<	x	≤	17.5
Caa2	17.5	<	x	≤	18.5
Caa3	18.5	<	x	≤	19.5
Ca	19.5	<	x	≤	20.5
C	x	>	20.5		

Source: Moody's.

For example, an issuer with an aggregate weighted factor score of 11,7 would have a Ba2 scorecard-indicated outcome.

Concerning the final rating, the below criteria is used by Moody's to assign each scorecard outcome and therefore, the credit rating (concerning the Telecommunications sector):

Table 6: Qualitative and quantitative factors and scale by their outcome (I), Telecommunications sector.

	Sub-factor Weight	Aaa	Aa	A	Baa	Ba	B	Caa	Ca
Factor 1 Scale (12.5%)									
Revenue (USD Billion)*[1]	12.5%	≥100	50-100	25-50	12.5-25	5-12.5	2-5	0.5-2	<0.5
Factor 2 Business Profile (27.5%)									
Business Model, Competitive Environment and Technical Positioning - Diversified Carriers	12.5%	Strong geographically diversified incumbent national provider of full suite of integrated services to a broad customer base with wireline and wireless segments exposed to very limited competitive challenges; and very successful international expansion; and very low technology risk.	National incumbent provider of full suite of integrated services to a broad customer base with wireline and wireless segments exposed to limited competitive challenges; and successful international expansion; and low technology risk.	National incumbent provider of full suite of integrated services to a broad customer base wireline and wireless segments exposed to increasing competitive challenges; and. moderate international expansion; and low to moderate technology risk. OR Regional provider*[2] of full suite of integrated services to a broad customer base with wireline and wireless segments exposed to moderate competitive challenges; and moderate expansion outside of home market with typically about 50% to 60% of sales in one market, country or region; and low to moderate technology risk.	National provider of full suite of integrated services to a fairly broad customer base and substantial competitive challenges; and moderate technology risk. OR Regional provider of full suite of integrated services to a fairly broad customer base and increasing competitive challenges and limited expansion outside of home market with typically about 60% to 80% of sales in one market, country or region; and moderate technology risk.	Regional provider of full suite of integrated services to a narrow customer base with increasing competitive challenges; or typically about 80% to 90% of sales in one market, country or region; or moderate to high technology risk.	Regional provider of full suite of integrated services to a narrow customer base with increasing competitive challenges; or typically more than 90% of sales in one market, country or region; or high technology risk.	Provider of full suite of integrated services highly dependent on access to incumbent's network; or very high technology risk.	Provider of full suite of integrated services with very limited access to incumbent's network; or extremely high technology risk; or high probability of disruption in service because of the poor quality of network.

Source: Moody's

Table 7: Qualitative and quantitative factors and scale by their outcome (II), Telecommunications sector.

	Sub-factor Weight	Aaa	Aa	A	Baa	Ba	B	Caa	Ca
						Moderate to high technology risk.			
Regulatory Environment	7.5%	Regulatory framework is fully developed, has a very long-track record of being extremely predictable and stable, and is extremely supportive of Return on Investment (ROI) for incumbent telecom providers and is very unlikely to change; and regulatory body is located in a highly rated sovereign with very strong institutional framework and effectiveness or strong independent regulator with unquestioned authority over telecom regulation that is national in scope; and very unlikely awards of new operating concessions.	Regulatory framework is fully developed, has a long track record of being very predictable and stable, and is highly supportive of ROI for incumbent telecom providers and is unlikely to change; and regulatory body is typically located in a high to moderate rated sovereign with strong institutional framework and effectiveness or strong independent regulator with authority over most telecom regulation that is national in scope; and unlikely awards of new operating concessions.	Regulatory framework is fully developed, is very predictable and stable in balancing the interests of the incumbent telecom providers and the new comers but with less track-record and is highly supportive of ROI for incumbent telecom providers; and regulatory body is a sovereign, sovereign agency or independent regulator with authority over most telecom regulation that is national in scope; and unlikely awards of new operating concessions.	Regulatory framework is fully developed, has a short track-record of being predictable and stable in overall supporting the interests of the incumbent telecom providers while being somewhat more supportive to new entrants, still allowing an adequate ROI for incumbent telecom providers; and regulatory body is a sovereign, sovereign agency or independent regulator with authority over most telecom regulation that is national in scope with change in administration having some potential to alter outlook; and potential awards of limited new operating concessions.	Regulatory framework is a) well-developed, with evidence of some inconsistency or unpredictability in the way framework has been applied, or framework is new and untested, but based on well-developed and established precedents, or b) jurisdiction has history of independent and transparent regulation in other sectors; or regulatory environment may sometimes be challenging and politically charged; or regulatory support for increased facilities and non-facilities based competition. OR Regulation generally favors new market entrants. OR Likely awards of new operating concessions. OR Regulatory bodies in active deliberations to negatively alter the regulatory framework.	Regulatory framework is developed, but there is a high degree of inconsistency or unpredictability in the way the framework has been applied; or regulatory environment is consistently challenging and politically charged; or there is no consistent track record of independent and transparent regulation. Jurisdiction has a history of difficult or less supportive regulatory decisions, or regulatory authority has been or may be challenged or eroded by political or legislative action.; or regulatory support for non-facilities based competition. OR Regulation strongly favors new market entrants. OR Regulatory change to have strong negative impact on the regulatory framework.	Regulatory framework is not developed, is unclear, is undergoing substantial change or has a history of being unpredictable or adverse to telecom operators; or regulatory body lacks a consistent track record or appears unsupportive, uncertain, or highly unpredictable; or may face high risk of significant government intervention in operations or markets; or strong regulatory support for non-facilities based competition. OR Regulation is highly unbalanced towards favoring new market entrants.	Regulatory framework or regulatory body carry extremely high risk for the business continuity of telecom operators.

Source: Moody's

Table 8: Qualitative and quantitative factors and scale by their outcome (III), Telecommunications sector.

	Sub-factor Weight	Aaa	Aa	A	Baa	Ba	B	Caa	Ca
Market Share	7.5%	Company is the principal player in the local market and in most of the regions where it operates. OR Company has monopoly-type presence in its local region.	Company is a clear market leader in the local market and holds competitive positions in all regions where it operates. OR Company is the principal player and very strong market leader in its local region.	Company is a very solid competitor in the local market and holds competitive positions in most of the regions where it operates. OR Company is a clear market leader in its local region.	Company is a well-positioned competitor in its local market and holds competitive positions in many regions where it operates. OR Company is a very solid competitor in its local region.	Company is a mid to lower-tier competitor in its local market and holds competitive positions in some of the markets where it operates. OR Company is a well-positioned competitor in its local region.	Company is a small competitor in its local market and holds minor competitive positions in other markets. OR Company is a mid to lower-tier competitor in its local region.	Company is a small competitor in its local market.	Company is a start-up with no track record.
Factor 3: Profitability and Efficiency (10%)									
Revenue Trend and Margin Sustainability	10%	On a sustainable basis: Strong revenue growth AND Exceptional margin levels.	On a sustainable basis: Moderate revenue growth AND Very high margin levels.	On a sustainable basis: Slight revenue growth AND High margin levels.	On a sustainable basis: Stable revenues AND Good margin levels.	Expectation of: Slight, sustained decline in revenues OR Sustained moderate margin levels.	Expectation of: Moderate, sustained decline in revenues OR Sustained modest margin levels.	Expectation of: Strong decline in revenues OR Sustained weak margin levels.	Expectation of: Steep decline in revenues OR Sustained very weak or extremely unpredictable margin levels.
Factor 4: Leverage and Coverage (35%)									
Debt / EBITDA (x)*[3]	15%	≤ 0.5	0.5-1	1-2	2-2.75	2.75-3.75	3.75-5.5	5.5-8	> 8
RCF / Debt (%)*[4]	10%	≥ 60	45-60	35-45	25-35	20-25	10-20	5-10	< 5
(EBITDA-CAPEX)/ Interest Expense (x)*[5]	10%	≥ 8	6.5-8	5-6.5	3.5-5	2-3.5	1-2	0.5-1	< 0.5

Source: Moody's

Table 9: Qualitative and quantitative factors and scale by their outcome (IV), Telecommunications sector.

	Sub-factor Weight	Aaa	Aa	A	Baa	Ba	B	Caa	Ca
Factor 5: Financial Policy	**(15%)**								
Financial Policy	15%	Expected to have extremely conservative financial policies (including risk and liquidity management); very stable metrics; public commitment to very strong credit profile over the long term	Expected to have very stable and conservative financial policies (including risk and liquidity management); stable metrics; minimal event risk that would cause a rating transition; public commitment to strong credit profile over the long term	Expected to have predictable financial policies (including risk and liquidity management) that preserve creditor interests. Although modest event risk exists, the effect on leverage is likely to be small and temporary; strong commitment to a solid credit profile	Expected to have financial policies (including risk and liquidity management) that balance the interest of creditors and shareholders; some risk that debt funded acquisitions or shareholder distributions could lead to a weaker credit profile	Expected to have financial policies (including risk and liquidity management) that tend to favor shareholders over creditors; above average financial risk resulting from shareholder distributions, acquisitions or other significant capital structure changes	Expected to have financial policies (including risk and liquidity management) that favor shareholders over creditors; high financial risk resulting from shareholder distributions, acquisitions or other significant capital structure changes	Expected to have financial policies (including risk and liquidity management) that create a material risk of debt restructuring in varied economic environments	Expected to have financial policies (including risk and liquidity management) that create elevated risk of debt restructuring even in healthy economic environments

Source: Moody's

4.2. Probability of Default Analytical Models

4.2.1 Z-Score model

The Z-Score model, commonly referred to as the Altman Z-Score, was developed by Professor Edward I. Altman in 1968. Although Altman *et al.* have subsequently modified the original Z-Score model to create the Z'-Score Model, the Z"-Score Model, and the Zeta Model, the Z-Score model is still a common component of many credit rating systems.

The Z-Score is constructed from six accounting values and one market-based value. These seven values are combined into five ratios which are the pillars that comprise the Z-Score. The five pillars are combined using the equation below to result in each company's Z-Score (Altman 2002).

$$Z - Score = 1.2\beta_1 + 1.4\beta_2 + 3.3\beta_3 + 0.6\beta_4 + 1.0\beta_5 \tag{6}$$

where

$$\beta_1 = \frac{Working\ Capital}{Total\ Assets}$$

$$\beta_2 = \frac{Retained\ Earnings}{Total\ Assets}$$

$$\beta_3 = \frac{EBIT\ (Earnings\ Before\ Interest\ and\ Taxes)}{Total\ Assets}$$

$$\beta_4 = \frac{Market\ Value\ of\ Equity}{Book\ Value\ of\ Total\ Liabilities}$$

$$\beta_5 = \frac{Sales}{Total\ Assets}$$

This formula appeals to the practitioner's intuition because each pillar describes a different and relevant aspect (from the point of view of its credit health) of a company. Liquidity, cumulative profitability, asset productivity, market based financial leverage, and capital turnover are addressed by the five ratios respectively. The Z-Score presumes that each ratio is linearly related to a company's probability of bankruptcy.

The Working capital/Total assets ratio is a measure of the net liquid assets of the firm relative to the total capitalization. Working capital is defined as the difference between current assets and current liabilities. When a firm is experiencing consistent operating losses, current assets will decrease in relation to total assets. In different analysis, β_1 proved to be more valuable than the current ratio and the quick ratio. This ratio explicitly considers liquidity and size dimensions. The Retained Earnings/Total assets ratio refers to the earned surplus of a firm over its entire life. This measure of cumulative profitability over time is one of the two (the other is the use of the market value of equity, instead of the book value) "new" ratios evaluated by Altman for the latest Z-Score model. It considers implicitly the age of the firm due to its cumulative nature and the use of leverage in order to finance the asset growth of the firm. The Earnings before interest and taxes/Total assets ratio is a measure of the true productivity or profitability of the assets of a firm. It is not affected by any tax or leverage factors. It reflects the earning power of the assets that determines the value of assets. In a bankrupt sense, insolvency occurs when the total liabilities exceed this fair value. The Market value equity/Book value of total liabilities ratio shows how much the assets of a firm can decline in value (measured by market value of equity plus debt) before the liabilities exceed the assets and the firm becomes insolvent. This ratio adds a market value dimension to the model. The Sales/Total Assets ratio is the standard capital-turnover ratio illustrating the sales generating ability of the assets of a firm. It refers to the capability to deal with competitive conditions that the management have.

For the case of private firms (PF), the Z-score formula was recalibrated and the new β_i apply as below:

$$Z - Score_{\ PF} = 3.25 + 6.56\beta_1 + 3.26\beta_2 + 6.72\beta_3 + 1.05\beta_4 \tag{7}$$

including a constant parameter and wiping out the Sales / Total Assets ratio to reduce the industry effect. In this case, the ratio for β_4 is Book Value of Equity / Book Value of Total Liabilities.

Once the Z-Score has been obtained, the following matrix can be used to assign an estimated rating to the corresponding Z-Score.

Table 10: Equivalence between Z-Score and Rating

Z-Score	Equivalent Rating
> 8.15	AAA
7.6	AA+
7.3	AA
7.0	AA-
6.85	A+
6.65	A
6.4	A-
6.25	BBB+
5.85	BBB
5.65	BBB-
5.25	BB+
4.95	BB
4.75	BB-
4.5	B+
4.15	B
3.75	B-
3.2	CCC+
2.5	CCC
1.75	CCC-
0	D

Source: Altman *et al*, 2004.

4.2.2 Ohlson model

The Ohlson model is based on a logistic regression (Ohlson 1980), and, unlike the Z-Score model, it directly represents the probability of default within the next two years for a given firm. It uses ratios with accounting metrics that are similar to Almant's Z-Score but includes dummy variables to get into the output the impact of the effects of leverage and financial losses.

The Ohlson model is as follows:

$$O - Score = -1.32 - 0.407X_1 + 6.03X_2 - 1.43X_3 + 0.0757X_4 - 1.83X_5 - 2.37X_6 - 1.72X_7 + 0.285X_8 - 0.521X_9 \quad (8)$$

Ratios or predictors which were selected for the probabilistic model of bankruptcy are the following ones:

$$X_1 = \log\left(\frac{Total\ Assets}{Gross\ National\ Product}\right)$$

$$X_2 = \frac{Total\ Liabilities}{Total\ Assets}$$

$$X_3 = \frac{Working\ Capital}{Total\ Assets}$$

$$X_4 = \frac{Current\ Liabilities}{Current\ Assets}$$

$$X_5 = \frac{Funds\ from\ Operations}{Total\ Liabilities}$$

$$X_6 = \frac{Net\ Income}{Total\ Assets}$$

$$X_7 = 1\ if\ TL > TA; 0\ otherwise$$

$$X_8 = 1\ if\ Net\ Income_t\ and\ Net\ Income_{t-1} < 0; 0\ otherwise$$

$$X_9 = \frac{NI_t - NI_{t-1}}{|NI_t| + |NI_{t-1}|}$$

The final O-score is translated into the default probability for the next two years following the below logistic function:

$$PD_{Ohlson} = \frac{e^{O-Score}}{1 + e^{O-Score}} \tag{9}$$

4.3. KMV Structural model

This model was initially proposed by Merton (1974) and then adjusted for practical implementation by Kealhofer, McQuown and Vasicek – KMV (1984, 1989). It is currently used by Moody's (2003), Refinitiv or Bloomberg (with some adjustments and improvements) for short term Default Probability estimation. It can be seen as a set of equations constructed in order to obtain the credit risk embedded in the equity price of a company.

The idea behind this model is that equity prices are a good predictor of a company's net assets value performance, and that this fact can be linked to the concept of liquidity and leverage management. It estimates the default risk using the relationship between equity, assets and liabilities.

4.3.1 Concepts and preliminary basis

The model assumes that a company will default when the value of its assets is not sufficient to pay the debts that the company should settle in the short-medium term. In this sense, when the value of the assets decreases below the value of the debts, the company's value is zero or near zero. The probability that this event occurs is the probability of default of the company. At this stage, two aspects should be considered:

1) The need of estimating the probability of the value of assets decreasing below the value of the liabilities a given period.

2) The need of relating default probabilities and credit ratings.

The model proposed herein is based on a methodology that estimates the probabilities of default based on the company's equity and its financial statements. The company shares and/or the peers' equity market will act as the predictor of the company assets performance and its volatility. These two items are critical in order to estimate the probability of the assets having a value lower that the debt value.

The debt value is the other key factor: the higher the debt book value, the higher the probability of assets going down below such a debt book value, for a certain timeframe.

4.3.2 Model theory (I): lognormal property of equity prices and Montecarlo simulation

The main model concern is obtaining the probability of the assets value decreasing below the debt value in the near future. In this sense, we need to know how the assets can perform "*forward-looking*" (i.e., the values they can take in the future).

For this purpose, Montecarlo (MC) framework is generally used in the market to simulate future movements of an asset (equities, foreign currency rates, interest rates, commodities) based on the normality property assumed in the returns, and the implied lognormality that market-quoted asset prices show. Therefore, the main inputs needed are:

- The assets' annualized volatility. This volatility can be obtained from the volatility of the equity market value of the entity. If company's shares were not publicly traded, we could use similar traded companies to estimate this input.
- The annualized expected return of the assets.

Montecarlo method is based on the following assumption: an asset value moves with uncertainty in the market; this is, it is stochastic by nature. However, although assets are understood to follow a stochastic process, its expected returns and volatility define its expected value and confidence intervals on a given timeframe. The stochastic process that allows to simulate an asset movement on a given period is also known as a generalized Wienner process, and can be noted as:

$$dS = \mu S dt + \sigma S dZ \tag{10}$$

where S is the asset price, μ is the asset drift (computed as the average annualized return); σ is the instantaneous volatility (standard deviation) at time t for the asset price, and Z is a standard Brownian motion, which provides the process with stochastic property and follows a Normal distribution (0,1). The discrete-time version of the model is:

$$\Delta S = \mu S \Delta t + \sigma S\, Z\sqrt{\Delta t} \tag{11}$$

Following the Ito's lemma (see, for instance, Brigo and Mercurio, 2006) and discretizing, we have:

$$\ln(S\,(t+\Delta t)) = \ln(S\,(t)) + \mu\Delta t - \frac{\sigma^2}{2}\Delta t + \sigma\big(Z(t+\Delta t) - Z(t)\big) \tag{12}$$

and, in terms of the generic asset price jump from t to t + Δt:

$$S(t+\Delta t) = S(t)\, e^{\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma Z\sqrt{\Delta t}} \tag{13}$$

This is, (13) is the equation that defines the asset price movement simulated from t to $t+\Delta t$, based on the annualized asset volatility σ, drift μ and the stochastic value that Z takes for each simulation jump.

4.3.3 Model theory (II): the Company as a call option and the equity-assets relationship

Assuming that the equity price follows the process stated in (11), one can jump to the next model assumption: the company can be seen as a call option, in the sense that when the assets value decreases below the debt value, the company's value is near zero. This property is based on the following facts and assumptions:

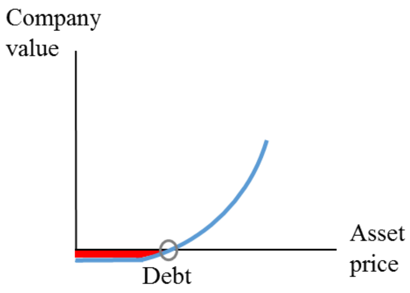
1) an option price can be simulated following (13) as in the Black-Scholes-Merton option pricing framework,

2) the company's assets value is expected to follow the same behavior as the equity has by (12); this is, lognormally distributed, with adjusted annualized volatility and drift. This assumption is consistent as the equity movements will impact the asset movements assuming constant liabilities, but the movement proportion will not be the same, so that drift and volatility should be adjusted,

3) debt value is assumed as a constant for the simulation period.

The Merton's credit risk model assumes the analogy of a company value (its equity market value) and a call option on its assets value, as the figure below suggests:

Figure 5. The company value seen as a call option



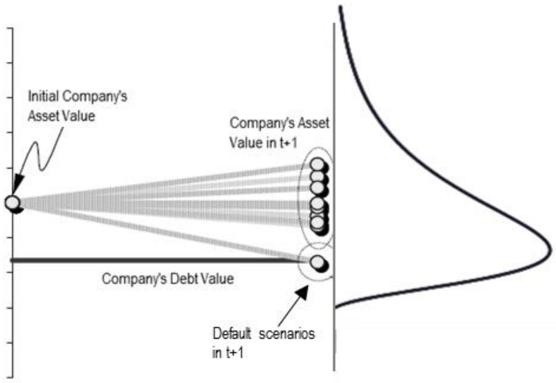
Source: Compiled by the author.

In the Black-Scholes-Merton framework, an asset future path can be therefore simulated by the following model, which is the model shown in (22) but adapting the model inputs for the asset value diffusion process:

$$V(t+\Delta t) = V(t)\, e^{\left(\mu - \frac{\sigma_V^2}{2}\right)\Delta t + \sigma_V Z \sqrt{\Delta t}} \tag{14}$$

where $V(t)$ is the company's asset value today; μ is the drift, understood as the asset annualized growth, σ_V is the asset volatility and Z is a standard Brownian motion. Under these premises, equation (14) can be used to simulate the asset pathways over a given timeframe in order to calculate the percentage of simulations that lead the asset price to end up below the debt book value over a given timeframe (so-called time-to-default, usually one year) to get the probability of default.

Figure 6. Merton's default model - simulation scheme



Source: Merton (1984)

Black-Scholes-Merton model for option pricing provides also with an analytical solution for the simulation framework explained above[16]. A call option value, in this case, the company's expected value (equity) depending on contingent underlying (the assets value), can be calculated. Hence, from the Black-Scholes pricing model, the probability of default can be calculated deriving from (23): Black-Scholes model defines a call option price as:

$$Call_{European} = S\,\Phi(d_1) - Ke^{-rt}\Phi(d_2) \tag{15}$$

where *S* is the equity price, *K* is the strike, *r* is the risk-free rate, and:

$$d1 = \frac{Ln\left(S/_K\right) + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}} \tag{16}$$

$$d2 = \frac{Ln\left(S/_K\right) + (r - \frac{\sigma^2}{2})t}{\sigma\sqrt{t}} = d1 - \sigma\sqrt{t} \tag{17}$$

So, in the case of a company's expected value, the above equations turn into the following ones:

$$Equity\ value = V\,\Phi(d_1) - De^{-\mu t}\Phi(d_2) \tag{18}$$

$$d1 = \frac{Ln\left(V/_D\right) + (\mu + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}} \tag{19}$$

$$d2 = \frac{Ln\left(V/_D\right) + (\mu - \frac{\sigma^2}{2})t}{\sigma\sqrt{t}} = d1 - \sigma\sqrt{t} \tag{20}$$

$\Phi(d_2)$ is the analytical solution for the probability of an asset price being higher than the strike price; this is, the probability of exercising the option. $d2$ is the Asset Distance to Default in number of standard deviations. $\Phi(d_2)$ is therefore the probability of the underlying being higher than the strike price, in this case the debt book value. This is,

[16] See, for example, Black and Scholes (1973) or Hull (2012) for obtaining the analytical solution from the generalized Wienner process.

$$\Phi(d_2) = Survival\ Probability \tag{21}$$

hence,

$$1 - \Phi(d_2) = Default\ Probability \tag{22}$$

When using and calibrating models like (18), two factors are intrinsically critical: the company's asset growth and the volatility of the asset. The company's assets growth represents the average return expected for the assets, in such a way that the higher the drift, the higher the expected asset value and therefore the lower default probability. It can be estimated as the annualized return the assets have had during the last 5 years.

Regarding asset volatility, it is clear that it is a factor that is not observable, and not very reliable given the frequency at which financial statements are issued. However, as previously described, assets' volatility is affected by the equity value. Hence, one needs to figure out the value of assets volatility given by the equity volatility: this calculation relies on the Black-Scholes differential equation:

$$rf = \frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} \tag{23}$$

where f is the derivative price on a contingent underlying S which follows the stochastic process and r is the risk-free rate. (23) can be approximated by a Taylor series expansion giving

$$\Delta f = \frac{\partial f}{\partial t}\Delta t + \frac{\partial f}{\partial S}\Delta S + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\Delta S^2 + \frac{1}{2}\frac{\partial^2 f}{\partial t^2}\Delta t^2 + \frac{\partial^2 f}{\partial S \partial t}\Delta S \Delta t + \cdots \tag{24}$$

(24) states the relationship between the derivative price and the risk factors involved in its pricing. The first term on the right-hand side states how much the price of the derivative changes for each change in a time unit. This partial derivative is known as Theta (Θ). The second one, Delta (Δ), relates the price change of the derivative with the underlying price change. The third one, Gamma (Γ), is the second partial derivative of the price of the derivative with respect to the underlying price, to capture the convexity effect, as can be seen in **Figure 5**. Subsequently, additional and cross-partial derivatives can be computed. Ignoring the Theta term, and option price change can be understood as the following:

$$\Delta f = Delta\ \Delta S + \frac{1}{2} Gamma\ \Delta S^2 \tag{25}$$

Hence, we can establish the relationship between asset and equity volatility absolute quantities as

$$\sigma_{equity} Equity_{t_0} = \Delta_{Eq|V}\ \sigma_V V_0 + \frac{1}{2}\Gamma_{Eq|V}\ \sigma_V V_0^2 \tag{26}$$

where σ_{equity} is the historical or implied annualized equity market price volatility, $Equity_{t_0}$ is the company's equity market price at the calculation moment, $\Delta_{Eq|V}$ is the delta of the Equity on the company's assets, $\Gamma_{Eq|V}$ is the gamma in the same context, V_0 is the company's assets value, and σ_V is the assets volatility to be calibrated. Knowing that on European options, the Black-Scholes framework gives

$$Delta_{call} = \Phi(d_1) \tag{27}$$

$$Gamma_{call} = \frac{\Phi(d_1)}{S_0\sigma\sqrt{t}} \tag{28}$$

we can calibrate σ_V by rearranging the terms in (26), and get the volatility to be used in (19) and (20).

4.3.4 *Model implementation: from default probabilities to short-term Credit Ratings and additional considerations*

Following Refinitiv database, the below table relates the 1-year probability of default and the rating letter assigned.

Table 11: Implied 1y Probability of Default & Rating

Probability of Default (Lower Limit)	Probability of Default (Upper Limit)	Implied Letter Rating
0,0000%	0,0010%	AAA
0,0010%	0,0020%	AA+
0,0020%	0,0040%	AA
0,0040%	0,0080%	AA-
0,0080%	0,0150%	A+
0,0150%	0,0250%	A
0,0250%	0,0380%	A-
0,0380%	0,0540%	BBB+
0,0540%	0,0730%	BBB
0,0730%	0,1110%	BBB-
0,1110%	0,1870%	BB+
0,1870%	0,3060%	BB
0,3060%	0,4720%	BB-
0,4720%	0,8700%	B+
0,8700%	1,5600%	B
1,5600%	2,5000%	B-
2,5000%	3,6900%	CCC+

Source: Refinitiv, 2021

Some topics to be considered when implementing this model are:

1) Entities are generally more likely to default when their asset value reaches a certain critical level somewhere between the value of total liabilities and the value of short-term debt. Therefore, in practice, using only the short-term debt or the total liabilities as a strike might not be an accurate measure of the actual probability of default. The strike selection will also depend on the debt structure and the leverage ratio sensitivity, among others. However, a widespread solution is to set the strike, so-called Default Point (DPT), as

$$DPT = Short\ Term\ Debt + 0.5\ Long\ Term\ Debt \tag{29}$$

2) Unlike in the Merton concept, the KMV μ is no longer a risk-free rate related return, but the expected rate of the return of the company's asset. This is, the relative logarithmic return between $Assets_{t-1}$ and $Assets_t$.

3) The Distance-to-Default equation can be approximated by

$$\text{DTD} = \frac{\text{E}(V_t) - DPT}{\sigma} \tag{30}$$

when drift is very low and time-to-default (t) is also short, being $\text{E}(V_t) = V_t\, e^{\mu t}$.

The below example is shown in turn to clarify the model implementation.

Say that, once the company's latest balance sheet has been analyzed, we have the following information:

- Total Assets: 40.000.000€
- Short-term debt book value: 15.000.000€
- Long-term debt book value: 18.000.000€
- Drift: 0.8%, annualized
- Asset volatility already calibrated: 16%
- Time-to-default: 1 year

We consider the DPT = 15m € + 0.5·18m € = 24m €

Thus, we use the above information to compute $d2$:

$$d2 = \frac{Ln\left({}^{40}/_{24}\right) + (0.008 - \frac{0.16^2}{2})1}{0.16\sqrt{1}} = 3.16$$

so that

$$1 - \Phi(d_2) = 1 - 99.92\% = 0.08\%$$

Following **Table 11**, the 1-year default probability leads to an estimated credit rating BBB-. This is an example on how to use financial and market information within the model. Obviously, further financial and accounting analysis is highly recommended to accurately set the risk factors within the model. This is, there could be some items which maybe could be adjusted or not considered, for instance, longest-term debt.

4.4. Exposure projection for IFRS 13 CVA estimation

As previously discussed in section **3.2.1**., a derivative can have positive or negative values for both counterparties throughout its life span. Therefore, for the CVA calculation, it is necessary to model the exposure in the future, assuming that it is not constant, opposite to the case of a bond or a loan which are expected to pay coupons and notional like an "amortized cost". This is understood as the potential exposure amount, which determines the amount of CVA for each counterparty to be subtracted to the current derivative Mark-to-market.

4.4.1 Potential exposure

Depending on the type of derivative, the risk factors that affect its value will be different. For example, if we are long on a currency forward, the evolution of exchange and interest rates will be the factors to be estimated; if we buy an IRS, we must estimate the evolution of the corresponding interest rates, while if we are exposed to an equity swap, both interest rates and the price of the corresponding equity underlying must be projected.

Therefore, the EAD will not be "flat" but will be different in the many different moments into the future. So, the Fair Value of the derivative would be given by the sum of the current exposure and the future default risk exposure: **MtM - CVA**[17]:

$$FV_{Derivative} = MtM_t - CVA = MtM_t - (1 - R)\int_0^T E_t^{\mathbb{Q}}\left[(DF(0,t)\,V(t)^+\right] \cdot PD_c(t)dt \tag{31}$$

where $E_t^{\mathbb{Q}}[(DF(0,t) \cdot V(t)^+]$ is the expected discounted value of the derivative's positive exposure $V(t)^+$ under a probability measure $\mathbb{Q}$; $PD_c(t)$ is the conditional probability of default at *t*; and R is the estimated Recovery Rate. As the above expression is "continuous", this means that in practice, the discretized CVA expression would be:

$$CVA = (1 - R)\sum_{t=0}^{T} DF(0,t)\,V(t)^+\,\Delta PD_{t-1,t} \tag{32}$$

Our concern lies in a robust estimate of the potential risk: for a given current MtM, we need to project its future value to estimate the exposure that we will have to manage. In words, estimate the exposure of the given derivative at every future time until maturity. Hence, we need to project that expected discounted value of the derivative's positive exposure: $V(t)^+$ (also called Expected Positive Exposure or EPE) at any future point *t* in the above expression. Usually, the points on which the EPE is computed and multiplied by the conditional default probability ΔPD are assumed monthly or quarterly, matching with the times when coupons of the derivative are paid off.

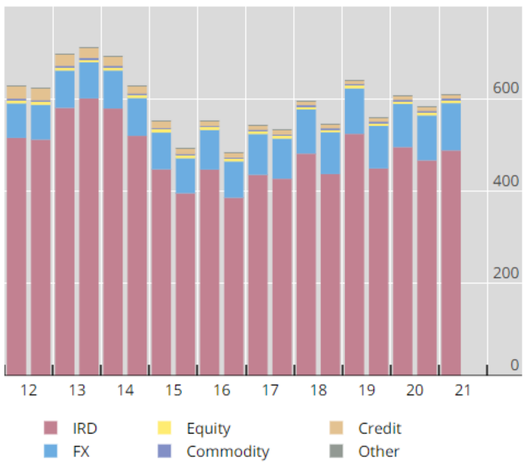
To project the EPE, we must determine the market risk factors (interest rates, exchange rates, equity returns, volatility, etc.) that affect the valuation of the derivative and project its evolution over time. This evolution can be determined by generating numerous 'evolution' scenarios for the corresponding risk factors, according to different stochastic processes and models (Montecarlo, Vasicek, Hull&White, LMM, etc.) that best fit the historical distribution of the risk factor values, as well as adjusting due to the macroeconomic situation that affects the evolution of these factors.

[17] For the sake of simplicity, we will work with the assumption that only CVA is taken as counterparty risk, not considering the own default risk (DVA). However, as seen in section 3.2., DVA should also be considered, therefore the own PD and also the Expected Negative Exposure (i.e., the Expected Positive Exposure for the counterparty) should be computed as DVA inputs.

4.4.2 Projecting interest rates-linked exposure

Although there are many different risk factors to which a derivative can have exposure, in this section we will focus on two different models to project interest rate exposures, as interest rate derivatives are, by far, the most relevant derivatives in the OTC market. In the figure below it is shown the global OTC market notional outstanding, split by asset class[18].

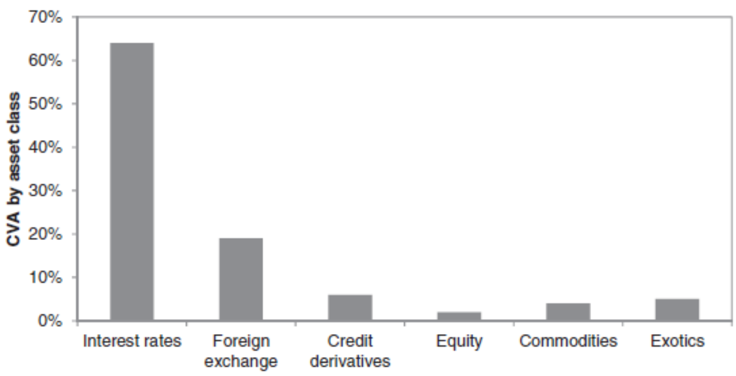
Figure 7. Notional amount outstanding (USD trillions), OTC derivatives



Source: BIS, 2021

Also, it should be known that, although most of corporate and investment banking players have exposure to many different risk factors, interest rate risk is common among many types of hedges and strategies. Likewise, a relevant portion of the CVA amount in the financial sector arises from the hedging trades sold to corporates and non-financial counterparties, which vastly uses IRSs and Cross-Currency swaps as hedging instruments.

Figure 8. CVA amount share of global OTC derivatives by asset class



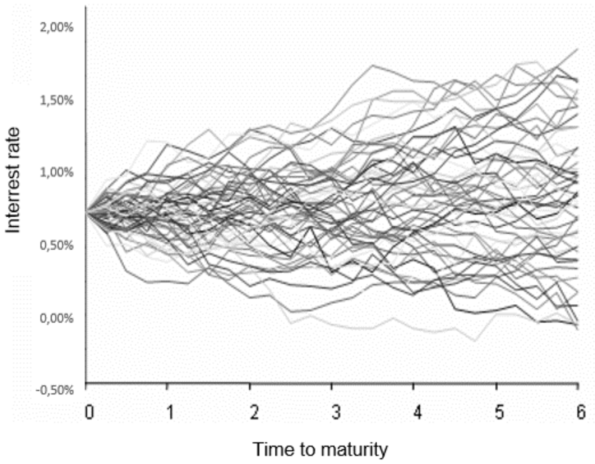
Source: Solum Financial, 2013

Focusing on the potential models to be applied, if we take as an example the valuation of the future exposure of a EURIBOR 3M-linked IRS, we should establish as a risk factor the evolution

[18] IRD means Interest-Rate Derivative

of the market EURIBOR 3M to model forward rates, and also the discounting curve (e.g., ESTR swap curve). Therefore, the curves evolution would have to be projected, so we should generate projection scenarios of those underlyings, and then translate them into the corresponding future MtM at every single future point (e.g., every three months) to maturity.

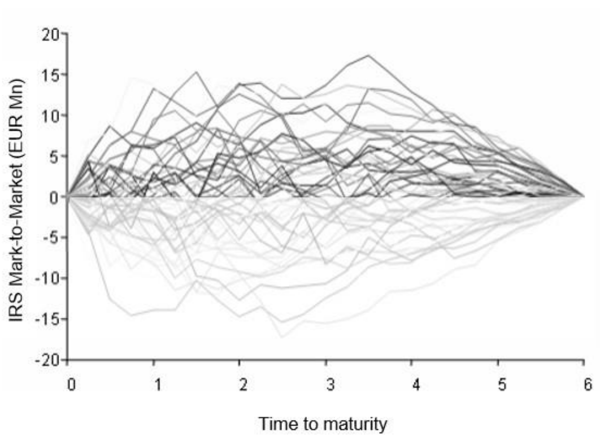
Figure 9. Example of interest rate simulation paths



Source: compiled by the author

Applying each interest rate projected scenario to, for instance, an Interest-Rate Swap (IRS) valuation, we would obtain a series of MtM scenarios as follows:

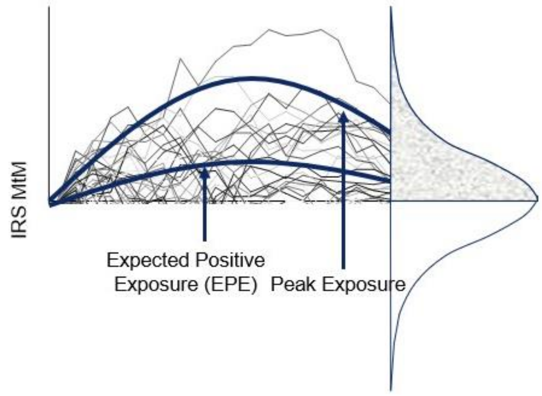
Figure 10. Example of interest rate swap MtM simulation paths



Source: compiled by the author

where we would have to keep in mind the positive regions, which represent the positive exposure in each time period (i.e., if the derivative has positive value, then we have counterparty risk as the counterparty "owes" value to us). As we need to estimate the expected positive exposure (EPE), then the average of the exposure at any time *t* would represent that EPE. Other exposure profiles, like the Peak Exposure (representing the 99^{th} percentile of the MtM future distribution) can be used for calculating additional metrics, in this case the CVA VaR.

Figure 11. Example of interest rate swap MtM simulation paths, EPE and Peak Exposure



Source: compiled by the author

Therefore, the steps to compute the Expected Positive Exposure would be:

- Choice of the corresponding risk factors for the derivative price
- Scenario generation according to a well-fitted stochastic model
- Valuation of the derivative in all scenarios
- Compute the average of the positive exposures at any calculation time *t* to obtain the vector of EPEs to be used to compute the CVA amount.

4.4.3 Libor Market Model

When generating the scenarios of interest rates, a well-known Montecarlo-based simulation methodology is widely used among risk management practitioners. The model is known as *Lognormal Forward-Libor Model*, or *Libor Market Model* (LMM) (Brace, Gatarek and Musiela, 1997), and can be very useful in the field of CVA or Montecarlo VaR when simulating interest rates, for the reasons listed below:

- This model simulates the entire forward curve and discount factors from current forward quotes, so this feature allows to directly price floating interest rate products;

- The model risk factors are directly observed in the market (or they can be calibrated from quoted data), opposite to, for instance, models like *Hull-White* or *Cox-Ingersoll-Rox* (CIR), for which risk factors like mean-reversion should be calibrated depending on time series length;

- This model is derived from the Black-76 option pricing framework. Therefore, quoted Black volatilities can be used to strip the volatility surface, so this model is also consistent with the prices directly observed in the market;

- The model entails using the so-called *forward volatilities*, which depend on cap forward volatilities quoted in EUR, GPB or USD option markets. This fact provides the model with soundness in terms of market expectations when generating future scenarios, being compliant with the *forward-looking* requirements under IFRS.

The LMM, in terms of risk-neutral dynamics, can be written as

$$dF_k(t) = \mu_k(t)F_k(t)dt + \sigma_k(t)F_k(t)dZ_k(t) \tag{33}$$

where $F_k(t)$ is each generic forward rate bucket $F(t;T_{k-1},T_k)$; μ_k is the forward rate drift (computed as the average forward rate annual return for simplification purposes)[19]; $\sigma_k(t)$[20] is the instantaneous volatility at time t for the forward rate F_k, which will be the caplet forward volatility; and $Z_k(t)$ is a standard Brownian motion. Under the probability measure of the numeraire $P(t, t+\Delta t)$ (used to compute $F_k(t)$), the lognormal behaviour of $F_k(t)$ will be determined by Ito's formula, as

$$d\ln\left(F_k(t)\right) = \mu_k(t)dt - \frac{\sigma_k(t)^2}{2}dt + \sigma_k(t)dZ_k(t) \tag{34}$$

so that discretizing we have

$$\ln\left(F_k(t+\Delta t)\right) = \ln\left(F_k(t)\right) + \mu_k(t)\Delta t - \frac{\sigma_k^2(t)}{2}\Delta t + \sigma_k(t)\big(Z_k(t+\Delta t) - Z_k(t)\big) \tag{35}$$

and, in terms of the generic forward rate jump from t to Δt:

$$F_k(t+\Delta t) = F_k(t)\, e^{\left(\mu_k - \frac{\sigma_k^2}{2}\right)\Delta t + \sigma_k Z_k \sqrt{\Delta t}} \tag{36}$$

As EUR rates have had many forward rate tenors in negative regions so far, the market provides prices for option volatilities in the Black environment as well. This entails including the shift α so as negative rate scenarios can be avoided when applying the model[21] (Beinker and Stapper, 2012).

$$F_k(t+\Delta t) = \left(F_k(t) + \alpha\right) e^{\left(\mu_k - \frac{\sigma_k^2}{2}\right)\Delta t + \sigma_k Z_k \sqrt{\Delta t}} - \alpha \tag{37}$$

Therefore, we can generalize for the entire forward curve simulation the above expression. This means that we can simulate each forward rate tenor $F_k(t)$ to which a swap has exposure over time, until each forward rate in the floating leg is paid. Moreover, discount factors are simulated based on the simulated forwards, following their risk-neutral relationship

[19] The drift can be inputted in the model as a volatility-dependent factor, but it has been assumed as a constant annualized return for each forward bucket, for simplification purposes.

[20] Caplet volatilities are subject of a time-variance treatment as $\sigma_k^2(t)$ is a function of time in a Montecarlo simulation framework, and therefore σ_k does not exactly represent each caplet volatility at t_0. For further information on this treatment, see Hull (2012) or Brigo & Mercurio (2006).

[21] The Black-Scholes model and the derived simulation models, like LMM, depend on the lognormal relationship between the underlying price and the strike price. This means that when one of them turns negative, the model would not work. Hence, this drawback can be saved applying a "shift" to the underlying forward rate to make it positive, before simulating or valuing an option under the Black-76 environment.

$$P(0, t + \Delta t) = \frac{P(0, t)}{1 + F_k(t + \Delta t)} \tag{38}$$

Implied ATM caplet volatilities calibration

As explained in above paragraphs, $\sigma_k(t)$ represents the caplet volatility for each $F_k(t)$ to be simulated. However, the options market only provides with volatilities implied in caps per strike and maturity, not volatilities for every single caplet composing the cap (i.e., not for each forward rate we need to simulate). Then, we need to carry out a bootstrapping process to calibrate the caplet volatilities implied in the cap volatility ATM skew.

Under the Black-76 model, the call option (cap) value is

$$N \, \Delta t \, P(0, t_{k+1}) \, [(F_k + \alpha)\Phi(d_1) - (K_k + \alpha)\Phi(d_2)] \tag{39}$$

whereas the put option (floor) value is

$$N \, \Delta t \, P(0, t_{k+1}) \, [-(F_k + \alpha)\Phi(-d_1) + (K_k + \alpha)\Phi(-d_2)] \tag{40}$$

where

$$d_1 = \frac{Ln\left({}^{(F_k + \alpha)}\!\big/_{(K_k + \alpha)}\right) + \frac{\sigma_k^2}{2} t_k}{\sigma_k \sqrt{t_k}}$$

$$d_2 = d_1 - \sigma_k \sqrt{t_k}$$

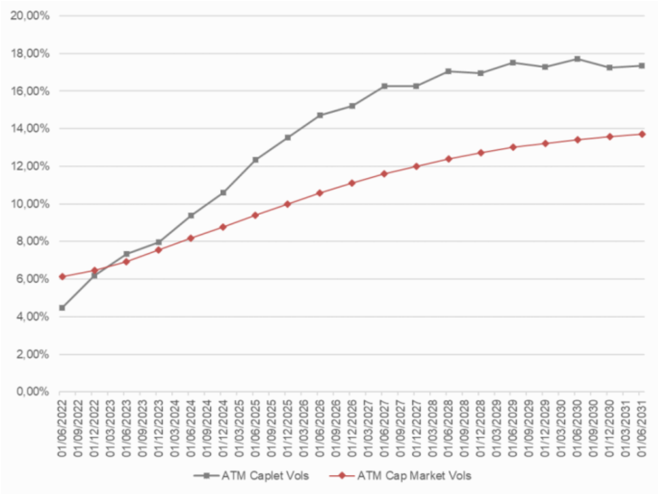
and F_k is the forward interest rate at time 0 for the period between time t_k and t_{k+1}, K_k is the strike, α is the curve shift (3% given the market convention for EUR), σ_k is the shifted Black cap volatility for t_k and K_k, Δt is the accrual for each caplet/floorlet (payment accrual), $P(0, t_{k+1})$ is the discount factor, and N is the notional amount. Namely,

$$Cap^{MKT}\left(0; K + \alpha; \sigma_j{}^{cap}\right) = \sum_{k=1}^{j} Caplet\left(F_k(0); \Delta t; K + \alpha; \sigma^{caplet_k | \sigma_j{}^{cap}}\right) \tag{41}$$

where $\sigma^{caplet_k | \sigma_j{}^{cap}}$ is the caplet volatility for each caplet that makes their sum equal to the hypothetical market Cap price with constant $\sigma_j{}^{cap}$ for a maturity *j*. Although there is a unique volatility for a cap strike and maturity, there could be different caplet volatilities depending on the option maturity, as they represent, in average, such a cap volatility. With this framework, each entire caplet volatility time structure would be stripped iteratively, from the first caplet – which will be equal to the cap – to the latest one based on a target cap price and maturity $Cap^{MKT}\left(0; K + \alpha; \sigma_j{}^{cap}\right)$. For European option pricing, either $\sigma_j{}^{cap}$ or its corresponding $\sigma^{caplet_k | \sigma_j{}^{cap}}$ can be used (as caplets/floorlets are accepted to be priced using a unique cap/floor volatility for a certain maturity as well), but this issue matters regarding forward curve simulation.

For instance, the EOD 29/12/2021 shifted Black forward volatilities quoted for caps and their corresponding caplet volatilities calibrated following (41) on a cap maturity in 2030 and $K_k = 0\%$ are shown below:

Figure 12. EUR6M Cap and Caplet vols, K 0%, Cap Maturity Dec '30



Source: Bloomberg and compiled by the author

As it can be seen, caplet volatilities are different to the ones from cap market. It is totally convenient to use caplet vols in the LMM so that each forward rate is simulated under its own expected volatility.

4.4.4 Swaption Mark-to-Market as a proxy for an IRS Potential Exposure

Following Figure 7 and Figure 8, it is well noted that interest-rate derivatives are the ones most used across the financial world. Within this asset class, IRSs represent a vast amount of those outstanding derivatives, both cleared and bilateral. Therefore, in this subsection we cover one modelling solution particularly used by consulting firms and non-financial companies to project the Expected Exposure for this kind of derivatives.

An interest rate swap is a contractual agreement entered into between two counterparties where they agree to exchange fixed for variable interest rates, periodically, for an agreed period of time and with a notional amount of principal. The principal amount is "notional" as there is no need to exchange actual amounts of principal (although in Cross-Currency Swap this practice is common). However, the notional amount is required in order to compute the actual cash amounts that will be periodically exchanged. The valuation of an IRS is pretty simple: on the one side we have the floating leg (the leg which is paid or received by a counterparty which is composed by floating payments), whose payments are projected with the forward rates corresponding to the contractual floating reference rate (e.g., EURIBOR 3M), whereas the fixed leg is simply the array of fixed payments upon the fixed rate already specified in the contract. The difference between the discounted value of the sum of payments from each leg is the IRS mark-to-market (depending on the direction and amount of the payments, one counterparty will have positive MtM whereas the another will have negative MtM). Therefore, the problem arises not when the MtM should be calculated at time 0, but when it should be projected into the future. One solution would perfectly be the LMM, explained in section 4.4.3. However, there already exists a closed formula to estimate the EPE for a given IRS.

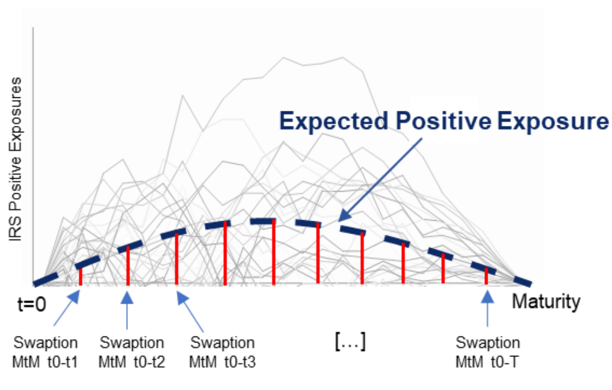
Swaptions and the relationship with IRSs

A Swaption (Swap Option) is an option that reserves the right to purchase an interest-rate swap at a prescribed time in the future, with a fixed interest rate for the fixed leg and a prescribed floating rate for the floating leg. The holder of such a European call option has the right, but not the obligation to pay fixed in exchange for variable interest rate. Therefore, this option is also known as “Payer Swaption”. The holder of the equivalent put option has the right, but not the obligation to receive interest at a fixed rate (Receiver Swaption) and pay floating.

In words, a swaption is an option on a forward interest rate. Like interest rate swaps, swaptions are used to mitigate the effects of unfavorable interest rate fluctuations at a future date. The premium paid by the holder of a swaption be considered as insurance against interest rate movements. In this way, businesses can guarantee limits in interest rates.

If we consider the case of a company that will start hedging its debt six months from now. The debt will mature in five years, at a floating interest rate payable every six months. This company can protect itself against rising interest rates by purchasing a payer swaption. By paying a *premium*, the company obtains the right to receive variable payments (e.g., EURIBOR 6M) to pay a predetermined fixed interest rate i_{fixed} for a 5-year period. The swap therefore begins six months from now (expiry date of the swaption). That previously mentioned *premium* is the swaption MtM, which gathers the expected future value of the underlying IRS. In other words, the swaption MtM represent the expected value of the swap at the swaption maturity. This is, the EPE for the underlying IRS six months from now. Subsequently, the value of the swaption can be used to project the IRS value into the future, allowing us to build an array of Expected IRS values, by concatenating several swaption MtMs with increasing maturities in a row, from now until our IRS expires.

Figure 13. Expected Positive Exposure profile for an IRS from swaption MtMs



Source: compiled by the author

As it can be seen in the figure above, the price at t=0 of each swaption with maturity [0, *tn*] represents the EPE for the IRS at that time, enabling us to build the profile of the IRS EPE and therefore, to compute the CVA following (32).

European Swaption pricing framework

The model usually used to value a European swaption assumes that the underlying swap rate at the maturity of the option is lognormal. This means that the typical Black-Scholes model can be used, with some adjustments.

The cash flows made to the buyer of a payer swaption at time T (maturity of the swaption) amounts to

$$\sum_{i=1}^{n} N \cdot P(T, t_i) \cdot \left(i_{FS} - i_{fixed}\right) \cdot (t_i - t_{i-1}) \tag{42}$$

where N is the notional amount, $P(T, t_i)$ is the discount factor from t_i to T, i_{fixed} is the fixed rate of the underlying swap and i_{FS} is the floating rate (forward swap rate). Therefore, its value today

$$P(0,T) \sum_{i=1}^{n} N \cdot P(T, t_i) \cdot \left(i_{FS} - i_{fixed}\right) \cdot (t_i - t_{i-1}) = \tag{43}$$

$$= \sum_{i=1}^{n} N \cdot P(0, t_i) \cdot \left(i_{FS} - i_{fixed}\right) \cdot (t_i - t_{i-1})$$

Taking the expected positive values, the value of a European swaption would be

$$N \cdot \left(i_{FS} - i_{fixed}\right)^{+} \cdot (t_i - t_{i-1}) \tag{44}$$

According to the Black model, the price of the European payer swaption at time 0 is

$$N \cdot A \cdot (t_i - t_{i-1}) \left[i_{fixed}\, \Phi(d_1) - i_{FS}\, \Phi(d_2)\right] \tag{45}$$

where

$$d_1 = \frac{Ln\left(\frac{i_{FS}}{i_{fixed}}\right) + \frac{\sigma_{FS}^2}{2} T}{\sigma_{FS}\sqrt{T}}$$

$$d_2 = d_1 - \sigma_{FS}\sqrt{T}$$

$$A = Annutity = \sum_{i=1}^{n} P(0, t_i)(t_i - t_{i-1})$$

σ_{FS} being the forward swap volatility for the forward swap (underlying) which level will depend on the forward swap strike, the swaption maturity and the underlying swap maturity, and i_{fixed} is indeed the strike of the swaption. Therefore, (45) will represent the price for every single swaption starting at t=0 and maturing at each of the node for which the EPE will be estimated, allowing us to have the expected value of the IRS at any time bucket in the future.

4.5. Conclusions

Credit rating models presented in this chapter has extensive use within the financial industry. However, there are some disadvantages that need to be outlined concerning the application to IFRS rules on the Credit Rating and the long-run PD estimation, which are the main reasons that motivated this doctoral research:

- *Credit rating agencies* apply many subjective explanatory variables to their models' outcome. The model described in section **4.1**. is just the base case but, as it can be noted, there are risk factors like *Business Model, Competitive Environment and Technical Positioning*, *Regulatory Environment* or *Financial Policy*, among others, that are subjective and variable among agencies and over time. Therefore, applying this model would just provide a limited information on the credit quality of a company. Also, there is limited, non-updated information on the modelling particularities, as these agencies do not fully disclose their rating-assignment methodologies.

- *Analytical models* like the ones presented in section **4.2**. are a good initial measure in terms of global scale. However, they are static (they have not been recalibrated to current economic environment, so they are not *forward-looking*) and are not sectorial-specific. The PD for a given timeframe can be quite dissimilar for companies belonging to same rating notch but to different sectors and geographies.

- *Structural models* like KMV provides an actual estimation of the short-term PD for a given company, as it directly uses its assets and liabilities amounts and calculates the probability that the former falls below the level of the latter. Nonetheless, there exist two main drawbacks when applying this type of models:

    - Although long-run PDs can be extrapolated from the 1Y PD initially calculated, this process entails many assumptions in terms of liabilities level over time, as well as on the volatility used for assets projection (if the company is not publicly traded, volatility should be proxied with sectorial peers).

    - The modeling set up, including estimation of drift and asset volatility, can be complex and research on sectorial peers needs to be carried out, with no guarantees that the market information can be extrapolated to the analyzed company.

Consequently, none of these models completely comply with the requirements listed in section **1.3.1.** Although Structural models or Credit Rating agency models are used in a wide extent, there exist yet some limitations to be avoided in the development of the model in Chapter 5, particularly concerning the *forward-looking* requirement, the capability of being fitted to different sectors and geographies, and the capability to be used by non-expert practitioners so that they can have a model framework ready to be implemented to comply with audit requirements.

In this chapter, two modelling solutions for the CVA Exposure concerning IFRS 13 have been presented. Although it is not objective of discussion in this doctoral thesis, it is needed to bear in mind the relevancy of the exposure projection in the CVA calculation, which is different to the EAD presented for the ECL calculation in the IFRS 9 framework. The Libor Market Model and the Black model for swaptions are relatively straightforward to implement (when some assumptions are made i.e., no correlation between forward rates exists for LMM) and are directly fed with interest rate curves and volatilities quoted in the market, so that the input data are reliable and observable (although some further interpolation and bootstrapping techniques might be implemented concerning the volatilities used in their usage).

CHAPTER 5: PROPOSED MODEL TO ESTIMATE CREDIT RATING AND PD UNDER IFRS 9: FRS MODEL

As previously discussed, the duty of estimating a PD for a company which has no liquid credit instruments nor credit rating is relatively hard. Hence, the risk-based metrics like PD or YTM need to be somewhat estimated via modeling. Also, under the rules of IFRS 9 and having in mind the best market practices, not only these metrics should be modelled but should also include, or be aligned with, several material aspects:

1. Specifically focused on complying with IFRS 9 ECL requirements. The IFRS-9 PD should be based not only on historical information but should also consider *forward-looking* information.

2. Able to be applied to non-quoted/non-rated entities.

3. Comparatively easy to implement, so that entities can use it in order to comply with IFRS-9 ECL requirements, particularly in terms of liquid input data availability.

4. Providing an output of a credit rating in the same scale as the credit rating issued by CRAs, so that the corresponding PD may be obtained from information derived from comparable companies.

Therefore, in the research performed throughout the doctoral period I have tried to comply with all of the above requirements in the widest possible extent when developing the model. The model has been named “Financial Ratios Scoring” (FRS) model. The FRS model is partially based on Duan *et al*. (2018) and Ivanovic *et al*. (2015) for instance, in the sense that uses similar information as input, but the treatment and subsequent modelling is intended to go beyond so that the model calibration is aligned to audited financial information from comparable companies and also dependent on the sector-specific issues.

The main model input is the information obtained from the counterparty financial statements (i.e., the main inputs are financial ratios). According to the values of several key balance sheet and profit and loss accounting ratios, the company is allocated in a certain position (score) within a consistent distribution of companies that possess an official credit rating (issued by a rating agency or quoted by relevant financial vendors), and which belong to the same or similar sectors. The position within that distribution is related to a certain credit rating (the official rating of companies with a similar score), and therefore this credit rating can be linked to a expected default probability.

According to Cappon *et al*. (2018), credit ratings are “opinions” issued by rating agencies regarding the credit worthiness of corporate, municipal, and sovereign borrowers. Agencies generally avoid claiming that credit ratings predict probabilities of default. Nevertheless, they do publish detailed default studies which show historical ratings migration and default events as a function of the initial rating and time horizon. Analysts and risk managers routinely use default study data as estimates of default probabilities. In practice, it is assumed that a rating generally matches a range of default probabilities.

The FRS model is intensive in terms of data collection (as we will see, it is necessary to create a distribution of sector companies). However:

- It can be considered to be highly consistent since the model’s inputs are calibrated with the financial information of companies which do have an agency rating.

- It is not as intensive in terms of data sources and input data treatment, as other structural models (see Chapter 4).

Among the ratios considered by the model (which may also vary from one sector to another), those with higher relevance in terms of credit risk are those related to debt and interest coverage, leverage or liquidity. In other words, the relative debt level of a company is generally the factor with most influence on its credit risk. Growth and profitability are also considered but linked to liabilities and equity.

The model proposed uses quantitative data as its main inputs (financial ratios) are obtained from the entity’s financial statements. In theory, qualitative data is not directly used in the model, fundamentally due to the following factors:

- Qualitative factors or metrics are difficult to measure and to model due to several reasons such as the fact that the same information is not available for all entities, and they entail a significant level of subjectivity, etc. In this sense, as the model aims to be both robust and relatively easy to implement at the same time, it does not consider subjective qualitative factors (at least not directly).

- In recent years, financial and market information (pure quantitative factors) has tended to be more reliable. This makes quantitative factors more effective when estimating a probability of default or assigning a credit rating. In fact, in terms of default events and recovery rates, quantitative models have been taking new assumptions into account and covering recent scenarios (by way of example see Moody’s reports on default risk and recovery rates - Moody’s, 2017).

As will be observed throughout the conclusions, when the model is well calibrated and financial ratios used as inputs are representative enough, the explanatory power for certain ratios is highly related with the criteria used by rating agencies in terms of ratios used to assess on the credit risk within a given sector.

5.1. Methodology and model theory development

As previously stated, the FRS model is based on reflecting the position (score) of a company within a representative group of rated companies, so as to provide the company with a credit rating in line with its associated score.

With regards to this score:

- It may also be considered as a "percentile" in the model context (in fact, it is a percentile within a sectorial group). It is configured on a basis that value "1" represents the worst position and "100" the best position within a financial metric for a given sector or peer group.

- It will depend on the financial ratios selected, and therefore on the position of each financial ratio within its group (hereinafter also named as "distribution").

- Therefore, the model will need to be composed by exogenous variables (financial ratios) with enough explanatory power. For this, Stepwise AIC optimization technique is chosen, in order to select the most representative ratios for a given sector.

- It should be noted that the exogenous variables are transformed and used in the model as percentiles (as explained, all type of rations ranging from 1 to 100). This is done this way to avoid variable transformation. When the ratios are translated to percentiles, we avoid any problem with variables under different distribution regimes (e.g., working with variables in absolute differences or log differences instead of variables in levels).

The construction of the FRS model consists of five main steps:

- Step 1 – Definition of potential financial ratios
- Step 2 – Calculation of peers' general score
- Step 3 – Calculation of the specific score for each potential financial ratio for all the peers
- Step 4 – Model calibration: regression, variable selection through a variable-optimization method (in this case, Stepwise AIC will be used)
- Step 5 – Obtaining the model credit rating and the expected PD for the company

5.1.1 Step 1 – Definition of potential financial ratios

In this first step, a group of key financial ratios is defined as potential explanatory variables of the credit letter for the sector to which the company belongs. Generally, these ratios are related to metrics such as coverage, leverage, liquidity, profitability, and growth.

Initially, it is recommended to use a wide range of ratios as a first step, before optimizing the model. These ratios are widely used by analysts (Fazzini, 2018) and by rating agencies (see Moody's 2017b, for example), since they represent the key financial dimensions that act as drivers for a rating profile. They are easy to calculate using the financial information included in the public financial statements issued by companies.

Nevertheless, it should be noted that additional ratios could be included for specific sectors according to the nature of their business, such as Passenger Load in the commercial airlines sector, or Loan Default Rate in the banking sector, etc. As will be explained in subsequent sections, the intrinsic characteristics of a sector are disclosed when calibrating the ratio weights, hence to a certain extent the "sector" variable is covered by this methodology.

Table 12: Example of set of Ratios that can be used in the FRS model

Financial Ratio
Pretax Income /Sales
Debt / EBITDA
Free Funds from Ops / Debt
EBIT/ Interest Expense
EBITDA / Assets
Return on Equity
Net Margin
Return on Assets
Interest Exp. / Sales
Debt / Equity
Debt / Assets
Cash / Total Debt
Short Term Debt /Total Debt
Quick Ratio

Source: Compiled by the author.

Some relevant ratios that are used as risk factors by the main CRAs models are the following:

- "Interest expense/Sales" and "EBITDA/Interest Expense" (coverage ratios) focus on to what extent interest expenses related to debt are "covered" by income from normal business operations. The higher the interest expense in relation to sales or EBITDA, the weaker the financial position of the company. In other words, the ratio analyzes to what extent the entity generates sufficient resources in order to be able to pay the interests related to external debt.

    - In the first ratio, the higher the level, the lower the coverage (less sales income is available to pay the interest expense).

    - In the second ratio, the higher the ratio level, the higher the generated surplus (and the higher the coverage).

  In general terms, the model places much importance on coverage ratios as a default event is usually understood as the situation in which a company is not able to entirely pay the short-term debt. In this sense, coverage ratios can act as signals of credit problems.

- Pre-tax income as a percentage of sales. It provides a metric on the company's effectiveness with regard to the cost structure and its capability to reach yield premiums in comparison with peer companies. The capital-intensive nature of the transportation industry makes it important to include interest expense when considering profitability, as capital costs are as relevant as operating costs. Therefore, while this ratio may be relevant for modelling purposes, correlation and significance should also be checked.

- Financial leverage and coverage metrics are indicators of a company's financial capacity and long-term viability. Financial flexibility is critical to this sector as it indicates the degree of stress a company would suffer during an economic downturn. In addition, leverage affects a company's ability to reinvest in the business, as a highly leveraged company may not have the same access to capital (new funds) as other companies with a lower leverage level. Furthermore, leverage partly affects the capacity to deal with changing market conditions in the highly cyclical business operations to which this type of company may be exposed. Financial leverage and coverage are additionally represented in the model by the following ratios:

    - **Debt/EBITDA** ratio is an indicator of debt serviceability and leverage, and is commonly used in this sector as a proxy for comparative financial strength.

    - **Funds from Operations (Free Cash Flows from Operations minus Capex) to Debt** is an indicator of a company's ability to repay principal on its outstanding debt. This ratio compares cash flow generation from operations before working capital movements to outstanding debt.

    - **EBIT to Interest Expense** is an indicator of a company's ability to cover its ongoing costs of borrowing.

- "(Liabilities - Cash & Securities)/Assets" statically analyzes the company's leverage level (or relative debt level). It compares the assets (that could be used to pay the debt) with the net debt (net of cash and liquid securities). The higher the ratio result, the higher the relative debt level (and the higher the credit risk).

- "Retained Earnings/Liabilities" compares the company's result with its debt level. It analyzes the company's leverage level more dynamically. The higher the ratio result, the lower the credit risk.

- "Current Assets/Current Liabilities" is known as working capital. Depending on the sector involved, the interpretation of the result may vary. Generally speaking, the higher the ratio, then the higher the liquidity level. Nevertheless, in the retail sector, a low ratio may be interpreted in a positive way, i.e. the entity is being financed by its suppliers (the average collection period is lower than the average payment period).

- "Cash & Securities/Current Assets" analyzes to what extent current assets are composed of liquidity (the higher the ratio level, the higher the liquidity level).

- ROA and ROE analyzes the profitability of the company. They calculate return in relation to assets (ROA), and return in relation to equity (ROE).

5.1.2 Step 2 – Calculation of peers' general score

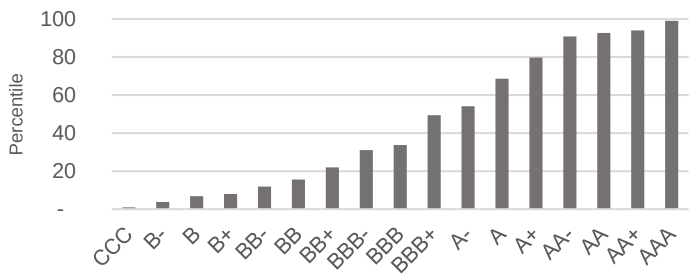
This step consists in creating a database including a portfolio of companies ("peers") which possess an official credit rating (issued by a rating agency) and giving a general score to each of them.

Where possible, companies included in the database should belong to the same sector and country as the company under analysis and should have recently been rated by a relevant credit rating agency (i.e., Moody's, Fitch or S&P). Alternatively, given the limited number of rated companies over the sectors, the database can also be created by using the credit letter issued by Refinitiv or Bloomberg upon their internal models.

In some cases, it can be difficult to find peers since companies are highly diversified and act in many different industries and markets at the same time. Nevertheless, it is recommended the inclusion of as many peers as possible.

A score is assigned to each peer company in the portfolio, and each company is ranked according to its position (a percentile between 1 and 100) within the entire portfolio of companies. This position represents the general score. By way of example, for a specific real case (in a specific sector), we included the information required in a database in order to build the cumulative distribution function, deriving the following figure:

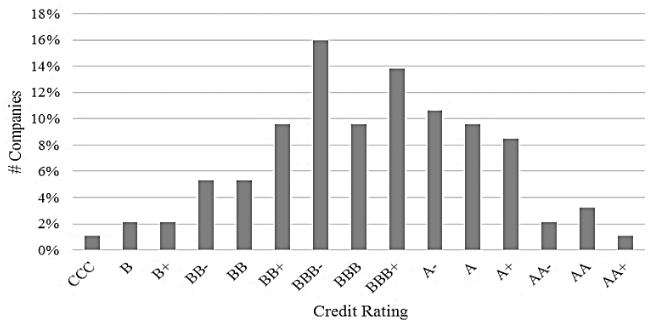
Figure 14. Example of Score distribution per Credit Rating



Source: Compiled by the author.

As it can be seen in the above distribution, the credit rating is directly related to the score ("position" or "percentile") within the distribution. This distribution was created using Refinitiv information on rated companies. Certain companies with an equal rating are scored slightly differently according to their outlook, size and debt coverage. In the example above, this means that there are 13 companies with the same rating (BBB-) between percentile 25 and 37. This is normal given the fact that there are more companies rated between BBB- and BBB+ than in any other rating bucket. **Figure 14** above represents a cumulative distribution of 63 rated companies. Its corresponding probability density function is shown in **Figure 15**:

Figure 15. Example of Density function of Credit Rating



Source: Compiled by the author.

5.1.3 *Step 3 – Calculation of the specific score for each financial ratio for all peers*

In this step, it is needed to assign a score to each peer company in the portfolio in relation to each ratio. In other words, each company has on the one hand a general score (step 2), and on the other a specific score for each ratio (step 3).

Therefore:

- We calculate every ratio included in Table 12 for all of the companies in the sample.
- Hence, a distribution for each ratio according to the results is created.
- Then, each company is given a score (percentile) for each ratio depending on its position within the distribution.

5.1.4 *Step 4 – Panel data construction and Model calibration: regression and variable selection through Stepwise AIC*

A percentile-composed matrix is prepared which shows the relationship between the comparable companies' rating, their general score, and the score given to each ratio within the entire peer sample.

In order to make the model "temporal unbiased", it is necessary to build a panel data matrix given that the modelled variable (the general score) depends not only on the selected variables but also on its inherent variation. Consequently, the peer sample is structured in a cross-sectional matrix (panel data), in which the rows present the values for each company within the sectorial group as of December of several past years. The following table presents an example:

Table 13: Example of Scoring Panel Data (general and specific scores)

Company	General Score		Specific Score for each ratio				
	Agency Rating	General Score	ROE	ROA	EBITDA / Interest Expense	Net Debt / Assets	Etc.
Company A Year T-3	BBB-	25	37	40	28	48	…
Company A Year T-2	BBB+	52	65	61	54	65	…
Company A Year T-1	BB+	16	12	7	20	22	…
Company A Year T	BBB-	25	15	12	32	29	…
Company B Year T-3	AA+	94	95	97	86	56	…
Company B Year T-2	AAA	98	96	98	87	67	…
Company B Year T-1	AA	93	94	95	85	58	…
Company B Year T	AA+	96	92	96	84	61	…
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Company N Year T	BB+	13	6	9	16	4	…

Source: Compiled by the author.

The above table should be fed with the following inputs:

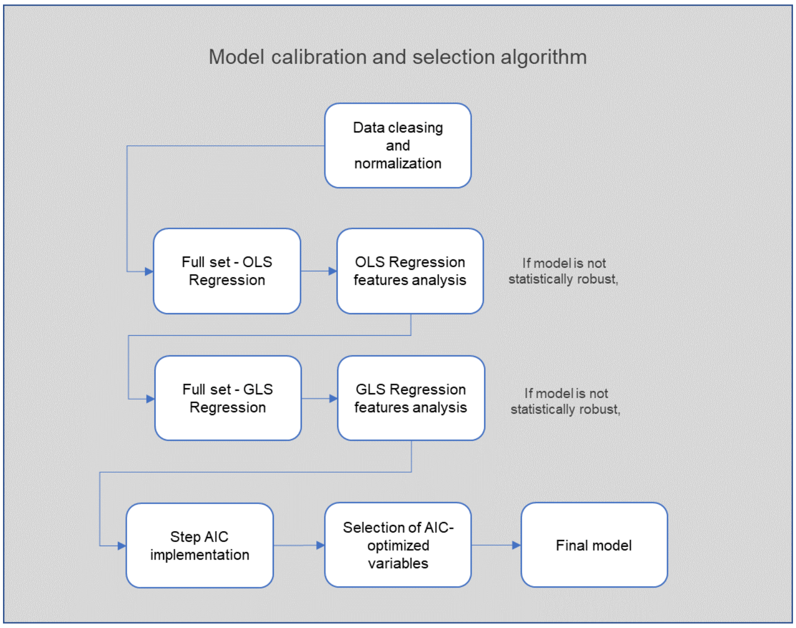
- The name of each peer company in the portfolio.
- The rating and general score of each peer company in the portfolio.
- The specific score of each peer company in the portfolio for each ratio.

Once the Panel data is built, then the following algorithm to create the regression model over the sectorial ratios is followed:

5.1.4.1 Model selection algorithm

Selection algorithm consists of several steps that are presented in the following scheme and described in detail below. This algorithm describes the main steps to be taken in turn to select the best model in terms of regressors and their representative within the model. The steps cover an initial OLS model calibration with all available explanatory variables (ratios) and then continues to end up with the final, optimal model:

Figure 16. Model selection algorithm



Source: compiled by the author

1. Data Cleansing and normalization

First of all, it should be noted that there could be incomplete information for some of the companies within the sample, as well as companies with financial statements containing rare figures. Subsequently, a data cleansing and normalization process should be applied to the database before continuing with the model optimization. The following steps would be followed:

1. Identify missing values from historical financial statements
2. Delete companies for which financial variables are rare or non-existing
3. Homogenize rating information. Some companies could be rated by Moody´s, whose rating scales is different to the one used by S&P and Fitch. Also, some companies could be rated with short-term rating values, which are in a different scale than long-term rating values.
4. Delete the companies that belong to geographies too dissimilar in comparison with the whole dataset, in order to avoid geographical noise (e.g., when most of companies in the dataset belong to Europe or America, it is convenient to exclude a company that could be based on Vietnam or Madagascar, for example.).
5. Use the same currency and FX rate for those financial variables in foreign currency.
6. Use international scale to translate those ratings from local scale. For instance, Argentina or Peru have local rating scale provided by CRAs whose rating notches do not match with the ones used in Europe or America, which are by default international-scaled ratings.

2. Full dataset - OLS regression

Due to the fact that the panel data is structured with many entities and over several years, the paradigm of traditional time-series analysis is subject to more profound changes. In this case, we have several sets of endogenous variables to be used to model the General Score, since there are several individual entities for which the explanatory variables may present different behaviors. That is to say:

$$y_{ij} = \alpha + x_{1ij}\beta_{1ij} + x_{2ij}\beta_{2ij} + \cdots + x_{nij}\beta_{nij} \ldots + \eta_{ij} + \varepsilon \qquad (46)$$

where y_{ij} is the dependent variable (General Score) for the individual i and timeframe of observation j; x_{nij} is the value for the explanatory variable n (Specific Score) for a given individual i of the observation period j; β_{nij} is the coefficient of each explanatory variable in the same context; and η_{ij} represents the potential unobservable, correlated differences for the values of each explanatory variable x_n at each observation node in j.

Given the heterogeneity of the entities within our sample, it is possible that entities with similar characteristics may display different behaviors. In fact, it is possible that a single entity may present different behaviors for the given timeframe data set. Therefore, first of all it is necessary to analyze the existence of any potential unobservable factors which may influence consistency in the output parameters and may entail autocorrelation and heteroskedasticity in the residuals.

These unobservable factors (which may be either fixed or stochastic for a given data set) are expected to result in biased model coefficients for a given timeframe and entity, hence this must be treated accordingly.

One solution is to assume that all α and each coefficient β_{nij} are constant for the entire data set and for each individual i. In this case, it would be possible to calibrate the model via Ordinary Least Squares (OLS). However, this method could lead to problems of autocorrelation and heteroskedasticity, given that the error variance may vary among individuals or even for a given timeframe and individual. In turn, this problem would be solved by calibrating the model via a Generalized Least Squares method (GLS).

Alternatively, it may be assumed that intercept α varies among individuals and over time. Therefore, it is necessary to verify that the source of said unobservable factors is the difference occurring in intercept α. In this case, the model should be transformed as follows, where each explanatory variable will be the deviation with respect to its average, for each individual:

$$(y_{ij} - \bar{y}_i) = \sum_{n=1}^{N} \beta_{nij}\,(x_{nij} - \bar{x}_{ni}) + \varepsilon \qquad (47)$$

where y_{ij} is the dependent variable for the individual i, and $\bar{x}_{ni}$ is the time average of each explanatory variable for each individual.

3. OLS Regression analysis

In order to determine which solution should be adopted, we need to firstly ran a regression calibrated via OLS over all of the several variables available in our data set. This means that, with high probability, we are going to end up, in a first stage, with model that incorporates heteroskedasticity and correlated residuals, and secondly, with too many variables with low explanatory power and highly correlation between each other. However, this previous step is necessary in order to identify what are the potential issues that the initial data set can bring into the output estimation.

The main regression tests performed over the model are the following:

- *Parameter representativeness*: it is checked what p-value and t-Student value have each explanatory variable, so that the main representative variables can be identified.

- *Normality in residuals*, mainly via Q-Q Plot and Jarque-Bera test, Shapiro-Wilk test and Anderson-Darling test. 5% threshold value holds for hypothesis acceptation/rejection for each of the tests. If the p-value of a test result for normality of residuals is higher than 5%, it is considered that the test fails to reject the null hypothesis that the errors are normally distributed. Since three tests are used for testing normality of residuals, it is required to have p-values for all the tests to be higher than 5% in order to establish the fact that residuals are normally distributed and thus, the model can be considered as a valid method for related predictions.

- *Heteroskedasticity*: Heteroskedasticity in errors is tested with the Breusch-Pagan test. As the result of F-statistic, if p-value is greater than 5%, it is considered that the test fails to reject the null hypothesis that the results having homoskedasticity.

- *Multicollinearity*: correlation between explanatory variables is checked, as it would affect the resulting model and may provide with spurious results and arbitrary goodness-of-fit values. Also, VIF (Variance-inflation Factor) is computed for each variable. If VIF is over a value of "10", the variable will be a candidate to be removed, although it is fully recommended to wait until the final step in the model calibration (Stepwise AIC) as in that step many of the initial variables are directly removed.

- *Autocorrelation*: an additional assumption that residuals should not be serially correlated is tested in order to establish consistency and asymptotic normality. The assumption tested and respective tests used are the Ljung-Box test and the Breusch-Godfrey test. 5% threshold holds true for p-values of the Ljung-Box test and the Breusch-Godfrey test for serial correlation of residuals. If both tests provide p-values greater than 5% thus, fail to reject the null hypotheses that residuals are not serially correlated so that additional solutions need to be assessed in order to fix this problem. Also, Partial ACF plots are used to assess on the presence of autocorrelation in residuals, in order to detect not only autocorrelation, but the lag between correlated errors. If presence of autocorrelation is found, then it would be necessary to recalibrate the model in order to include the autoregressive process, specifying the correlation structure to fit. As indicated above, the use of Generalized Least-Squares (GLS) can mitigate this issue.

4. Full dataset GLS regression and regression model analysis

A GLS regression extends the Ordinary Least-Squares (OLS) estimation of the normal linear model by providing for possibly unequal error variances and for correlations between different errors. A common application of GLS estimation is to time-series regression, in which it is generally implausible to assume that errors are independent.

If we recall the OLS equation, in the standard linear model:

$$\mathrm{E}(y|\mathrm{X}) = \mathrm{X}\upbeta \tag{48}$$

then

$$y = \mathrm{X}\upbeta + \varepsilon \tag{49}$$

where y is the n x 1 response vector; X is an n x k +1 model matrix; β is a k + 1 x 1 vector of regression coefficients to estimate; and ε is an n x 1 vector of errors. Assuming that $\varepsilon \sim N_n(0,\ \sigma^2 \mathrm{I_n})$, or at least that the errors are uncorrelated and equally variable, leads to the familiar OLS estimator of

$$\mathrm{b}_{OLS} = (\mathrm{X'X})^{-1}\mathrm{X'}y \tag{50}$$

with a covariance matrix

$$Var(\mathrm{b}_{OLS}) = \sigma^2(\mathrm{X'X})^{-1} \tag{51}$$

We can then assume that $\varepsilon \sim N_n(0, \Sigma)$, where the error covariance matrix Σ is symmetric and positive-definite. If Σ is known, the log-likelihood for the model is as follows:

$$\log L\,(\beta) = -\frac{n}{2}\log 2\pi - \frac{1}{2}\log(det\ \Sigma) - \frac{1}{2}(y - \mathrm{X}\upbeta)'\Sigma^{-1}(y - \mathrm{X}\upbeta) \tag{52}$$

which is maximized by the GLS estimator of β,

$$\mathrm{b}_{GLS} = (\mathrm{X}'\Sigma^{-1}\mathrm{X})^{-1}\mathrm{X}'\Sigma^{-1}y \tag{53}$$

with a covariance matrix

$$Var(\mathrm{b}_{GLS}) = (\mathrm{X}'\Sigma^{-1}\mathrm{X})^{-1} \tag{54}$$

In the real application, the error covariance matrix Σ is not known, and must be estimated from the data along with the regression coefficients.

Correlated errors

We assume that the process generating the regression errors is stationary, that is to say that all errors have the same expectation (already assumed to be 0) and the same variance (σ^2), while the covariance of two errors depends only upon their separation s in time:

$$\mathrm{C}(\varepsilon_t, \varepsilon_{t+s}) = \mathrm{C}(\varepsilon_t, \varepsilon_{t-s}) = \sigma^2 \rho_s \tag{55}$$

where ρ_s is the error named autocorrelation at lag "s".

Therefore, the error covariance matrix can be assumed to be as follows:

$$\mathbf{\Sigma} = \sigma^2 \begin{bmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{n-1} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{n-2} \\ \rho_2 & \rho_1 & 1 & \cdots & \rho_{n-3} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \rho_{n-1} & \rho_{n-2} & \rho_{n-3} & \cdots & 1 \end{bmatrix} = \sigma^2 \mathbf{P} \tag{56}$$

If we knew the values of σ^2 and the ρ_s , then we could apply this result in order to find the GLS estimator of $\boldsymbol{\beta}$ in a time-series regression but, of course, these are generally unknown parameters. Furthermore, the high number of different ρ_s makes this impossible to estimate unless a structure of autocorrelated errors has already been defined. In a case such as this, a common solution is to define an autoregressive process for the errors AR(p) as follows:

$$\varepsilon_t = \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2} + \cdots + \phi_p \varepsilon_{t-p} + \nu_t \tag{57}$$

where ν_t is the random shock assumed to be a Gaussian White Noise, and $\rho_n = \phi_p$.

Therefore, if we know which is the last lag where autocorrelation may exist, we can then specify it in a GLS calibration. This is a milestone when calibrating the model that is solved once the residual partial autocorrelation is analyzed, as it will be seen in subsequent sections.

Once the model has been adapted to the autocorrelation structure, the same tests as in the previous subsection 3 are carried out.

5. Stepwise by AIC and selection of AIC optimal variables

Stepwise regression is a method of fitting regression models in which the choice of predictive variables is carried out by an automatic procedure. In each step, a variable is considered for addition to or subtraction from the set of explanatory variables based on some prespecified criterion (Harrel, 2001; Knecht, 2005). Usually, this takes the form of a forward, backward, or combined sequence of F-tests or t-tests.

The frequent practice of fitting the final selected model followed by reporting estimates and confidence intervals without adjusting them to take the model building process into account has led to calls to stop using stepwise model building altogether or to at least make sure model uncertainty is correctly reflected. Alternatives include other model selection techniques, such as adjusted R^2, Akaike information criterion (AIC), Bayesian information criterion (BIC), etc. In the FRS model case, AIC is the metric selected used as it combines a robust selection criterion with model risk uncertainty.

Other variable-selection techniques, like Ridge regression or LASSO[22], were also candidates to be used instead of Stepwise selection technique. Unlike Stepwise, LASSO regression is a regularization technique. This means that this model uses a *shrinkage* process, through which the data values are shrunk towards a central point as the mean. The LASSO procedure encourages sparse models (i.e., models with fewer parameters). This type of regression is well-suited for models showing high levels of multicollinearity. However, LASSO uses a tuning parameter to penalize the number of parameters in the model. One can fix the tuning parameter or use a complicated iterative process to choose this value. Therefore, Stepwise has been finally chosen as we need to optimize the model directly by eliminating those non-representative variables rather than using the tuning parameter. The reason behind this is that one of the initial hypotheses assumes that the optimal model would be composed only by those relevant financial variables but not specifying them initially, hence the final variables should not be affected by any penalization process. There should only be variables with explanatory power as such.

Akaike-information Criterion for Stepwise model optimization

The AIC criterion is an unbiased estimator of prediction error and thereby relative quality of statistical models for a given set of data. Given a collection of models for the data, AIC estimates the quality of each model, relative to each of the other models. Thus, AIC provides a means for model selection.

AIC is founded on information theory. When a statistical model is used to represent the process that generated the output, the representation will almost never be exact, so some information will be lost by using the model to represent the process. AIC estimates the relative amount of information lost by a given model: the less information a model loses, the higher the quality of that model.

In estimating the amount of information lost by a model, AIC deals with the trade-off between the goodness of fit of the model and the simplicity of the model. In other words, AIC deals with both the risk of overfitting and the risk of underfitting about the relative expected discrepancy

[22] "LASSO" stands for *Least Absolute Shrinkage and Selection Operator*. It is a statistical formula for the regularization of data models and feature selection.

In literature on "model inference"[23] another two additional information criteria are used as well: the BIC (Bayesian information criterion), and the TIC (Takeuchi information criterion).

TIC requires a large sample, which usually tends to be difficult to obtain in this context. Moreover, although the BIC criterion is based on Bayesian theory instead of a "frequentist procedure", its formulation is very similar to the AIC, the difference is the penalty term for the number of parameters of each model. BIC often penalizes more the models that have more parameters in comparison with the AIC, so is often more useful than the AIC in nested models.

If we have a statistical model of some data. Let k be the number of estimated parameters in the model. Let $\mathcal{L}$ be the maximum value of the likelihood function for the model. Then the AIC value of the model is

$$AIC(Model) = -2\,ln(\mathcal{L}) + 2k \tag{58}$$

The number of inputs or parameters of each model corresponds to the number of parameters entering the calibration. Given a data sample the model that has lower AIC will have a lower expected information loss, so it will be better than a model with greater loss of information.

When using a particular model, under the assumption that errors (ϵ) are independent and identically distributed with a normal24 distribution with zero mean and σ^2 variance, $N(0,\ \sigma^2)$, we can define the density function (g) for each model based on the normal distribution function.

$$g\ (Output_i/\theta) == \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{1}{2}\frac{[Output_i - (Output \mid Model,\ \ Input)_i]^2}{\sigma^2}\right) \tag{59}$$

where $\theta = (Input,\ \sigma)$

If we define the density function in terms of the error (ϵ), it remains as

$$\mathrm{g}\left(Output_i/\theta\right) = \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{1}{2}\left[\frac{\epsilon_i}{\sigma}\right]^2\right) \tag{60}$$

The likelihood function for each model would be the product of the density function defined for each of the sample observation (we assume *n* observations within a sample)

$$\mathcal{L}\ (\theta/(Output \mid Model,\ \ Input)) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{1}{2}\left[\frac{\epsilon_i}{\sigma}\right]^2\right) = \tag{61}$$

[23] See Anderson & Burnham (2004): "Multimodel Inference. Understanding AIC and BIC in Model Selection".

[24] Assuming that errors or other distribution models are (uniform, exponential, Poisson, Gamma, etc.) it should calculate the likelihood function, the AIC and the corresponding AICc.

$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^{N} exp\left(-\frac{1}{2}\sum_{i=1}^{N}\left[\frac{\epsilon_i}{\sigma}\right]^2\right)$$

The maximum likelihood function for each model remains as

$$\mathcal{L}\left(\hat{\theta}\right) = \left(\frac{1}{\sqrt{2\pi}\hat{\sigma}}\right)^{N} exp\left(-\frac{1}{2}N\right) \tag{62}$$

And the logarithm of the maximum likelihood function remains as

$$ln\left(\mathcal{L}\left(\hat{\theta}\right)\right) = -\frac{1}{2}N\,ln(\hat{\sigma}^2) - \frac{1}{2}N\,ln(2\pi) - \frac{1}{2}N \tag{63}$$

In this case the AIC would be

$$AIC = -2\left[-\frac{1}{2}N\,ln(\hat{\sigma}^2) - \frac{1}{2}N\,ln(2\pi) - \frac{1}{2}N\right] + 2K \tag{64}$$

$$AIC = N\,[ln(\hat{\sigma}^2) + ln(2\pi) + 1] + 2K \tag{65}$$

If the calibration process is performed minimizing the mean-squared error (MSE) for the differences between the real General Score and the General Score as output of each model, then, the variance of the error sample for each model corresponds to the MSE, applying it to the AIC, remains as:

$$AIC = N\,[ln(MSE) + ln(2\pi) + 1] + 2K \tag{66}$$

For small samples, we can apply a correction[25] term. This correction term depends on the density function we assume for errors. If errors are i.i.d. $N(0,\ \sigma^2)$, the corrected AIC could be defined as:

$$AIC_c = AIC + \frac{2K(K+1)}{N-K-1} \tag{67}$$

where N is the size of sample and K the number of parameters used by the model.

To calculate the probability distribution based on the AIC we need to calculate the "Akaike weights" (weight of evidence) that gives the probability that the model is the best model for the data (in terms of K-L divergence). The Akaike weights can be calculated from model likelihood for a given sample. The likelihood of a given model for a given sample can be defined as[26]:

$$\mathcal{L}\,(Model/data) \propto exp\left(-\frac{1}{2}\Delta_{Model}\right) \tag{68}$$

[25] Overall, this correction term should be used unless $N/K >\approx 40$ for the model with highest *K*. Moreover, this correction term only makes sense if the number of parameters is sufficiently less than the number of observations. Samples with a small number of observations and comparatively large number of parameters, the AIC_c penalty could be excessive.

[26] Where ∞ means "*proportional to*".

where $\Delta_{Model} = AIC(Model) - AIC_{min}$ and AIC_{min} is the lowest AIC value of all the considered models.

Approaches for stepwise regression

There are three main approaches when selecting variables and models in a Stepwise AIC process:

- *Forward selection*, which involves starting with no variables in the model, testing the addition of each variable using a chosen model fit criterion, adding the variable (if any) whose inclusion gives the most statistically significant improvement of the fit, and repeating this process until none improves the model to a statistically significant extent.

- *Backward elimination,* which involves starting with all candidate variables, testing the deletion of each variable using a chosen model fit criterion, deleting the variable (if any) whose loss gives the most statistically insignificant deterioration of the model fit, and repeating this process until no further variables can be deleted without a statistically significant loss of fit.

- *Bidirectional elimination,* a combination of the above, testing at each step for variables to be included or excluded.

Therefore, in order to select the appropriate variables from the full set of ratios, Bidirectional elimination approach is used when the model has been implemented for the Telecommunications sector, as it will be seen in below sections.

Regression on the selection

As a next step, further reduction of the selection is performed based on the significance of the remaining variables. The following loop is performed:

1. Regression with all the variables left in the selection is performed
2. In case that poor significance is observed on a variable based on the p-value and predefined significance level, the variable is excluded from the selection. In case multiple variables are not significant, the one with the highest p-value is removed first.

   Steps 1 and 2 are repeated until all the variables remaining in the selection are significant based on the p-value.

6. Final model form and composition

Once the Stepwise AIC process has been run on both OLS and GLS model forms, the best model in terms of regression tests and better AIC with a maximum number of variables is selected (this limits the potential multicollinearity problems). When dealing with panel datasets, the most possible outcome is to have a Stepwise-optimized model calibrated with GLS with the form of AR(*p*), covering the potential autocorrelation found in the initial regression models, if any.

5.1.5 Step 5 – Obtaining the model credit rating and the expected PD for the company

The final output of the model will be of the following form:

$$Company\ General\ Score = \alpha + X_1\beta_1 + X_2\beta_2 + \cdots + X_n\beta_n + \varepsilon \tag{69}$$

Where $X_{1,2,\ldots,n}$ represent the Score for all the X ratios selected in the optimal model, and $\beta_{1,2,\ldots,n}$ represent the coefficient given to the corresponding ratio score by the AIC-based optimization process.

By following the cumulative distribution shown in Figure 14, then the general, final score can be directly associated to its most probable credit letter. However, the Expected Credit Loss and the CVA and DVA metrics are needed to be fed with the PD for a given timeframe. Therefore, it is needed to map all the potential credit letters to its implied PD cumulative curve.

As one of the IFRS rules is that, when using variables to calculate the Expected Credit Loss, the output should be *forward-looking*, historical PDs cannot be used. Conversely, implied PDs in Credit Default Swaps or liquid senior bonds can be used in this regard. The usage of PDs implied in market quotes provides the model output with robustness in terms of default market future expectations for a given entity, sector, or rating letter.

Therefore, it is necessary to build a matrix of PD cumulative curve which provide the default rate per rating letter and tenor. In this research I have used the PD curves that are quoted by Refinitiv. Refinitiv provides, through its SECTORCDS function, the forward default probability rates implied in market Credit Default Swaps spreads. Refinitiv provides several tables with PD curves constructed upon different single name CDSs, according to sector, rating, and geography. This market information already implicitly incorporates a forward-looking approach.

For instance, the BBB-rated CDS spread curve for the Telecommunications sector in Europe, together with its implied default probability curve and estimated Recovery Rate is the following:

Figure 17. BBB-rated CDS spread curve, Telecommunications sector, 17/03/2022

0#BBBTELCDBMK= BBB TEL BMK

Credit Default Swap Curve

Issuer	Mid Spread	Def Prob	DV01	RecovRate	EODComp Date
BBB TEL 6M	32.30	0.14		38.79	17MAR22
BBB TEL 1Y	37.06	0.47		38.79	17MAR22
BBB TEL 2Y	46.87	1.36		38.79	17MAR22
BBB TEL 3Y	57.79	2.62		38.79	17MAR22
BBB TEL 4Y	69.61	4.27		38.79	17MAR22
BBB TEL 5Y	82.08	6.34		38.79	17MAR22
BBB TEL 7Y	101.89	11.00		38.79	17MAR22
BBB TEL 10Y	117.74	17.77		38.79	17MAR22
BBB TEL 20Y	126.86	34.49		38.79	17MAR22
BBB TEL 30Y	133.00	48.86		38.79	17MAR22

Source: Refinitiv

This CDS curve is built as an average of single-name CDSs quoted on senior unsecured bonds, for the relevant sector and geography, and it is a reliable source for PDs containing *forward-looking* default expectations that can be used in several fields of counterparty credit risk, e.g., for ECL or CVA calculation.

Table 14 therefore is composed by all the sectorial PDs implied in CDS spread curves for every single rating letter and is used in the model implementation shown in section 5.2. For those intermediate notches, linear interpolation has been used:

Table 14: CDS-implied cumulative PDs (%) and Recovery Rates (%), Telecommunications sector, 31/12/2021

Rating	6M	1Y	2Y	3Y	4Y	5Y	7Y	10Y	20Y	30Y	Recovery Rate
AAA	**0,07**	**0,15**	**0,40**	**0,76**	**1,33**	**2,15**	**4,37**	**7,72**	**16,07**	**25,52**	**40,00**
AA+	0,07	0,15	0,41	0,81	1,42	2,26	4,43	7,80	16,78	26,54	40,00
AA	**0,06**	**0,14**	**0,41**	**0,85**	**1,51**	**2,36**	**4,48**	**7,88**	**17,49**	**27,55**	**40,00**
AA-	0,07	0,17	0,48	0,99	1,73	2,70	5,09	8,80	19,38	30,21	39,91
A+	0,08	0,19	0,55	1,12	1,96	3,04	5,69	9,71	21,26	32,88	39,81
A	**0,09**	**0,22**	**0,62**	**1,26**	**2,18**	**3,38**	**6,30**	**10,63**	**23,15**	**35,54**	**39,72**
A-	0,11	0,27	0,77	1,54	2,67	4,13	7,61	12,62	26,62	39,71	39,61
BBB+	0,14	0,32	0,91	1,83	3,16	4,89	8,92	14,62	30,09	43,88	39,49
BBB	**0,16**	**0,37**	**1,06**	**2,11**	**3,65**	**5,64**	**10,23**	**16,61**	**33,56**	**48,05**	**39,38**
BBB-	0,25	0,61	1,74	3,41	5,80	8,65	14,83	22,66	42,25	57,06	39,13
BB+	0,35	0,84	2,42	4,70	7,96	11,67	19,44	28,71	50,93	66,08	38,87
BB	**0,44**	**1,08**	**3,10**	**6,00**	**10,11**	**14,68**	**24,04**	**34,76**	**59,62**	**75,09**	**38,62**
BB-	0,78	1,89	5,01	9,09	14,20	19,61	29,54	40,74	65,43	79,06	37,73
B+	1,13	2,70	6,91	12,17	18,29	24,55	35,04	46,71	71,24	83,02	36,83
B	**1,47**	**3,51**	**8,82**	**15,26**	**22,38**	**29,48**	**40,54**	**52,69**	**77,05**	**86,99**	**35,94**
B-	2,47	5,26	11,58	18,78	26,14	33,00	43,43	54,26	75,34	83,89	29,37
CCC	**3,46**	**7,01**	**14,33**	**22,29**	**29,90**	**36,51**	**46,32**	**55,82**	**73,62**	**80,78**	**22,80**

Source: Refinitiv

5.2. Model implementation and performance measurement

So as to assess on the modelling expected outcome, a representative set of companies within a given sector has been selected in order to apply the model. The outcome is expected to be similar to the overall ratings, or much closer to those provided by the market (credit rating agencies - CRAs). Also, not only the outcome but the variables used in the model are expected to be similar to the ones used by CRAs.

The chosen sector is “Telecommunications”, for which overall ratings are issued by Moody’s, S&P or Fitch. Following the steps detailed throughout previous section, the following ratios and the inherent percentile within the sample for each company and ratio are computed: *Pre-tax Income to Sales; Debt to EBITDA; Funds from Operations to Debt; EBIT to Interest Expense; Return On Equity; Net Margin; Return On Assets; EBITDA to Interest Expense; Debt to Equity; Debt to Assets, Cash to Total Debt; Short Term Debt to Total Debt;* and *Quick Ratio*.

In terms of sectorial companies, I directly took all the companies provided as “peers” of Telefónica, S.A., by Refinitiv, in turn to ensure comparability. Some of these peers have no information published to compute some of the ratios, so previously these companies were deleted from the dataset.

The following companies were finally chosen from the sample in order to calibrate the model factors, since these peers possess the most liquid, updated financial information in line with the financial statements date used (31/12/2020):

Table 15: Sectorial companies used to implement the FRS model for Telecommunications sector

Company Names used in the model construction	
BCE Inc	Singapore Telecommunications Ltd
BT Group PLC	Solusi Tunas Pratama Tbk PT
Chorus Ltd	Sri Lanka Telecom PLC
Cogent Communications Holdings Inc	Sunrise Communications Group AG
Deutsche Telekom AG	Swisscom AG
EI Towers SpA	Talktalk Telecom Group Ltd
Emirates Telecommunications Group Company PJSC	Tata Communications Ltd
Far Eastern New Century Corp	Tata Teleservices (Maharashtra) Ltd
Hellenic Telecommunications Organization SA	Telecom Argentina SA
Iliad SA	Telecom Italia SpA
Koninklijke KPN NV	Telefonica SA
KT Corp	Telekom Austria AG
Lumen Technologies Inc	Telekom Malaysia Bhd
Nippon Telegraph and Telephone Corp	Telenet Group Holding NV
NOS SGPS SA	Telia Company AB
Ooredoo QPSC	Telus Corp
Orange SA	True Corporation PCL
PLDT Inc	Verizon Communications Inc
Proximus NV	Vodafone Group PLC
Rostelekom PAO	Windstream Holdings Inc
Saudi Telecom Company SJSC	Zayo Group Holdings Inc
Shaw Communications Inc	Ziff Davis Inc

Source*:* Refinitiv, compiled by the author

The following figure shows the general and particular percentiles for each sample company and ratio within the sectorial database used:

Table 16: Sector companies and their ratio percentiles from the sample

Company	Global Percentile	PreTax Income / Sales	Debt / EBITDA	Funds from Ops / Debt	EBIT / Int Expense	EBITDA / Assets	ROE	Net Margin	ROA	EBITDA / INTEREST EXPENSE	D/E	D/A	Cash to Debt	ST DEBT / TOTAL DEBT	QR
BCE Inc_2017	38	87	58	54	73	67	71	81	80	58	54	47	9	23	7
BCE Inc_2018	35	87	56	54	70	63	68	79	77	56	57	49	5	31	14
BCE Inc_2020	63	75	46	50	57	45	63	74	64	44	53	46	2	66	27
BT Group PLC_2017	52	57	78	75	74	65	88	61	73	66	47	80	63	28	25
BT Group PLC_2018	35	62	73	72	78	64	85	63	78	71	49	73	75	44	51
BT Group PLC_2019	28	64	63	58	74	55	87	68	76	67	41	64	79	56	75
BT Group PLC_2020	25	58	35	43	64	39	60	59	55	64	37	30	76	49	77
Chorus Ltd_2018	27	70	33	1	25	32	44	64	33	17	27	32	6	95	16
Chorus Ltd_2019	25	44	20	2	20	25	31	43	20	13	22	23	37	38	8
Chorus Ltd_2020	38	39	22	6	19	32	33	42	20	15	20	16	0	48	2
Cogent Communications Holdings Inc_2017	8	33	16	41	23	92	4	26	47	9	6	2	82	86	98
Cogent Communications Holdings Inc_2018	52	43	16	45	26	95	3	44	60	11	5	1	82	87	99
Cogent Communications Holdings Inc_2019	64	56	11	44	29	85	4	54	63	11	4	3	87	89	100
Cogent Communications Holdings Inc_2020	12	15	3	36	13	60	9	15	16	6	2	1	81	87	99
Deutsche Telekom AG_2017	50	34	53	71	45	47	58	37	47	42	35	57	28	54	32
Deutsche Telekom AG_2018	50	35	47	73	47	41	39	23	27	56	32	53	27	49	34
Deutsche Telekom AG_2019	26	52	44	71	49	66	63	38	44	60	25	29	29	46	38
Deutsche Telekom AG_2020	33	47	33	51	43	53	59	32	29	46	18	20	37	60	67
Emirates Telecommunications Group Company PJSC_2017	90	94	95	97	86	56	84	88	88	84	89	96	96	32	78
Emirates Telecommunications Group Company PJSC_2018	91	98	96	98	87	67	83	89	90	84	91	96	98	6	73
Emirates Telecommunications Group Company PJSC_2019	91	93	94	95	85	58	81	90	88	82	88	93	98	15	76
Emirates Telecommunications Group Company PJSC_2020	91	96	92	96	84	61	80	92	89	80	86	89	97	4	70
Hellenic Telecommunications Organization SA_2017	19	24	85	21	37	74	20	18	22	57	78	81	95	5	59
Hellenic Telecommunications Organization SA_2018	19	47	89	51	71	82	41	37	50	74	81	84	94	11	61
Hellenic Telecommunications Organization SA_2019	19	19	88	81	39	92	52	41	57	73	66	72	92	5	52
KT Corp_2017	38	20	91	88	56	62	22	20	29	75	88	89	88	21	70
KT Corp_2018	54	25	92	93	68	51	29	24	42	78	92	92	94	29	85
KT Corp_2019	50	22	89	80	67	43	26	22	35	81	89	92	88	43	80
KT Corp_2020	69	22	85	84	67	48	26	22	38	81	85	87	89	27	82
Lumen Technologies Inc_2017	16	18	4	25	16	6	15	16	14	15	44	27	4	91	58
Lumen Technologies Inc_2018	5	4	5	44	5	8	8	5	5	6	34	24	4	88	31
Lumen Technologies Inc_2019	15	2	2	38	2	2	1	2	2	2	25	19	21	76	28
Lumen Technologies Inc_2020	16	6	9	43	7	12	6	6	6	9	23	19	3	73	10
Nippon Telegraph and Telephone Corp_2017	99	72	94	92	98	50	46	55	60	98	95	95	73	22	87
Nippon Telegraph and Telephone Corp_2018	99	77	96	95	99	54	50	60	70	99	96	98	67	22	84
Nippon Telegraph and Telephone Corp_2019	78	74	91	81	100	40	50	58	64	100	94	94	71	9	86
Nippon Telegraph and Telephone Corp_2020	99	71	87	70	99	37	49	56	61	99	91	91	70	4	71
NOS SGPS SA_2018	31	60	78	92	83	85	67	66	80	89	64	59	1	25	26
NOS SGPS SA_2019	35	61	70	90	82	87	70	67	74	88	51	42	2	62	37
NOS SGPS SA_2020	38	37	50	89	75	73	48	50	52	86	43	28	38	61	51
Orange SA_2017	97	36	29	27	46	25	36	30	34	39	38	37	78	19	33
Orange SA_2018	50	40	30	22	46	26	37	31	33	40	36	35	61	14	35
Orange SA_2019	51	58	29	30	57	35	54	51	48	57	30	25	68	37	64
Orange SA_2020	74	54	30	37	54	29	73	74	68	61	35	32	70	32	63
Proximus NV_2019	82	51	86	65	85	79	66	53	66	94	71	79	43	71	40
Proximus NV_2020	80	73	88	78	91	91	84	71	87	95	69	75	40	72	44
Saudi Telecom Company SJSC_2017	79	95	99	100	98	78	74	96	94	97	100	99	99	50	90
Saudi Telecom Company SJSC_2018	71	97	100	99	96	74	75	96	95	96	9	100	100	75	93
Saudi Telecom Company SJSC_2019	89	94	99	99	95	75	78	95	92	95	99	99	96	51	89
Saudi Telecom Company SJSC_2020	64	92	98	98	97	77	78	94	93	96	99	98	99	18	92
Shaw Communications Inc_2017	44	80	76	56	58	49	47	75	58	49	84	82	44	98	43
Shaw Communications Inc_2018	23	21	51	17	26	15	14	15	15	29	83	81	35	93	21
Shaw Communications Inc_2019	35	83	63	49	61	39	58	85	72	45	80	74	77	20	47
Shaw Communications Inc_2020	31	84	61	69	62	51	56	81	67	54	73	66	46	78	62
Singapore Telecommunications Ltd_2017	72	78	71	89	81	16	68	97	92	72	96	88	25	13	24
Singapore Telecommunications Ltd_2018	91	98	90	91	88	49	80	99	98	83	98	91	22	39	33
Singapore Telecommunications Ltd_2019	88	70	69	91	81	13	54	94	86	68	97	90	26	37	49
Singapore Telecommunications Ltd_2020	75	71	54	80	76	15	21	51	43	67	90	83	36	12	30
Swisscom AG_2017	70	86	81	77	88	84	86	84	91	87	67	70	32	29	36
Swisscom AG_2018	66	85	84	74	89	80	81	82	89	91	76	76	31	53	60
Swisscom AG_2019	64	78	72	68	90	75	82	87	91	93	65	63	19	40	49
Swisscom AG_2020	70	84	75	76	91	77	73	86	85	92	74	67	25	59	63
Talktalk Telecom Group Ltd_2018	6	6	4	40	4	10	2	6	4	7	18	16	23	64	37
Talktalk Telecom Group Ltd_2019	10	11	23	53	13	30	56	19	39	16	21	20	33	91	19
Talktalk Telecom Group Ltd_2020	10	50	57	52	51	94	98	70	94	36	26	13	23	69	15
Telecom Italia SpA_2017	49	51	25	36	36	20	29	46	30	20	49	39	57	47	54
Telecom Italia SpA_2018	10	5	6	35	6	4	9	5	6	13	47	40	44	30	30
Telecom Italia SpA_2019	7	56	25	33	37	22	25	39	24	26	45	34	48	58	56
Telecom Italia SpA_2020	10	49	20	19	34	19	90	100	97	29	59	36	65	50	68
Telefonica SA_2017	51	49	35	37	50	52	77	43	49	50	20	22	50	42	29
Telefonica SA_2018	53	65	39	39	64	59	74	49	51	63	22	29	53	36	43
Telefonica SA_2019	44	28	27	57	40	33	28	18	18	48	19	26	56	35	46
Telefonica SA_2020	49	30	26	53	44	35	61	25	25	59	12	21	55	39	81
Telekom Austria AG_2017	35	45	68	60	72	81	67	58	70	78	53	49	40	99	65
Telekom Austria AG_2018	49	42	77	64	73	83	51	46	56	79	60	62	6	68	46
Telekom Austria AG_2019	49	60	54	47	75	78	64	57	61	77	44	25	13	26	54
Telekom Austria AG_2020	49	65	71	75	77	82	69	65	75	77	54	52	42	16	35
Telenet Group Holding NV_2017	21	31	18	50	24	88	6	36	41	19	4	6	5	74	2
Telenet Group Holding NV_2018	19	76	17	63	42	95	5	70	73	21	1	2	8	68	4
Telenet Group Holding NV_2019	14	73	18	55	38	93	5	67	65	20	3	4	10	65	4
Telenet Group Holding NV_2020	19	79	21	64	47	96	2	82	84	33	2	5	8	65	3
Telia Company AB_2017	38	67	32	57	55	13	51	80	63	38	77	68	80	82	96
Telus Corp_2017	60	81	49	47	68	65	79	75	78	54	40	35	16	61	12
Telus Corp_2018	65	82	51	42	69	57	72	76	77	53	50	47	12	77	41
Telus Corp_2019	60	81	37	34	63	46	76	80	71	46	36	30	11	73	39
Telus Corp_2020	40	64	27	33	49	27	53	61	53	42	40	33	18	74	42
True Corporation PCL_2017	49	17	36	20	23	8	16	17	15	32	73	84	51	2	13
True Corporation PCL_2018	50	44	36	35	52	11	30	34	27	55	58	78	50	3	20
True Corporation PCL_2019	50	27	12	10	33	9	25	30	22	26	39	55	71	43	60
True Corporation PCL_2020	52	12	8	22	12	12	16	13	13	14	15	14	29	23	19
Verizon Communications Inc_2017	49	85	53	46	70	68	93	73	81	47	24	39	7	84	64
Verizon Communications Inc_2018	38	80	57	70	65	64	92	78	83	44	29	50	13	77	65
Verizon Communications Inc_2020	32	89	50	73	77	40	89	87	82	62	32	56	60	80	91
Vodafone Group PLC_2017	26	29	42	68	50	21	7	3	4	68	84	71	81	16	23
Vodafone Group PLC_2018	25	46	52	79	66	29	38	71	62	73	85	69	80	20	25
Vodafone Group PLC_2019	28	4	7	59	4	4	8	4	5	22	78	42	92	71	95
Vodafone Group PLC_2020	27	15	19	61	15	24	11	7	9	34	55	22	78	45	72
Windstream Holdings Inc_2017	1	2	1	14	1	1	99	2	2	1	6	5	1	83	17
Windstream Holdings Inc_2018	1	5	14	16	8	88	97	4	3	5	5	4	14	0	0
Windstream Holdings Inc_2019	1	1	0	5	1	0	100	1	1	0	1	12	12	70	53
Ziff Davis Inc_2017	70	88	67	87	61	73	71	85	85	33	71	56	84	100	97
Ziff Davis Inc_2018	60	77	64	94	59	70	64	73	79	36	72	60	67	98	94
Ziff Davis Inc_2019	38	75	48	87	60	47	75	88	84	41	61	51	86	15	77

Source: Compiled by the author; Refinitiv.

It should be highlighted that the global score (percentile) for each company was assigned by a random number within a confidence interval of percentile for the corresponding company rating, in order to test the capacity of the model to cover the existing percentile dispersion within the same notch, particularly for ratings from BB+ to BBB+. The way this percentile dispersion works is presented in section 5.3.

5.2.1 *Full dataset OLS Regression and analysis*

The first step is to run an OLS regression on the entire panel data set. The output on the linear model, following (69) is the below:

Table 17: OLS Regression statistics – Total Sample of Ratio percentiles

Estimator	Coefficient	t value	Pr(>|t|)	Significance
IncomeToSales	0.525415	2.026	0.0458	*
DtEBITDA	-0.165742	-0.673	0.5028	
FFOpsToDebt	0.008746	0.074	0.9414	
EBITtoInterest	0.512611	1.266	0.2089	
EBITDAtoAssets	-0.157808	-1.233	0.2209	
ROE	-0.018395	-0.211	0.8332	
NetMargin	-0.516259	-1.437	0.1543	
ROA	0.231903	0.710	0.4797	
EBITDAtoInterest	0.053609	0.187	0.8518	
DtoE	0.056223	0.419	0.6766	
DtoA	0.139830	0.601	0.5493	
CashToDebt	0.082911	0.777	0.4391	
STDebtToDebt	0.012775	0.164	0.8699	
QuickRatio	0.047713	0.406	0.6854	

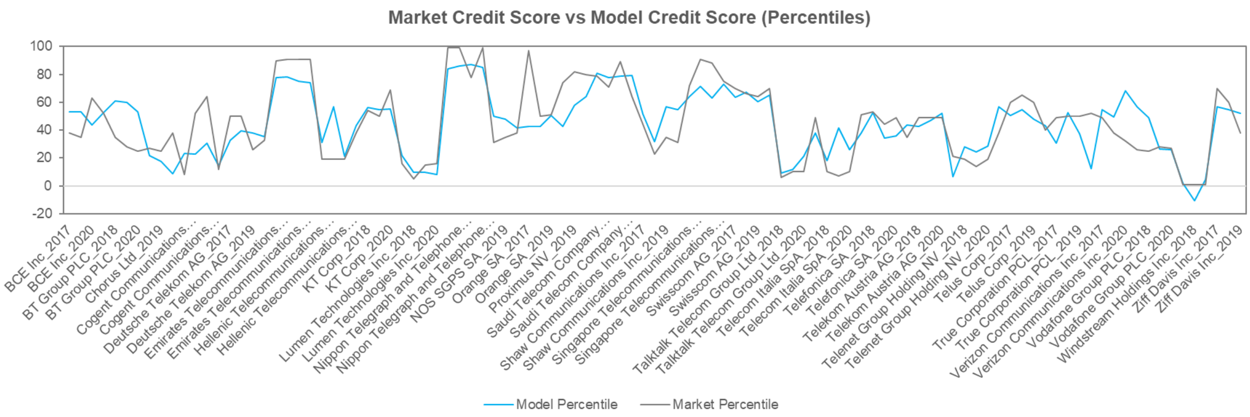
Residual standard error	17.75			
Adjusted R-squared	0.8872			
F-Statistic	58.28			
p-value	<2.2e-16			
Significance codes: 0 '***', 0.001 '**', 0.01 '*', 0.05 '.', 0.1 ' '				

Source: Compiled by the author

The results show that the explanatory power of these regressors altogether is poor, with only Income to Sales as a good significant variable with a 95% of confidence. Although R^2 is relatively high (0.8872), F-value is not as high as desired in a multivariate linear model (58.28) and therefore there could be spurious conclusions from this.

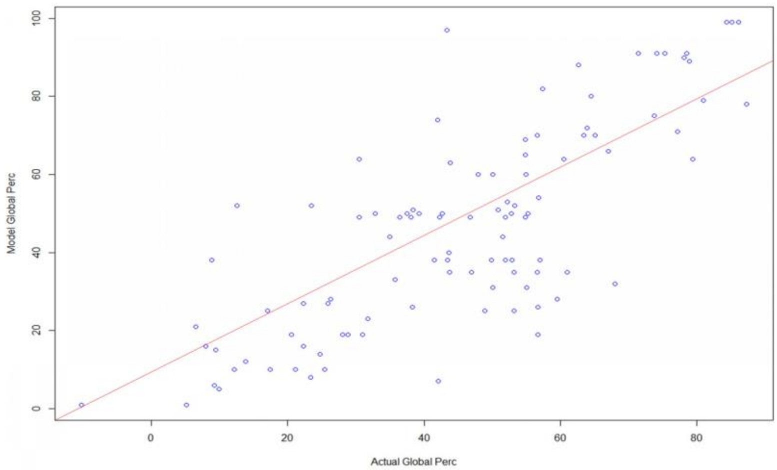
Figure 18 and Figure 19 below show how the model performs in terms of performance and goodness of fit. There is high dispersion over the average, and some of the estimations are out of the real values (e.g., one estimation with percentile < 0). This means that the model needs to be improved in terms of regressors used and the performance for near-to-boundary estimations.

Figure 18. Actual Percentile vs. Model Predicted Percentile – Total Sample of Ratio dataset.



Source: Compiled by the author

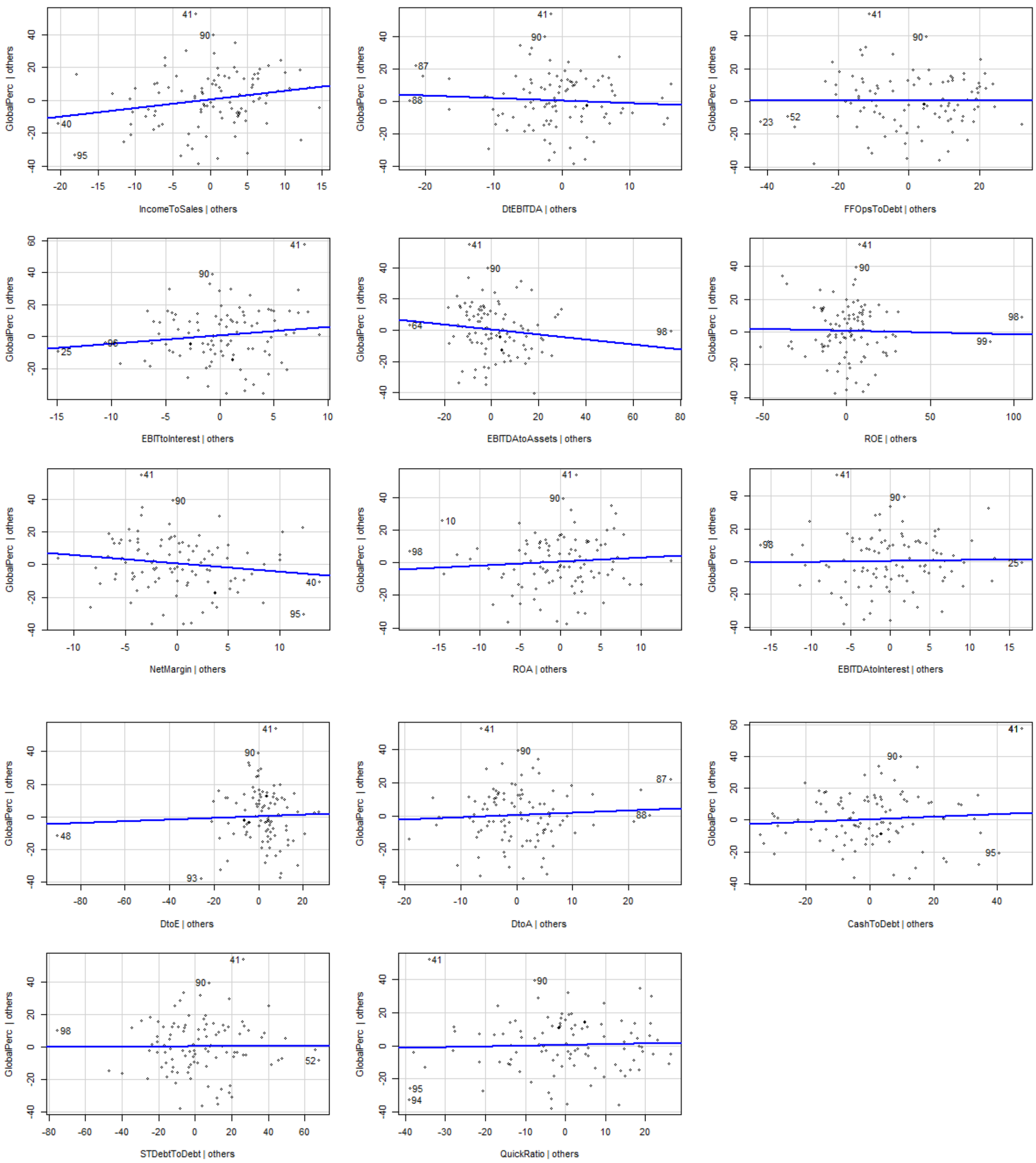
Figure 19. OLS goodness of fit– Total Sample of Ratio dataset



Source: Compiled by the author

As it can be seen below, there is no conclusion when dealing with too many variables in the model, as no explanatory relationship is even found when regressing each of them with the rest as exogenous variables:

Figure 20. Added-variable plots for Total Sample of Ratio dataset

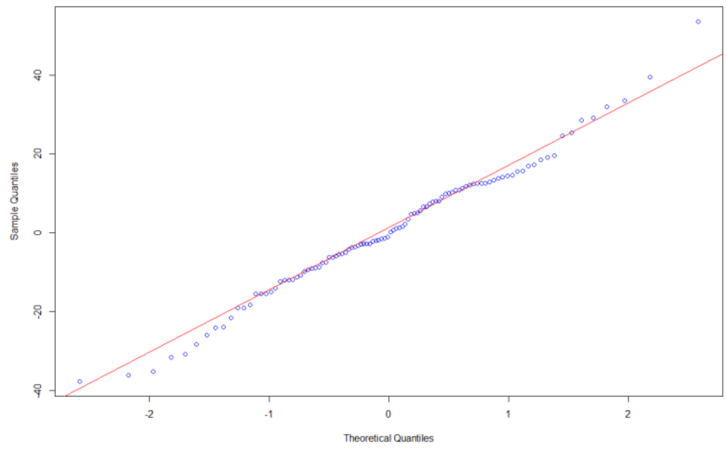


Source: compiled by the author

In terms of heteroskedasticity identification, it should also be noted that evidence of heteroskedasticity was not found as per the result of the Breusch-Pagan test (assuming a constant linear relationship), with a p-value of 0.093.

Concerning normality in residuals, both Jarque-Bera and Anderson-Darling tests resulted in p-values over 5%. Also, Normal Q-Q plot is the following, upon which the assumption of normality would not be rejected neither:

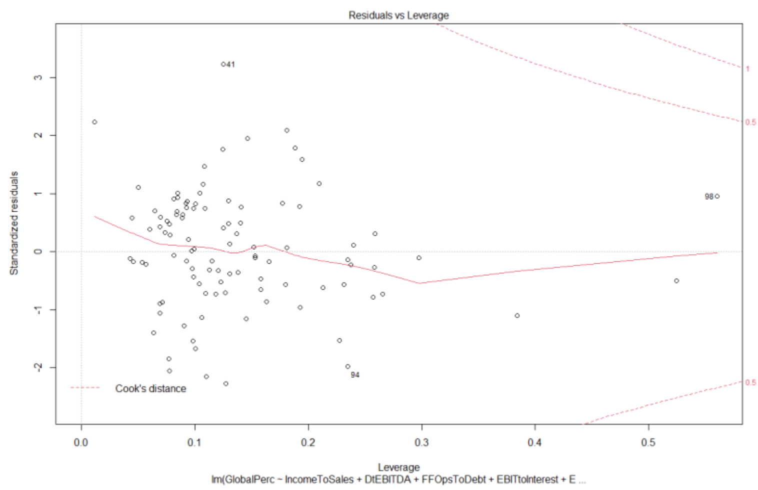
Figure 21. Normal Q-Q Plot – Total Sample of Ratio dataset



Source: Compiled by the author

In addition, we verified that no outliers were identified among the sample, hence they all fall within the Cook distance lines:

Figure 22. Cook's distance Plot – Total Sample of Ratio dataset



Source: Compiled by the author

However, when testing Multicollinearity, several issues arise from using too many variables, some of them similar by nature. In this sense, Table 18 below summarizes the Variance Inflation Factor (VIF) for each of the regressors show that for many of them, it is found a high correlation with all the others regressors (values over 10), particularly Income to Sales, EBITDA to Interest, Net Margin and ROA:

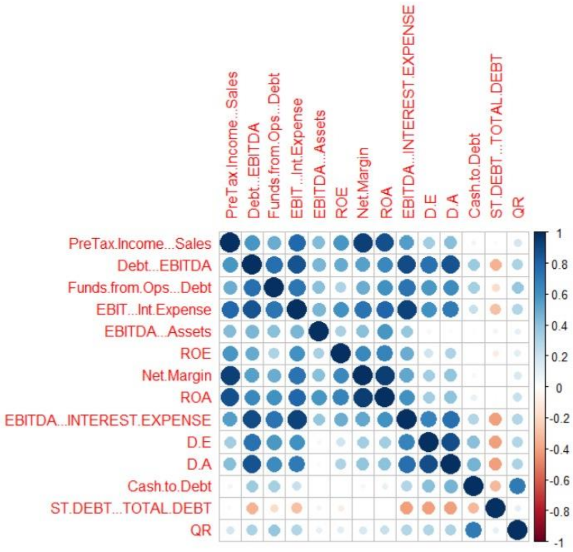
Table 18: VIF matrix – Total Sample of Ratio dataset

IncomeToSales	DtEBITDA	FFOpsToDebt	EBITtoInterest	EBITDAtoAssets	ROE	NetMargin
81,33	68,88	18,59	205,02	17,66	8,34	154,3
EBITDAtoInterest	**DtoE**	**DtoA**	**CashToDebt**	**STDebtToDebt**	**QuickRatio**	**ROA**
97,71	20,79	62,64	12,61	61,05	14,97	132,05

Source: Compiled by the author

Likewise, the correlation matrix for the entire dataset of ratios and historical figures shows that for many pairs of ratios there is a relevant correlation found, hence several regressors are to be deleted as explanatory variables:

Figure 23. Correlation heatmap – Total Sample of Ratio dataset

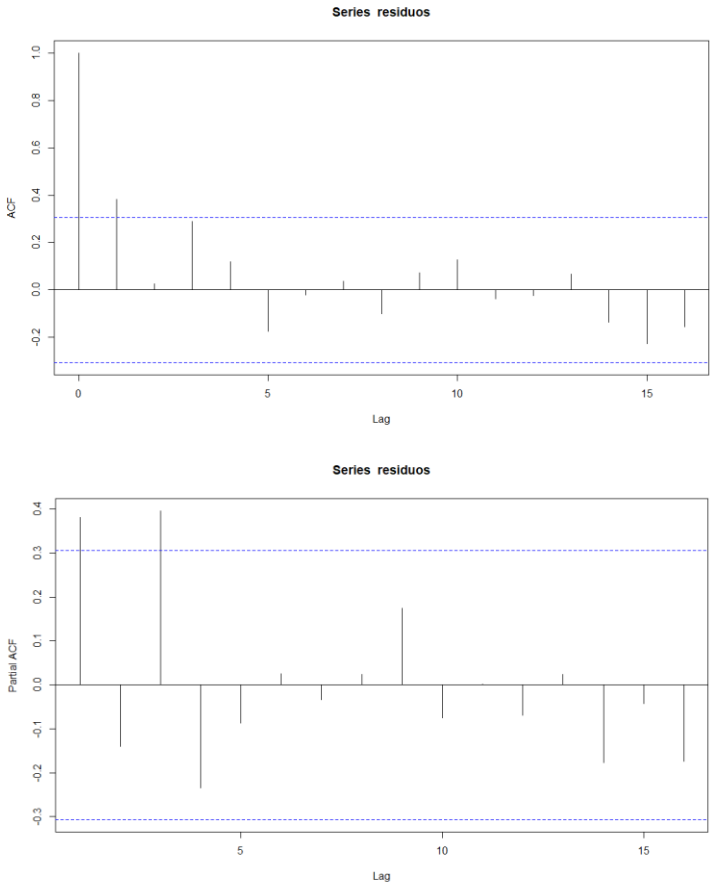


Source: Compiled by the author

Furthermore, as indicated at the beginning of this section, the use of panel data with OLS methods may lead to models being biased due to the existence of autocorrelation.

Due to the fact that the errors are unobservable in the linear model, particularly as regards panel data where relevant variables can be missed, the detection method should focus on the best available estimator, i.e., the residuals created in the regression. Therefore, we first performed the analysis by checking residual autocorrelation and partial autocorrelation plots:

Figure 24. Residual autocorrelation and partial autocorrelation – Total Sample of Ratio dataset.



Source: Compiled by the author

The dashed horizontal lines on the plots correspond to approximately 95% confidence limits. The general pattern of the autocorrelation and partial autocorrelation functions is suggestive of an autoregressive process of order 3 - AR(3) -.

Additionally, a Ljung-Box test was performed in order to quantitatively analyze the weight of the autocorrelation for the given lags. Assuming the autoregressive process is of order 3, the statistic is 10.32, which is above that expected for a 95% confidence interval. This is another indicator of existing correlation in residuals.

5.2.2 Full dataset GLS Regression and analysis

As previously explained, Generalized Least Squares regression model can deal when residual autocorrelation appears. The GLS model on the same regressors has been calibrated with a maximum-likelihood estimation and specifying an AR(3) correlation structure, with the following outcome:

Table 19: GLS Regression statistics – Total Sample of Ratio percentiles

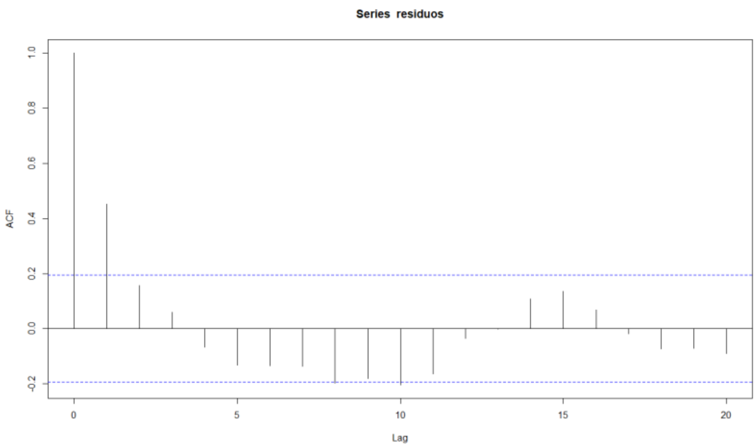
Estimator	Coefficient	t value	Pr(>|t|)	Significance
IncomeToSales	0.6089629	2,6502015	0.0095	
DtEBITDA	0.0191481	0.0713090	0.9433	
FFOpsToDebt	0.0587320	0.4482574	0.6551	
EBITtoInterest	0.0814704	0.2239797	0.8233	
EBITDAtoAssets	-0.1927676	-15.213.856	0.1317	
ROE	0.0110666	0.1088378	0.9136	
NetMargin	-0.3438124	-11.174.034	0.2669	
ROA	0.1161067	0.4080666	0.6842	
EBITDAtoInterest	0.1406262	0.5078302	0.6128	
DtoE	0.0964402	0.7580663	0.4504	
DtoA	0.1431725	0.6227102	0.5351	
CashToDebt	0.1411497	12031034	0.2322	
STDebtToDebt	0.0206404	0.2593770	0.7960	
QuickRatio	-0.0858752	-0.7106527	0.4792	

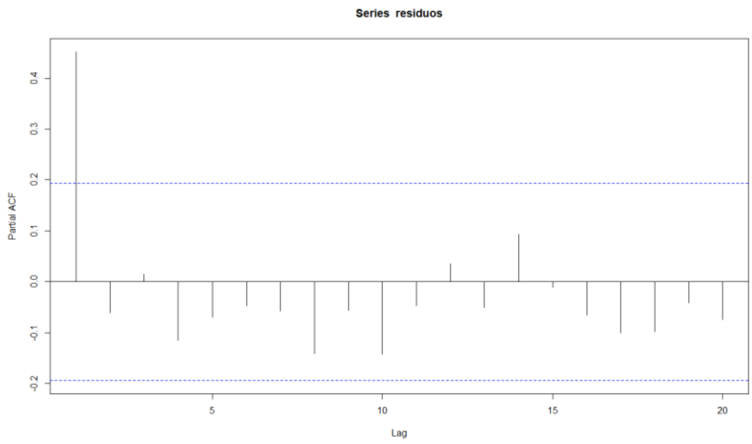
Correlation Structure	ARMA(3,0)			
Phi_1 | Phi_2 | Phi_3	0.488178181	-0.060436550	0.009269673	
Residual standard error	17.90			
Adjusted R-squared	0.90			
AIC | BIC | Loglikelihood	882,48	929,73	-423,24	
p-value	<2.2e-16			
Significance codes: 0 '***', 0.001 '**', 0.01 '*', 0.05 '.', 0.1 ' '				

Source: Compiled by the author

The adjusted R^2 is 0.9002. Normality in residuals and homoskedasticity are still present in the residuals, and the residual autocorrelation seems to be fixed by specifying an AR(3) correlation structure:

Figure 25. Residual autocorrelation and partial autocorrelation for GLS regression with AR(3) – Total Sample of Ratio dataset





Source: compiled by the author

Nevertheless, in this case none of the coefficients are significative, so that we still need to apply the Stepwise AIC optimization to choose the best regressors between the entire ratio dataset.

5.2.3 *Stepwise AIC and selection of AIC-optimized variables*

As specified in above paragraphs, a Stepwise AIC bidirectional elimination approach is used to wipe out all the non-significant variables. Table 20 and Table 21 represent the output of Stepwise AIC processes under OLS and GLS for a maximum number of 6 variables to be selected:

Table 20: Regression statistics for OLS-optimized model via Stepwise AIC

Estimator	Coefficient	t value	Pr(>|t|)	Significance
IncomeToSales	0.45385	2.456	0.01581	*
EBITtoInterest	0.65703	6.182	1.5e-08	***
EBITDAtoAssets	-0.17457	-2.681	0.00862	**
NetMargin	-0.29009	-1.768	0.08019	·
CashToDebt	0.16315	3.289	0.00140	**

Residual standard error	17.16			
Adjusted R-squared	0.8946			
F-Statistic	174.20			
p-value	<2.2e-16			
Significance codes: 0 '***', 0.001 '**', 0.01 '*', 0.05 '·', 0.1 ' '				

Source: compiled by the author

Table 21: Regression statistics for GLS-optimized model via Stepwise AIC

Estimator	Coefficient	t value	Pr(>|t|)	Significance
IncomeToSales	0.5860172	3,858881	0.0002	***
EBITDAtoAssets	-0.1386863	-1,837675	0.0692	.
NetMargin	-0.2057614	-1,536059	0.1278	
EBITDAtoInterest	0.2409340	1,911286	0.0590	.
DtoA	0.2561716	2,359679	0.0203	*
CashToDebt	0.0911577	1,409418	0.1619	

Correlation Structure	ARMA(3,0)			
Phi_1 | Phi_2 | Phi_3	0.467995435	-0.069395920	-0.001527835	
Residual standard error	17.22			
Adjusted R-squared	0.91			
AIC | BIC | Loglikelihood	868,18	894,43	-424,09	
p-value	<2.2e-16			
Significance codes: 0 '***', 0.001 '**', 0.01 '*', 0.05 '·', 0.1 ' '				

Source: compiled by the author

As it can be seen in above tables, the optimized models provide the following ratios as the main explanatory variables:

- Income to Sales
- EBIT to Interest
- EBITDA to Interest
- EBITDA to Assets
- Cash to Debt
- Net Margin

Both OLS and GLS keep adjusted R^2 figures rounding 0.90, but this time the final explanatory variables are significant as per their p-values and t-values (with the exception of *Net Margin* and *Cash to Debt* for GLS which are within the optimized model although are not as significant as desired, however they have been included due to the optimizing AIC-based algorithm and having in mind that for GLS, a correlation structure is defined).

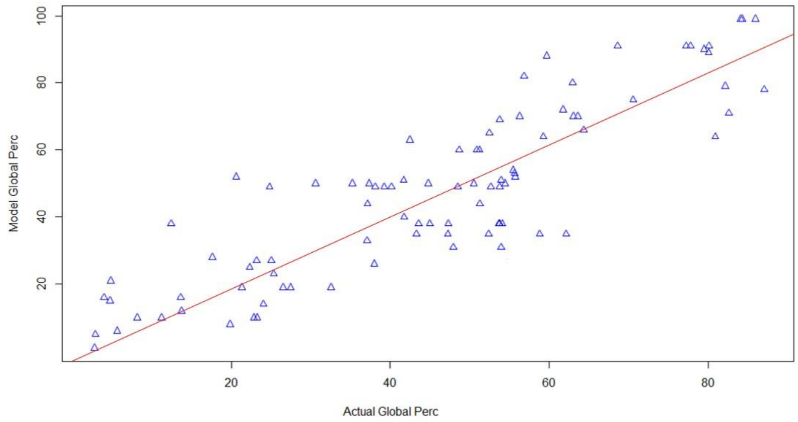
5.2.4 *Final Optimized Models*

As previously seen, most of regressors are shared between the two models and coefficients are similar. Below are the final optimized regression models for both OLS and GLS methods, following the output summarized in Table 20 and Table 21:

$$\begin{aligned} &Company\ General\ Score\ _{Telco-OLS} = \\ &\qquad = IncToSales\ 0.45385 + EBITtoInt\ 0.6570 \\ &\qquad - EBITDAtoAssets\ 0.17457 - NetMargin\ 0.2901 \\ &\qquad + CashtoDebt\ 0.1632 \end{aligned} \quad \text{(Model A)}$$

$$\begin{aligned} &Company\ General\ Score\ _{Telco-GLS} \\ &\qquad = IncToSales\ 0.5860 - EBITDAtoAssets\ 0.1386 \\ &\qquad - NetMargin\ 0.2057 + EBITDAtoInt\ 0.2409 + DtA\ 0.2561 \\ &\qquad + \ CashtoDebt\ 0.0912 \end{aligned} \quad \text{(Model B)}$$

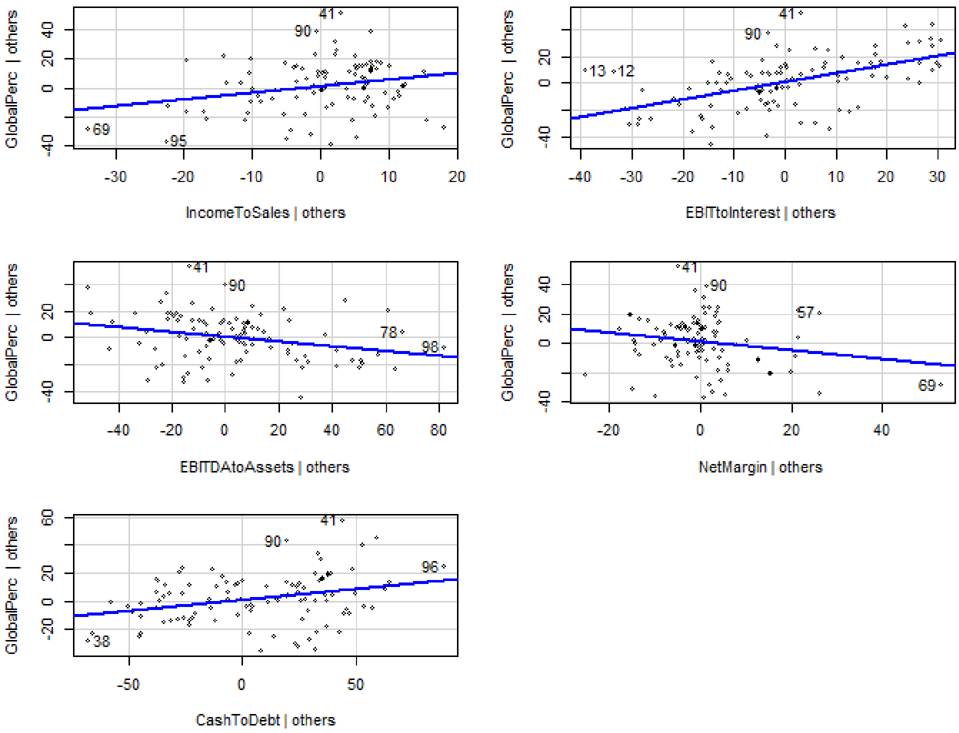
Figure 26. GLS goodness of fit plot– Optimal variables under Stepwise AIC selection



Source: Compiled by the author

Normality in residuals and homoskedasticity are maintained in the new model versions, as expected. On the other hand, the new model structure improves the correlation between regressors, although it is not completely removed. Below, added-value plots, correlation heatmap and VIF values for optimal variables is shown:

Figure 27. Added-variable plots for OLS-optimized model



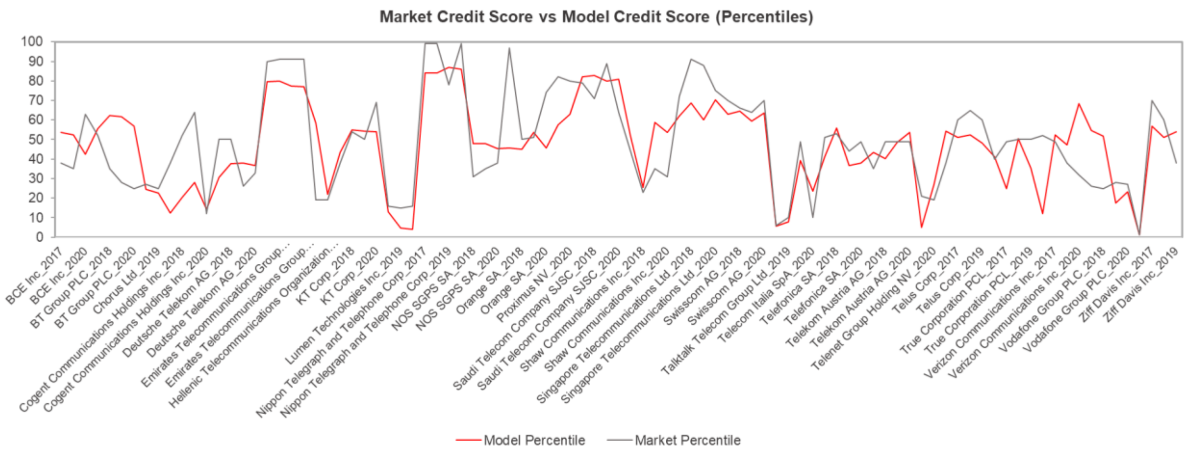
Source: compiled by the author

Figure 28. Correlation matrix heatmap – Optimal model variables under Stepwise AIC selection

Source: compiled by the author

The model performance is displayed below. The optimized model reaches a good grade of accuracy when estimating the scores and their derived impact on expected credit rating letter and probabilities of default:

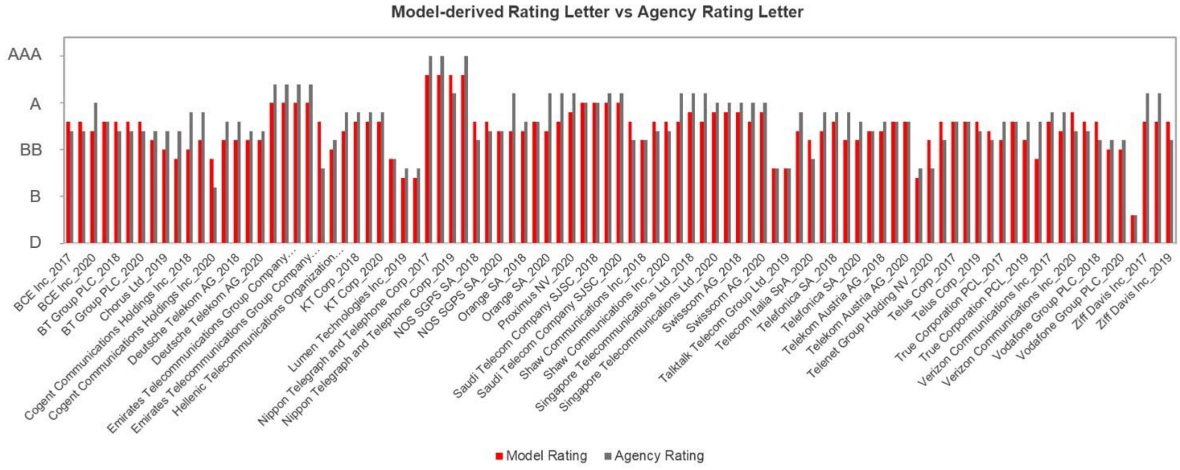
Figure 29. Model performance - Actual Market Percentiles vs. Model Predicted Percentiles under GLS – Optimal variables under Stepwise AIC selection



Source: compiled by the author

The derived rating letters from the modeled Global Scores (percentiles) show a good estimation power for the model in terms of pure rating notch. Below is the rating letter estimation performance under the AIC-optimized model:

Figure 30. Model performance for Agency Rating Letter vs. Modelled Rating Letter under GLS – Optimal variables with Stepwise AIC selection

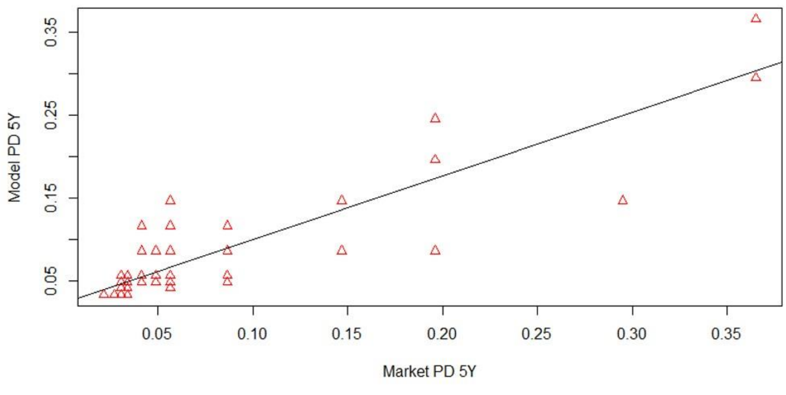


Source: compiled by the author

5.2.5 *Ultimate model output: implied Probabilities of Default as input for IFRS ECL and CVA figures*

The model performance can be ultimately measured throughout the implied Probabilities of Default in the modeled rating letters. The estimated PD is directly interpolated from the modeled global scores (and subsequently, from rating letters) by using **Table 14**. Below the regression plots and regression statistics are detailed:

Figure 31. Regression plot for Market CDS-implied 5Y PDs vs. Modeled CDS-implied 5Y PDs under GLS – Optimal variables with Stepwise AIC selection.



Source: compiled by the author

Statistics for the above regression (5Y CDS-derived PD modeled from the General Score vs 5Y CDS-derived PD from Agency ratings) are relevant. Linear coefficient of modeled PD is 0.9576, and the R^2 reaches 0.9098, with robust F-statistic and p-value figures. Average of cumulative 5Y PD obtained from the entire set of companies is 6.74%, whereas the total average cumulative 5Y PD modeled from estimated Global Scores is 7.24%.

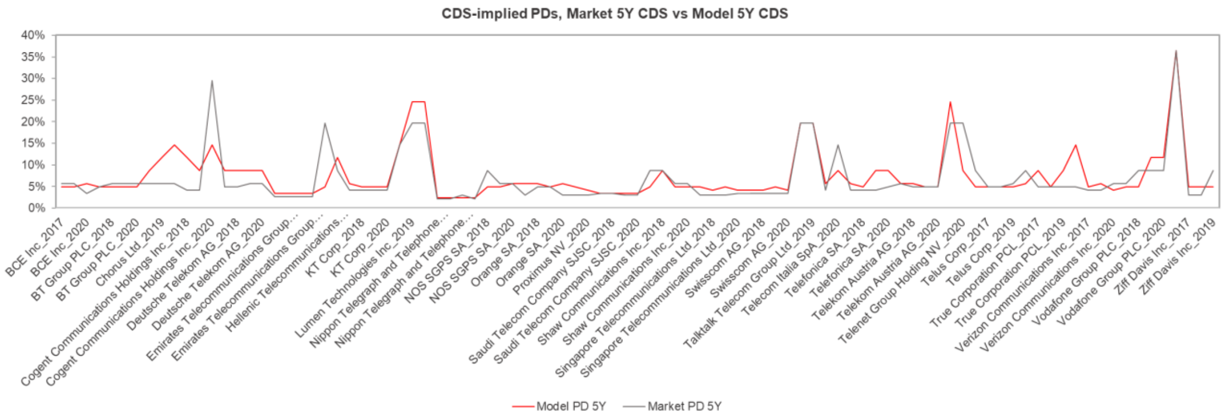
Table 22: Regression statistics for Market CDS-implied 5Y PDs vs. Modeled CDS-implied 5Y PDs under GLS – Optimal variables with Stepwise AIC selection

Estimator	Coefficient	t value	Pr(>|t|)	Significance
Model PD 5Y	0.95757	30.65	<2e-16	***

Residual standard error	0.03114			
Adjusted R-squared	0.9098			
F-Statistic	939.20			
p-value	<2.2e-16			
Significance codes: 0 '***', 0.001 '**', 0.01 '*', 0.05 '.', 0.1 ' '				

Source: compiled by the author

Figure 32. Model performance for Market CDS-implied 5Y PDs vs. Modelled CDS-implied 5Y PDs under GLS – Optimal variables with Stepwise AIC selection



Source: compiled by the author

5.2.6 *Starting hypothesis checkpoint and conclusion: explanatory variables used by Rating Agencies are aligned with the ones used by the optimized model*

As it has been observed, when the model variables are optimal the output from the model (measured either in General Scores, in rating letters or even in PDs) is robust and consistent. Therefore, my starting hypothesis was that, if the model uses optimal variables, and the output is aligned to the sample actual ratings, therefore those optimal explanatory variables should be similar in nature to the ones used in the rating agencies criteria, at least in a certain extent (due to the fact that qualitative factors are also used and they are not present in the model, at least explicitly).

If we recall the information published by Moody´s in Table 2, it can be noted that the main quantitative metrics or ratios used by Moody's in their credit rating assessment for the Telecommunications sector are the same as, or quite similar to, the optimal variables used in the model implementation shown for such a sector:

Table 23: financial metrics used by Moody´s in rating assignment criteria vs GLS optimal variables

Financial metric used in Moody´s credit rating assessment	Weight in the model output		Model optimal variables
Revenue	12.5%	→	*Income / Sales*
Revenue Trend and Margin Sustainability	10%	→	*Net Margin*
Debt / EBITDA	15%		-
Retained Cash / Debt	10%	→	*Cash / Debt*
(EBITDA-CAPEX) / Interest Expense	10%	→	*EBIT&EBITDA / Interest Expense*
	57.5%		

Source: Moody´s and compiled by the author

The only metric which is not shared at all between Moody's criteria and the optimized model is *Debt/EBITDA*, which is out of the AIC-optimal variables set for both OLS and GLS models, although *Debt/Assets* is a ratio of similar interpretation which is used in GLS model. As a conclusion, it is demonstrated that, with a high degree of certainty, the initial hypothesis is met. The model replicates in a relevant extent the quantitative criteria used by the rating agency, which is a conclusion to take into consideration also in the sense that the model could be applicable as long as the agency criteria does not change in a substantial way.

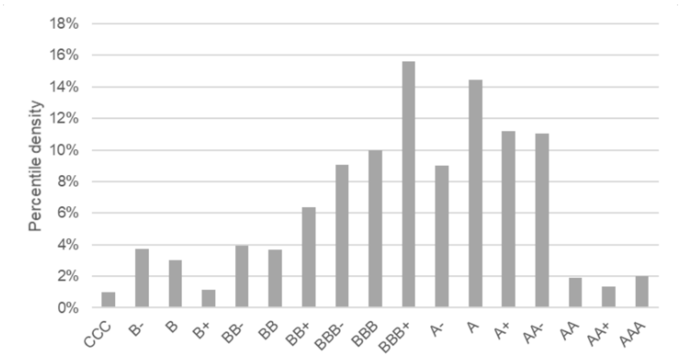
Also, it is worth mentioning that the model has been implemented by using a heterogeneous company dataset, including several geographies, currencies, and a wide range of ratings from different agencies. This means that the model assumptions are robust to cover companies from different countries and credit quality, as it is demonstrated in some extent in section 5.4. *Back-testing* / *Out-of-sample testing*. However, it should be noted that the range of applicability could be limited for companies from particular jurisdictions (e.g., from least developing countries), for companies whose local rating could be difficult to be translated to a global rating scale, or for companies in geographies with special situations (e.g., close-to-default countries), as there are many other qualitative factors present in the rating assessment that are not directly contemplated in financial ratios.

5.3. Model dataset distribution testing

As it was outlined in the model implementation section 5.1., there exists a range of percentiles for a given rating notch. This means that companies with different percentiles but within the same range of percentiles for a given rating notch would be rated with the same letter. For instance, following the distribution of companies and percentiles provided by Refinitiv, a company which is ranked 41 would have a rating of BBB, but another company with score 47 will be rated BBB as well.

For the telecommunications sector it was previously explained that the assignment of the global percentile from the rating letter was done by applying a random process. The randomness upon which a company global percentile is assigned was initially applied by using the following sample distribution:

Figure 33. In – sample distribution function for Telecommunication sector

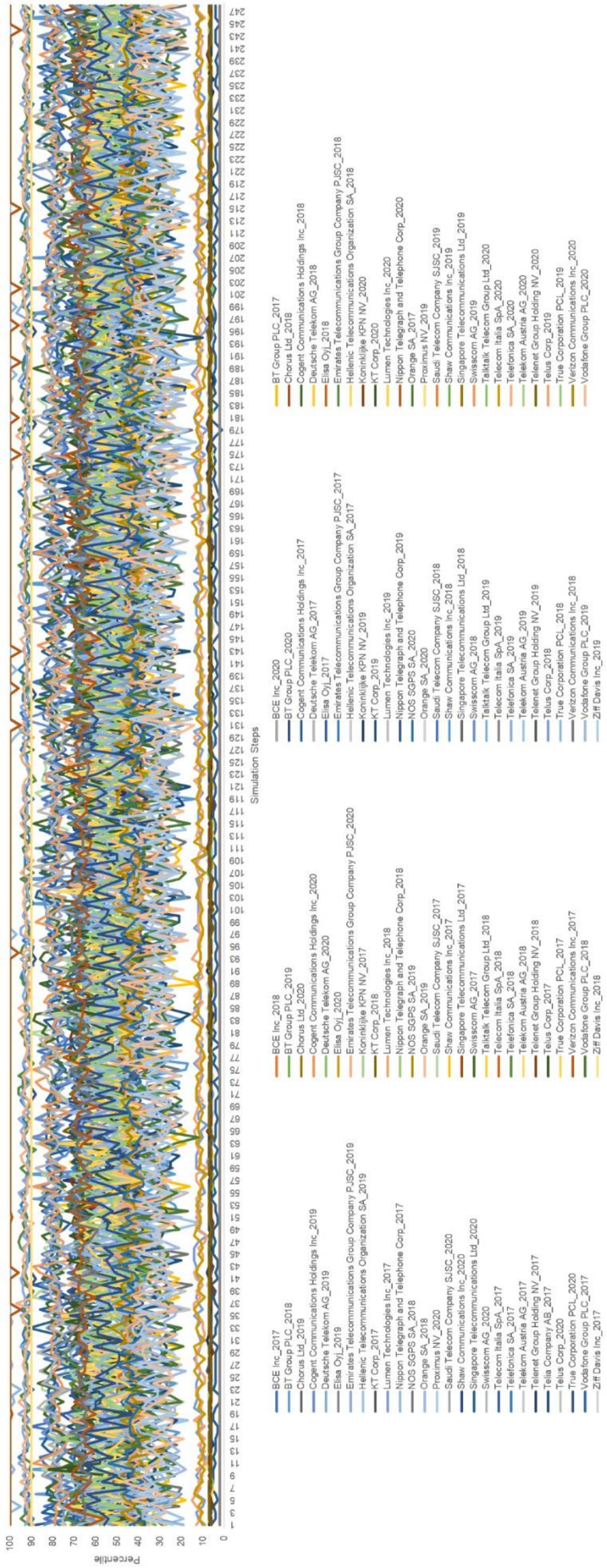


Source: Compiled by the author; Refinitiv.

By way of simplification, it is assumed that the percentile distribution follows therefore a normal distribution function, so that the random expected dispersion within each rating notch has been applied following a normal distribution and a standard deviation according the above distribution. This makes sense following the historical distributions of rating letters given by the main CRAs (see, for instance, Hirk; 2020[27], for further detail on rating letters historical distribution functions) which follow quasi-normal distribution as shown in **Figure 33**. The figure below illustrates the simulation process applied to every company's score from the starting percentile level (the average for its corresponding agency rating). It has been performed by computing many random scenarios for each company based on a standard deviation upon a normally-distributed Brownian motions:

[27] Hirk, R (2020): "Multivariate ordinal models in credit risk: Three essays". Available on https://epub.wu.ac.at/7508/

Figure 34: Percentile dispersion simulation calculated from its Probability Density Function



Source: compiled by the author

It may be noted that, generally speaking, higher dispersion occurs in mid-to-lower quality notches (i.e., for scores at the beginning of the simulation below 60), with both the highest investment grades and highly speculative grades remaining almost unchanged, as expected according to the below rating migration matrix:

Table 24: Average One-Year Alphanumeric Rating Migration Rates, 1983-2017

From\To	Aaa	Aa1	Aa2	Aa3	A1	A2	A3	Baa1	Baa2	Baa3	Ba1	Ba2	Ba3	B1	B2	B3	Caa1/2/3/ Default
Aaa	91,08%	5,42%	2,38%	0,56%	0,28%	0,15%	0,02%	0,06%	0,00%	0,02%	0,01%	0,01%	0,00%	0,00%	0,00%	0,00%	0,00%
Aa1	1,73%	80,96%	8,21%	6,10%	1,50%	0,94%	0,19%	0,13%	0,08%	0,01%	0,04%	0,00%	0,01%	0,04%	0,03%	0,01%	0,04%
Aa2	1,06%	4,35%	78,22%	10,25%	3,59%	1,68%	0,40%	0,09%	0,16%	0,07%	0,03%	0,02%	0,00%	0,03%	0,01%	0,02%	0,02%
Aa3	0,16%	1,05%	4,19%	80,75%	8,58%	3,66%	0,85%	0,24%	0,25%	0,13%	0,03%	0,03%	0,02%	0,01%	0,00%	0,00%	0,05%
A1	0,05%	0,10%	1,01%	5,12%	81,14%	7,78%	2,88%	0,67%	0,48%	0,22%	0,19%	0,13%	0,05%	0,06%	0,02%	0,01%	0,10%
A2	0,06%	0,03%	0,21%	1,06%	5,78%	80,62%	7,46%	2,67%	1,04%	0,39%	0,18%	0,14%	0,17%	0,06%	0,03%	0,01%	0,10%
A3	0,05%	0,05%	0,10%	0,31%	1,58%	6,33%	79,96%	6,95%	2,81%	0,92%	0,38%	0,16%	0,13%	0,11%	0,04%	0,02%	0,12%
Baa1	0,03%	0,03%	0,08%	0,12%	0,21%	1,68%	6,75%	79,62%	7,17%	2,44%	0,65%	0,36%	0,24%	0,28%	0,06%	0,04%	0,24%
Baa2	0,04%	0,04%	0,02%	0,07%	0,18%	0,59%	2,05%	6,52%	80,39%	6,60%	1,41%	0,66%	0,47%	0,34%	0,20%	0,09%	0,32%
Baa3	0,03%	0,01%	0,02%	0,04%	0,08%	0,18%	0,50%	1,93%	8,59%	78,66%	4,90%	2,17%	1,04%	0,74%	0,30%	0,25%	0,58%
Ba1	0,02%	0,00%	0,02%	0,02%	0,16%	0,13%	0,22%	0,76%	2,58%	9,97%	73,05%	5,09%	4,27%	1,62%	0,64%	0,54%	0,93%
Ba2	0,00%	0,00%	0,02%	0,03%	0,09%	0,12%	0,17%	0,39%	0,72%	3,81%	7,91%	72,37%	6,82%	3,80%	1,34%	0,95%	1,48%
Ba3	0,00%	0,01%	0,01%	0,01%	0,07%	0,18%	0,18%	0,10%	0,47%	0,81%	2,85%	6,63%	73,52%	7,39%	3,24%	1,89%	2,64%
B1	0,01%	0,01%	0,02%	0,01%	0,05%	0,03%	0,08%	0,10%	0,22%	0,32%	0,73%	2,88%	6,44%	73,96%	6,17%	4,45%	4,53%
B2	0,00%	0,01%	0,00%	0,01%	0,02%	0,02%	0,10%	0,13%	0,14%	0,27%	0,21%	0,68%	2,06%	7,26%	71,83%	7,94%	9,32%
B3	0,01%	0,00%	0,02%	0,00%	0,03%	0,03%	0,06%	0,03%	0,05%	0,11%	0,14%	0,22%	0,63%	2,27%	6,27%	71,56%	17,75%
Caa1	0,00%	0,01%	0,00%	0,00%	0,00%	0,02%	0,00%	0,02%	0,00%	0,03%	0,07%	0,13%	0,22%	0,40%	1,35%	7,68%	90,08%

Source*:* Moody's (2018); Compiled by the author.

This is a testing process used to demonstrate that the random dispersion assumed among company scores is logical for modelling purposes.

5.4. Back-testing

Back-testing is performed to measure the accuracy and effectiveness of the model. In general, back testing is a technique to compare the model outcome based on historical data with the actual realization. As the model is not intensive in historical data depth but on the data quality, the back-testing is focused on two main techniques: *Out-of-sample testing* and *Cross-validation*.

5.4.1 Out-of-sample testing

To test the predictive ability of the model, 3 companies belonging to the Telecommunication sector are randomly selected from different geographies, rating agencies and currencies, and with financial information cut-off date as of different years as well. This way we are testing the model by using several dimensions in data (i.e., several CRAs, years, currencies, and geographies in the same dataset). If the model is robust enough, it should ensure a certain level of accuracy when modeling the global percentiles of companies which are not in the initial model calibration dataset.

Firstly, we calculate the financial ratios score for the three companies on the optimized GLS model regressors, for which optimal ratios are: *PreTax Income/Sales*; *EBITDA/Assets*, *Net Margin*; *EBITDA/Interest Expense*; *Debt/Assets* and *Cash/Debt*. The table below summarizes the information used to feed the model:

Table 25: Out-of-sample tested companies and ratio percentiles for optimal regressors under GLS

				Percentiles					
Company	**Agency global-scale rating**	**Agency**	**Information Year**	PreTax Income / Sales	EBITDA / Assets	Net Margin	EBITDA / Int. Expense	Debt / Assets	Cash / Debt
Ooredoo QPSC	**A-**	**Fitch**	Dec 2018	59	35	40	32	43	90
Sunrise Communications Group AG	**BBB-**	**S&P**	Dec 2020	20	44	25	69	40	41
Telekom Malaysia Bhd	**BBB+**	**Fitch**	Dec 2020	67	70	68	47	61	91

Source: Refinitiv (31/05/2022), compiled by the author

Then, the optimized GLS model (Model B) is applied with the percentiles of the corresponding ratios for the three companies:

Table 26: Optimal ratios and coefficients, GLS model

Ratio percentile	**AIC-optimized coefficients under GLS**
PreTax Income / Sales	0,5860
EBITDA / Assets	-0,1386
Net Margin	-0,2057
EBITDA / Int. Expense	0,2409
Debt / Assets	0,2561
Cash / Debt	0,0912

Source: compiled by the author

As can be seen below, the application of the model to those companies retrieves a good estimation power, particularly for *Sunrise Communications* and *Telekom Malaysia*, for which the model provides the same rating notch as estimation, whereas for *Ooredoo QPSC* there is a difference of 2 notches.

Table 27: Out-of-sample back-testing output

			Model Output and differences					
Company	Agency global-scale rating	Agency	Model Percentile	Model Rating	Diff. Notches	Agency rating-implied 5Y PD (%)	Model rating-implied 5Y PD (%)	Diff. (%)
Ooredoo QPSC	**A-**	**Fitch**	48,29	**BBB+**	2	4,13	4,89	0,75
Sunrise Communications Group AG	**BBB-**	**S&P**	31,01	**BBB-**	0	8,65	8,65	0,00
Telekom Malaysia Bhd	**BBB+**	**Fitch**	50,62	**BBB+**	0	4,89	4,89	0,00

Source: Refinitiv (31/05/2022), compiled by the author

The relevant impact of the model accuracy is on the IFRS 9 and 13 output, which is the PD, for which the explanatory power is relatively high for the 5Y PD estimation.

5.4.2 Cross-validation process

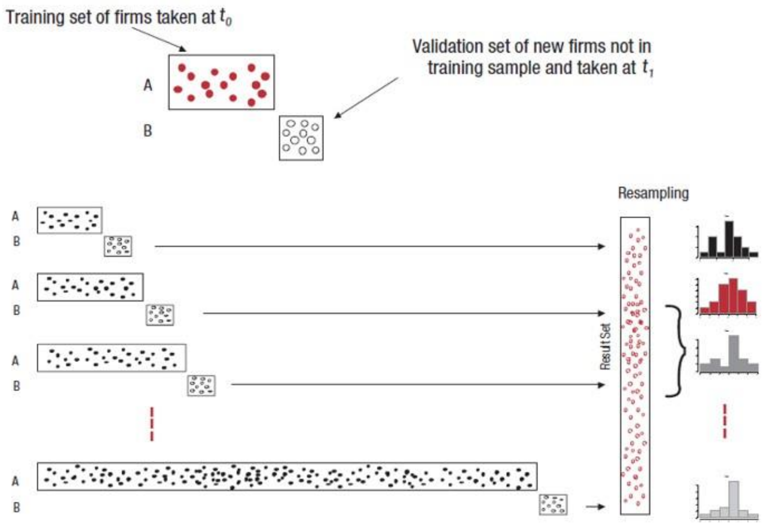
We have already seen that the model replicates in a relevant way the actual rating letter for a significant set of companies. Likewise, the model has demonstrated a robust predictive power for some out-of-sample companies. Now, it is needed to test the model accuracy with a combination of in-sample and out-of-sample companies, by randomly changing the dataset used to calibrate it. This way the performance and predictive power of the model is tested by altering the sample with multiple variations, sizes, and combinations of different subsets of inputs, i.e., by implementing cross-validation or resampling methods.

The fundamental principle behind these cross-validation techniques consists of dividing the data into two sets:

- the training set: used to train (i.e., build) the model.

- the testing set (or validation set): used to test (i.e., validate) the model by estimating the prediction error on the general percentile with a sample of the entire company population used initially.

Figure 35. Example on how cross-validation technique works by resampling the model input data



Source: Moody's (2011)

Three main methods are used for cross-validating the model performance to assess its predictive power:

- Leave One Out - Cross Validation: LOOCV
- Bootstrapping
- Repeated K-Folds

The main outputs to be analyzed from these methods are the following:

- The *R-squared* (R^2), representing the squared correlation between the observed outcome values and the values predicted by the model. The higher the adjusted R^2, the better the model.

- *Root Mean Squared Error* (RMSE) which measures the average prediction error made by the model in predicting the outcome for an observation. That is, the average difference between the observed known outcome values and the values predicted by the model. The lower the RMSE, the better the model.

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n}} \tag{70}$$

- *Mean Absolute Error* (MAE) which is an alternative to the RMSE that is less sensitive to outliers. It corresponds to the average absolute difference between observed and predicted outcomes. The lower the MAE, the better the model.

$$\text{MAE} = \frac{\sum_{i=1}^{n} |y_i - \hat{y}_i|}{n} \tag{71}$$

In the above equations, y_i denotes the i_{th} observation in the sample where $\hat{y}_i$ denotes the i_{th} prediction of the model and n is the number of observations/predictions. In order to prevent any possible overfitting, it is assessed whether there is considerable difference between the prediction errors of training data and the prediction errors of test data for each validation subset, as well as the average error results for all of the subset assessed.

Although the initial model output is the company global score, the goal of the model is to predict the PD to be used in the IFRS framework. Therefore, the accuracy of the model will be measured in terms of 5Y PD, so that we can ensure the accuracy of the output to be used in as an input for ECL or CVA figures.

Regarding the techniques used to test the model estimation power, they have been implemented in the following way:

- **LOOCV**: the sample is split into two sections, one of *n-1* data points which is used to reproduce a regression for predicting the value of the remaining data points, for each of which a regression of *n-1* is calibrated. Also, the LOOCV has been implemented under a Stepwise AIC optimization approach, for which I leave the process to select from 1 to 7 maximum number of variables, so that there will be three dimensions of randomness: type of regressor, number of regressors and degrees of freedom. The output averages are the following:

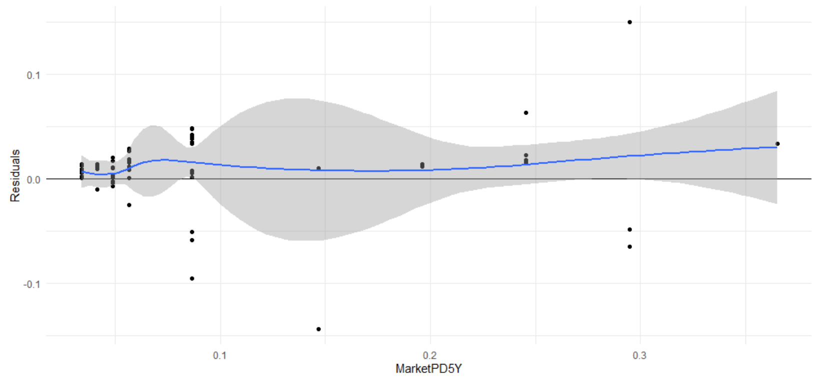
  - RMSE: 0.03411986
  - R2: 0.7281695
  - MAE: 0.02076997

  with an optimal number of variables = 5.

- **Bootstrapping**: this method randomly selects a sample of n observations from the original data set. This subset is then used to evaluate the model. In this case, the sampling is performed with replacement, which means that the same observation can occur more than once in the bootstrap data set. It has been also performed with Stepwise AIC. This provides the advantage of having a large number of potential subsets to simulate data samples. 1,000 scenarios are simulated with the following average output:

  - RMSE: 0.03186692
  - R2: 0.7972403
  - MAE: 0.01903913

Figure 36. Residuals fitting for Bootstrap resampling on Predicted 5Y Probability of Default



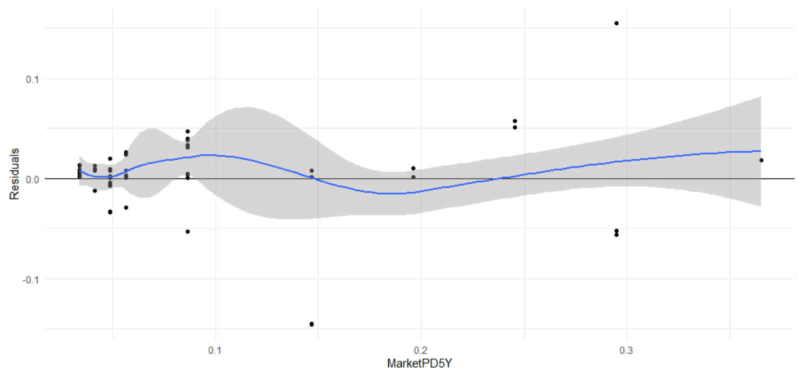
Source: compiled by the author

- **Repeated K-folds:** this method divides the data into *k* buckets of almost equal size. Of these *k* folds, one is used as a validation set while the others are involved in calibrating the regression. In this regard, 1,000 regression folds are simulated so as to be able to generate sufficient prediction scenarios, in order to confirm whether the model's predictive power is robust enough. This method can be considered less unbiased than the above since it uses random data for both regression subsamples, including thousands of combinations of training and validation data sets.

  For a K-fold implementation with the sample divided into 10 buckets, the average output for the Stepwise AIC-based k-fold is as follows:

  - RMSE: 0.02669951
  - R2: 0.8835198
  - MAE: 0.01684332

Figure 37. Residuals fitting for Repeated K-Folds on Predicted 5Y Probability of Default



Source: compiled by the author

As shown above, the cross-validation process has provided with robust results. This means that, using many different samples in terms of components and size, the model is robust enough, so that the explanatory variables chosen in the optimization process are representative for a good estimation power with the current input dataset.

CHAPTER 6: PROPOSED IBR MODELS UNDER IFRS 16

Previous chapter has been fully dedicated to present and develop the modelling framework on the estimation of the credit rating for a company with the focus on obtaining the PD to be applied in the IFRS 9 and also, in the CVA estimation under IFRS 13. In this section we go through the methodology development of two models that cover the counterparty risk impact on the IFRS 16 space and provide engines to deal with this aspect when valuing leasing contracts with non-standard LGDs.

As explained in introductory sections, entities that use leasing products in their economic activity must discount future lease payments to value the leased asset or liability. The discount rate is generally understood as the lessee's IBR (Incremental Borrowing Rate). IFRS 16 states the IBR must consider that the hypothetical loan is collateralized by the leased asset. In this regard, there is a lack of accounting and finance literature focused on analyzing how the IBR should be calculated taking into consideration both the counterparty credit risk of the lessee and the quality of the collateral.

The starting hypothesis is that this quality is mainly determined by the underlying asset's expected LGD (Loss-Given Default) so that the relationship between the IBR and the LGD could be modelled. In this chapter, two quantitative models based on CDS spreads and liquid bond prices are proposed, so that the IBR can be estimated given the lessee credit rating and collateral-linked LGD. The methodology relies on the quantitative relationship existing between CDS spreads and bonds yields with the LGD implied in their market prices.

6.1. Theoretical basis

The proposed models are developed within a default risk pricing framework, and the relationship between default probability, LGDs/Recovery Rates and yield-to-maturities/CDS spreads. The main objective is to develop a framework that allows an existing "standard" borrowing rate (the standard IBR/YTM) - Step 1 as explained in Section **3.3** - to be adjusted in order to obtain a new rate that implicitly reflects the expected recovery rate of the underlying asset (collateral) - Step 2 as explained in Section **3.3** -. In line with the previous literature (Section **2.1**), the adjustment to the initial yield should be performed in such a way that higher recovery rates will entail lower yields, and vice-versa.

Before introducing the models, consideration must be given to the fact that entities need to obtain two specific data prior to applying the model:

1. A “standard” IBR/YTM for a certain date (or a standard curve if we consider several maturities). Generally, as we will see, this standard IBR/YTM assumes a recovery rate of 40%.

2. The expected recovery information for the leased assets.

With regards to the appropriate standard discounting curve, if the lessee maintains issued quoted debt or bonds, then this curve can be constructed using this information. Should this not be the case, additional analyses should be undertaken in order to calibrate a curve that can be associated with the lessee’s rating and sector under a standard seniority (usually senior unsecured debt).

Recovery information (i.e., the Recovery Rate) is the other critical data needed to be obtained. Under normal circumstances, standard market information related to loan and bonds recovery rates may be easily obtained as regards the main sectors, covering historical data on LGDs. This information is usually provided by the main CRAs. Most quoted debt instruments (and their linked standard yield curves) are senior unsecured bonds, with a standard recovery rate of approximately 40%, according to historical performance adopted to price standard credit-linked instruments (bonds, CDS and other credit derivatives) by market conventions.

In Table 28 below, the average recovery rates for debt instruments that defaulted in recent years (according to data from Moody’s) are presented, and I compare them with historical averages. The information provided on the recovery rates is categorized by priority position (from 1st lien bank loans to junior, unsecured and subordinated securities). It can be seen that over the past three decades, recovery rates have generally been correlated with seniority in the debt structure of the issuer. In this regard, seniority refers to a higher average recovery rate. For example, first lien bank loans (loans with higher seniority) have the highest average recovery rate (around 67%). This result is logical given their secured nature and their seniority within the debt structure, and is coherent with previous literature (see Chapter 3).

Table 28: Average corporate debt recovery rates measured by trading prices

Priority Position	Issuer-weighted recoveries		
	2017	2016	1983-2017
1st Lien Bank Loan	69.04%	75.05%	67.07%
2nd Lien Bank Loan	17.87%	22.50%	30.38%
Sr. Unsecured Bank Loan	9.00%	n.a.	45.87%
1st Lien Bond	62.43%	48.72%	53.62%
2nd Lien Bond	52.75%	34.07%	45.18%
Sr. Unsecured Bond	53.85%	31.45%	37.74%
Sr. Subordinated Bond	38.00%	36.72%	31.10%
Subordinated Bond	74.38%	24.50%	32.05%
Jr. Subordinated Bond	17.50%	0.63%	22.79%

Source: Moody’s (2018)

The recovery data shown above is based on trading prices “at default” or “post default”. An alternative recovery measure is based on ultimate recoveries, i.e., the value that creditors recover once the default event is resolved (Table 29). For example, in the case of issuers filing for bankruptcy, the ultimate recovery is the present value of the cash or securities that creditors actually receive when the issuer's bankruptcy is legally finalized, typically one to two years after the initial default date.

Table 29: Average Corporate debt Recovery Rates measured by ultimate recoveries, 1987-2017

Priority Position	Emergence Year			Default Year		
	2017	2016	1987-2017	2017	2016	1987-2017
Loans	81.3%	72.6%	80.4%	80.2%	78.3%	80.4%
Senior Secured Bonds	52.3%	35.9%	62.3%	57.5%	46.9%	62.3%
Senior Unsecured Bonds	54.1%	11.7%	47.9%	47.4%	29.2%	47.9%
Subordinated Bonds	4.5%	6.6%	28.0%	n/a	8.0%	28.0%

Source: Moody's (2018)

From **Table 28** and **Table 29**, it can be seen that senior, unsecured bonds are mostly assumed to have an average recovery rate, from a historical perspective, of approximately 40%. This recovery rate is aligned with the multiple recovery rates present on the credit market by issuer's rating for Senior, Unsecured bonds, for a relevant time horizon:

Table 30: Average Senior Unsecured Bond Recovery Rates by Year Prior To Default, 1983-2017

Issuer's rating	Year 1	Year 2	Year 3	Year 4	Year 5
Aaa		3.33%	3.33%	61.88%	69.58%
Aa	37.24%	39.02%	38.08%	43.95%	43.18%
A	30.36%	42.57%	44.97%	44.49%	44.17%
Baa	42.89%	44.16%	43.99%	43.79%	43.52%
Ba	44.63%	43.30%	42.13%	41.60%	41.59%
B	37.62%	36.77%	37.21%	37.71%	38.36%
Caa-C	38.10%	38.43%	38.50%	38.83%	38.86%
Investment Grade	40.04%	43.33%	43.96%	44.11%	43.86%
Speculative Grade	38.34%	38.19%	38.31%	38.66%	38.99%
All Rated	38.40%	38.47%	38.71%	39.11%	39.45%

Source: Moody's (2018)

The fact that a senior, unsecured bond is expected to have a recovery rate of approximately 40% is relevant to the model proposal, as will be subsequently explained in Sections 3.2 and 3.3.

This background serves as the underlying basis for the model. As may be expected, lease collaterals generally have different recovery rates (not necessarily 40%), given their specific nature, expected value, asset usage and expected amortization. **Table 31** below summarizes real data gathered from leasing collaterals and their respective recovery rates:

Table 31: Estimated Recovery Rates for leasing contracts

Leased asset	Recovery Rate
Vehicles	60.47%
Machinery	50.91%
ICT	11.79%
Equipment	33.96%
Other	53.98%

Source: Hartmann-Wendels *et al*. (2014) and Ou *et al*. (2013)

Nonetheless, as previously stated, quoted products do not exist in markets that are linked to the different recovery rates associated with the various and plausible underlying assets backing a leasing contract. Hence the methodology proposed in this dissertation aims to address this issue. In this context, we need a model framework that deals with the main risk factors involved, namely yield-to-maturities, credit ratings, recovery rates, credit spreads, default probabilities and updated market information.

6.2. Model hypotheses

As previously outlined, it is difficult to find traded products that provide a wide range of implied recovery rates. The use of traded products whose price depends directly on yield-to-maturities (or credit spreads) and recovery rates is fully recommended. This will allow us to calibrate yields or spreads associated with different recovery rates (depending on the collateral backing the leasing contract).

The main model hypotheses and assumptions are as follows:

1. Credit-linked traded products will be used to analyze how their prices (in terms of fair value, yield-to-maturity or credit spread) change when recovery rates change, assuming a deterministic, static default probability curve for any recovery rate/product tranche seniority (Jarrow and Turnbull, 1995; Duffie and Singleon, 1999; Chiang and Tsai, 2010).

2. The model presented assumes that the lessee is a company (a legal entity), with public financial information for which a corporate rating can be estimated. Hence, this framework does not apply to an individual as a lessee.

3. Seniority and collateral type are the most important determinants of recovery rates at default: higher seniority/more liquid collateral imply higher recovery at default. In the models presented, standard recoveries are approximately 40% since the bonds used for calculation are generally senior unsecured bonds, as well as the curves used in this regard (Moody's, 2018).

4. Neither long-term averages nor moving averages of loss-given default are predictors of current losses-given default per se. This is due to the cyclical nature of LGD. Therefore LGDs, and subsequently recovery rates, can vary for the same product/seniority/collateral

depending on the state of the economy. This also includes geographical issues. In short, the LGDs assumed for a given issuer/issuance may vary throughout time (EBA[28], 2017).

5. Secured debt is less sensitive to the default risk and to the general state of the economy than unsecured debt. This applies to the majority of the different classes of assets and represents the pivotal point of the models introduced. More specifically, issuances with more liquid collaterals behind them are expected to have higher recovery rates than those with less liquid collaterals (Benmelech and Bergman, 2009; Cerquerio *et al*., 2016, Duo and Meder, 2020; Lara-Rubio *et al*., 2016; Matias and Dias, 2015, Moody's, 2018).

6. Fixed income and credit markets are assumed to be the most reliable sources of information, including updates in every risk factor used. The relationship between LGDs, YTMs and default risk is understood through market instruments (Schonbucher, 2003).

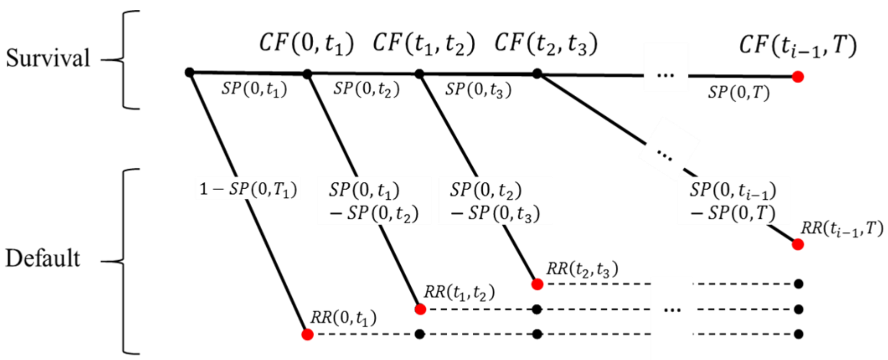
7. The model does not cover lease contract liquidity and sovereign risk, as explained in the following subsections.

6.3. Bond price-based model: Methodology, model theory development and implementation

The first modelling proposal is based on the pricing of traded debt instruments issued by the lessee (or similar peers in terms of rating and sector) following a default-tree model.

A binomial tree can be constructed in order to calculate the debt instrument's value at each tree node[29], taking into account the conditional default probabilities $[SP_i - SP_{i-1}]$ existing at each node i. Namely,

Figure 38. Default-tree model algorithm



Source: compiled by the author

[28] EBA: European Banking Authority.
[29] See, for instance, Castagna, A., and Fede, F. (2013).

where $CF(t_{i-1}, t_i)$ is the risk-free cash-flow to be paid by the instrument; $SP(0, t_i)$ is the survival probability of the product between t_0 and t_i; $1 - SP(0, t_i)$ is the default probability for the same period; and $RR(t_{i-1}, t_i)$ is the estimated recovery rate of the debt instrument for each period.

This tree shows that bonds payments have a survival probability at each node t_i, but they are complemented with their default probability at the same node, where the payment value will be the only estimated recovery. That is to say, at every node t_i a default event may occur, or the obligor will continue until the next date t_{i+1}. Following default, the non-defaulted path continues (indicated by the upper continuation of the tree), but the defaulted security only earns, for a certain node, its recovery payoff and ceases to exist from there on (represented by dashed lines). The sum of the payment scenarios (indicated by red dots), weighted by the probability of their occurrence, is equal to the instrument fair value. The aforementioned also means that the probabilities attached to the branches of the tree are only the conditional default and survival probabilities at this node as seen from t = valuation date. Therefore, under the above model, the defaultable debt instrument price is as follows:

$$\left.\mathit{Fair\ Value}_{t_0} = \sum_{i=1}^{T} CF(t_{i-1}, t_i)\, P(0, t_i)\, SP(0, T) + \right\} \text{Default-free cash-flows}$$

$$\left. + \sum_{i=1}^{T} [RR(t_{i-1}, t_i) CF(t_{i-1}, t_i)[SP(0, t_{i-1}) - SP(0, t_i)]\,]P(0, t_i) \right\} \text{Sum of the defaultable cash-flows recovery} \tag{72}$$

where CF is the default-free cash flow at each node i, and $P(0, t_i)$ is the risk-free discount factor.

We can assume (given the objective of the model) that default event (hazard) rates λ_{t_i} for different predefined time intervals [t_{i-1}; t_i] of the instrument life are deterministic and constant, so that the instrument survival probability between each time interval is:

$$SP[0, t_i] = e^{(-\lambda t_i)} \tag{73}$$

Hence, if we already know the market price (fair value) of the product and the recovery rate linked to its seniority, we can carry out an initial calibration of the factor λ to the instrument market price. This is the market price (fair value) as shown in (72). Once the hazard rate has been calibrated, then all the risk factors of this model have been defined: $\mathit{Bond\ market\ value} = f[CF, \lambda, RR,\ P(0, t_i)]$. Therefore, we can analyze the sensitivity of the price to the recovery rate as follows:

- We already possess a bond’s market price with the implied hazard rate, and we will assume that the hazard rates are constant, and therefore SP only increases over time.

- Once the default-tree has been constructed, the initial recovery rate *RR* may be changed to the chosen recovery rate estimated upon the collateral backing the leasing contract.

- Hence, this change in the *RR* implies a change in the bond price and, therefore, in the bond price sensitivity to the Recovery Rate, assuming that there is no immediate correlation between Recovery Rates and probability of default (although it does exist in the long-term).

- The price change can be translated into the Yield-to-Maturity or curve change, following the general framework of bond pricing:

$$Bond\ market\ value = \sum_{i=1}^{n} CF_{t_i}\, B(0, t_i) \tag{74}$$

where

$$B(0, t_i) = \frac{1}{(1 + YTM)^{t_i}}$$

The new bond price will be given by $f[CF, \lambda, RR(new), P(0, t_i)]$ following (74), and subsequently the implied change in the Yield-to-Maturity (ΔYTM) will be obtained by calibrating the YTM to the new bond price, following (72) and (73).

6.3.1 *Default tree implementation example*

Our scenario assumes a senior unsecured bond corresponding to a certain issuer that matures in 2025. We know that today's bond market mid-price is 99.50%, paying a semiannual coupon of 3% with bullet amortization. The coupon payments will be made on every 30th September and every 31st March, and the valuation date is 30/09/2021. The nodes of the tree correspond to the payment times, for simplification purposes. $P[0, t_i]$ will be constructed by using the €STR curve as of the valuation date.

Assuming that the recovery rate for the bonds is 40%, the default tree can be constructed as of 30/09/2021, while the hazard rate required to obtain the bond market price (99.50%) must be calibrated, following (72) and (73), against the market price. Calibrating λ_i results in an implied hazard rate of 6.0875%, and the final tree would be as follows:

Table 32: Default tree scenarios and bond Net Present Value for a standard RR = 40%

Tree scenarios		Cash-flows NPV	Scenario probability conditional to λ_i
default	31/03/2022	40.27%	3.031%
default	30/09/2022	41.90%	2.955%
default	30/03/2023	43.54%	2.834%
default	30/09/2023	45.17%	2.793%
default	30/03/2024	46.81%	2.679%
default	30/09/2024	48.44%	2.626%
default	30/03/2025	50.07%	2.504%
default	30/09/2025	51.68%	2.469%
no default	30/09/2025	114.57%	78.110%
		Bond market value **99.50%**	

Source: compiled by the author

As a result, we obtain the bond market price which has been computed from the tree by using its corresponding recovery rate (40%) and by calibrating its implied hazard rate.

Table 32 above represents the value of each tree branch and its probability occurrence, in line with **Figure 38** and (72). For instance, the first default node has a cash-flow NPV of 40.27% (*40% of notional recovery * risk-free discount factor*), with a default probability of 3.031% ($1 - SP[0, t_1]$). The second node works in the same way, and now includes the first coupon previously received at 31/03/2022 plus the recovery rate of the notional to be received at the next default date scenario (30/09/2022), all risk-free discounted and weighted by its conditional default probability of 2.955% ($SP[0, t_2] - SP[0, t_1]$). The rest of the defaulting branches represent the same scenario (i.e., cash-flows received until default plus the notional recovery rate at default date, all risk-free discounted and weighted by their conditional probability rate). The final node represents the survival scenario, where no Recovery Rate occurs, and only the total cash-flows, including notional, are risk-free discounted, as shown in (72).

In order to assess the sensitivity of the bond price to the recovery rate, only a change in the recovery rate of the tree is required. For instance, assuming the bond has a recovery rate of 50%, then the tree would return the following figures:

Table 33: Default tree scenarios and bond Net Present Value for a new RR = 50%.

Tree scenarios		Cash-flows NPV	Scenario probability conditional to λ_i
default	31/03/2022	50.34%	3.031%
default	30/09/2022	52.00%	2.955%
default	30/03/2023	53.66%	2.834%
default	30/09/2023	55.31%	2.793%
default	30/03/2024	56.98%	2.679%
default	30/09/2024	58.63%	2.626%
default	30/03/2025	60.28%	2.504%
default	30/09/2025	61.90%	2.469%
no default	30/09/2025	114.57%	78.110%
		Bond market value **101.72%**	

Source: compiled by the author

As can be seen, the net present value for any single branch of the tree has increased, given the higher expected recovery rate, whereas the default and no default probability for each branch remain constant (the bond seniority and the issuer are the same).

As previously mentioned, in the case that the objective of the model implementation is to assess the change in the instrument YTM, then firstly the original bond YTM should be computed. In this specific situation, the original bond market price (99.50%) provides a YTM of 3.1563%. With the new bond market price (101.72%), the corresponding YTM is 2.5668%, meaning that the change in YTM or ΔYTM = -0.5895%.

This is a practical example to show how the model works, providing information on the change in the YTM resulting from a shift in the Recovery Rate of a given asset, which is precisely the main objective of this study with regard to IBR computation for leased assets. That is to say, the change arising in an instrument's YTM following a change in the Recovery Rate can be applied to the IBR of a leased asset with similar issuer/borrower (in this case, the lessee) rating, and maturity. This will be applied to an entire set of bonds with liquid prices in turn to test the model performance.

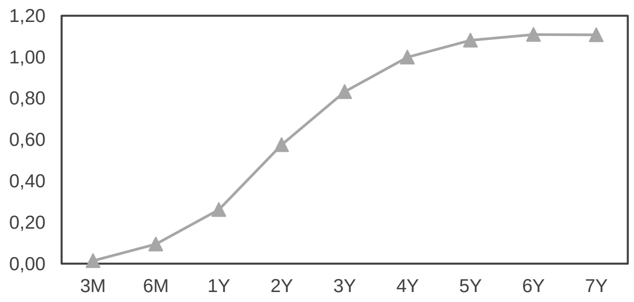
6.3.2 Specific aspects of leasing contracts

In terms of leasing contracts and IBR estimation, many companies do not have credit ratings nor liquid bonds issued in order to estimate the standard IBR (Step 1 as explained in Section **6.1**). If a company is not able to estimate the standard IBR, then it cannot calculate the change in the standard IBR if said IBR is applied to a different recovery rate (Step 2 in Section **6.1**).

In these cases, the most frequent solution is to estimate a theoretical credit rating for the issuer and to use sectorial bond prices or yield curves from comparable issuers (with a similar rating and maturity). Therefore, it is critical to use a model that provides a consistent rating for the lessee. In this regard, the proposed model in Chapter 5 covers this issue in a relevant extent.

By way of example, a leasing contract for which the IBR must be estimated has machinery as the underlying asset (the collateral). The lessee has been estimated to have a BB rating and belongs to the "basic materials" sector. The company has no liquid bonds nor similar debt instruments quoted on the market. First of all, a standard IBR curve is required, representing the company credit risk. Bloomberg and Refinitiv provide liquid indexed yield curves for many sectors and geographies. In this case, for the Basic Materials sector, the BB yield curve provided by Refinitiv (RIC *0#BBEURMATBMK=*) is as follows:

Figure 39. Basic Materials sector, BB-rated standard YTM curve (%), 30/09/21



Source: Refinitiv

The company's leasing contract matures in 5 years (September 2026), and therefore the YTM (IBR) required pertaining to liquid bonds maturing in 5 years is approximately 1.10%. The recovery rate for these bonds is assumed to be 40% (since they are senior, unsecured vanilla bonds), and the average mid-price is 106.81%, paying an average coupon of 2.587%. for that maturity. See next table for further information on the Refinitiv curve constituents:

Table 34: Basic Materials sector, BB-rated YTM curve bond constituents, 30/09/21

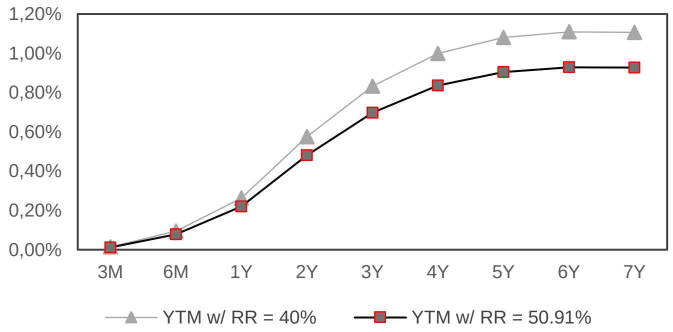
Issuer Name	Coupon (%)	Maturity	Bid	Ask	Swap Spread	Asset Swap	ISIN
SEALED AIR	4.50	15/09/2023	107.259	107.674	70.5	76.1	XS1247796185
BALL	4.38	15/12/2023	109.331	109.630	56.8	58.6	XS1330978567
BALL	0.88	15/03/2024	101.109	101.488	79.7	77.6	XS2080317832
WIENERBERGER	2.00	02/05/2024	104.672	105.073	57.9	58.7	AT0000A20F93
CROWN EURO	2.63	30/09/2024	104.957	105.412	102.7	103.8	XS1490137418
TITAN GLOBAL	2.38	16/11/2024	103.413	104.413	145.6	144.8	XS1716212243
CROWN EURO	3.38	15/05/2025	107.745	108.007	121.2	125.4	XS1227287221
WIENERBERGER	2.75	04/06/2025	107.516	108.216	86.4	89.7	AT0000A2GLA0
METINVEST	5.63	17/06/2025	105.889	107.389	415.4	420.7	XS2056722734
CROWN EURO	**2.88**	**01/02/2026**	**106.750**	**107.121**	137.8	141.1	XS1758723883
SYNGENTA FIN	**3.38**	**16/04/2026**	**109.834**	**110.162**	129.8	135.3	XS2154325489
BALL	**1.50**	**15/03/2027**	**102.486**	**104.486**	121.2	120.5	XS2080318053
TITAN GLOBAL	2.75	09/07/2027	105.955	106.827	178.3	179.9	XS2199268470
SYNGENTA FIN	1.25	10/09/2027	101.021	101.425	122.9	121.7	XS1199954691
ASHLAND SVC	2.00	30/01/2028	104.183	104.690	142.5	143.3	XS2103218538
VERALLIA	1.63	14/05/2028	103.490	103.966	117.9	118.3	FR0014003G27

Source: Refinitiv

Using this information, we are able to calibrate a default-tree similar to the one shown in **Table 32**, obtaining an implied hazard rate of 2.7365% and with the tree already prepared for a shift in the recovery rate.

As previously stated, the leased asset (used as collateral) is a machinery-type asset. Based on historical data from Hartmann-Wendels *et al*. (2014) seen in **Table 31**, this asset has a recovery rate of approximately 50.91%. Therefore, all that is required is a shift from 40% to 50.91% to be made in the Recovery Rate used in the tree and, as a result, the new bond price would be 108.25%, meaning a ΔYTM = -0.1789% or -17.89 basis points, thus decreasing from the original YTM (1.10%) to the new lower YTM (0.9211%). This change could hence be applied proportionally to all available, liquid maturities of the standard YTM curve for a given sector and rating in order to make the adjustment required, thereby resulting in a new YTM curve adapted to the required recovery rate. Figure below simulates this shift from the original YTM curve to the new curve adapted to the machinery recovery rate, for all available maturities:

Figure 40. Basic Materials sector, BB-rated standard and shifted YTM curves, 30/09/2021



Source: Refinitiv and Compiled by the author

6.3.3 A practical example

I have performed several analyses using quoted bonds, applying the model by using prices of bonds issued by companies that maintain quoted bonds with different recovery rates. The starting YTM used is the one belonging to the senior unsecured bond, and subsequently we analyze whether it correctly predicts the change in YTM for a similar bond in terms of contractual features but with a different recovery rate.

The bonds contractual data (duration, currency, coupon frequency and type, market conventions, etc.) pertaining to each of the issuers must be similar enough for them to be comparable, thus permitting the main driver behind the differences seen in their YTMs and credit spreads to be their implied LGD (or implicitly, the Recovery Rate), due to the difference in the credit tranche.

In order to illustrate the assessment of the model, two isolated use cases have been performed (which have been subsequently extended to a wider sample). The first test has been carried out using quoted bonds issued by BBVA (BBVA.MC). I selected three bonds which were highly similar to each other in terms of issuer, currency and duration, but which belonged to different seniority tranches:

Table 35: Several outstanding bonds for BBVA, SA, for several seniority tranches, 30/09/21

Issuer	ISIN	Maturity	Coupon	Currency	Seniority	Issuance Rating	Implied Recovery Rate
BBVA, SA	XS2013745703	21/06/2026	1.000%	EUR	Senior Unsecured	BBB+ (FTC)	40%
BBVA, SA	ES0413211915	22/11/2026	0.875%	EUR	Senior Secured (Covered Bond)	Aa1 (Moody's)	65%
BBVA, SA	XS1562614831	10/02/2027	3.500%	EUR	Subordinated Unsecured	Baa2 (Moody's)	30%

Source: Refinitiv

In this case, we have a standard bond (Sr. Unsecured) with an implied market recovery rate of 40%. Furthermore, BBVA has issued other bonds with similar maturity and currency but belonging to the Senior Secured and Subordinated Unsecured tranches, with recovery rates of approximately 65% and 30% respectively, in line with historical data from Moody's (**Table 28** and **Table 29**).

If we use the proposed default-tree model and simulate the impact on the YTM by changing the original recovery rate as per the Sr. Unsecured note by the recoveries for the other tranches, we obtain the results shown in the table below:

Table 36: Several outstanding bonds for BBVA, SA, for several seniority tranches, and model outputs, 30/09/2021

Issuer	ISIN	Maturity	Coupon	Currency	Seniority	Issuance Rating	Implied Recovery Rate	Market Price	Model Price	Actual YTM	Actual ΔYTM (from Sr. Unsec. Bond)	Model YTM	Model ΔYTM (from Sr. Unsec. Bond)
BBVA, SA	XS2013745703	21/06/2026	1.000%	EUR	Senior Unsecured	BBB+	40%	104.30%	104.30%	0.0795%		0.0795%	
BBVA, SA	ES0413211915	22/11/2026	0.875%	EUR	Senior Secured (Covered Bond)	Aa1	65%	105.50%	105.50%	-0.1980%	-0.2775%	-0.1650%	-0.2445%
BBVA, SA	XS1562614831	10/02/2027	3.500%	EUR	Subordinated Unsecured	Baa2	30%	115.71%	115.71%	0.5030%	+0.4235%	+0.4709%	+0.3914%

Source: Refinitiv and Compiled by the author

As it can be seen, we obtain similar results when comparing the actual YTM for any single bond with the YTM obtained when replacing the original Sr. Unsecured bond recovery rate in the default-tree model (40%) by the corresponding recoveries for the covered bond and the subordinated, unsecured note.

This effect can be seen in the case of CaixaBank (CABK.MC), for example. In the table below three outstanding bonds with similar contractual details have been taken, with seniority constituting the sole notable difference between them:

Table 37: Several outstanding bonds for CaixaBank, for several seniority tranches, 30/09/2021

Issuer	ISIN	Maturity	Coupon	Currency	Seniority	Issuance Rating	Implied Recovery Rate
CaixaBank	ES0213307053	09/07/2026	0.750%	EUR	Senior Unsecured	A- (FTC)	40%
CaixaBank	XS2013574038	19/06/2026	1.375%	EUR	Senior Non-Preferred	BBB+ (FTC)	35%
CaixaBank	ES0440609339	11/01/2027	1.250%	EUR	Senior Secured (Covered bond)	AAA (FTC)	65%

Source: Refinitiv

The results when shifting the recovery rate in the default-tree model from 40% to 35% and 65% achieve a change in the YTM similar to those directly seen in the quoted YTMs:

Table 38: Several outstanding bonds for CaixaBank, for several seniority tranches, and model outputs, 30/09/21

Issuer	ISIN	Maturity	Coupon	Currency	Seniority	Issuance Rating	Implied Recovery Rate	Market Price	Model Price	Actual YTM	Actual ΔYTM (from Sr. Unsec. Bond)	Model YTM	Model ΔYTM (from Sr. Unsec. Bond)
CaixaBank	ES0213307053	09/07/2026	0.750%	EUR	Senior Unsecured	A-	40%	102.81%	102.81%	0.1290%		0.1235%	
CaixaBank	XS2013574038	19/06/2026	1.375%	EUR	Senior Non-Preferred	BBB+	35%	104.83%	104.83%	0.2910%	+0.1620%	0.2539%	+0.1304%
CaixaBank	ES0440609339	11/01/2027	1.250%	EUR	Senior Secured	AAA	65%	107.28%	107.28%	-0.1724%	-0.3014%	-0.1451%	-0.2685%

Source: Refinitiv, compiled by the author

6.3.4 *Model implementation and Performance measurement*

In order to assess the robustness of the model along with its predictive power, the above analysis is done for a sample of outstanding bonds issued in EUR, USD and GBP. I extensively researched Refinitiv to locate issuers that have issued more than one bond with different estimated recovery rates (belonging to different seniority tranches) but issued in the same currency, with similar duration and paying a similar coupon. In other words, the bonds under analysis would be so similar in nature that the main explanatory variable for the gap between their YTMs would have to be the seniority tranche. This is necessary in order to analyze whether the model correctly predicts the change in YTM when a change in recovery rate occurs.

A sample with outstanding bonds from the world's principal bond markets is constructed. Table 39 shows the number of bonds initially included in the sample by exchange market. Only fixed rate bonds are included, issued by corporates or financial institutions, with maturity dates between 2026 and 2040. For this reason (i.e., the fact that we can only use issuers that have issued more than one bond with different estimated recovery rates), several bond markets with hundreds of potential bonds have been analyzed, so that to build a database with sufficient bonds to test model outputs. For the most part, the potential population of quoted bonds is expected to be highly limited and to belong to financial entities. More specifically, using a manual selection process, 91 bonds issued by 43 issuers (Table 40) are used, all of which complied with the criteria of same issuer, maturity year and different debt seniority (unsecured vs. subordinated/non-preferred/mortgage/secured).

Table 39: number of bonds initially included in the sample by exchange market

Exchange	Bonds
Deutsche Boerse AG	2,853
Dublin	1,380
Euronext.liffe Paris	771
London	1,264
Luxembourg	1,927
NYSE	3,191
Singapore	596
Vienna Stock Exchange	341
TOTAL	**12,323**

Source: Compiled by the author

Table 40: Bonds used for model testing, 30/09/2021

Bond	Maturity	Seniority	ISIN	Coupon (%)	Mod. Duration
Bayerische Landesbank 2016 1.4% 29/06/29	2029	Senior Unsecured	DE000BLB32E3	1.4	7.3428
Bayerische Landesbank 2018 1 3/4% 17/10/28	2028	Subordinated Unsecured	DE000BLB6TV4	1.75	6.5154
Dekabank Deutsche Girozentrale 2018 1.33% 25/01/30	2030	Senior Unsecured	DE000DK0PDR8	1.33	7.8796
Dekabank Deutsche Girozentrale 2020 1 1/4% 01/04/30	2030	Senior Non-Preferred	DE000DK0T1B2	1.25	8.0626
Dekabank Deutsche Girozentrale 2020 1.1% 25/11/30	2030	Subordinated Unsecured	DE000DK0T2A2	1.1	8.6042
Deutsche Bank AG 2011 4 1/4% 14/09/26	2026	Senior Unsecured	DE000DB7XNA3	4.25	4.5877
Deutsche Bank AG 2016 4.2% 15/06/26	2026	Subordinated Unsecured	DE000DL19S19	4.2	4.209
Aareal Bank AG 2018 0.8% 05/09/28	2028	Mortgage	DE000A2E4CE8	0.8	6.8061
Aareal Bank AG 2019 0.87% 28/06/29	2029	Senior Non-Preferred	DE000A2E4CV2	0.87	7.4798
ABN Amro Bank NV 2016 1% 13/04/31 CBB16	2031	Senior Secured	XS1394791492	1	9.1608
ABN Amro Bank NV 2021 1% 02/06/33 Regulation S	2033	Senior Non-Preferred	XS2348638433	1	10.9892
Altice Financing SA 2021 4 1/4% 15/08/29 Regulation S	2029	Senior Secured	XS2373430425	4.25	6.6425
Altice France Holding 2020 4% 15/02/28 Regulation S	2028	Senior Unsecured	XS2138140798	4	5.5535
Argentum Capital SA 2019 2.1% 19/01/26	2026	Senior Secured	XS1947921075	2.1	4.0979
Argentum Capital SA 2020 1.7% 27/01/27	2027	Senior Unsecured	XS2090803466	1.7	5.0289
Bayerische Landesbank 2016 1/2% 24/03/26	2026	Mortgage	DE000BLB3Z54	0.5	4.4736
Bayerische Landesbank 2017 0.55% 09/08/27	2027	Senior Unsecured	DE000BLB43N1	0.55	3.3615
Banque Nationale de Paris Paribas SA 2016 2 1/4% 11/01/27 Regulation S	2027	Subordinated Unsecured	XS1470601656	2.25	4.9962
Banque Nationale de Paris Paribas SA 2018 1 1/8% 11/06/26 Regulation S	2026	Senior Non-Preferred	XS1748456974	1,125	4.6158
Commerzbank AG 2018 7/8% 06/06/28	2028	Mortgage	DE000CZ40MV5	0.875	6.542
Commerzbank AG 2019 0.85% 15/08/29	2029	Senior Non-Preferred	DE000CZ40N95	0.85	7.6003
Deutsche Bank AG 2015 1 3/4% 09/04/35	2035	Senior Unsecured	DE000DB7XLM2	1.75	11.9416
Deutsche Bank AG 2018 1.405% 04/11/33	2033	Mortgage	DE000DL19T91	1,405	11.165
DZ Bank AG Deutsche 2016 0.67% 18/05/27	2027	Senior Secured	DE000DG4T8R7	0.67	5.5727
DZ Bank AG Deutsche 2017 0.725% 21/06/27	2027	Senior Unsecured	DE000DG4UAZ5	0.725	5.6353

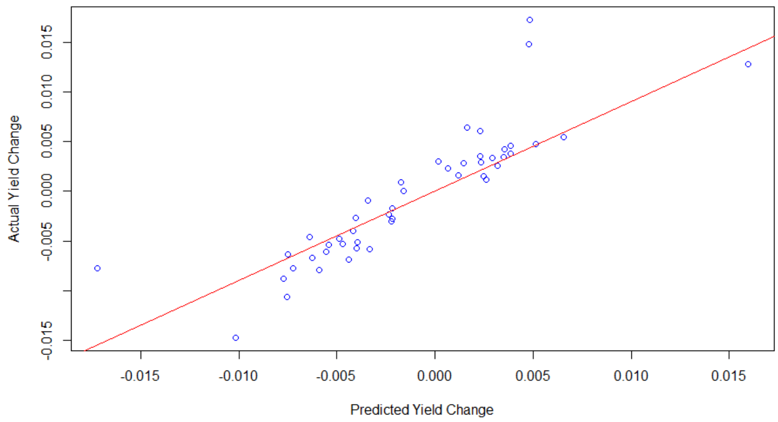
DZ Bank AG Deutsche 2020 1/2% 18/02/27	2027	Subordinated Unsecured	DE000DDA0V15	0.5	5.2993
Elia Transmission 2013 3 1/4% 04/04/28 Regulation S	2028	Senior Unsecured	BE0002432079	3.25	5.9756
Elia Transmission 2014 3% 07/04/29	2029	Unsecured	BE0002466416	3	6.8425
Landesbank Hessen 2016 1.06% 08/11/27	2027	Senior Unsecured	DE000HLB2KP8	1.06	5.8954
Landesbank Hessen 2017 0.997% 30/11/27	2027	Senior Secured	DE000HLB2NE6	0.997	6.0167
Landesbank Hessen 2017 1.745% 20/02/35 Regulation S	2035	Senior Unsecured	XS1567856445	1.745	11.9509
Landesbank Hessen 2018 1 1/4% 19/09/33	2033	Senior Secured	DE000HLB4U71	1.25	11.1228
Landesbank Saar 2015 1 1/4% 23/12/26	2026	Senior Unsecured	DE000SLB5862	1.25	5.0716
Landesbank Saar 2015 1% 15/05/26	2026	Senior Secured	DE000SLB3917	1	4.5692
Landesbank Saar 2017 0.83% 28/09/26	2026	Mortgage	DE000SLB1358	0.83	4.9253
Muenchener Hypothekenbank 2017 5/8% 07/05/27 1762	2027	Mortgage	DE000MHB18J6	0.625	5.5605
Muenchener Hypothekenbank 2019 1/2% 08/06/26	2026	Senior Non-Preferred	DE000MHB61E7	0.5	4.6539
Norddeutsche Landesbank 2013 2.13% 24/04/28	2028	Senior Secured	DE000NLB1LD6	2.13	6.2117
Norddeutsche Landesbank 2014 2 1/2% 23/05/28 1766	2028	Senior Non-Preferred	DE000NLB8CQ2	2.5	6.154
Unicredit Bank AG 2018 1% 29/03/28	2028	Senior Unsecured	DE000HVB29D7	1	6.3414
Unicredit Bank AG 2019 7/8% 11/01/29	2029	Mortgage	DE000HV2ARM0	0.875	7.0988
WCFS und Ifbk Hessen 2016 5/8% 10/06/26	2026	Senior Unsecured	DE000A1R0162	0.625	4.6814
WCFS und Ifbk Hessen 2018 7/8% 14/06/28	2028	Senior Non-Preferred	DE000A2DAF36	0.875	6.5746
BARCLAYS PLC 2016 4 3/8% 12/01/26 S	2026	Senior Unsecured	US06738EAN58	4.375	5.2162
BARCLAYS PLC 2016 5.2% 12/05/26 S	2026	Subordinated Unsecured	US06738EAP07	5.2	5.3163
FORD MOTOR COMPANY 1992 9.95% 15/02/32 P02/95	2032	Unsecured	US345370BH27	9.95	7.7727
FORD MOTOR COMPANY 1998 8.9% 15/01/32 S	2032	Senior Unsecured	US345370BV11	8.9	7.8014
HCA INCORPORATED 2016 5 1/4% 15/06/26 S	2026	First Lien	US404119BT57	5.25	5.4084
HCA INCORPORATED 2018 5 3/8% 01/09/26 S	2026	Senior Unsecured	US404121AH82	5.375	5.453
MORGAN STANLEY 2014 4.35% 08/09/26 F	2026	Subordinated Unsecured	US6174467Y92	4.35	5.6565
MORGAN STANLEY 2016 3 7/8% 27/01/26 F	2026	Senior Unsecured	US61746BDZ67	3.875	5.326
STATE STREET CORP. 2020 2.4% 24/01/30 S	2030	Senior Unsecured	US857477BG73	2.4	8.7868
STATE STREET CORP. 2021 2.2% 03/03/31 S	2031	Senior Subordinated Unsecured	US857477BP72	2.2	8.9012
WELLS FARGO & CO 2005 5 1/2% 01/08/35 S	2035	Subordinated Unsecured	US929903AM44	5.5	11.0967
WELLS FARGO & CO 2005 5 3/8% 07/02/35 S	2035	Senior Unsecured	US949746JM44	5.375	10.9647
ALLGEMEINE SPARK. 2015 1.13% 16/02/27 2	2027	Senior Secured	AT000B101076	1.13	6.7712
ALLGEMEINE SPARK. 2017 1.4% 05/07/27 4	2027	Senior Unsecured	AT000B101274	1.4	6.9756
BANK FUER TIROL 2017 1.72% 10/03/27 7	2027	Senior Unsecured	AT0000A1U834	1.72	6.7522
BANK FUER TIROL 2018 1.325% 22/03/28 3	2028	Senior Secured	AT0000A20BV7	1.325	7.5126
BANK OF AM CORP 2008 7% 31/07/28 REG.S	2028	Senior Unsecured	XS0379947236	7	6.7953
BANK OF AM CORP 2008 8 1/8% 02/06/28 REG.S	2028	Subordinated	XS0365909125	8.125	6.3112
BQ.FEDV.DCM.SA 2017 1.43% 05/04/29 REG.S	2029	Senior Unsecured	XS1591784639	1.43	8.5542
BQ.FEDV.DCM.SA 2019 1 7/8% 18/06/29	2029	Subordinated	FR0013425162	1.875	8.4953

CANADA HSG.TST.NO.1 2021 1.4% 15/03/31 97	2031	Second Lien	CA13509PHS52	1.4	9.246
CANADA HSG.TST.NO.1 2021 1.6% 15/12/31 101	2031	Senior Unsecured	CA13509PHW64	1.6	9.12
CMWL.BK.OF AUS. 2010 4.3% 13/08/32 REG.S	2032	Senior Unsecured	XS0532303186	4.3	9.6195
CMWL.BK.OF AUS. 2012 3.994% 13/02/30 REG.S	2030	Senior Secured	XS0745915826	3.994	8.6623
COMPAGNIE DE FNCMT. 2012 2.915% 14/12/32 580	2032	Secured	FR0011370378	2.915	11.1328
COMPAGNIE DE FNCMT. 2013 3.05% 22/08/33	2033	Senior Secured	FR0011553684	3.05	11.4642
COOPERATIEVE RABO. 2016 1.43% 01/09/36 REG.S	2036	Senior Unsecured	XS1484005985	1.43	14.6276
COOPERATIEVE RABO. 2018 1.595% 08/03/38 REG.S	2038	Mortgage	XS1785456713	1.595	15.7994
CREDIT AGRICOLE S A 2015 2.129% 10/09/27 REG.S	2027	Senior Unsecured	XS1288342493	2.129	6.9997
CREDIT AGRICOLE SA 2016 2.3% 24/10/26 Q	2026	Subordinated	FR0013192762	2.3	6.2032
CTRY.GDN.HDG.CTD. 2020 4.2% 06/02/26 REG.S	2026	Senior Unsecured	XS2210960022	4.2	5.91
CTRY.GDN.HDG.CTD. 2020 5 1/8% 14/01/27 REG.S	2027	First Lien	XS2100725949	5.125	5.7579
DEUTSCHE APOTH.UND 2016 0.825% 17/06/26 REG.S	2026	Senior Unsecured	XS1434566250	0.825	6.1414
DEUTSCHE APOTH.UND 2017 3/4% 05/10/27 REG.S	2027	Mortgage	XS1693853944	0.75	7.4473
DWR CYMRU FNG.UK 2020 1 3/8% 31/03/33 REG.S	2033	Senior Secured	XS2115092442	1.375	12.164
DWR CYMRU FNG.UK 2021 2 3/8% 31/03/34 REG.S	2034	Subordinated Secured	XS2328412064	2.375	12.95
ERSTE GROUP BANK AG 2010 4.41% 21/04/30 932	2030	Mortgage	AT000B008248	4.41	7.7974
ERSTE GROUP BANK AG 2014 4.46% 20/12/29 1329	2029	Subordinated	AT0000A18991	4.46	8.2058
LA BANQUE PTLE.SA 2016 2 1/4% 05/10/28	2028	Subordinated	FR0013207354	2.25	7.7892
LA BANQUE PTLE.SA 2018 2% 13/07/28 61	2028	Senior Non-Preferred	FR0013349099	2	7.7412
LANDESBANK BWTG. 2001 5.7% 28/02/31 226	2031	Senior Non-Preferred	XS0125912336	5.7	8.6898
LANDESBANK BWTG. 2001 6.19% 30/06/31 236	2031	Subordinated	XS0131928391	6.19	8.7209
LLOYDS BANK PLC 2015 1.326% 23/04/30 REG.S	2030	Senior Unsecured	XS1220089590	1.326	9.3407
LLOYDS BANK PLC 2016 1.35% 01/02/31 REG.S	2031	Senior Secured	XS1354465566	1.35	10.2776
NAT AUS BK LTD 2015 7/8% 19/02/27 REG.S	2027	Senior Secured	XS1191309720	0.875	6.7783
NAT AUS BK LTD 2016 0.655% 15/03/27 REG.S	2027	Senior Unsecured	XS1490954978	0.655	6.8853
RAIFF.LB.TIROL AG 2016 1.175% 20/07/26	2026	Senior Unsecured	AT0000A1MBY1	1.175	6.1406
RAIFF.LB.TIROL AG 2017 0.95% 15/11/27	2027	Senior Secured	AT0000A1Z080	0.95	6.75

Source: Refinitiv, compiled by the author

Once the theoretical YTM change was computed for each pair of bonds following the tree model explained previously, a regression analysis on the model output is performed (i.e. the theoretical, predicted ΔYTM vs. the actual ΔYTM currently seen in the market for each pair of bonds) as of same banking day (30/09/2021). Figure 41 below contains a summary of the main results for an Ordinary-Least Squares regression.

Figure 41. OLS regression, Predicted ΔYTM vs Actual ΔYTM, 30/09/2021

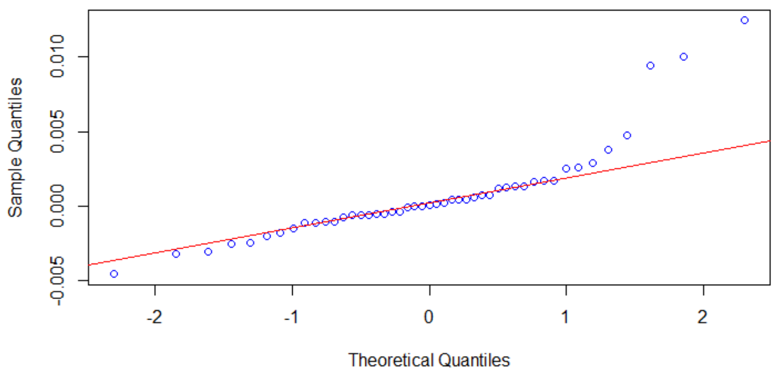


Source: Compiled by the author

The statistical outputs reflect a robust goodness-of-fit of the predicted ΔYTM compared to the actual one. For an OLS linear regression with the modeled YTM as the explanatory variable, R2 reaches 0.7491, taking into account that there are certain outliers identified in the series which penalize the final outcome. The estimator is 0.90011, being significative with a p-value of 2.08e-15 and t-test value of 11.72 for 46 degrees of freedom.

It should also be noted that no evidence of heteroskedasticity in the Breusch-Pagan test has been identified. However, full normality in residuals cannot be assumed as per the Jarque-Bera test output and Q-Q plot, due to the four outliers already identified in the regression outcome:

Figure 42. Normal Q-Q Plot, Predicted ΔYTM vs Actual ΔYTM, 30/09/2021

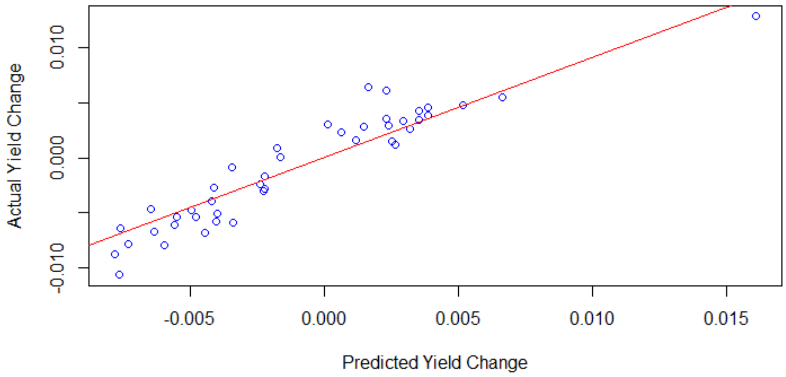


Source: Compiled by the author

The abovementioned outliers within the bond sample are issuances from *Ford*, *Deutsche Bank*, *Altice Financing* and *Erste Group*. Some of these issuances have in common a low rating score (speculative grade), with particular emphasis on the subordinated bonds from *Deutsche Bank* and *Erste Group*. The difference between the predicted ΔYTMs and the actual ones seen between the pair of bonds when one bond is located in the subordinated tranche is relatively high (more than 100 bp), considering that the model does not capture some issuance-specific situations. This means that while the model accurately predicts the expected direction and amount of the ΔYTM arising from a change in the expected Recovery Rate, in certain specific cases when the change in the tranche level for issuances from the same entity is relatively high (e.g., when the pair of bonds is composed by a secured note and a subordinated unsecured bond), then the market requires an additional risk premium to the issuance in the lower tranche.

If we eliminate the outliers from the sample, we obtain the regression output shown below in Figure 43.

Figure 43. OLS regression without outliers, Predicted ΔYTM vs Actual ΔYTM, 30/09/2021

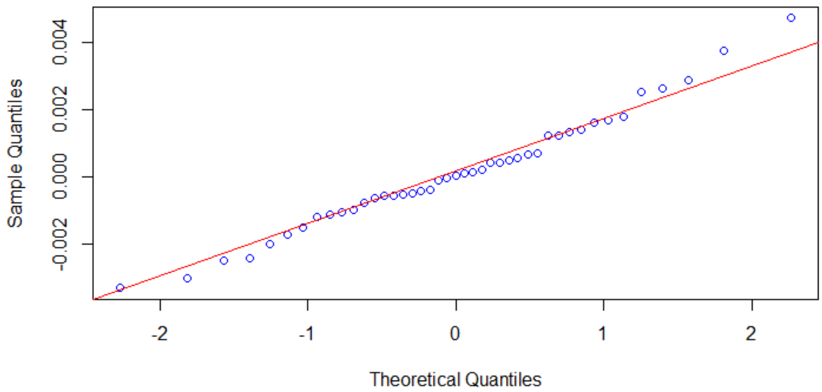


Source: Compiled by the author

The goodness-of-fit of the predicted ΔYTM increases considerably, with R^2 reaching 0.8929. For an OLS linear regression with the modeled YTM as the explanatory variable, the estimator is 0.90685, with the t-test value equal to 18.71 and p-value < 2e-16.

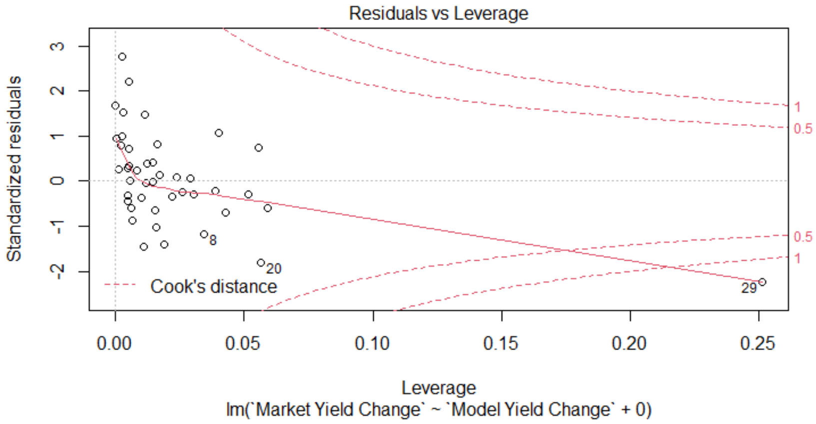
It is now possible to assume the normality in residuals with a Jarque-Bera test output of 1.1749 and a p-value = 0.5557, with the following Q-Q plot and Cook's distance:

Figure 44. Normal Q-Q Plot for regression residuals w/out outliers, Predicted ΔYTM vs Actual ΔYTM, 30/09/2021



Source: Compiled by the author

Figure 45. Cook's distance for regression residuals w/out outliers, Predicted ΔYTM vs Actual ΔYTM, 30/09/2021



Source: Compiled by the author

Although we have assessed the model robustness for the widest possible population as per the market database, we also need to assess the model's predictive power for additional random cases and over different sample sizes, using subsets of the current population in order to predict out-of-sample ΔYTMs.

6.3.5 Out-of-sample model testing and training

We have already seen that the model replicates the actual YTM change seen in the bond sample for a range of different shifts in the recovery rate to a high degree of confidence. Likewise, the model is also tested by carrying out out-of-sample testing techniques, i.e., cross-validation or resampling methods.

I have used the same three methods for cross-validating the model performance as in section 5.4.:

1) Leave One Out - Cross Validation: LOOCV
2) Bootstrapping
3) Repeated K-Folds

Hence regarding the outputs of model estimation power, below are the main results obtained:

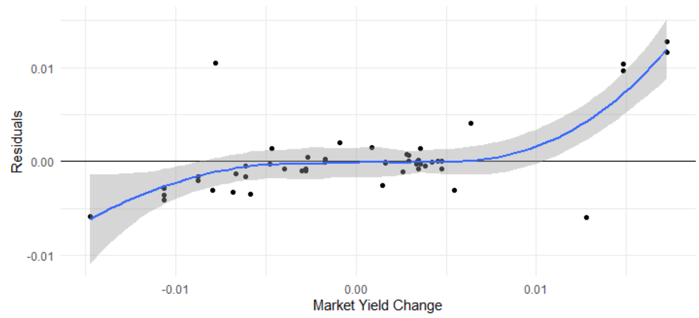
1) **LOOCV**:
   - RMSE: 0.00346
   - R^2: 0.71182
   - MAE: 0.00221

   Excluding the abovementioned outliers, the values are as follows:

   - RMSE: 0.00184
   - R^2: 0.87102
   - MAE: 0.00138

2) **Bootstrapping** with 1,000 scenarios:
   - RMSE: 0.00345
   - R^2: 0.80007
   - MAE: 0.00231

Figure 46. Residuals fitting for Bootstrap resampling on Predicted ΔYTM, 30/09/2021

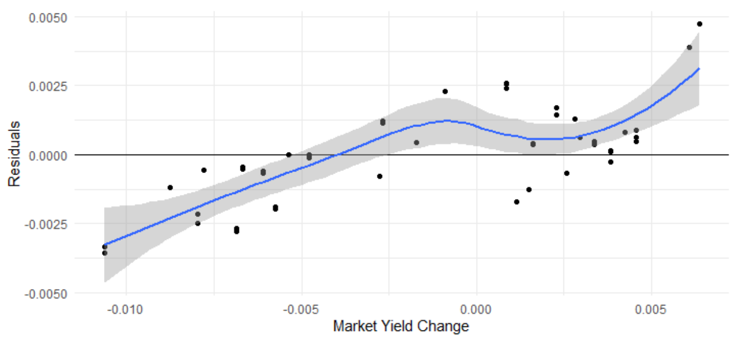


Source: Compiled by the author

Excluding the outliers, the values are the following:

- RMSE: 0.00186
- R^2: 0.89716
- MAE: 0.00142

Figure 47. Residuals fitting for Bootstrap resampling on Predicted ΔYTM, without outliers, 30/09/2021

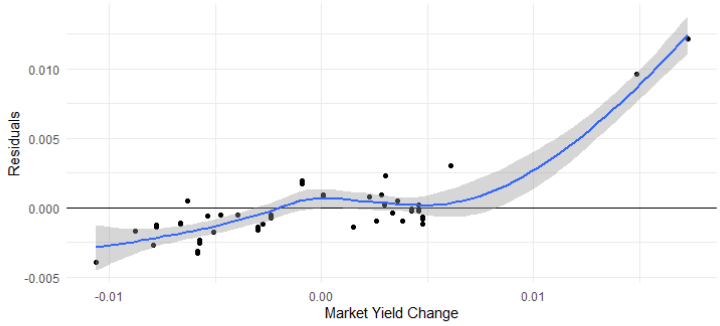


Source: Compiled by the author

3) **Repeated K-folds**: a simulation of 1,000 regression folds is done, with the sample divided into 10 buckets, and the outputs are as follows:

   - RMSE: 0.00296
   - R^2: 0.9041
   - MAE: 0.00221

Figure 48. Residuals fitting for Repeated K-Folds on Predicted ΔYTM, 30/09/2021

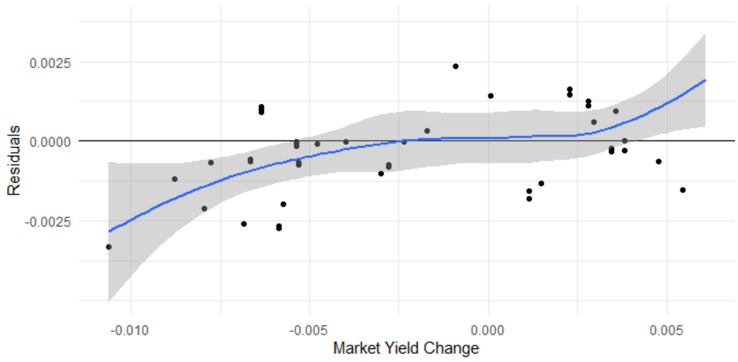


Source: Compiled by the author

Excluding the outliers, the values are:

- RMSE: 0.00170
- R^2: 0.93541
- MAE: 0.00138

Figure 49. Residuals fitting for Repeated K-Folds on Predicted ΔYTM, without outliers, 30/09/2021



Source: Compiled by the author

As shown above, the cross-validation process has provided with robust results. This means that, using many different samples in terms of components and size, the model is robust enough, so that the explanatory variables chosen in the optimization process are representative for a good estimation power with the current input dataset.

6.4. CDS price-based model: Methodology, model theory development and implementation

In the proposed model, we analyze the sensitivity of a credit-linked instrument in relation to the recovery rate. CDS prices are used, with the default probability embedded in a CDS price, and the CDS spread sensitivity to changes in the expected recovery rate. The theory is similar to the one presented for the Bond-price model in section 6.3, in the sense that we measure the sensitivity of a fixed-income product price to changes made to the Recovery Rate. However, in this case we will propose using quoted CDS spreads as a metric directly providing the change in the YTM.

In this context, we firstly need to understand the model framework for standard CDS pricing.

6.4.1 CDS pricing framework

A standard CDS is a contract in which one counterparty (the protection buyer) pays a regular fee (the CDS spread), while the other counterparty (the protection seller) must pay a default payment if a credit event occurs with respect to a reference entity. The default payment is designed to approximate the loss that the holder of a bond issued by the reference entity would suffer at the default event. Therefore, it is necessary to calculate the default event's probability of occurrence by using a specific distribution function (Harb and Louhichi, 2017).

Default event models, just like many other models used to infer occurrence probability, may be understood to follow an intensity-based process N: an event probability with an occurrence rate λ for a time period $(T - t) = \Delta\mathrm{t}$. That is,

$$P[\mathrm{N}(t+\Delta t)-\mathrm{N}(t)=1]=\lambda\Delta\mathrm{t} \tag{75}$$

so that

$$P[\mathrm{N}(t+\Delta t)-\mathrm{N}(t)=0]=1-\lambda\Delta\mathrm{t} \tag{76}$$

We subdivide the interval [t, T] into n subintervals of length $\Delta\mathrm{t} = (T - t)$. In each of these subintervals, the process N has a jump with probability $\lambda\Delta\mathrm{t}$. If we conduct *n* independent binomial experiments, each with a probability of $\lambda\Delta\mathrm{t}$ for a "jump" outcome, the probability of no jump at all in [*t*, *T*] is given by

$$P[\mathrm{N}(T)=\mathrm{N}(t)]=(1-\lambda\Delta\mathrm{t})^{n}=\left(1-\frac{1}{n}\lambda(T-t)\right)^{n} \tag{77}$$

And since $\left(1-\frac{x}{n}\right)^{n} \rightarrow e^{x}$ if $n \rightarrow \infty$, this converges to a Poisson process with no event between each subinterval n:

$$P[\mathrm{N}(T)=\mathrm{N}(t)]=e^{(-\lambda(T-t))} \tag{78}$$

Translated into default probabilities, and considering different occurrence (hazard) rates λ_i for different predefined time intervals $[t_{i-j}, t_i]$ of the instrument life, the instrument survival probability between t and $t + \Delta t$ is

$$SP[t, t + \Delta t] = e^{(-\lambda_i \Delta t)} \tag{79}$$

and therefore, the cumulative PD in the same context will be

$$PD[t, t + \Delta t] = 1 - e^{(-\lambda_i \Delta t)} = 1 - SP[t, t + \Delta t] \tag{80}$$

If we assume that there exist different hazard rates for different time intervals, we will obtain a discretized distribution of hazard rates for each time interval as follows:

$$Hazard\ rate_{\Delta t_i} = \begin{cases} \lambda_1, & if\ \Delta t_i \in [0, t_1) \\ \lambda_2, & if\ \Delta t_i \in [t_1, t_2) \\ \lambda_3, & if\ \Delta t_i \in [t_2, t_3) \\ \vdots & \vdots \\ \lambda_T, & if\ \Delta t_i \in [t_{T-1}, t_T) \end{cases} \tag{81}$$

From this we will obtain the Survival Probability Curve (SPC), i.e., the cumulative survival rate to be used in the CDS pricing framework:

$$SPC_{0,T} = \begin{cases} SP[0, t_1) = e^{-\lambda_1 \Delta t_1}, & if\ T \in [0, t_1) \\ SP[0, t_2) = e^{-\lambda_1 \Delta t_1 - \lambda_2 \Delta t_2}, & if\ T \in [0, t_2) \\ SP[0, t_3) = e^{-\lambda_1 \Delta t_1 - \lambda_2 \Delta t_2 - \lambda_3 \Delta t_3}, & if\ T \in [0, t_3) \\ \vdots & \vdots \\ SP[0, t_T) = e^{-\lambda_1 \Delta t_1 - \lambda_2 \Delta t_2 - \lambda_3 \Delta t_3 - \cdots - \lambda_{T-1} \Delta_{T-1} - \lambda_T \Delta_T} & if\ T \in [0, t_T) \end{cases} \tag{82}$$

where Δt_i is the time interval for each λ_i.

Returning to the CDS note pricing, the protection buyer pays the CDS spread as an insurance fee in order to hedge the potential default of a reference entity. This implies therefore that the protection buyer pays the CDS related to said reference entity over a predefined period of time. The present value of the protection "leg" – assuming that the potential default events occur between every two payment dates – is estimated as follows in (83) and (84) below:

$$Prot.Leg = N \cdot Spread \cdot\ \Delta t \sum_{i=1}^{T} SP[0, t_i]\, P(0, t_i)\ + \frac{1}{2}\left[SP[0, t_{i-1}] - SP[0, t_i]\right] P(0, {}^{(t_{i-1} + t_i)}/_{2}) \tag{83}$$

where N is the contract notional; $Spread$ is the spread of the CDS for the predefined period of time (maturity) i; $SP[0, t_i]$ is the cumulative survival probability at each payment time, with the particularity that each payment time will have different hazard rates. In other words, the CDS is modeled with as many different hazard rates as payment nodes. This implies that the survival probability curve gains convexity and adapts each payment interval to the expected default probability that exists within it.

Conversely, the default leg is expected to be as follows:

$$Default\ leg = N\,(1-R)\sum_{i=1}^{T}\left[SP[0,t_{i-1}] - SP[0,t_i]\right] P(0, {}^{(t_{i-1}+t_i)}/_{2}) \tag{84}$$

with $SP[0,t_{i-1}] - SP[0,t_i]$ representing the conditional probability in the CDS life-time filtration that the default time of the entity underlying the CDS will occur in the middle of the interval $(t_{i-1}, t_i]$ (given survival until t_{i-1}); and where R is the recovery rate, assumed constant for the entire CDS extension.

Therefore, the CDS price at any point in time will be the difference between the two legs (from the protection buyer's point of view):

$$CDS\ price = \ Default\ leg - Protection\ leg \tag{85}$$

The CDS is usually priced at par at inception, meaning that

$$Default\ leg = Protection\ leg \quad \ if\ \ t = 0 \tag{86}$$

with a spread that makes the protection leg equal to the default leg. Hence, if we solve for the equilibrium spread, leaving the notional out of this scope, the aforementioned equation becomes the following:

$$Spread = (1-R)\frac{\sum_{i=1}^{T}\left[SP[0,t_{i-1}] - SP[0,t_i]\right] P(0, {}^{(t_{i-1}+t_i)}/_{2})}{\Delta t \sum_{i=1}^{T}\ SP[0,t_i]\, P(0,t_i)\ + \frac{1}{2}\left[SP[0,t_{i-1}] - SP[0,t_i]\right] P(0, {}^{(t_{i-1}+t_i)}/_{2})} \tag{87}$$

The term in the numerator is the cumulated probability of default for the CDS life extension, whereas the term in the denominator is the cumulated survival probability, which may also be understood as the CDS price sensitivity to a bp (basis point) of spread, $Sp01$:

$$Spread = (1-R)\frac{Cumulated\ PD}{Sp01} \tag{88}$$

Assuming that we already know the underlying bond recovery rate R, the next step in this method will be to calibrate the different hazard rates. This process will entail that each λ_i will be calibrated starting from a standard initial period, for instance 6 months, in such a way that the Survival Probability nodes are defined from the earliest to the latest, arriving at the equality shown in the equation above.

Using this framework, the sensitivity of the spread to the Recovery Rate is relatively easy to analyze. The basis will also consist of finding fixed vanilla bonds issued by entities similar to the lessee in terms of rating, currency and sector. However, we also need to find CDSs which are similar in this same context.

Bonds and CDSs used for the analysis need to be unquestionably linked because the CDSs spread will determine the market YTM. If we find a liquid vanilla bond for which a quoted CDS curve exists (or, at least, a CDS curve for its rating and sector), we can then price the bond price with accuracy by using the formula below:

$$Bond\ price = \sum_{i=1}^{n} CF_i \frac{1}{(1 + rf_i + sp_i)^i} \tag{89}$$

where rf_i is the risk-free rate and sp_i is the market CDS spread at each cash-flow CF_i payment date i. Hence, if a senior unsecured vanilla bond has a market price x, this price can be accurately replicated by using an adequate risk-free curve and a related CDS spread curve in terms of sector, rating and currency. The CDS spread represents, in terms of credit risk, the premium over the risk-free rate required by the market to buy the underlying bond. Therefore, the CDS spread can be understood to be the portion of yield with idiosyncratic credit risk. Thus, we can deduce why this spread should be added to the risk-free rate in order to price the underlying bond, given that it consists of the premium paid off in the CDS trade due to the default probability.

The previous assumption comes from the fact that a CDS is designed so that a combined position of a CDS with a defaultable bond issued by a counterparty is very well-hedged against default risk and should therefore trade close to the price of an equivalent default-free bond. This means that the sum of the CDS and the risk-free bond should be equal to the defaultable bond price.

6.4.2 A practical example

Below we analyze how the bond price changes as well as the subsequent change in the YTM once the recovery rate changes at the same time.

By way of example, imagine that our lessee is a company within the transportation and logistics sector. It needs to arrange a leasing contract maturing in 4 years, with technical electric equipment as collateral, and with an average expected recovery rate of approximately 33% in line with the information presented in **Table 31**. The rating of the company is BBB, and it does not have liquid debt instruments quoted on the market; therefore, we should be able to find comparable peers with liquid bonds and credit default swaps.

The Refinitiv EUR BBB Transportation CDS curve reference and its risk factors are as follows, as of 20/01/2022 (RIC *0#BBBTRACDBMK*=):

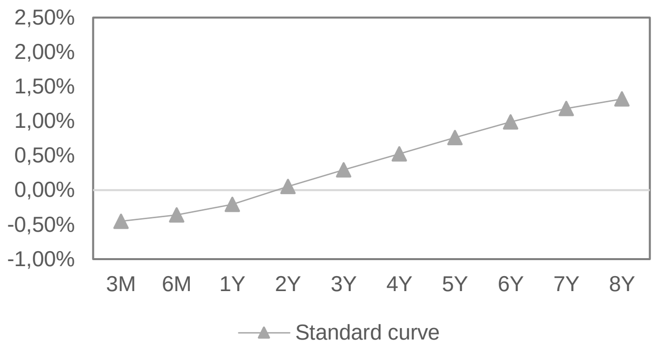
Table 41: EUR BBB Transportation sector CDS index curve, 20/01/2022

		Bid Spread (bps)	Default Prob. (%)	Sp01	Recovery Rate (%)
6M	20/07/2022	17.26	0.12	416.25	40.00
12M	20/01/2023	20.37	0.31	922.22	40.00
2Y	20/01/2024	33.46	1.08	1918.89	40.00
3Y	20/01/2025	46.76	2.28	2894.20	40.00
4Y	20/01/2026	64.06	4.18	3836.55	40.00
5Y	20/01/2027	81.87	6.66	4743.69	40.00
7Y	20/01/2029	105.40	11.79	6446.88	40.00
10Y	20/01/2032	123.12	18.91	8725.03	40.00
20Y	20/01/2042	137.41	36.59	14493.10	40.00
30Y	20/01/2052	147.21	51.80	18288.80	40.00

Source: Refinitiv and compiled by the author.

For this rating, sector and currency, the bond market gives the following YTM curve (RIC *0#BBBEURTRABMK*=):

Figure 50. EUR BBB Transportation sector YTM curve, 20/01/2022



Source: Refinitiv and Compiled by the author.

According to the bonds constituting the 4y tenor for the above curve as at valuation date, the average market price is approximately 103.35%, with an average YTM of 0.52%, and an average maturity date of 17/06/2026 with an annual coupon of 1.38%. Using this structure, we firstly replicate the price by using the BBB Transportation CDS curve quoted by Refinitiv (Table 41), and subsequently by using the Euribor 6M zero coupon curve plus the CDS spreads in order to verify that the CDS curve to be used fits the expected pricing:

Table 42: EUR BBB Transportation sector 4Y Maturing bond pricing

		20/01/2022	20/01/2023	20/01/2024	20/01/2025	20/01/2026
Coupon	1.39%		1.408%	1.408%	1.411%	101.408%
Discount Factor	$\frac{1}{(1+rf_i+sp_i)^i}$		100.272%	99.821%	98.782%	97.485%
					Market price	103.46%
					Price using CDS spread	103.07%

Source: Refinitiv and compiled by the author

As expected, the convergence between the market price and the price derived from using the CDS spreads proves sufficiently quantitative for modelling purposes. This can only occur if the CDS curve is sufficiently representative in terms of rating, sector and geography as regards the bonds used to calibrate the yield curve. There is a discrepancy of only 39 bps in price, but it should be noted that the constituents of the Refinitiv CDS curve are not the same as those constituting the yield curve, apart from other slight differences in market conventions.

Once this convergence has been checked, the next step required is to measure the sensitivity of the bond price to the recovery rate. To this end, firstly we need to measure the expected change in the credit spreads arising from a change in the recovery rate. Hence, as per equation 88, the change in the recovery rate from 40% to 33.96% entails the following increase in the credit spread curve (given a constant PD and *Sp01*):

Table 43: BBB Transportation CDS curve adjusted with a Recovery Rate = 33.96% under the proposed model, 20/01/2022

	New Mid Spread with RR = 33.96% (bps)	Change (bps)
6M	17.36	1.53
12M	21.13	1.87
2Y	35.79	3.17
3Y	52.63	4.66
4Y	75.95	6.72
5Y	99.01	8.76
7Y	132.98	11.77
10Y	159.55	14.12

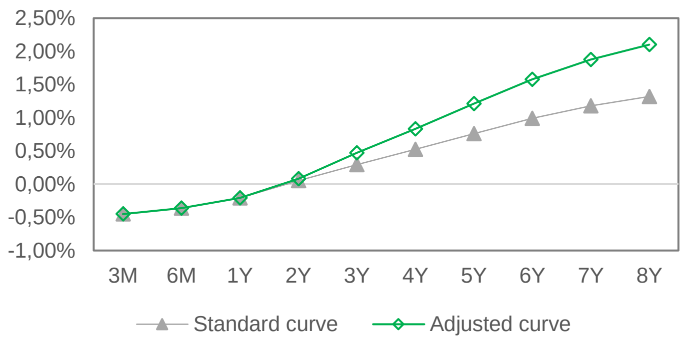
Source: compiled by the author.

This shift means that the bond will be priced at 102.47% which, translated into the YTM change, results in a shift of + 24.35 bps (from 0.5240% to 0.7675%).

This process should also be carried out for the representative maturities in the yield curve. The necessary similarity between the bonds compounding the yield curve and the CDSs used for the bond pricing regarding rating and sector is clear.

As a result, the new curve is as follows:

Figure 51. EUR BBB Transportation sector standard and adjusted YTM curves (%), 20/01/2022



Source: Refinitiv and compiled by the author.

The adjusted curve is higher due to the fact that the lower recovery rate should be compensated via an increase in the return for the lessor, assuming that the default rate does not change.

Certain assumptions for this framework related to the CDS valuation should be taken into consideration:

- As mentioned above, a CDS is designed as a combined position of a CDS with a defaultable bond issued by a counterparty.

- In order to ease the computations, we assume that each payoff occurs at the time of each default.

- The delivery option which is embedded in a CDS with physical delivery is ignored. We consider that CDSs hedge the corresponding defaultable bonds, and therefore the same recovery rates are intrinsic for the bonds and the hedging CDS. If this were not the case, the necessary matching between yield and corresponding CDS curves under sector and rating would not occur.

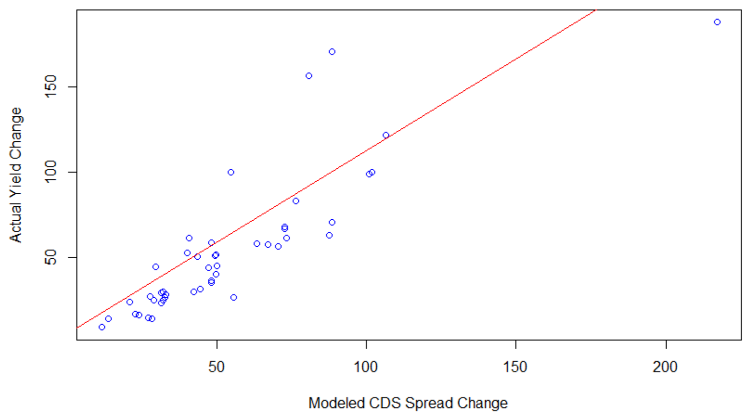
- It is assumed that the CDS is triggered by an individual obligors' default (despite the fact that the obligor is a basket in terms of the model, we consider it as a whole for the purpose of sector/rating CDS curve treatment).

6.4.3 *Model implementation and Performance measurement*

In order to test the initial hypothesis and to corroborate the model robustness, several analyses have been performed by using quoted bonds with CDSs issued on them. As in the case of the Bond-price model, I have searched for issuers that maintain quoted bonds with different recovery rates (due to different seniority levels, different guarantees, etc.) and similar maturities for the same sample shown in **Table 40** of section **6.3**, but also having CDSs issued. The sample decreases in 6 inputs as there were no CDSs quoted for every company in the sample. Then it is checked whether the model predicts the change in the bonds' YTM in response to a change in the estimated recovery rate.

Once the theoretical YTM change was computed for each pair of bonds following the CDS spread change model outlined in the previous section, a regression analysis on the model output is performed – i.e., the theoretical predicted ΔYTM as per the CDS spread model change vs. the actual ΔYTM currently seen in the market for each pair of bonds – as of same banking day (20/01/2022). **Figure 52** below contains a summary of the main results for an Ordinary-Least Squares regression:

Figure 52. OLS regression, Predicted ΔYTM via CDS Spread change vs Actual ΔYTM, 20/01/2022 (bps)

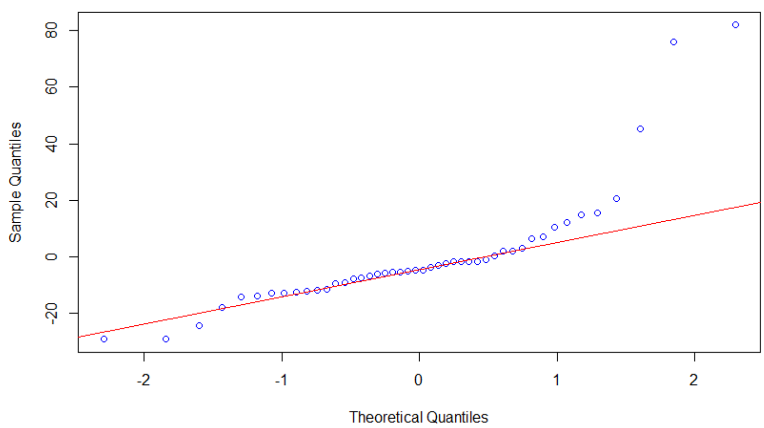


Source: Compiled by the author.

The statistical outputs reflect a robust goodness-of-fit of the predicted ΔYTM compared to the actual one. For an OLS linear regression with the modeled YTM as the explanatory variable, R^2 reaches 0.7251, taking into account that there are certain outliers identified in the series which penalize the final outcome. The estimator is 1.0786, being significative with a p-value of 6.4e-14 and t-test value of 10.773 for 44 degrees of freedom.

It should also be noted that no evidence of heteroskedasticity in the Breusch-Pagan test (assuming linear relationship) has been identified. However, full normality in residuals cannot be assumed as per the Jarque-Bera test output and Q-Q plot, due to the four outliers already identified in the regression outcome.

Figure 53. Normal Q-Q Plot, Predicted ΔYTM via CDS Spread change vs Actual ΔYTM, 20/01/2022

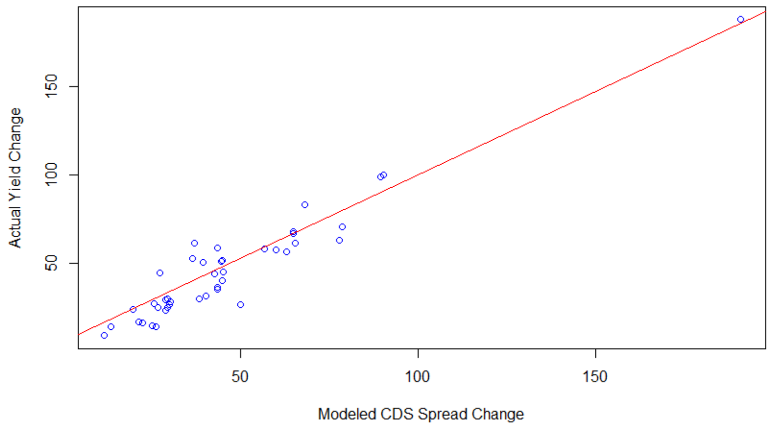


Source: Compiled by the author.

As in the case of the model presented in section 6.3., the abovementioned outliers within the bond sample are issuances from *Deutsche Bank*, *Bayerische Landesbank* and *Erste Group*.

If we eliminate the outliers from the sample, we obtain the regression output shown in the figure below.

Figure 54. OLS regression without outliers, Predicted ΔYTM via CDS Spread change vs Actual ΔYTM, 20/01/2022 (bps)

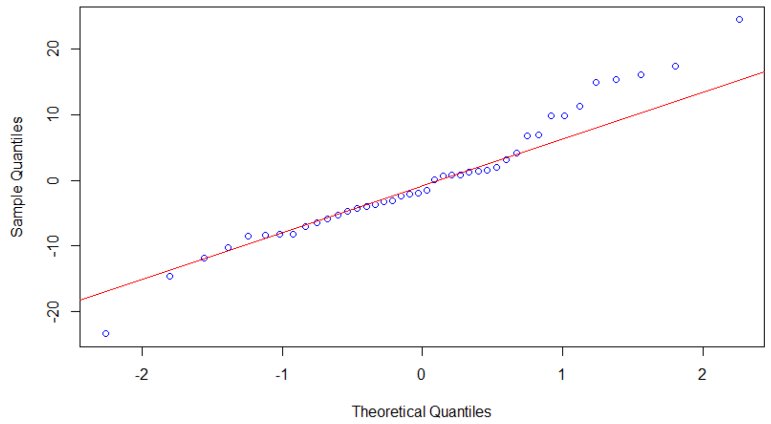


Source: Compiled by the author.

The goodness-of-fit of the predicted ΔYTM increases considerably, with R^2 reaching 0.9118. For an OLS linear regression with the modeled YTM as the explanatory variable, the estimator is 0.9418, with the t-test value equal to 20.336 and p-value < 6.49e-16.

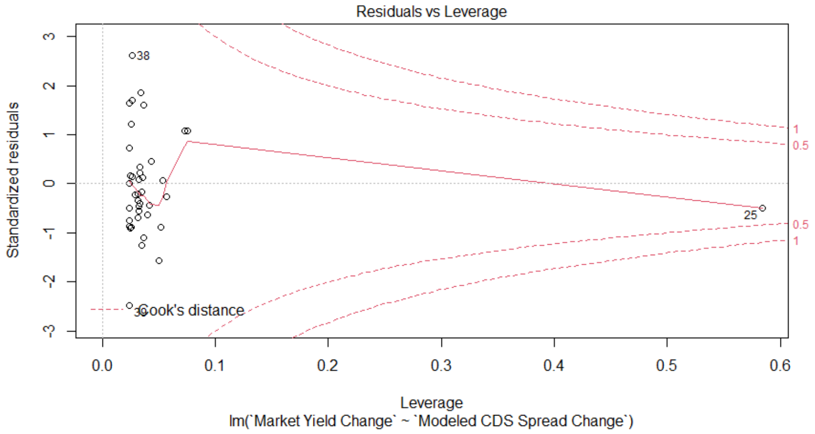
It is now possible to assume the normality in residuals with a Jarque-Bera test output of 1.3397 and a p-value = 0.5118, with the following Q-Q plot and Cook's distance:

Figure 55. Normal Q-Q Plot for regression residuals w/out outliers, Predicted ΔYTM via CDS Spread change vs Actual ΔYTM, 20/01/2022



Source: Compiled by the author.

Figure 56. Cook's distance for regression residuals w/out outliers, Predicted ΔYTM via CDS Spread change vs Actual ΔYTM, 20/01/2022



Source: Compiled by the author.

Although I have assessed the model robustness for the widest possible population as per the market database, we also need to assess the model's predictive power for additional random cases and for different sample sizes, using subsets of the current population in order to predict out-of-sample ΔYTMs.

6.4.4 Out-of-sample model testing and training

We have already seen that the model replicates the actual YTM change seen in the bond sample for a range of different shifts in the recovery rate to a high degree of confidence. Likewise, we also tested model performance and predictive power by carrying out out-of-sample testing techniques, i.e., cross-validation or resampling methods.

I have used the same three methods for cross-validating the model performance as in section 5.4.:

1) Leave One Out - Cross Validation: LOOCV
2) Bootstrapping
3) Repeated K-Folds

Hence regarding the outputs of model estimation power, below are the main results:

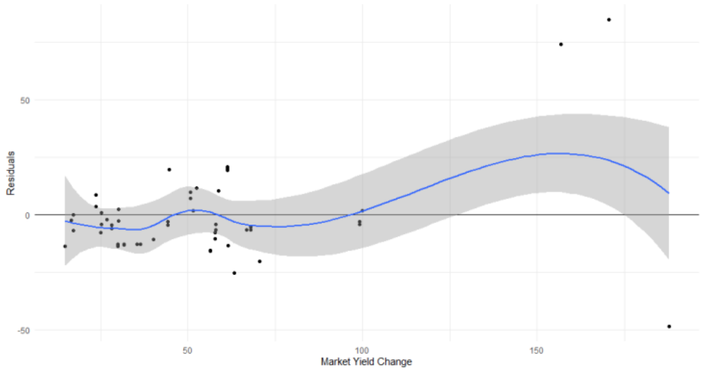
1) **LOOCV**: The output averages are as follows:
   - RMSE: 23.1102
   - R^2: 0.6796
   - MAE: 14.0505

   If we exclude the abovementioned outliers, however, the values are as follows:

   - RMSE: 9.6217
   - R^2: 0.9050
   - MAE: 7.4199

2) **Bootstrapping**: We have simulated 1,000 scenarios with the following output averages:
   - RMSE: 22.6849
   - R^2: 0.7425
   - MAE: 14.5983

Figure 57. Residuals fitting for Bootstrap resampling on Predicted ΔYTM, 20/01/2022

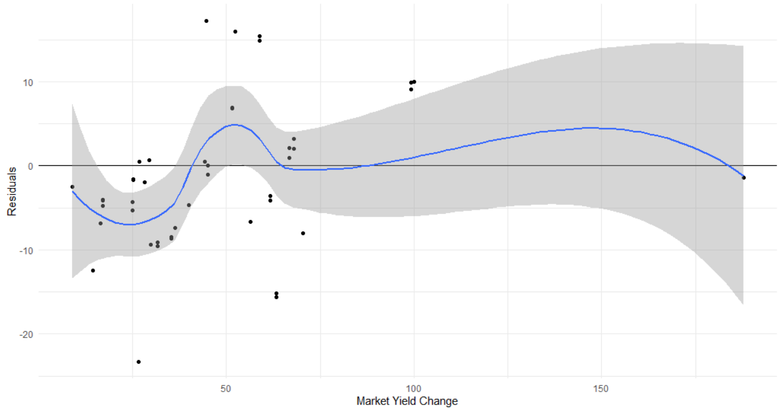


Source: Compiled by the author.

If we exclude the outliers, the values are as follows:

- RMSE: 9.9791
- R^2: 0.08567
- MAE: 7.7946

Figure 58. Residuals fitting for Bootstrap resampling on Predicted ΔYTM, without outliers, 20/01/2022



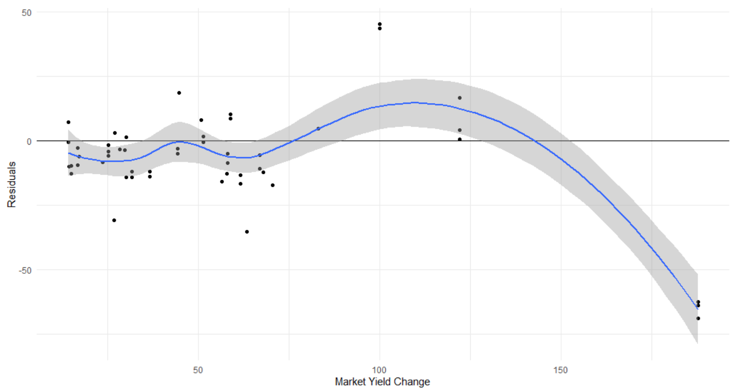
Source: Compiled by the author.

3) **Repeated K-folds**: I have implemented a simulation of 1,000 regression folds with the sample divided into 10 buckets, and the outputs are as follows:

   - RMSE: 19.3329
   - R^2: 0.8275

- MAE: 14.0294

Figure 59. Residuals fitting for Repeated K-Folds on Predicted ΔYTM, 20/01/2022

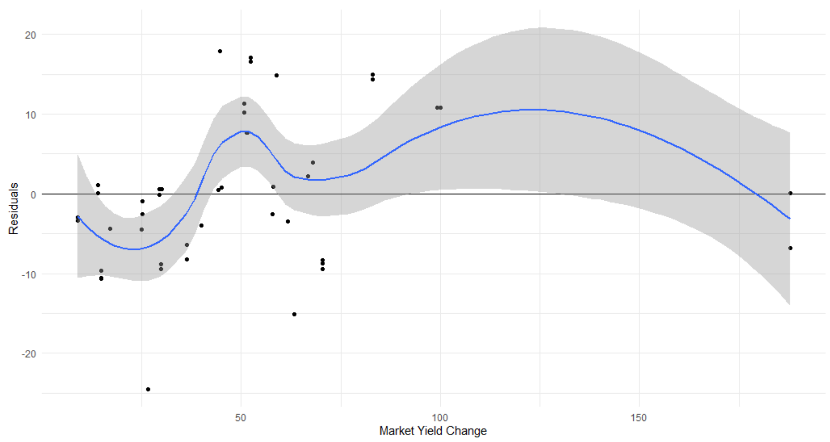


Source: Compiled by the author.

If we exclude the outliers, the values are as follows:

- RMSE: 8.9749
- R^2: 0.8614
- MAE: 7.4221

Figure 60. Residuals fitting for Repeated K-Folds on Predicted ΔYTM, without outliers, 20/01/2022



Source: Compiled by the author.

CHAPTER 7: CONCLUSIONS AND FUTURE LINES OF RESEARCH

Counterparty credit risk is the main financial risk to be monitored by financial and non-financial institutions, worldwide. It entails a huge impact in areas as diverse as Business, Finance, Risk management, Funding & Liquidity management, Treasury, Trading, Solvency control, Accounting, Reporting, etc.

Concerning valuation and accounting matters, counterparty credit risk is present throughout IFRS rules, with emphasis on a particular way under IFRS 9, IFRS 13 and IFRS 16. Under the IFRS 9, entities must estimate the PD (Probability of Default) for all financial assets (and other elements) not measured at Fair Value through Profit or Loss in order to compute the Expected Credit Loss for those assets. Also, regarding the potential impact that a modification in a debt instrument terms (i.e., debt restructuring) may have under IFRS 9, the original debt could have to be derecognized and replaced with the present value of the modified debt, which should be computed by discounting its cash-flows with a robust, liquid yield curve according to the company´s credit quality and instrument seniority. Likewise, under IFRS 13 framework, the expected counterparty credit risk should be incorporated to the value of a derivative which is measured at Fair Value. In this case, the derivative credit risk will be determined for both counterparty (CVA – Credit Value Adjustment) and own credit risk (DVA – Debt Value Adjustment). Therefore, the counterparty credit quality (and subsequent PD) and the own PD for the entire life of the instrument should be estimated.

7.1. Credit Rating and Probability of Default estimation model

It is widely known that financial and non-financial companies face many information-related issues when computing PD. In some cases, the inputs required (PD or the bond interest rate/YTM) can be directly estimated from observable market information, such as CDS spread quotes or the issuer’s bond price quotes. In other cases, however, this information is not available. The counterparty whose credit quality needs to be estimated may not have quoted credit-linked instruments, nor a credit rating issued by an independent rating agency. In such cases, entities need to implement a methodology for internally estimating the credit quality (credit rating) of a company as a basis for obtaining a PD or a YTM/discount rate curve, and also a method to correctly calibrate the adjustments needed on those PD or discount curves due to some particularities of the asset or the counterparty. This is specifically relevant in the case of IFRS 9 for ECL estimation, IFRS 13 for Fair Value and "exit price" computation, and under IFRS 16 IBR estimation as well.

There is a line of research in which authors propose models for obtaining a credit rating, to use it in the event that there is no official credit rating available. The first historical work was that by Altman (1968), which used five financial ratios to predict bankruptcy. Since then, many authors have also proposed models in which financial variables are used for estimating credit risk. See, for example, Merton (1974); Kaplan and Urwitz (1979); Ohlson (1980); Ederington (1985); Longstaff and Schwartz (1995); Duffee (1999); and Kamstra *et al.* (2001). More recently, Creal *et al.* (2014), Tsay and Zhu (2017) and Jansen and Fabozzi (2017) have incorporated to the financial literature some interesting developments in the field of credit rating estimation for certain group of sectors and products.

However, there are still present in the financial literature several issues concerning the PD modelling for accounting and IFRS reporting purposes, with a relatively global application. Few proposed models for obtaining an internally developed credit rating fulfil most of the relevant criteria, among others:

a) Specifically addressed to accounting purposes

b) Specifically focused on complying with IFRS 9 expected loss requirements. The IFRS 9 PD should be based not only on historical information but should also consider *forward-looking* information.

c) Able to be applied to non-quoted/non-rated entities. Few models have mainly been developed for non-quoted companies (Beever, 1968; Ohlson, 1980; Campbell *et al*., 2008, etc.), however they do not provide a full-scale solution to make it aligned to changes in rating agency criteria or sectorial economic situation over time.

d) Comparable, so that the results can be compared to market or credit rating information.

e) Able to be applied to one specific counterparty/company within a given sector and jurisdiction

f) Able to be implemented by updated obtaining public information from an updated market/sectorial framework

g) Able to be extrapolated into a Rating Letter, a PD rate, a yield-to-maturity curve, or a credit spread. This fact leads to a solution for lack of counterparty credit information under the IFRS 13 and IFRS 16 frameworks as well.

In this regard, the first objective of the doctoral research presented in Chapter 5 is to fulfil the above criteria, in line with IFRS rules and also aiming to overcome the shortcomings found in the financial literature. Therefore, I have presented a model that provide with a robust output (as a credit rating, a PD or even as a discount rate) to be used as input needed to impairment calculation (Expected Credit Loss) and debt restructuring valuation figures under IFRS 9, as well as CVA and DVA metrics to be estimated under IFRS 13.

The model provides the output via a regression scheme which retrieves a theoretical credit rating for a counterparty as a first, necessary step when estimating the PD or the discounting curve. The model is new in a certain extent in comparison with other academic models in several aspects, such as the size and composition of the database used to calibrate the model variables (financial ratios percentiles within a sectorial distribution, for several years in a row) and the fact that is intended to provide a "*forward-looking*" risk approach. The assumption that can be taken as an initial hypothesis is that historical financial ratios are a reliable source of information to estimate a rating letter when those are efficiently combined, with no necessity of qualitative nor additional company´s management-related information. It is demonstrated that, with a granular sectorial database and by applying optimization in variables via Stepwise AIC process, the model output is reliable and robust to estimate the credit rating of a given company. Therefore, once the database is deep and representative enough, the model can be implemented and used for different sector and geographies, with a *forward-looking* approach and able to cover the changes in rating criteria throughout time, hence available to be used for accounting and reporting purposes under different audit exercises.

The model has been tested by comparing its output for entities already given with an official credit rating from CRAs (Moody's, Fitch, or Standard and Poor's). Therefore, we obtain a unified framework which incorporates a firm's specific features along with its sectorial and regional factors, and which enables market assessments of credit risk to be incorporated into the book value of financial assets.

7.2. Leasing valuation and IBR estimation model

IFRS 16 is the new lease standard that has been applied since fiscal year 2019. These standards introduce a capitalization model to be applied by the lessee for most lease transactions. The model implies calculating the initial value of a lease asset and a lease liability by discounting lease payments over the lease term. Subsequently, the leased asset is subject to depreciation and impairment, and lease liability is basically recognized as a financial liability at amortized cost.

For the most part, entities are using what the standards call the Incremental Borrowing Rate (IBR) for discounting future lease payments. One of the factors that the IBR should consider is the collateral that the leased asset represents for the lessor. In the case that the lessee defaults, the lessor repossesses the leased asset and has the possibility of selling the asset or leasing the asset to a different counterparty in order to recover at least part of the hypothetical loan.

Previous works have shown that the use of discount rates across firms under IFRS is both inconsistent and arbitrary (Schneider et al., 2017; Blum and Thérond, 2019; Michelon et al., 2020). Therefore, the second objective of my research has been to cope with the necessity of having a robust modelling scheme to estimate the IBR according to market, observable data, with a high degree of confidence.

In this dissertation, I proposed two quantitative models for estimating the IBR while taking the applicable LGD into consideration. Assuming that the initial ("standard") curve is obtained from senior unsecured bonds (Step 1), the model are useful to estimate the IBR adjusted in line with the quality of the collateral (i.e., considering a recovery rate different to the general recovery rate assumed in the initial curve) (Step 2).

The models are based on the analysis of the percentage change of the CDS spread and Yield-to-maturity assumed for the lessee when the recovery rate changes. It can be implemented by entities that need to maintain several discount curves for the leased asset. The models use quoted CDS and bond market information as its main reliable source and are based on standard valuation models. In this way, the outcomes are aligned to market standards and quoted information, which is the basis for estimating fair values beyond vanilla securities.

As a summary, it can be said that there is a modelling gap in the accounting and finance literature when analysing how the IBR should be calculated taking into consideration both the counterparty credit risk of the lessee and the quality of the collateral. The starting hypothesis in this regard is that this quality is mainly determined by the underlying asset's expected LGD so that the relationship between the IBR and the LGD could be modelled. By performing an empirical analysis using almost 100 quoted bonds, it is demonstrated that the modelling results are statistically robust and demonstrates that the relationship between CDS spreads or bonds yield-to-maturity and the LGD (or Recovery Rate) implied in their market prices can be translated (for a relevant portion of use cases) as a sensitivity measure to estimate the IBR for a lease contract, by pivoting from a standard market yield curve.

Moreover, it is demonstrated that the model functions by using real market data of quoted bonds, i.e., by applying the models to a real sample of quoted bonds and CDS prices, and subsequently analyse whether the model predicts the change in YTM when a change in the recovery rate occurs.

Also, it is worth noting that the model presented is also applicable in many other contexts, such as estimating the fair value of a loan/bond that includes an asset as a collateral (for accounting, trading, valuation, or other purposes). In this case, the model can be used to adjust the discount curve and correctly reflect the higher (or lower) recovery rate expected from the asset. Another potential use would be the calculation of the interest rate of a collateralized loan transaction between a lender and a borrower; in this case the model can be used for adjusting the standard interest rate to the collateral value, calculate additional liquidity margins, etc.

7.3. Comments and Modelling limitations

As the FRS model relies on accounting information, two main model limitations are related to financial statements manipulation and the fact that qualitative information is not considered.

In relation to earning manipulation, Alissa *et al.* (2013) identify firms that deviate from expected credit ratings and demonstrate that these empirically estimated credit rating deviations are associated with earnings management activities. Their results suggest that firms below or above their expected credit ratings may be able to successfully achieve a desired upgrade or downgrade through the use of earnings management. Therefore, if the financial information is manipulated, the obtained credit rating would also be different from the correct one. Nevertheless, the model assumes that, generally, the financial information used is correct. The model main objective is not to detect financial statements fraud situations but using them to estimate the credit rating following market, public information.

On the other hand, in general, default risk (credit risk) can be measured in three different ways: using quantitative data, using qualitative data or using a combination of both. Quantitative data includes equity prices, credit markets data, financial instruments quotes, other financial data, etc. Qualitative data includes entity's structure, how the entity is perceived by the market, business estimations, business plans, information regarding the entity's governance and risk appetite; etc. As already stated, the model does not consider qualitative data as such. Although the optimization method is able to cope with this, it should be mentioned that CRAs do use qualitative data and company managerial information as inputs for their rating models. Therefore, there is a potential gap to be noted in the model estimation power.

Concerning the IFRS 16 models, there are several considerations to bear in mind. The first one is related to the liquidity risk attached to any leasing contract, namely the fact that quoted debt is much more liquid than a leasing contract. A bond holder can sell the bonds under certain clauses in a relatively liquid market, where the yield bid-ask spread may vary but the price is formally set. However, in the case of a leasing contract, the lessor – who is financing the lessee on a given timeframe – has no actual information on its asset price (the leasing contract), nor any certainty as to the collateral recovery (should a default event take place). Furthermore, the collateral value is expected to decrease over time, in terms of amortization and use effects. Thus, an additional liquidity spread, dependent on the nature of the collateral, the contract extension and the collateral expected degree-of-use may be added to both the standard and the adjusted curve.

Likewise, another limitation is the plausible liquidity squeeze in the bond prices, or the lack of bonds for several maturities, sectors, ratings or even currencies. In several sectors, and also as regards non-investment grade ratings, information pertaining to bond prices or YTMs is not always as liquid as desired. The bid-ask spreads are usually reasonably wide, and not many tenors are quoted for representative curves on Bloomberg or Refinitiv. Therefore, certain assumptions should be made in this regard: the extrapolation of bond spread change from investment grades to non-investment grades; the extrapolation of rating curves from liquid sectors to sectors without sufficient information; the extrapolation of the curve slope from the short and medium-term tenor to longer tenors for which there is no information on bonds, etc.

Moreover, it should be noted that several risk factors not entirely covered by the model (e.g., sovereign risk, currency risk) also exist, and therefore the reference data and the market information chosen may contain factors that could distort the results. Hence the importance of using market data and securities as similar as possible to the leasing contract under analysis is paramount.

Last but not least, one further specific limitation to the model should be noted. Certain situations arising from credit events and poorly collateralized assets may mean that the model does not capture the entire actual YTM change. For instance, the change in YTM seen between secured debt and subordinated debt for entities with low credit ratings could not be fully captured by the model because these entities may suffer from incremental spreads required by the market to compensate the "collateral" risk.

7.4. Future lines of research

The credit rating estimation accuracy depends on many potential risk factors, both qualitative and quantitative, and also on the availability of information from relevant sources. The credit rating model proposed in this dissertation aims to estimate credit rating letters from pure quantitative information (financial ratios from financial statements). Whereas the output results can be considered robust in terms of estimation power, there is a clear path of development that can be incorporated in the modelling framework in the future:

a. Qualitative information is not considered, at least directly, in the model development. Therefore, additional research could be performed to set a trustworthy database that can provide inputs in this regard, in line with the credit rating assignment process of credit rating agencies.

b. Database per sector and geography with international-scaled rating letters and ratios: the model can be fed with as many different sectorial information as possible, so that an intensive research work should be done to get the model ready for estimating the credit rating for almost any given company from every sector and geography.

c. Additional optimization methods might be explored, including neural networks, random forests and other machine learning techniques, once a reliable database with qualitative information and classification criteria is developed.

Also, with regards to the estimation models for IBR, it should be noted that additional bond and CDS population can be incorporated whenever possible. The models directly provide with changes in YTMs based on pricing formulas extensively used in the bond and CDS market, so there is no relevant way of improvement in the modeling side but in the input data:

a. Bond database should be frequently updated, taking into consideration that potential bond samples might need to be wiped out from the population due to liquidity shortcomings and misrepresentative, as exposed in above paragraphs.

b. Estimated LGDs used in the models to calibrate the changes in YTMs should be revisited accordingly, due to the fact that LGDs can change over time, depending on the state of the economy and additional asset features.

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